

SUPPLEMENTARY DATA

UNIT NADARAJAH AND HAGHIGHI DISTRIBUTION: PROPERTIES AND APPLICATIONS IN QUALITY CONTROL

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ABSTRACT. In practice, the data related to rates and proportion may have excess of ones wherein the beta distribution does not fit well. To deal with the inflation of ones, this article introduces unit Nadarajah and Haghighi distribution. Besides deriving statistical properties of the proposed distribution, several estimation methods are discussed. In particular, maximum likelihood estimation, least squares estimation, weighted least squares estimation, maximum product of spacing, minimum spacing absolute distance estimation, minimum spacing absolute log-distance estimation, Cramér-Von-Mises, Anderson-Darling method and right-tail Anderson-Darling method are considered. Using real data sets, it is shown that the new distribution outperforms some well-known existing distributions. Furthermore, the application of the proposed distribution in quality control is also discussed. A control chart using unit Nadarajah and Haghighi distribution is constructed and its performance is evaluated using the average run length.

Keywords: Anderson-Darling method; Control chart, Cramér-Von-Mises Estimation Method; Maximum Likelihood Estimation, Root mean squared error; Weighted Least Squared Estimation.

AMS 2010 Classification: 62F10, 62P10.

S.1. SUPPLEMENTARY DATA

This file contains the additional tables.

TABLE S.1. Simulation results for $\alpha=0.5$ and $\lambda=2.0$

| n | Est. | MLE | LSE | WLS | PCE | MPS | MSADE | MSALDE | CVM | AD | RAD |
|-----|-------------------|--------------------|-----------------------|---------------------|----------------------|---------------------|-----------------------|-----------------------|--------------------|---------------------|---------------------|
| 20 | Bias(α) | 3.218 ⁷ | -1.400 ⁵ | -0.116 ¹ | 25.879 ¹⁰ | 0.648 ² | -1.356 ⁴ | -1.328 ³ | 2.342 ⁶ | 5.638 ⁸ | 1.088 ⁹ |
| | RMSE(α) | 6.599 ⁷ | 1.400 ² | 0.785 ¹ | 27.146 ¹⁰ | 3.002 ⁵ | 1.492 ³ | 1.584 ⁴ | 4.908 ⁶ | 18.195 ⁸ | 25.055 ⁹ |
| | Bias(λ) | 0.698 ⁴ | 626.252 ¹⁰ | 1.649 ⁷ | -0.400 ³ | 1.649 ⁸ | -0.362 ¹ | -0.331 ¹ | 1.202 ⁵ | 1.373 ⁶ | 1.865 ⁹ |
| | RMSE(λ) | 1.646 ⁴ | 656.253 ¹⁰ | 2.924 ⁸ | 0.400 ³ | 2.898 ⁷ | 0.690 ² | 0.894 ³ | 2.608 ⁴ | 2.866 ⁸ | 6.561 ⁹ |
| | Dabs | 0.169 ⁵ | 0.209 ⁷ | 0.168 ² | 0.386 ¹⁰ | 0.169 ⁶ | 0.328 ⁸ | 0.329 ⁹ | 0.168 ³ | 0.168 ⁴ | 0.167 ¹ |
| | Dmax | 0.267 ⁶ | 0.326 ⁷ | 0.251 ¹ | 0.589 ⁸ | 0.252 ² | 0.786 ⁹ | 0.786 ¹⁰ | 0.265 ⁴ | 0.260 ³ | 0.260 ⁵ |
| | Total | 33 ⁶ | 41 ⁸ | 20 ¹ | 42 ^{9.5} | 30 ^{4.5} | 28 ² | 30 ^{4.5} | 29 ³ | 35 ⁷ | 42 ^{9.5} |
| 40 | Bias(α) | 1.376 ⁴ | -1.400 ⁷ | -0.084 ¹ | 27.044 ¹⁰ | 0.125 ² | -1.397 ⁵ | -1.399 ⁶ | 1.339 ³ | 1.675 ⁸ | 5.221 ⁹ |
| | RMSE(α) | 4.233 ⁷ | 1.400 ² | 0.666 ¹ | 27.693 ¹⁰ | 1.826 ⁵ | 1.410 ⁴ | 1.402 ³ | 3.566 ⁶ | 8.117 ⁸ | 14.934 ⁹ |
| | Bias(λ) | 0.649 ⁴ | 611.574 ¹⁰ | 1.076 ⁷ | -0.400 ³ | 1.171 ⁹ | -0.397 ¹ | -0.399 ² | 0.915 ⁵ | 0.985 ⁶ | 1.149 ⁸ |
| | RMSE(λ) | 1.134 ⁴ | 625.250 ¹⁰ | 1.651 ⁶ | 0.400 ¹ | 1.681 ⁸ | 0.443 ³ | 0.406 ² | 1.651 ⁷ | 1.626 ⁵ | 2.271 ⁹ |
| | Dabs | 0.166 ¹ | 0.207 ⁷ | 0.167 ⁵ | 0.400 ¹⁰ | 0.168 ⁶ | 0.328 ⁸ | 0.328 ⁹ | 0.167 ³ | 0.167 ⁴ | 0.167 ² |
| | Dmax | 0.259 ⁴ | 0.327 ⁷ | 0.253 ² | 0.613 ⁸ | 0.251 ¹ | 0.841 ¹⁰ | 0.841 ⁹ | 0.261 ⁵ | 0.257 ³ | 0.262 ⁶ |
| | Total | 24 ² | 43 ^{9.5} | 22 ¹ | 42 ⁸ | 31 ⁵ | 31 ⁵ | 31 ⁵ | 29 ³ | 34 ⁷ | 43 ^{9.5} |
| 60 | Bias(α) | 0.589 ³ | -1.400 ^{7.5} | -0.091 ² | 27.524 ¹⁰ | -0.072 ¹ | -1.400 ⁶ | -1.400 ^{7.5} | 0.794 ⁵ | 0.596 ⁴ | 2.919 ⁹ |
| | RMSE(α) | 2.534 ⁶ | 1.400 ^{4.5} | 0.597 ¹ | 27.964 ¹⁰ | 1.217 ² | 1.400 ³ | 1.400 ^{4.5} | 2.704 ⁷ | 4.364 ⁸ | 10.090 ⁹ |
| | Bias(λ) | 0.641 ⁴ | 607.577 ¹⁰ | 0.918 ⁷ | -0.400 ² | 1.014 ⁹ | -0.399 ¹ | -0.400 ³ | 0.833 ⁵ | 0.878 ⁶ | 0.986 ⁸ |
| | RMSE(λ) | 0.963 ⁴ | 616.405 ¹⁰ | 1.297 ⁵ | 0.400 ¹ | 1.333 ⁷ | 0.412 ³ | 0.400 ² | 1.347 ⁸ | 1.298 ⁶ | 1.713 ⁹ |
| | Dabs | 0.166 ¹ | 0.207 ⁷ | 0.167 ⁴ | 0.405 ¹⁰ | 0.168 ⁶ | 0.327 ⁹ | 0.327 ⁸ | 0.167 ² | 0.167 ⁵ | 0.167 ³ |
| | Dmax | 0.256 ⁴ | 0.327 ⁷ | 0.253 ² | 0.621 ⁸ | 0.250 ¹ | 0.864 ⁹ | 0.864 ¹⁰ | 0.258 ⁵ | 0.255 ³ | 0.260 ⁶ |
| | Total | 22 ² | 46 ¹⁰ | 21 ¹ | 41 ⁸ | 26 ³ | 31 ⁴ | 35 ⁷ | 32 ^{5.5} | 32 ^{5.5} | 44 ⁹ |
| 80 | Bias(α) | 0.257 ⁴ | -1.400 ⁷ | -0.096 ¹ | 27.738 ¹⁰ | -0.161 ² | -1.400 ⁸ | -1.398 ⁶ | 0.506 ⁵ | 0.218 ³ | 1.861 ⁹ |
| | RMSE(α) | 1.639 ⁶ | 1.400 ³ | 0.550 ¹ | 28.074 ¹⁰ | 0.857 ² | 1.400 ⁴ | 1.410 ⁵ | 2.162 ⁷ | 2.713 ⁸ | 7.719 ⁹ |
| | Bias(λ) | 0.642 ⁴ | 604.034 ¹⁰ | 0.837 ⁷ | -0.400 ² | 0.938 ⁹ | -0.400 ³ | -0.398 ¹ | 0.781 ⁵ | 0.817 ⁶ | 0.881 ⁸ |
| | RMSE(λ) | 0.884 ⁴ | 610.504 ¹⁰ | 1.125 ⁵ | 0.400 ¹ | 1.171 ⁷ | 0.400 ² | 0.437 ³ | 1.174 ⁸ | 1.128 ⁶ | 1.417 ⁹ |
| | Dabs | 0.166 ¹ | 0.207 ⁷ | 0.167 ² | 0.408 ¹⁰ | 0.167 ⁶ | 0.328 ⁸ | 0.328 ⁹ | 0.167 ³ | 0.167 ⁵ | 0.167 ⁴ |
| | Dmax | 0.254 ⁴ | 0.327 ⁷ | 0.253 ² | 0.625 ⁸ | 0.250 ¹ | 0.879 ⁹ | 0.879 ¹⁰ | 0.257 ⁵ | 0.254 ³ | 0.258 ⁶ |
| | Total | 23 ² | 44 ⁹ | 18 ¹ | 41 ⁸ | 27 ³ | 34 ^{6.5} | 34 ^{6.5} | 33 ⁵ | 31 ⁴ | 45 ¹⁰ |
| 100 | Bias(α) | 0.128 ³ | -1.400 ⁸ | -0.102 ² | 27.901 ¹⁰ | -0.185 ⁴ | -1.400 ⁸ | -1.400 ⁸ | 0.336 ⁵ | 0.095 ¹ | 1.263 ⁶ |
| | RMSE(α) | 1.193 ³ | 1.400 ⁵ | 0.518 ¹ | 28.176 ¹⁰ | 0.676 ² | 1.400 ⁵ | 1.400 ⁵ | 1.803 ⁷ | 2.024 ⁸ | 6.022 ⁹ |
| | Bias(λ) | 0.640 ⁴ | 602.929 ¹⁰ | 0.798 ⁷ | -0.400 ¹ | 0.887 ⁹ | -0.400 ^{2.5} | -0.400 ^{2.5} | 0.758 ⁵ | 0.786 ⁶ | 0.839 ⁸ |
| | RMSE(λ) | 0.840 ⁴ | 608.204 ¹⁰ | 1.038 ⁵ | 0.400 ¹ | 1.079 ⁷ | 0.400 ^{2.5} | 0.400 ^{2.5} | 1.085 ⁸ | 1.045 ⁶ | 1.286 ⁹ |
| | Dabs | 0.166 ¹ | 0.207 ⁷ | 0.167 ² | 0.410 ¹⁰ | 0.167 ⁶ | 0.327 ^{8.5} | 0.327 ^{8.5} | 0.167 ³ | 0.167 ⁴ | 0.167 ⁵ |
| | Dmax | 0.254 ⁴ | 0.327 ⁷ | 0.253 ² | 0.628 ⁸ | 0.250 ¹ | 0.888 ^{9.5} | 0.888 ^{9.5} | 0.256 ⁵ | 0.254 ³ | 0.257 ⁶ |
| | Total | 19 ^{1.5} | 47 ¹⁰ | 19 ^{1.5} | 40 ⁸ | 29 ⁴ | 36 ^{6.5} | 36 ^{6.5} | 33 ⁵ | 28 ³ | 43 ⁹ |

