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Supplementary data

Robust Design of Loss-Based Ideal Repetitive Group Sampling Plan under Uncertainty of Input Parameters

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Appendix A (PSO calibration)

Prior to optimizing M2, the L_9 orthogonal array design is employed to configure specific PSO factors. As illustrated in Table S.1, we explore three levels for each factor. Following the L_9 design, nine trials are meticulously planned for implementation. Subsequent to optimizing each trial with k=3 iterations, the signal-to-noise (S/N) ratio for the j^{th} trial is computed using the following formula [38]:

$$\left(\frac{S}{N}\right)_j = -10\log\left(\frac{1}{3}\sum_{k=1}^{3} OBF_{jk}^2\right), \quad j = 1,2,\ldots,9 \tag{S.1}$$

where OBF_{jk} represents the k^{th} optimal OBF for the j^{th} trial. The results of ratios are presented in Table S.1. The highest S/N ratio for each factor in Table S.2 identifies its optimal level. Consequently, we set the values w=0.8, (c_1, c_2)=(1.5, 2.5), N_P=20, and N_I=100. By applying this combination and repeating the optimization procedure three times, we consistently achieved an OBF of 363.5964. In comparison to the trials outlined in Table S.1, the efficacy of this optimal combination is supported by its superior OBFs and an S/N ratio of -51.2124.

Appendix B (Sensitivity Analysis)

Taguchi's orthogonal-array design is a method of designing experiments that usually requires only a fraction of the full factorial combination. Thus, it allows to analyze many factors with few runs. Taguchi designs are balanced, that is, no factor is weighted more or less in an experiment, thus allowing factors to be analyzed independently of each other. This method has recently been applied in control chart design [43]. Similarly, it is employed to analyze the influence of input parameters on

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the solutions of M2. Seven variables are considered as independent factors with E_L serving as the response variable. Table S.3 illustrates the planned levels for these variables. The allocation of independent variable levels to the L_{27} design, along with the optimization outcomes, is presented on the left and right sides of Table S.4, respectively. If the full factorial design were used, it would have 3^7=2187 runs. Whereas, L_{27} array requires only 27 runs. In Table S.5, Delta value represents the distinction between the highest and lowest E_L values. The Ranking row indicates that changes in K levels have the most substantial and least impact on E_L. Figure S.1 further supports this observation, illustrating that lower values of E_L correspond to specific levels of input parameters. E_L diminishes with the reduction of K, N, and AQL parameters. Conversely, a decline in α^U results in an increase in E_L. Optimal results are achieved by selecting a value for β^U within its upper and lower limits, leading to a reduction in E_L.

Table S.1. Experimental design of L_9 orthogonal array for PSO factors

Trial	w	(c_1, c_2)	N_P	N_I	OBF_{j1}	OBF_{j2}	OBF_{j3}	S/N
1	0.8	(1.5, 2.5)	20	50	363.59914	363.59967	363.59812	-51.21245
2	0.8	(2.0, 2.0)	50	100	363.59644	363.59644	363.59644	-51.21239
3	0.8	(2.5, 1.5)	80	150	363.59644	363.59644	363.59644	-51.21239
4	1.0	(1.5, 2.5)	50	150	363.59644	363.59644	363.59644	-51.21239
5	1.0	(2.0, 2.0)	80	50	363.60885	363.61088	363.62376	-51.21282
6	1.0	(2.5, 1.5)	20	100	363.59644	363.59644	363.59644	-51.21239
7	1.2	(1.5, 2.5)	80	100	363.59644	363.59644	363.59644	-51.21239
8	1.2	(2.0, 2.0)	20	150	363.59644	363.59644	363.59644	-51.21239
9	1.2	(2.5, 1.5)	50	50	363.66422	363.72169	363.64242	-51.21430

Table S.2. S/N ratios for various levels of PSO factors (the optimal level for each factor is bolded)

Factors	Level 1	Level 2	Level 3
w	**-51.2124**	-51.2125	-51.2130
(c_1, c_2)	**-51.2124**	-51.2125	-51.2130
N_{Pop}	**-51.2124**	-51.2130	-51.2125
N_{Itr}	-51.2132	**-51.2124**	-51.2124

Table S.3. Planning of factor levels for the sensitivity analysis

Factor	A	B	C	D	E	F	G
Notation	α^U	β^U	K	N	AQL	C_{ins}	C_{pr}
Level 1	0.01	0.01	1	500	0.010	3	12
Level 2	0.05	0.05	10	1000	0.015	7	25
Level 3	0.10	0.10	100	2500	0.025	15	50

Table S.4. Optimal outcomes obtained from trials generated using the L_{27} design under Γ=0 in the practical example

