Supplementary Data

Evaluation of Solid Waste Disposal Strategies in Chennai Using Complex q-Rung Orthopair Fuzzy Yager Aggregation Techniques

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Example 2.1. Let $\mathcal{C}_1 = \langle(0.9,0.4),(0.6,0.7)\rangle$, $\mathcal{C}_2 = \langle(0.7,0.2),(0.5,0.3)\rangle$, and $\mathcal{C}_3 = \langle(0.8,0.3),(0.4,0.6)\rangle$ represent the Cq-ROFNs. Let $\omega_1 = 0.3$, $\omega_2 = 0.5$, and $\omega_3 = 0.2$ be the weights associated with the Cq-ROFNs $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{C}_3$, respectively. Given that $q=3$ and $\beta=2$. From Equation (3), we have $\mathbb{S}(\mathcal{C}_1) = 0.55850$, $\mathbb{S}(\mathcal{C}_2) = 0.54975$ and $\mathbb{S}(\mathcal{C}_3) = 0.56475$. Based on Equation (7), we find $\lambda_1 = 1$, and from Equation (8), we get $\lambda_2 = \mathbb{S}(\mathcal{C}_1) = 0.55850$ and $\lambda_3 = \mathbb{S}(\mathcal{C}_1)\mathbb{S}(\mathcal{C}_2) = 0.3070354$. Therefore, $\sum_{k=1}^{3}\omega_k\lambda_k = 0.6406571$, and from Equation (6), we have $\delta_1 = 0.4682692$, $\delta_2 = 0.4358806$ and $\delta_3 = 0.0958501$.

Assume that Cq-ROFYPWA$(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3) = \oplus_{i=1}^{3}\delta_i\,\mathcal{C}_i = \langle(\sigma,\eta),(\tau,\gamma)\rangle$.

Using Equation (9), we compute:

$$\sigma = \left[1-\min\left\{1,\left(\sum_{i=1}^{3}\delta_i\left(1-\sigma_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}$$

$$= \left[1-\min\left\{1,\left(0.4682692\left(1-0.9^3\right)^2+0.4358806\left(1-0.7^3\right)^2+0.0958501\left(1-0.8^3\right)^2\right)^{1/2}\right\}\right]^{1/3}$$

$$= 0.7961576.$$

Similarly, we can calculate $\eta, \tau,$ and γ using Equation (9). The final result is:

$$\text{Cq-ROFYPWA}\left(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3\right) = \langle(0.7961576,\ 0.3291887),\ (0.5544466,\ 0.6254823)\rangle.$$

**** Proof of Theorems***

Proof of Theorem 2.2. Given that $\mathfrak{R}_i = \mathfrak{R}$ for all $\mathfrak{R} = \sigma,\eta,\tau,\gamma,$ Equation (9) becomes

$$\text{Cq-ROFYPWA}\left(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_m\right) = \oplus_{i=1}^{m}\delta_i\,\mathcal{C}_i$$

$$= \left\langle \begin{array}{l} \left(\left[1-\min\left\{1,\left(\sum_{i=1}^{m}\delta_i\left(1-\sigma_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}, \left[1-\min\left\{1,\left(\sum_{i=1}^{m}\delta_i\left(1-\eta_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}\right), \\ \left(\left[\min\left\{1,\left(\sum_{i=1}^{m}\delta_i\left(\tau_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}, \left[\min\left\{1,\left(\sum_{i=1}^{m}\delta_i\left(\gamma_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q}\right) \end{array} \right\rangle. \tag{12}$$

The expression in Equation (12) simplifies to $\langle(\sigma,\eta),(\tau,\gamma)\rangle$. Consequently, we have Cq-ROFYPWA$(\mathcal{C}_1,\mathcal{C}_2,\ldots,\mathcal{C}_m) = \mathcal{C}$.

Proof of Theorem 2.3. For each index $i = 1,2,\ldots,m,$ the subsequent inequalities are valid

$$\min_i \mathfrak{R}_i \le \mathfrak{R}_i \le \max_i \mathfrak{R}_i \quad \text{for } \mathfrak{R} = \sigma, \eta,$$
$$\max_i \mathfrak{R}_i \ge \mathfrak{R}_i \ge \min_i \mathfrak{R}_i \quad \text{for } \mathfrak{R} = \tau, \gamma.$$

Consequently, we obtain the following

$$\min_i \mathfrak{R}_i \le \left[1-\min\left\{1, \left(\sum_{i=1}^{m} \delta_i \left(1-\mathfrak{R}_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q} \le \max_i \mathfrak{R}_i \;\; for\; \mathfrak{R} = \sigma, \eta,$$
$$\max_i \mathfrak{R}_i \ge \left[1-\min\left\{1, \left(\sum_{i=1}^{m} \delta_i \left(\mathfrak{R}_i^{\,q}\right)^{\beta}\right)^{1/\beta}\right\}\right]^{1/q} \ge \min_i \mathfrak{R}_i \;\; for\; \mathfrak{R} = \tau, \gamma.$$

This implies that $\mathbb{S}\left(\mathcal{C}^-\right) \le \mathbb{S}$(Cq-ROFYPWA($\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m$))$\le \mathbb{S}\left(\mathcal{C}^+\right)$, by Equation (3) and Equation (9). Thus, the proof is concluded according to Definition 2.6.