TABLE S.2. Simulation results for $\alpha=1.5$ and $\lambda=2.0$

| n | Est. | MLE | LSE | WLS | PCE | MPS | MSADE | MSALDE | CVM | AD | RAD |
|-------|-------------------|---------------------|-----------------------|---------------------|----------------------|---------------------|-----------------------|-----------------------|---------------------|---------------------|----------------------|
| 20 | Bias(α) | 4.041 ⁵ | -1.400 ³ | 0.273 ¹ | 9.274 ⁷ | 1.882 ⁴ | -1.399 ² | 13.684 ⁹ | 4.110 ⁶ | 9.352 ⁸ | 13.755 ¹⁰ |
| | RMSE(α) | 9.070 ⁶ | 1.400 ² | 0.952 ¹ | 9.447 ⁷ | 4.608 ⁴ | 1.401 ³ | 15.056 ⁸ | 6.740 ⁵ | 21.421 ⁹ | 25.657 ¹⁰ |
| | Bias(λ) | -1.140 ⁶ | 674.605 ¹⁰ | -0.249 ² | -1.689 ⁷ | -0.256 ³ | -1.898 ⁸ | 12.48 ⁹ | -0.680 ⁵ | -0.547 ⁴ | -0.113 ¹ |
| | RMSE(λ) | 1.694 ¹ | 711.801 ¹⁰ | 2.095 ⁵ | 1.694 ² | 2.034 ⁴ | 1.903 ³ | 13.948 ⁹ | 2.123 ⁶ | 2.247 ⁷ | 5.756 ⁸ |
| | Dabs | 0.163 ² | 0.243 ⁸ | 0.165 ⁴ | 0.077 ¹ | 0.164 ³ | 0.661 ¹⁰ | 0.350 ⁹ | 0.167 ⁶ | 0.165 ⁵ | 0.169 ⁷ |
| | Dmax | 0.251 ⁶ | 0.392 ⁸ | 0.243 ² | 0.152 ¹ | 0.245 ³ | 0.982 ¹⁰ | 0.894 ⁹ | 0.250 ⁵ | 0.248 ⁴ | 0.252 ⁷ |
| Total | | 26 ⁴ | 41 ⁸ | 15 ¹ | 25 ³ | 21 ² | 36 ⁶ | 53 ¹⁰ | 33 ⁵ | 37 ⁷ | 43 ⁹ |
| 40 | Bias(α) | 3.288 ⁵ | -1.400 ³ | 0.390 ¹ | 9.027 ⁹ | 1.177 ² | 2.301 ⁴ | 14.483 ¹⁰ | 3.255 ⁶ | 4.949 ⁷ | 8.958 ⁸ |
| | RMSE(α) | 6.782 ⁶ | 1.400 ² | 0.892 ¹ | 9.334 ⁷ | 3.317 ³ | 6.290 ⁵ | 15.630 ⁹ | 5.717 ⁴ | 13.220 ⁸ | 18.185 ¹⁰ |
| | Bias(λ) | -1.160 ⁷ | 660.486 ¹⁰ | -0.766 ³ | -0.050 ¹ | -0.700 ² | 2.321 ⁸ | 14.396 ⁹ | -0.964 ⁶ | -0.905 ⁵ | -0.781 ⁴ |
| | RMSE(λ) | 1.404 ³ | 681.658 ¹⁰ | 1.291 ² | 2.238 ⁷ | 1.258 ¹ | 6.427 ⁸ | 15.361 ⁹ | 1.518 ⁵ | 1.435 ⁴ | 1.863 ⁶ |
| | Dabs | 0.165 ³ | 0.241 ⁸ | 0.165 ¹ | 0.172 ⁷ | 0.165 ² | 0.495 ¹⁰ | 0.341 ⁹ | 0.166 ⁵ | 0.165 ⁴ | 0.167 ⁶ |
| | Dmax | 0.251 ⁴ | 0.394 ⁷ | 0.245 ¹ | 0.401 ⁸ | 0.246 ² | 0.894 ⁹ | 0.927 ¹⁰ | 0.251 ⁵ | 0.249 ³ | 0.252 ⁶ |
| Total | | 28 ³ | 40 ⁷ | 9 ¹ | 39 ⁶ | 12 ² | 44 ⁹ | 56 ¹⁰ | 31 ^{4.5} | 31 ^{4.5} | 40 ^{7.5} |
| 60 | Bias(α) | 1.185 ³ | -1.400 ^{7.5} | 0.433 ² | 11.598 ¹⁰ | 0.840 ¹ | 3.757 ⁶ | 14.823 ^{7.5} | 2.707 ⁵ | 3.090 ⁴ | 6.693 ⁹ |
| | RMSE(α) | 7.820 ⁶ | 1.400 ^{4.5} | 0.853 ¹ | 11.715 ¹⁰ | 2.564 ² | 5.926 ³ | 16.090 ^{4.5} | 5.008 ⁷ | 9.006 ⁸ | 14.483 ⁹ |
| | Bias(λ) | -1.190 ⁴ | 659.309 ¹⁰ | -0.917 ⁷ | 4.788 ² | -0.834 ⁹ | 3.788 ¹ | 14.995 ³ | -1.054 ⁵ | -1.007 ⁶ | -0.942 ⁸ |
| | RMSE(λ) | 1.348 ⁴ | 674.518 ¹⁰ | 1.185 ⁵ | 5.370 ¹ | 1.127 ⁷ | 5.905 ³ | 15.969 ² | 1.388 ⁸ | 1.300 ³ | 1.526 ⁶ |
| | Dabs | 0.168 ⁶ | 0.239 ⁷ | 0.165 ² | 0.308 ⁸ | 0.164 ¹ | 0.387 ¹⁰ | 0.349 ⁹ | 0.166 ⁴ | 0.165 ³ | 0.166 ⁵ |
| | Dmax | 0.254 ⁶ | 0.392 ⁷ | 0.246 ² | 0.853 ⁹ | 0.246 ¹ | 0.846 ⁸ | 0.940 ¹⁰ | 0.251 ⁴ | 0.249 ³ | 0.252 ⁵ |
| Total | | 31 ⁵ | 40 ⁷ | 10 ² | 49 ⁹ | 9 ¹ | 45 ⁸ | 57 ¹⁰ | 27 ⁴ | 26 ³ | 36 ⁶ |
| 80 | Bias(α) | -2.010 ⁵ | -1.400 ^{3.5} | 0.463 ¹ | 16.188 ¹⁰ | 0.655 ² | -1.400 ^{3.5} | 15.092 ⁹ | 2.337 ⁷ | 2.189 ⁶ | 5.437 ⁸ |
| | RMSE(α) | 11.790 ⁷ | 1.400 ^{2.5} | 0.831 ¹ | 16.272 ⁹ | 2.078 ⁴ | 1.400 ^{2.5} | 16.400 ¹⁰ | 4.480 ⁵ | 6.640 ⁶ | 12.201 ⁸ |
| | Bias(λ) | -1.239 ⁶ | 658.692 ¹⁰ | -0.995 ² | 8.496 ⁸ | -0.898 ¹ | -1.900 ⁷ | 15.299 ⁹ | -1.109 ⁵ | -1.063 ⁴ | -1.046 ³ |
| | RMSE(λ) | 1.366 ⁵ | 670.145 ¹⁰ | 1.166 ² | 8.638 ⁸ | 1.091 ¹ | 1.900 ⁷ | 16.294 ⁹ | 1.339 ⁴ | 1.258 ³ | 1.411 ⁶ |
| | Dabs | 0.172 ⁶ | 0.238 ⁷ | 0.165 ² | 0.328 ⁸ | 0.165 ¹ | 0.662 ¹⁰ | 0.354 ⁹ | 0.167 ⁵ | 0.166 ³ | 0.166 ⁴ |
| | Dmax | 0.260 ⁶ | 0.391 ⁷ | 0.247 ² | 0.946 ⁸ | 0.246 ¹ | 0.985 ¹⁰ | 0.948 ⁹ | 0.252 ⁵ | 0.250 ³ | 0.252 ⁴ |
| Total | | 35 ⁶ | 40 ^{7.5} | 10 ^{1.5} | 51 ^{1.5} | 10 ^{1.5} | 40 ^{7.5} | 55 ¹⁰ | 31 ⁴ | 25 ³ | 33 ⁵ |
| 100 | Bias(α) | -1.687 ⁴ | -1.400 ³ | -0.481 ¹ | 20.687 ¹⁰ | 0.579 ² | 5.231 ⁸ | 15.342 ⁹ | 2.062 ⁶ | 1.795 ⁵ | 4.617 ⁷ |
| | RMSE(α) | 10.846 ⁸ | 1.400 ² | 0.821 ¹ | 20.758 ¹⁰ | 1.773 ³ | 6.741 ⁶ | 16.672 ⁹ | 4.050 ⁴ | 5.555 ⁵ | 10.645 ⁷ |
| | Bias(λ) | -1.229 ⁶ | 657.475 ¹⁰ | -1.036 ² | 11.465 ⁸ | -0.941 ¹ | 5.451 ⁷ | 15.610 ⁹ | -1.135 ⁵ | -1.092 ⁴ | -1.090 ³ |
| | RMSE(λ) | 1.335 ⁵ | 667.818 ¹⁰ | 1.166 ² | 11.553 ⁸ | 1.088 ¹ | 6.805 ⁷ | 16.588 ⁹ | 1.317 ⁴ | 1.244 ³ | 1.373 ⁶ |
| | Dabs | 0.170 ⁶ | 0.238 ⁷ | 0.165 ² | 0.331 ⁹ | 0.165 ¹ | 0.319 ⁸ | 0.358 ¹⁰ | 0.166 ⁵ | 0.166 ³ | 0.166 ⁴ |
| | Dmax | 0.258 ⁶ | 0.391 ⁷ | 0.248 ² | 0.962 ¹⁰ | 0.246 ¹ | 0.830 ⁸ | 0.953 ⁹ | 0.252 ⁴ | 0.250 ³ | 0.252 ⁵ |
| Total | | 35 ⁶ | 39 ⁷ | 10 ² | 55 ^{9.5} | 9 ^{1.0} | 44 ⁸ | 55 ^{9.5} | 28 ⁴ | 23 ³ | 32 ⁵ |