L_{27} Design								Optimal Results			
Trial	A	B	C	D	E	F	G	z_1	z_2	z_3	E_L
1	1	1	1	1	1	1	1	48	2.25	1.90	2705.04
2	1	1	1	1	2	2	2	41	2.09	1.72	3078.74
3	1	1	1	1	3	3	3	35	1.87	1.48	3664.14
4	1	2	2	2	1	1	1	88	2.25	1.91	50214.77
5	1	2	2	2	2	2	2	67	2.09	1.72	53463.42
6	1	2	2	2	3	3	3	50	1.87	1.46	58238.11
7	1	3	3	3	1	1	1	133	2.25	1.92	250142.06
8	1	3	3	3	2	2	2	92	2.09	1.73	265337.62
9	1	3	3	3	3	3	3	69	1.87	1.48	287695.11
10	2	1	2	3	1	2	3	84	2.25	1.91	125719.17
11	2	1	2	3	2	3	1	68	2.09	1.72	133666.78

12	2	1	2	3	3	1	2	115	1.87	1.49	143448.18
13	2	2	3	1	1	2	3	75	2.25	1.91	50485.30
14	2	2	3	1	2	3	1	59	2.09	1.72	53869.28
15	2	2	3	1	3	1	2	500	1.81	1.40	55617.23
16	2	3	1	2	1	2	3	44	2.25	1.87	5473.91
17	2	3	1	2	2	3	1	35	2.08	1.67	6078.57
18	2	3	1	2	3	1	2	42	1.87	1.45	5928.03
19	3	1	3	2	1	3	2	72	2.25	1.91	101238.96
20	3	1	3	2	2	1	3	304	2.20	1.62	105427.10
21	3	1	3	2	3	2	1	97	1.87	1.48	114776.05
22	3	2	1	3	1	3	2	48	2.24	1.86	13550.16
23	3	2	1	3	2	1	3	57	2.09	1.71	13523.23
24	3	2	1	3	3	2	1	45	1.86	1.45	14773.21
25	3	3	2	1	1	3	2	53	2.25	1.89	26046.36
26	3	3	2	1	2	1	3	73	2.09	1.73	26649.09
27	3	3	2	1	3	2	1	55	1.87	1.47	28963.17

Table S.5. Impacts of independent parameters on E_L

Factor	A	B	C	D	E	F	G
Level 1	108282	81525	7642	27898	69508	72628	72799
Level 2	64476	40415	71823	55649	73455	73563	74190
Level 3	49439	100257	142732	138651	79234	76005	75208
Delta	58844	59842	135090	110753	9725	3377	2410
Rank	4	3	1	2	5	6	7

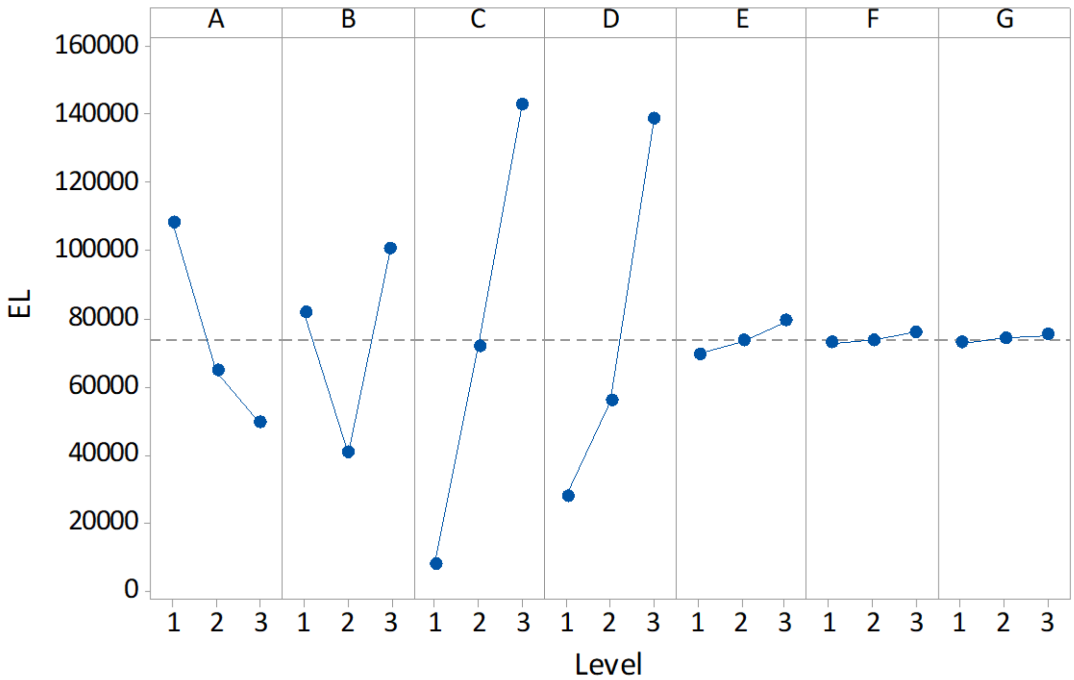

Figure S.1. Main effects of independent factors on E_L