Proof of Theorem 2.4. Given that $\mathcal{C}_i \le \mathcal{C}_i^{'}$ holds for $i = 1, 2, ..., m,$ as defined in Definition 2.2, we can deduce the following:

$$\mathbb{S}\left(\mathcal{C}_i\right) \le \mathbb{S}\left(\mathcal{C}_i^{'}\right) \text{for } i = 1, 2, ..., m.$$

Using these relationships, we establish the following inequalities:

$$\mathfrak{R}_i \le \mathfrak{R}_i^{'} \text{ where } \mathfrak{R} = \sigma, \eta,$$
$$\mathfrak{R}_i \ge \mathfrak{R}_i^{'} \text{ where } \mathfrak{R} = \tau, \gamma.$$

As a result, we conclude:

$$\left[1-\min\{1, (\sum_{i=1}^{m} \delta_i \left(1-\mathfrak{R}_i^{\,q}\right)^{\beta})^{\frac{1}{\beta}}\}\right]^{1/q} \le \left[1-\min\{1, (\sum_{i=1}^{m} \delta_i \left(1-\mathfrak{R}_i^{'\,q}\right)^{\beta})^{\frac{1}{\beta}}\}\right]^{1/q} \;\; for\; \mathfrak{R} = \sigma, \eta,$$
$$\left[1-\min\{1, (\sum_{i=1}^{m} \delta_i \left(\mathfrak{R}_i^{\,q}\right)^{\beta})^{\frac{1}{\beta}}\}\right]^{1/q} \ge \left[1-\min\{1, (\sum_{i=1}^{m} \delta_i \left(\mathfrak{R}_i^{'\,q}\right)^{\beta})^{\frac{1}{\beta}}\}\right]^{1/q} \;\; for\; \mathfrak{R} = \tau, \gamma.$$

This implies that $\mathbb{S}$(Cq-ROFYPWA($\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_m$)) $\le$ $\mathbb{S}$(Cq-ROFYPWA($\mathcal{C}_1^{'}, \mathcal{C}_2^{'}, ..., \mathcal{C}_m^{'}$)), by Equation (3) and Equation (9). Hence, we conclude the proof based on Definition 2.6.

Appendix A

Appendix A.1

Table A.1. Abbreviations and their full forms.

Abbreviation	Full form
AOs	Aggregation Operators
C&D	Construction and Demolition
CFS	Complex Fuzzy Sets
Bio-CNG	Bio-Compressed Natural Gas
Cq-ROFN	Complex q-Rung Orthopair Fuzzy Number
CIFS	Complex Intuitionistic Fuzzy Sets
CPyFS	Complex Pythagorean Fuzzy Sets
Cq-ROFS	Complex q-Rung Orthopair Fuzzy Sets
Cq-ROFYPOWA	Complex q-Rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Average
Cq-ROFYPOWG	Complexq-Rung Orthopair Fuzzy Yager Prioritized Ordered Weighted Geometric
Cq-ROFYPWA	Complex q-Rung Orthopair Fuzzy Yager Prioritized Weighted Average
Cq-ROFYPWG	Complex q-Rung Orthopair Fuzzy Yager Prioritized Weighted Geometric
GCC	Greater Chennai Corporation
IFS	Intuitionistic Fuzzy Sets
KWMC	Koyambedu Wholesale Market Complex
MCDM	Multi-Criteria Decision-Making
MG	Membership Grade
NMG	Non-membership Grade
OWA	Ordered Weighted Average
PRA	Prioritized Average
q-ROFS	q-Rung Orthopair Fuzzy Sets
SWDS	Solid Waste Disposal Strategies
TPD	Tonnes of Waste per Day
YTCN	Yager's t-conorm
YTN	Yager's t-norm

Appendix A.2

Table A.2. Generalization of Fuzzy Set Models to Complex q-Rung Orthopair Fuzzy Set.

Fuzzy Information	Fuzzy Number	Cq-ROFN	Condition ($q \geq 1$)
FS [83]	σ	$\langle(\sigma,0),(0,0)\rangle$	$0 \leq \sigma \leq 1$
IFS [14]	(σ,τ)	$\langle(\sigma,0),(\tau,0)\rangle$	$0 \leq \sigma+\tau \leq 1$
PyFS [81]	(σ,τ)	$\langle(\sigma,0),(\tau,0)\rangle$	$0 \leq \sigma^2+\tau^2 \leq 1$
q-ROFS [78]	(σ,τ)	$\langle(\sigma,0),(\tau,0)\rangle$	$0 \leq \sigma^q+\tau^q \leq 1$
CFS [53]	(σ,η)	$\langle(\sigma,\eta),(0,0)\rangle$	$0 \leq \sigma,\eta \leq 1$
CIFS [13]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma+\tau, \eta+\gamma \leq 1$
CPyFS [8]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma^2+\tau^2,\ \eta^2+\gamma^2 \leq 1$
Cq-ROFS [42]	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$\langle(\sigma,\eta),(\tau,\gamma)\rangle$	$0 \leq \sigma^q+\tau^q,\ \eta^q+\gamma^q \leq 1$