TABLE S.3. Simulation results for $\alpha=3.5$ and $\lambda=0.5$

| n | Est. | MLE | LSE | WLS | PCE | MPS | MSADE | MSALDE | CVM | AD | RAD |
|-----|-------------------|---------------------|------------------------|---------------------|---------------------|---------------------|-----------------------|-----------------------|--------------------|---------------------|----------------------|
| 20 | Bias(α) | 3.759 ⁵ | -3.400 ³ | -0.912 ¹ | 5.960 ⁷ | 1.008 ⁴ | -3.400 ² | -3.400 ⁹ | 4.157 ⁶ | 9.003 ⁸ | 12.338 ¹⁰ |
| | RMSE(α) | 8.115 ⁶ | 3.400 ² | 2.282 ¹ | 6.074 ⁷ | 6.680 ⁴ | 3.400 ³ | 3.400 ⁸ | 9.325 ⁵ | 25.405 ⁹ | 27.828 ¹⁰ |
| | Bias(λ) | 1.482 ⁶ | 1509.383 ¹⁰ | 3.258 ² | 0.288 ⁷ | 3.292 ³ | -0.400 ⁸ | -0.400 ⁹ | 2.514 ⁵ | 2.922 ⁴ | 4.143 ¹ |
| | RMSE(λ) | 3.207 ¹ | 1598.423 ¹⁰ | 5.411 ⁵ | 0.342 ² | 5.238 ⁴ | 0.400 ³ | 0.400 ⁹ | 4.814 ⁶ | 5.990 ⁷ | 14.414 ⁸ |
| | Dabs | 0.172 ² | 0.210 ⁸ | 0.168 ⁴ | 0.408 ¹ | 0.170 ³ | 0.331 ¹⁰ | 0.331 ⁹ | 0.168 ⁶ | 0.169 ⁵ | 0.166 ⁷ |
| | Dmax | 0.267 ⁶ | 0.338 ⁸ | 0.252 ² | 0.605 ¹ | 0.253 ³ | 0.798 ¹⁰ | 0.798 ⁹ | 0.261 ⁵ | 0.259 ⁴ | 0.259 ⁷ |
| | Total | 33 ⁶ | 39 ^{8.5} | 19 ¹ | 33 ⁶ | 29 ² | 31 ^{3.5} | 31 ^{3.5} | 33 ⁶ | 39 ^{8.5} | 43 ¹⁰ |
| 40 | Bias(α) | 1.774 ⁴ | -3.400 ⁶ | -0.954 ³ | -0.707 ² | -0.182 ¹ | -3.400 ^{7.5} | -3.400 ^{7.5} | 2.500 ⁵ | 3.512 ⁹ | 6.515 ¹⁰ |
| | RMSE(α) | 6.390 ⁷ | 3.400 ² | 2.064 ¹ | 4.403 ⁵ | 4.936 ⁶ | 3.400 ^{3.5} | 3.400 ^{3.5} | 7.747 ⁸ | 15.072 ⁹ | 18.595 ¹⁰ |
| | Bias(λ) | 1.311 ³ | 1479.768 ¹⁰ | 2.200 ⁶ | 4.754 ⁹ | 2.323 ⁷ | -0.400 ^{1.5} | -0.400 ^{1.5} | 1.973 ⁴ | 2.061 ⁵ | 2.545 ⁸ |
| | RMSE(λ) | 2.192 ³ | 1529.969 ¹⁰ | 3.353 ⁶ | 5.745 ⁹ | 3.295 ⁴ | 0.400 ^{1.5} | 0.400 ^{1.5} | 3.357 ⁷ | 3.346 ⁵ | 4.810 ⁸ |
| | Dabs | 0.167 ³ | 0.209 ⁷ | 0.167 ⁴ | NaN ¹⁰ | 0.168 ⁶ | 0.331 ^{8.5} | 0.331 ^{8.5} | 0.167 ² | 0.168 ⁵ | 0.166 ¹ |
| | Dmax | 0.260 ⁶ | 0.339 ⁷ | 0.254 ² | NaN ¹⁰ | 0.252 ¹ | 0.854 ^{8.5} | 0.854 ^{8.5} | 0.258 ⁵ | 0.256 ³ | 0.258 ⁴ |
| | Total | 26 ³ | 42 ⁹ | 22 ¹ | 45 ¹⁰ | 25 ² | 31 ⁵ | 31 ⁵ | 31 ⁵ | 36 ⁷ | 41 ⁸ |
| 60 | Bias(α) | 0.627 ² | -3.400 ⁷ | -1.049 ⁴ | -0.358 ¹ | -0.756 ³ | -3.400 ^{8.5} | -3.400 ^{8.5} | 1.416 ⁶ | 1.308 ⁵ | 3.815 ¹⁰ |
| | RMSE(α) | 4.889 ⁷ | 3.400 ³ | 1.951 ² | 1.529 ¹ | 3.867 ⁶ | 3.400 ^{4.5} | 3.400 ^{4.5} | 6.509 ⁸ | 9.887 ⁹ | 13.866 ¹⁰ |
| | Bias(λ) | 1.266 ³ | 1470.299 ¹⁰ | 1.878 ⁶ | 11.397 ⁹ | 2.002 ⁷ | -0.400 ^{1.5} | -0.400 ^{1.5} | 1.778 ⁷ | 1.819 ⁶ | 2.167 ⁸ |
| | RMSE(λ) | 1.861 ³ | 1509.415 ¹⁰ | 2.647 ⁵ | 11.710 ⁹ | 2.617 ⁴ | 0.400 ^{1.5} | 0.400 ^{1.5} | 2.788 ⁷ | 2.662 ⁶ | 3.612 ⁸ |
| | Dabs | 0.166 ¹ | 0.209 ⁷ | 0.167 ⁴ | 0.585 ¹⁰ | 0.168 ⁶ | 0.331 ^{8.5} | 0.331 ^{8.5} | 0.167 ³ | 0.167 ⁵ | 0.167 ² |
| | Dmax | 0.257 ⁴ | 0.340 ⁷ | 0.254 ² | 0.842 ⁸ | 0.251 ¹ | 0.878 ^{9.5} | 0.878 ^{9.5} | 0.257 ⁶ | 0.255 ³ | 0.257 ⁵ |
| | Total | 20 ¹ | 44 ¹⁰ | 23 ² | 38 ⁸ | 27 ³ | 34 ⁶ | 34 ⁶ | 34 ⁶ | 33 ⁴ | 43 ⁹ |
| 80 | Bias(α) | -0.057 ¹ | -3.400 ⁹ | -1.109 ⁵ | 1.133 ⁶ | -1.061 ⁴ | -3.400 ⁹ | -3.400 ⁹ | 0.743 ³ | 0.239 ² | 2.438 ⁷ |
| | RMSE(α) | 4.084 ⁷ | 3.400 ⁵ | 1.875 ² | 1.411 ¹ | 3.183 ³ | 3.400 ⁵ | 3.400 ⁵ | 5.619 ⁸ | 7.174 ⁹ | 11.097 ¹⁰ |
| | Bias(λ) | 1.247 ³ | 1466.142 ¹⁰ | 1.713 ⁶ | 16.729 ⁹ | 1.846 ⁷ | -0.400 ^{1.5} | -0.400 ^{1.5} | 1.657 ⁴ | 1.684 ⁵ | 1.924 ⁸ |
| | RMSE(λ) | 1.710 ³ | 1497.992 ¹⁰ | 2.295 ⁴ | 16.729 ⁹ | 2.299 ⁵ | 0.400 ^{1.5} | 0.400 ^{1.5} | 2.435 ⁷ | 2.310 ⁶ | 2.982 ⁸ |
| | Dabs | 0.165 ¹ | 0.209 ⁷ | 0.167 ⁴ | 0.637 ¹⁰ | 0.167 ⁶ | 0.331 ^{8.5} | 0.331 ^{8.5} | 0.167 ³ | 0.167 ⁵ | 0.167 ² |
| | Dmax | 0.255 ⁴ | 0.339 ⁷ | 0.253 ² | 0.938 ¹⁰ | 0.251 ¹ | 0.894 ^{8.5} | 0.894 ^{8.5} | 0.256 ⁶ | 0.255 ³ | 0.256 ⁵ |
| | Total | 19 ¹ | 48 ¹⁰ | 23 ² | 45 ⁹ | 26 ³ | 34 ^{6.5} | 34 ^{6.5} | 31 ⁵ | 30 ⁴ | 40 ⁸ |
| 100 | Bias(α) | -0.296 ³ | -3.400 ⁹ | -1.156 ⁴ | 2.411 ⁷ | -1.197 ⁵ | -3.400 ⁹ | -3.400 ⁹ | 0.283 ² | -0.243 ¹ | 1.580 ⁶ |
| | RMSE(α) | 3.559 ⁷ | 3.400 ⁵ | 1.826 ¹ | 2.465 ² | 2.766 ³ | 3.400 ⁵ | 3.400 ⁵ | 4.972 ⁸ | 5.848 ⁹ | 9.347 ¹⁰ |
| | Bias(λ) | 1.246 ³ | 1460.600 ¹⁰ | 1.630 ⁶ | 21.344 ⁹ | 1.7441 ⁷ | -0.400 ^{1.5} | -0.400 ^{1.5} | 1.599 ⁴ | 1.618 ⁵ | 1.823 ⁸ |
| | RMSE(λ) | 1.631 ³ | 1491.261 ¹⁰ | 2.116 ⁴ | 21.372 ⁹ | 2.117 ⁵ | 0.400 ^{1.5} | 0.400 ^{1.5} | 2.250 ⁷ | 2.138 ⁶ | 2.708 ⁸ |
| | Dabs | 0.166 ¹ | 0.209 ⁷ | 0.167 ⁴ | 0.650 ¹⁰ | 0.167 ⁶ | 0.331 ^{8.5} | 0.331 ^{8.5} | 0.167 ³ | 0.167 ⁵ | 0.167 ² |
| | Dmax | 0.255 ⁴ | 0.340 ⁷ | 0.253 ² | 0.964 ¹⁰ | 0.252 ¹ | 0.903 ^{8.5} | 0.903 ^{8.5} | 0.256 ⁵ | 0.254 ³ | 0.256 ⁶ |
| | Total | 21 ^{1.5} | 48 ¹⁰ | 21 ^{1.5} | 47 ^{9.0} | 27 ^{3.0} | 34 ^{6.5} | 34 ^{6.5} | 29 ^{4.5} | 29 ^{4.5} | 40 ⁸ |

TABLE S.4. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability for the upper-sided chart with $\alpha_0=0.75$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|------------|------------|------------|------------|-------------|------------|----------|------------|------------|
| ARL | 0.1 | | 69324.44 | 17331.109 | 13864.89 | 11554.07 | 9243.258 | 7702.715 | 6932.444 | 5332.649 | 4621.629 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.999 |
| Q1 | | | 19943.25 | 4985.706 | 3988.536 | 3323.756 | 2658.976 | 2215.789 | 1994.196 | 1533.964 | 1329.416 |
| Q2 | | | 48051.690 | 12012.663 | 9610.061 | 8008.326 | 6406.592 | 5338.769 | 4804.857 | 3695.964 | 3203.123 |
| Q3 | | | 96103.380 | 24025.326 | 19220.12 | 16016.65 | 12813.18 | 10677.54 | 9609.715 | 7391.928 | 6406.245 |
| ARL | 0.4 | | 17334.86 | 4333.715 | 3466.972 | 2889.143 | 2311.315 | 1926.095 | 1733.486 | 1333.451 | 1155.657 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 |
| Q1 | | | 4986.784 | 1246.588 | 997.242 | 831.011 | 664.779 | 553.959 | 498.549 | 383.466 | 332.318 |
| Q2 | | | 12015.260 | 3003.556 | 2402.775 | 2002.255 | 1601.735 | 1334.721 | 1201.214 | 923.931 | 800.694 |
| Q3 | | | 24030.52 | 6007.112 | 4805.551 | 4004.509 | 3203.469 | 2669.441 | 2402.429 | 1847.862 | 1601.388 |
| ARL | 0.5 | | 13868.89 | 3467.222 | 2773.777 | 2311.481 | 1849.185 | 1540.988 | 1386.889 | 1066.838 | 924.593 |
| CV | | | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 3989.686 | 997.314 | 797.822 | 664.828 | 531.834 | 443.171 | 398.839 | 306.766 | 265.845 |
| Q2 | | | 9612.833 | 2402.948 | 1922.289 | 1601.85 | 1281.411 | 1067.785 | 960.971 | 739.129 | 640.532 |
| Q3 | | | 19225.67 | 4805.897 | 3844.579 | 3203.7 | 2562.821 | 2135.569 | 1921.943 | 1478.258 | 1281.064 |
| ARL | 0.6 | | 11558.24 | 2889.559 | 2311.648 | 1926.373 | 1541.099 | 1284.249 | 1155.824 | 889.096 | 770.549 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 3324.954 | 831.131 | 664.876 | 554.039 | 443.203 | 369.312 | 332.366 | 255.633 | 221.529 |
| Q2 | | | 8011.214 | 2002.544 | 1601.966 | 1334.914 | 1067.862 | 889.827 | 800.809 | 615.927 | 533.758 |
| Q3 | | | 16022.43 | 4005.087 | 3203.931 | 2669.827 | 2135.723 | 1779.654 | 1601.619 | 1231.855 | 1067.515 |
| ARL | 0.9 | | 7707.158 | 1926.789 | 1541.432 | 1284.527 | 1027.621 | 856.351 | 770.716 | 592.859 | 513.811 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 2217.067 | 554.159 | 443.298 | 369.391 | 295.484 | 246.213 | 221.577 | 170.411 | 147.670 |
| Q2 | | | 5341.849 | 1335.202 | 1068.092 | 890.019 | 711.946 | 593.231 | 533.873 | 410.592 | 355.799 |
| Q3 | | | 10683.7 | 2670.405 | 2136.185 | 1780.039 | 1423.892 | 1186.461 | 1067.746 | 821.183 | 711.599 |
| ARL | 1 | | 6936.942 | 1734.236 | 1387.389 | 1156.157 | 924.926 | 770.772 | 693.695 | 533.611 | 462.463 |
| CV | | | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 1995.49 | 498.765 | 398.983 | 332.462 | 265.941 | 221.593 | 199.419 | 153.367 | 132.899 |
| Q2 | | | 4807.975 | 1201.734 | 961.318 | 801.041 | 640.763 | 533.912 | 480.486 | 369.525 | 320.208 |
| Q3 | | | 9615.951 | 2403.468 | 1922.636 | 1602.081 | 1281.526 | 1067.823 | 960.972 | 739.049 | 640.417 |
| ARL | 1.3 | | 5337.263 | 1334.316 | 1067.453 | 889.544 | 711.635 | 593.029 | 533.727 | 410.559 | 355.818 |
| CV | | | 1 | 1 | 0.999531 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 |
| Q1 | | | 1535.291 | 383.715 | 306.943 | 255.762 | 204.581 | 170.460 | 153.399 | 117.967 | 102.219 |
| Q2 | | | 3699.162 | 924.531 | 739.555 | 616.238 | 492.921 | 410.710 | 369.604 | 284.231 | 246.288 |
| Q3 | | | 7398.324 | 1849.061 | 1479.11 | 1232.477 | 985.843 | 821.420 | 739.209 | 568.462 | 492.575 |
| ARL | 1.5 | | 4626.294 | 1156.574 | 925.259 | 771.049 | 616.839 | 514.033 | 462.629 | 355.869 | 308.420 |
| CV | | | 1 | 1 | 0.999459 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 1330.758 | 332.582 | 266.037 | 221.673 | 177.309 | 147.734 | 132.946 | 102.233 | 88.583 |
| Q2 | | | 3206.356 | 801.329 | 640.994 | 534.104 | 427.214 | 355.954 | 320.324 | 246.323 | 213.434 |
| Q3 | | | 6412.712 | 1602.658 | 1281.988 | 1068.208 | 854.428 | 711.908 | 640.648 | 492.646 | 426.868 |
| ARL | 2 | | 3470.969 | 867.743 | 694.194 | 578.495 | 462.796 | 385.664 | 347.097 | 266.998 | 231.399 |
| CV | | | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 998.392 | 249.490 | 199.563 | 166.279 | 132.994 | 110.805 | 99.709 | 76.667 | 66.425 |
| Q2 | | | 2405.546 | 601.127 | 480.832 | 400.636 | 320.439 | 266.975 | 240.243 | 184.722 | 160.047 |
| Q3 | | | 4811.092 | 1202.253 | 961.664 | 801.271 | 640.879 | 533.95 | 480.486 | 369.445 | 320.093 |
| ARL | 2.5 | | 2777.775 | 694.444 | 555.555 | 462.963 | 370.370 | 308.642 | 277.778 | 213.676 | 185.186 |
| CV | | | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 | 0.998 | 0.997 |
| Q1 | | | 798.972 | 199.635 | 159.679 | 133.042 | 106.405 | 88.647 | 79.768 | 61.327 | 53.131 |
| Q2 | | | 1925.06 | 481.005 | 384.735 | 320.555 | 256.375 | 213.588 | 192.194 | 147.762 | 128.014 |
| Q3 | | | 3850.12 | 962.010 | 769.469 | 641.109 | 512.749 | 427.175 | 384.389 | 295.524 | 256.028 |
| ARL | 2.7 | | 2572.383 | 643.096 | 514.477 | 428.731 | 342.985 | 285.821 | 257.239 | 197.877 | 171.493 |
| CV | | | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 | 0.997 | 0.997 |
| Q1 | | | 739.885 | 184.863 | 147.862 | 123.194 | 98.527 | 82.082 | 73.859 | 56.782 | 49.192 |
| Q2 | | | 1782.694 | 445.414 | 356.262 | 296.827 | 237.392 | 197.769 | 177.958 | 136.811 | 118.523 |
| Q3 | | | 3565.387 | 890.827 | 712.523 | 593.654 | 474.784 | 395.538 | 355.915 | 273.621 | 237.046 |
| ARL | 3 | | 2315.644 | 578.911 | 463.129 | 385.941 | 308.753 | 257.295 | 231.565 | 178.127 | 154.377 |
| CV | | | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 | 0.997 | 0.997 |
| Q1 | | | 666.026 | 166.398 | 133.090 | 110.884 | 88.679 | 73.875 | 66.473 | 51.100 | 44.268 |
| Q2 | | | 1604.736 | 400.9246 | 320.67 | 267.167 | 213.665 | 177.996 | 160.162 | 123.122 | 106.659 |
| Q3 | | | 3209.471 | 801.848 | 641.34 | 534.335 | 427.329 | 355.992 | 320.324 | 246.243 | 213.319 |

TABLE S.5. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability for the Two-sided chart with $\alpha_0=0.75$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|-------|--------|--------|---------|---------|---------|----------|----------|----------|
| ARL | 0.1 | | 1.046 | 1.214 | 1.282 | 1.357 | 1.484 | 1.634 | 1.748 | 2.182 | 2.574 |
| CV | | | 0.211 | 0.420 | 0.469 | 0.513 | 0.571 | 0.623 | 0.654 | 0.736 | 0.782 |
| Q1 | | | 0.092 | 0.166 | 0.189 | 0.215 | 0.257 | 0.304 | 0.339 | 0.469 | 0.585 |
| Q2 | | | 0.222 | 0.399 | 0.458 | 0.519 | 0.619 | 0.732 | 0.816 | 1.131 | 1.409 |
| Q3 | | | 0.445 | 0.799 | 0.915 | 1.038 | 1.237 | 1.464 | 1.633 | 2.261 | 2.818 |
| ARL | 0.4 | | 1.133 | 1.820 | 2.221 | 2.778 | 4.099 | 6.515 | 9.312 | 36.066 | 117.186 |
| CV | | | 0.342 | 0.671 | 0.741 | 0.800 | 0.869 | 0.920 | 0.945 | 0.986 | 0.996 |
| Q1 | | | 0.134 | 0.361 | 0.481 | 0.645 | 1.029 | 1.726 | 2.532 | 10.231 | 33.568 |
| Q2 | | | 0.323 | 0.869 | 1.159 | 1.553 | 2.478 | 4.159 | 6.101 | 24.651 | 80.880 |
| Q3 | | | 0.646 | 1.739 | 2.317 | 3.107 | 4.957 | 8.319 | 12.203 | 49.301 | 161.761 |
| ARL | 0.5 | | 1.153 | 2.022 | 2.578 | 3.402 | 5.563 | 10.134 | 16.226 | 100.386 | 449.219 |
| CV | | | 0.364 | 0.711 | 0.782 | 0.840 | 0.906 | 0.949 | 0.969 | 0.995 | 0.999 |
| Q1 | | | 0.143 | 0.423 | 0.586 | 0.826 | 1.452 | 2.769 | 4.523 | 28.735 | 129.088 |
| Q2 | | | 0.343 | 1.016 | 1.412 | 1.992 | 3.498 | 6.672 | 10.897 | 69.235 | 311.028 |
| Q3 | | | 0.687 | 2.032 | 2.824 | 3.983 | 6.996 | 13.343 | 21.794 | 138.470 | 622.056 |
| ARL | 0.6 | | 1.172 | 2.226 | 2.960 | 4.117 | 7.465 | 15.641 | 28.183 | 270.085 | 1042.26 |
| CV | | | 0.383 | 0.742 | 0.814 | 0.870 | 0.931 | 0.968 | 0.982 | 0.998 | 0.999 |
| Q1 | | | 0.149 | 0.482 | 0.698 | 1.034 | 2.000 | 4.354 | 7.963 | 77.555 | 299.696 |
| Q2 | | | 0.361 | 1.162 | 1.682 | 2.491 | 4.819 | 10.491 | 19.186 | 186.862 | 722.093 |
| Q3 | | | 0.722 | 2.325 | 3.363 | 4.983 | 9.638 | 20.983 | 38.372 | 373.723 | 1444.186 |
| ARL | 0.9 | | 1.217 | 2.859 | 4.278 | 6.925 | 17.081 | 54.772 | 138.689 | 1073.366 | 1027.135 |
| CV | | | 0.423 | 0.806 | 0.875 | 0.925 | 0.970 | 0.991 | 0.996 | 0.999 | 0.999 |
| Q1 | | | 0.167 | 0.668 | 1.081 | 1.845 | 4.769 | 15.613 | 39.754 | 308.644 | 295.345 |
| Q2 | | | 0.402 | 1.610 | 2.603 | 4.445 | 11.489 | 37.617 | 95.785 | 743.654 | 711.609 |
| Q3 | | | 0.805 | 3.221 | 5.207 | 8.889 | 22.979 | 75.234 | 191.569 | 1487.307 | 1423.219 |
| ARL | 1 | | 1.230 | 3.078 | 4.779 | 8.124 | 22.168 | 81.414 | 223.498 | 1039.881 | 925.319 |
| CV | | | 0.433 | 0.822 | 0.889 | 0.936 | 0.977 | 0.994 | 0.998 | 0.999 | 0.999 |
| Q1 | | | 0.172 | 0.732 | 1.226 | 2.190 | 6.232 | 23.277 | 64.152 | 299.011 | 266.054 |
| Q2 | | | 0.414 | 1.765 | 2.953 | 5.277 | 15.016 | 56.085 | 154.570 | 720.444 | 641.036 |
| Q3 | | | 0.827 | 3.529 | 5.906 | 10.554 | 30.033 | 112.169 | 309.140 | 1440.888 | 1282.072 |
| ARL | 1.3 | | 1.265 | 3.765 | 6.495 | 12.713 | 46.493 | 236.475 | 610.052 | 821.154 | 711.943 |
| CV | | | 0.458 | 0.857 | 0.919 | 0.959 | 0.989 | 0.998 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.184 | 0.932 | 1.721 | 3.511 | 13.231 | 67.886 | 175.357 | 236.087 | 204.669 |
| Q2 | | | 0.443 | 2.245 | 4.146 | 8.461 | 31.879 | 163.565 | 422.509 | 568.834 | 493.135 |
| Q3 | | | 0.887 | 4.489 | 8.292 | 16.921 | 63.758 | 327.129 | 845.019 | 1137.667 | 986.269 |
| ARL | 1.5 | | 1.285 | 4.247 | 7.831 | 16.769 | 73.504 | 403.125 | 763.132 | 711.979 | 617.062 |
| CV | | | 0.471 | 0.874 | 0.934 | 0.969 | 0.993 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.191 | 1.072 | 2.106 | 4.679 | 21.002 | 115.828 | 219.396 | 204.679 | 177.374 |
| Q2 | | | 0.460 | 2.582 | 5.073 | 11.273 | 50.601 | 279.078 | 528.616 | 493.160 | 427.368 |
| Q3 | | | 0.920 | 5.164 | 10.146 | 22.546 | 101.203 | 558.156 | 1057.232 | 986.320 | 854.736 |
| ARL | 2 | | 1.328 | 5.551 | 11.940 | 31.541 | 195.962 | 664.877 | 687.592 | 534.092 | 462.879 |
| CV | | | 0.497 | 0.905 | 0.957 | 0.984 | 0.997 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.206 | 1.448 | 3.289 | 8.929 | 56.231 | 191.129 | 197.664 | 153.505 | 133.018 |
| Q2 | | | 0.496 | 3.489 | 7.925 | 21.514 | 135.483 | 460.511 | 476.256 | 369.858 | 320.497 |
| Q3 | | | 0.991 | 6.979 | 15.849 | 43.028 | 270.967 | 921.022 | 952.512 | 739.716 | 640.994 |
| ARL | 2.5 | | 1.364 | 7.002 | 17.329 | 55.069 | 370.370 | 606.075 | 555.289 | 427.351 | 370.371 |
| CV | | | 0.517 | 0.926 | 0.971 | 0.991 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.218 | 1.867 | 4.840 | 15.698 | 106.405 | 174.213 | 159.603 | 122.797 | 106.405 |
| Q2 | | | 0.525 | 4.498 | 11.662 | 37.824 | 256.375 | 419.759 | 384.551 | 295.870 | 256.375 |
| Q3 | | | 1.049 | 8.996 | 23.324 | 75.647 | 512.749 | 839.506 | 769.101 | 591.740 | 512.749 |
| ARL | 2.7 | | 1.377 | 7.625 | 19.893 | 67.475 | 426.794 | 567.078 | 514.367 | 395.724 | 342.961 |
| CV | | | 0.523 | 0.932 | 0.975 | 0.993 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 |
| Q1 | | | 0.222 | 2.046 | 5.578 | 19.267 | 122.637 | 162.994 | 147.829 | 113.699 | 98.519 |
| Q2 | | | 0.535 | 4.931 | 13.439 | 46.423 | 295.484 | 392.722 | 356.185 | 273.948 | 237.375 |
| Q3 | | | 1.070 | 9.862 | 26.879 | 92.846 | 590.969 | 785.444 | 712.369 | 547.896 | 474.751 |
| ARL | 3 | | 1.395 | 8.609 | 24.219 | 89.548 | 477.095 | 513.313 | 463.035 | 356.189 | 308.698 |
| CV | | | 0.532 | 0.940 | 0.979 | 0.994 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.228 | 2.329 | 6.823 | 25.617 | 137.108 | 147.527 | 133.063 | 102.326 | 88.663 |
| Q2 | | | 0.549 | 5.614 | 16.439 | 61.723 | 330.350 | 355.455 | 320.605 | 246.545 | 213.626 |
| Q3 | | | 1.099 | 11.227 | 32.877 | 123.446 | 660.700 | 710.910 | 641.209 | 493.090 | 427.253 |

TABLE S.6. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability of the lower-sided chart with $\alpha_0=1.00$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|--------|-------|-------|--------|--------|---------|----------|----------|------------|
| ARL | 0.1 | | 1.022 | 1.093 | 1.119 | 1.146 | 1.188 | 1.234 | 1.267 | 1.374 | 1.455 |
| CV | | | 0.146 | 0.291 | 0.326 | 0.356 | 0.398 | 0.436 | 0.459 | 0.522 | 0.559 |
| Q1 | | | 0.075 | 0.117 | 0.128 | 0.139 | 0.156 | 0.173 | 0.185 | 0.221 | 0.248 |
| Q2 | | | 0.179 | 0.281 | 0.309 | 0.336 | 0.376 | 0.417 | 0.445 | 0.533 | 0.596 |
| Q3 | | | 0.359 | 0.562 | 0.618 | 0.672 | 0.753 | 0.834 | 0.890 | 1.066 | 1.193 |
| ARL | 0.4 | | 1.071 | 1.357 | 1.485 | 1.634 | 1.911 | 2.273 | 2.576 | 3.962 | 5.558 |
| CV | | | 0.258 | 0.513 | 0.571 | 0.623 | 0.691 | 0.748 | 0.782 | 0.865 | 0.906 |
| Q1 | | | 0.106 | 0.215 | 0.257 | 0.304 | 0.388 | 0.496 | 0.586 | 0.989 | 1.450 |
| Q2 | | | 0.256 | 0.519 | 0.619 | 0.732 | 0.936 | 1.195 | 1.411 | 2.383 | 3.495 |
| Q3 | | | 0.512 | 1.038 | 1.238 | 1.465 | 1.872 | 2.391 | 2.822 | 4.766 | 6.989 |
| ARL | 0.5 | | 1.085 | 1.443 | 1.612 | 1.817 | 2.216 | 2.770 | 3.264 | 5.806 | 9.255 |
| CV | | | 0.279 | 0.554 | 0.616 | 0.671 | 0.741 | 0.799 | 0.833 | 0.909 | 0.944 |
| Q1 | | | 0.113 | 0.244 | 0.297 | 0.360 | 0.479 | 0.643 | 0.786 | 1.522 | 2.516 |
| Q2 | | | 0.272 | 0.587 | 0.716 | 0.868 | 1.155 | 1.548 | 1.895 | 3.668 | 6.062 |
| Q3 | | | 0.543 | 1.173 | 1.431 | 1.735 | 2.310 | 3.096 | 3.789 | 7.334 | 12.124 |
| ARL | 0.6 | | 1.097 | 1.528 | 1.743 | 2.012 | 2.559 | 3.369 | 4.135 | 8.618 | 15.854 |
| CV | | | 0.297 | 0.588 | 0.653 | 0.709 | 0.781 | 0.839 | 0.871 | 0.940 | 0.968 |
| Q1 | | | 0.118 | 0.271 | 0.337 | 0.419 | 0.581 | 0.817 | 1.039 | 2.332 | 4.416 |
| Q2 | | | 0.286 | 0.652 | 0.813 | 1.009 | 1.399 | 1.969 | 2.504 | 5.619 | 10.639 |
| Q3 | | | 0.571 | 1.305 | 1.625 | 2.017 | 2.799 | 3.938 | 5.007 | 11.239 | 21.278 |
| ARL | 0.9 | | 1.128 | 1.783 | 2.158 | 2.672 | 3.868 | 6.001 | 8.408 | 30.151 | 93.267 |
| CV | | | 0.337 | 0.663 | 0.732 | 0.791 | 0.861 | 0.913 | 0.939 | 0.983 | 0.995 |
| Q1 | | | 0.132 | 0.349 | 0.462 | 0.614 | 0.962 | 1.578 | 2.272 | 8.529 | 26.687 |
| Q2 | | | 0.319 | 0.842 | 1.113 | 1.478 | 2.317 | 3.802 | 5.474 | 20.551 | 64.301 |
| Q3 | | | 0.638 | 1.684 | 2.226 | 2.967 | 4.635 | 7.604 | 10.949 | 41.102 | 128.601 |
| ARL | 1 | | 1.138 | 1.868 | 2.304 | 2.919 | 4.415 | 7.251 | 10.653 | 46.718 | 176.786 |
| CV | | | 0.348 | 0.682 | 0.753 | 0.812 | 0.879 | 0.928 | 0.952 | 0.989 | 0.997 |
| Q1 | | | 0.136 | 0.375 | 0.505 | 0.686 | 1.120 | 1.939 | 2.918 | 13.295 | 50.714 |
| Q2 | | | 0.328 | 0.904 | 1.218 | 1.653 | 2.699 | 4.671 | 7.032 | 32.034 | 122.192 |
| Q3 | | | 0.657 | 1.809 | 2.435 | 3.306 | 5.397 | 9.342 | 14.063 | 64.069 | 244.384 |
| ARL | 1.3 | | 1.163 | 2.126 | 2.769 | 3.756 | 6.478 | 12.697 | 21.662 | 183.486 | 1377.041 |
| CV | | | 0.374 | 0.728 | 0.799 | 0.857 | 0.919 | 0.959 | 0.977 | 0.997 | 0.999 |
| Q1 | | | 0.146 | 0.453 | 0.642 | 0.929 | 1.716 | 3.507 | 6.087 | 52.642 | 396.006 |
| Q2 | | | 0.353 | 1.090 | 1.548 | 2.239 | 4.134 | 8.449 | 14.665 | 126.836 | 954.145 |
| Q3 | | | 0.705 | 2.181 | 3.095 | 4.478 | 8.268 | 16.899 | 29.331 | 253.671 | 1908.291 |
| ARL | 1.5 | | 1.178 | 2.300 | 3.104 | 4.399 | 8.287 | 18.344 | 34.768 | 476.096 | 6010.897 |
| CV | | | 0.388 | 0.752 | 0.823 | 0.879 | 0.938 | 0.972 | 0.986 | 0.999 | 0.999 |
| Q1 | | | 0.152 | 0.504 | 0.739 | 1.116 | 2.237 | 5.132 | 9.858 | 136.820 | 1729.083 |
| Q2 | | | 0.367 | 1.215 | 1.783 | 2.688 | 5.390 | 12.365 | 23.751 | 329.658 | 4166.089 |
| Q3 | | | 0.733 | 2.429 | 3.565 | 5.377 | 10.780 | 24.730 | 47.502 | 659.316 | 8332.179 |
| ARL | 2 | | 1.210 | 2.747 | 4.031 | 6.365 | 14.944 | 45.302 | 113.477 | 5868.797 | 335003.5 |
| CV | | | 0.417 | 0.797 | 0.867 | 0.918 | 0.966 | 0.989 | 0.996 | 0.999 | 1 |
| Q1 | | | 0.164 | 0.636 | 1.009 | 1.683 | 4.154 | 12.888 | 32.501 | 1688.204 | 96374.357 |
| Q2 | | | 0.396 | 1.531 | 2.431 | 4.055 | 10.008 | 31.053 | 78.309 | 4067.594 | 232206.385 |
| Q3 | | | 0.792 | 3.063 | 4.862 | 8.111 | 20.016 | 62.107 | 156.619 | 8135.187 | 464412.77 |
| ARL | 2.5 | | 1.238 | 3.213 | 5.102 | 8.939 | 26.155 | 109.818 | 370.370 | 85019.46 | 28976440 |
| CV | | | 0.438 | 0.829 | 0.897 | 0.942 | 0.981 | 0.995 | 0.999 | 1 | 1 |
| Q1 | | | 0.174 | 0.772 | 1.319 | 2.425 | 7.379 | 31.448 | 106.405 | 24458.43 | — |
| Q2 | | | 0.420 | 1.859 | 3.177 | 5.843 | 17.780 | 75.773 | 256.375 | 58930.65 | — |
| Q3 | | | 0.840 | 3.718 | 6.354 | 11.686 | 35.561 | 151.545 | 512.749 | 117861.3 | — |
| ARL | 2.7 | | 1.248 | 3.405 | 5.574 | 10.173 | 32.494 | 155.808 | 594.466 | 257929.1 | — |
| CV | | | 0.446 | 0.841 | 0.906 | 0.949 | 0.985 | 0.997 | 0.999 | 1 | 1 |
| Q1 | | | 0.1779 | 0.828 | 1.454 | 2.780 | 9.203 | 44.679 | 170.873 | 74201.43 | — |
| Q2 | | | 0.428 | 1.994 | 3.506 | 6.699 | 22.175 | 107.651 | 411.706 | 178782.5 | — |
| Q3 | | | 0.888 | 3.988 | 7.011 | 13.398 | 44.349 | 215.301 | 823.412 | 357565 | — |
| ARL | 3 | | 1.262 | 3.701 | 6.332 | 12.275 | 44.710 | 262.219 | 1208.825 | 1418697 | — |
| CV | | | 0.456 | 0.854 | 0.918 | 0.958 | 0.989 | 0.998 | 0.999 | 1 | 1 |
| Q1 | | | 0.183 | 0.913 | 1.674 | 3.385 | 12.718 | 75.292 | 347.613 | 408133.5 | — |
| Q2 | | | 0.441 | 2.200 | 4.032 | 8.157 | 30.643 | 181.409 | 837.547 | 983365.5 | — |
| Q3 | | | 0.882 | 4.401 | 8.065 | 16.314 | 61.286 | 362.819 | 1675.094 | 1966731 | — |

TABLE S.7. ARL, Cv and quartiles of the run length distribution assuming 0.0027 as the false alarm probability of the upper-sided chart with $\alpha_0=1.00$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|-----------|----------|----------|----------|-----------|-----------|----------|----------|----------|----------|------------|
| ARL | 0.1 | | 92472.54 | 23118.13 | 18494.507 | 15412.089 | 12329.67 | 10274.73 | 9247.254 | 7113.272 | 6164.836 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 26602.55 | 6650.529 | 5320.394 | 4433.638 | 3546.882 | 2955.711 | 2660.125 | 2046.217 | 1773.369 |
| Q2 | | | 64096.73 | 16023.92 | 12819.069 | 10682.499 | 8545.93 | 7121.551 | 6409.361 | 4930.198 | 4272.792 |
| Q3 | | | 128193.5 | 32047.85 | 25638.138 | 21364.999 | 17091.86 | 14243.1 | 12818.72 | 9860.396 | 8545.58422 |
| ARL | 0.4 | | 23121.88 | 5780.471 | 4624.377 | 3853.647 | 3082.918 | 2569.098 | 2312.188 | 1778.607 | 1541.459 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 6651.608 | 1662.794 | 1330.207 | 1108.481 | 886.756 | 738.939 | 665.031 | 511.529 | 443.306 |
| Q2 | | | 16026.52 | 4006.371 | 3205.027 | 2670.798 | 2136.569 | 1780.416 | 1602.34 | 1232.49 | 1068.111 |
| Q3 | | | 32053.04 | 8012.741 | 6410.055 | 5341.596 | 4273.139 | 3560.833 | 3204.68 | 2464.98 | 2136.223 |
| ARL | 0.5 | | 18498.51 | 4624.627 | 3699.701 | 3083.085 | 2466.468 | 2055.39 | 1849.851 | 1422.962 | 1233.234 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 5321.545 | 1330.278 | 1064.194 | 886.804 | 709.415 | 591.155 | 532.025 | 409.217 | 354.635 |
| Q2 | | | 12821.84 | 3205.201 | 2564.091 | 2136.685 | 1709.278 | 1424.341 | 1281.872 | 985.977 | 854.466 |
| Q3 | | | 25643.68 | 6410.401 | 5128.182 | 4273.370 | 3418.557 | 2848.682 | 2563.745 | 1971.951 | 1708.932 |
| ARL | 0.6 | | 15416.26 | 3854.064 | 3083.251 | 2569.376 | 2055.501 | 1712.917 | 1541.626 | 1185.866 | 1027.751 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 4434.837 | 1108.601 | 886.852 | 739.019 | 591.187 | 492.632 | 443.354 | 341.009 | 295.522 |
| Q2 | | | 10685.39 | 2671.087 | 2136.800 | 1780.609 | 1424.418 | 1186.957 | 1068.227 | 821.633 | 712.036 |
| Q3 | | | 21370.78 | 5342.174 | 4273.600 | 3561.218 | 2848.836 | 2373.914 | 2136.454 | 1643.266 | 1424.072 |
| ARL | 0.9 | | 10279.17 | 2569.793 | 2055.834 | 1713.195 | 1370.556 | 1142.13 | 1027.917 | 790.706 | 685.278 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 |
| Q1 | | | 2956.989 | 739.139 | 591.283 | 492.713 | 394.141 | 328.427 | 295.569 | 227.328 | 196.998 |
| Q2 | | | 7124.631 | 1780.898 | 1424.649 | 1187.149 | 949.651 | 791.318 | 712.151 | 547.729 | 474.652 |
| Q3 | | | 14249.26 | 3561.796 | 2849.298 | 2374.299 | 1899.301 | 1582.635 | 1424.303 | 1095.457 | 949.304 |
| ARL | 1 | | 9251.753 | 2312.938 | 1850.351 | 1541.959 | 1233.567 | 1027.973 | 925.176 | 711.674 | 616.784 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 2661.42 | 665.247 | 532.169 | 443.450 | 354.731 | 295.586 | 266.013 | 204.592 | 177.294 |
| Q2 | | | 6412.48 | 1602.86 | 1282.219 | 1068.458 | 854.697 | 712.189 | 640.936 | 492.948 | 427.175 |
| Q3 | | | 12824.96 | 3205.72 | 2564.437 | 2136.916 | 1709.394 | 1424.379 | 1281.872 | 985.896 | 854.351 |
| ARL | 1.3 | | 7117.886 | 1779.472 | 1423.577 | 1186.315 | 949.052 | 790.877 | 711.789 | 547.53 | 474.526 |
| CV | | | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 2047.544 | 511.778 | 409.394 | 341.138 | 272.881 | 227.377 | 204.625 | 157.371 | 136.369 |
| Q2 | | | 4933.396 | 1233.089 | 986.402 | 821.944 | 657.486 | 547.847 | 493.028 | 379.172 | 328.569 |
| Q3 | | | 9866.792 | 2466.178 | 1972.804 | 1643.888 | 1314.972 | 1095.694 | 986.056 | 758.344 | 657.139 |
| ARL | 1.5 | | 6169.501 | 1542.375 | 1233.900 | 1028.250 | 822.600 | 685.500 | 616.950 | 474.577 | 411.301 |
| CV | | | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 1774.711 | 443.569 | 354.827 | 295.665 | 236.504 | 197.062 | 177.342 | 136.384 | 118.179 |
| Q2 | | | 4276.026 | 1068.747 | 854.928 | 712.382 | 569.836 | 474.806 | 427.291 | 328.605 | 284.745 |
| Q3 | | | 8552.051 | 2137.493 | 1709.856 | 1424.764 | 1139.673 | 949.612 | 854.582 | 657.211 | 569.490 |
| ARL | 2 | | 4628.375 | 1157.094 | 925.675 | 771.396 | 617.117 | 514.264 | 462.838 | 356.029 | 308.559 |
| CV | | | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 1331.357 | 332.731 | 266.156 | 221.773 | 177.389 | 147.801 | 133.006 | 102.279 | 88.623 |
| Q2 | | | 3207.798 | 801.689 | 641.283 | 534.344 | 427.406 | 356.114 | 320.468 | 246.434 | 213.529 |
| Q3 | | | 6415.597 | 1603.38 | 1282.565 | 1068.689 | 854.812 | 712.228 | 640.936 | 492.868 | 427.059 |
| ARL | 2.5 | | 3703.699 | 925.925 | 740.740 | 617.284 | 493.827 | 411.523 | 370.370 | 284.901 | 246.914 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 1065.344 | 266.228 | 212.954 | 177.438 | 141.921 | 118.244 | 106.405 | 81.817 | 70.889 |
| Q2 | | | 2566.862 | 641.456 | 513.095 | 427.522 | 341.948 | 284.899 | 256.375 | 197.131 | 170.801 |
| Q3 | | | 5133.724 | 1282.911 | 1026.191 | 855.043 | 683.896 | 569.798 | 512.749 | 394.262 | 341.602 |
| ARL | 2.7 | | 3429.721 | 857.431 | 685.945 | 571.621 | 457.297 | 381.081 | 342.973 | 263.825 | 228.649 |
| CV | | | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 986.526 | 246.524 | 197.190 | 164.301 | 131.412 | 109.486 | 98.523 | 75.754 | 65.634 |
| Q2 | | | 2376.955 | 593.979 | 475.114 | 395.871 | 316.627 | 263.798 | 237.384 | 182.523 | 158.140 |
| Q3 | | | 4753.91 | 1187.958 | 950.228 | 791.741 | 633.254 | 527.596 | 474.766 | 365.046 | 316.281 |
| ARL | 3 | | 3087.249 | 771.812 | 617.45 | 514.542 | 411.634 | 343.028 | 308.725 | 237.481 | 205.817 |
| CV | | | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 | 0.998 |
| Q1 | | | 888.002 | 221.893 | 177.486 | 147.881 | 118.276 | 98.539 | 88.671 | 68.175 | 59.066 |
| Q2 | | | 2139.571 | 534.633 | 427.637 | 356.306 | 284.976 | 237.422 | 213.645 | 164.263 | 142.315 |
| Q3 | | | 4279.142 | 1069.266 | 855.274 | 712.613 | 569.952 | 474.844 | 427.291 | 328.526 | 284.629 |

TABLE S.8. ARL, Cv and quartiles of the run length distribution assuming 0.0027 as the false alarm probability of the two-sided chart with $\alpha_0=1.00$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|-------|-------|--------|--------|--------|---------|---------|---------|----------|
| ARL | 0.1 | | 1.024 | 1.103 | 1.133 | 1.163 | 1.212 | 1.265 | 1.302 | 1.428 | 1.524 |
| CV | | | 0.153 | 0.306 | 0.342 | 0.374 | 0.418 | 0.458 | 0.482 | 0.548 | 0.586 |
| Q1 | | | 0.077 | 0.121 | 0.134 | 0.146 | 0.165 | 0.184 | 0.197 | 0.239 | 0.269 |
| Q2 | | | 0.185 | 0.293 | 0.323 | 0.353 | 0.398 | 0.443 | 0.475 | 0.575 | 0.649 |
| Q3 | | | 0.369 | 0.585 | 0.646 | 0.706 | 0.795 | 0.887 | 0.949 | 1.151 | 1.299 |
| ARL | 0.4 | | 1.078 | 1.397 | 1.544 | 1.718 | 2.049 | 2.493 | 2.877 | 4.725 | 7.017 |
| CV | | | 0.268 | 0.533 | 0.593 | 0.647 | 0.716 | 0.774 | 0.808 | 0.888 | 0.926 |
| Q1 | | | 0.109 | 0.229 | 0.276 | 0.329 | 0.429 | 0.561 | 0.674 | 1.209 | 1.871 |
| Q2 | | | 0.264 | 0.551 | 0.664 | 0.795 | 1.035 | 1.352 | 1.623 | 2.915 | 4.509 |
| Q3 | | | 0.527 | 1.102 | 1.328 | 1.589 | 2.071 | 2.704 | 3.245 | 5.830 | 9.017 |
| ARL | 0.5 | | 1.092 | 1.492 | 1.688 | 1.929 | 2.412 | 3.106 | 3.745 | 7.289 | 12.579 |
| CV | | | 0.289 | 0.574 | 0.638 | 0.694 | 0.765 | 0.823 | 0.856 | 0.929 | 0.959 |
| Q1 | | | 0.116 | 0.259 | 0.320 | 0.394 | 0.537 | 0.740 | 0.926 | 1.949 | 3.473 |
| Q2 | | | 0.279 | 0.625 | 0.772 | 0.949 | 1.294 | 1.784 | 2.232 | 4.697 | 8.368 |
| Q3 | | | 0.559 | 1.250 | 1.544 | 1.898 | 2.588 | 3.568 | 4.463 | 9.394 | 16.737 |
| ARL | 0.6 | | 1.105 | 1.587 | 1.836 | 2.155 | 2.825 | 3.859 | 4.876 | 11.402 | 23.261 |
| CV | | | 0.308 | 0.608 | 0.675 | 0.732 | 0.804 | 0.861 | 0.892 | 0.955 | 0.978 |
| Q1 | | | 0.122 | 0.289 | 0.366 | 0.461 | 0.658 | 0.959 | 1.253 | 3.134 | 6.547 |
| Q2 | | | 0.294 | 0.697 | 0.881 | 1.112 | 1.586 | 2.311 | 3.019 | 7.551 | 15.774 |
| Q3 | | | 0.588 | 1.394 | 1.763 | 2.223 | 3.173 | 4.622 | 6.039 | 15.103 | 31.549 |
| ARL | 0.9 | | 1.138 | 1.872 | 2.311 | 2.931 | 4.439 | 7.303 | 10.735 | 46.459 | 161.613 |
| CV | | | 0.348 | 0.683 | 0.753 | 0.812 | 0.880 | 0.929 | 0.952 | 0.989 | 0.997 |
| Q1 | | | 0.136 | 0.377 | 0.507 | 0.689 | 1.127 | 1.953 | 2.942 | 13.221 | 46.349 |
| Q2 | | | 0.329 | 0.907 | 1.223 | 1.661 | 2.716 | 4.707 | 7.089 | 31.855 | 111.675 |
| Q3 | | | 0.657 | 1.815 | 2.445 | 3.322 | 5.432 | 9.414 | 14.178 | 63.709 | 223.349 |
| ARL | 1 | | 1.148 | 1.968 | 2.479 | 3.225 | 5.129 | 8.997 | 13.950 | 74.827 | 293.494 |
| CV | | | 0.359 | 0.701 | 0.772 | 0.831 | 0.897 | 0.943 | 0.963 | 0.993 | 0.998 |
| Q1 | | | 0.140 | 0.405 | 0.557 | 0.775 | 1.326 | 2.442 | 3.868 | 21.382 | 84.289 |
| Q2 | | | 0.338 | 0.977 | 1.342 | 1.868 | 3.196 | 5.883 | 9.319 | 51.519 | 203.088 |
| Q3 | | | 0.677 | 1.953 | 2.685 | 3.735 | 6.392 | 11.765 | 18.637 | 103.038 | 406.175 |
| ARL | 1.3 | | 1.174 | 2.257 | 3.019 | 4.232 | 7.787 | 16.634 | 30.399 | 281.180 | 774.348 |
| CV | | | 0.385 | 0.746 | 0.818 | 0.874 | 0.934 | 0.969 | 0.983 | 0.998 | 0.999 |
| Q1 | | | 0.151 | 0.491 | 0.715 | 1.067 | 2.093 | 4.639 | 8.601 | 80.747 | 222.622 |
| Q2 | | | 0.363 | 1.184 | 1.723 | 2.571 | 5.043 | 11.179 | 20.722 | 194.553 | 536.390 |
| Q3 | | | 0.726 | 2.368 | 3.447 | 5.143 | 10.087 | 22.359 | 41.445 | 389.105 | 1072.780 |
| ARL | 1.5 | | 1.189 | 2.453 | 3.410 | 5.017 | 10.173 | 24.811 | 50.541 | 515.646 | 794.908 |
| CV | | | 0.399 | 0.769 | 0.841 | 0.895 | 0.949 | 0.979 | 0.990 | 0.999 | 0.999 |
| Q1 | | | 0.157 | 0.549 | 0.829 | 1.294 | 2.780 | 6.992 | 14.395 | 148.198 | 228.537 |
| Q2 | | | 0.378 | 1.328 | 1.997 | 3.118 | 6.699 | 16.849 | 34.684 | 357.072 | 550.641 |
| Q3 | | | 0.755 | 2.648 | 3.994 | 6.237 | 13.398 | 33.698 | 69.369 | 714.144 | 1101.283 |
| ARL | 2 | | 1.223 | 2.959 | 4.503 | 7.451 | 19.188 | 64.445 | 162.827 | 687.865 | 617.053 |
| CV | | | 0.427 | 0.814 | 0.882 | 0.930 | 0.974 | 0.992 | 0.997 | 0.999 | 0.999 |
| Q1 | | | 0.169 | 0.698 | 1.146 | 1.996 | 5.375 | 18.396 | 46.698 | 197.743 | 177.371 |
| Q2 | | | 0.408 | 1.681 | 2.760 | 4.809 | 12.951 | 44.322 | 112.516 | 476.445 | 427.362 |
| Q3 | | | 0.815 | 3.362 | 5.521 | 9.619 | 25.901 | 88.645 | 225.032 | 952.889 | 854.723 |
| ARL | 2.5 | | 1.252 | 3.489 | 5.779 | 10.697 | 34.642 | 149.835 | 370.370 | 569.056 | 493.827 |
| CV | | | 0.449 | 0.845 | 0.909 | 0.952 | 0.985 | 0.997 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.179 | 0.852 | 1.514 | 2.931 | 9.821 | 42.961 | 106.405 | 163.563 | 141.921 |
| Q2 | | | 0.432 | 2.053 | 3.648 | 7.063 | 23.664 | 103.511 | 256.374 | 394.093 | 341.948 |
| Q3 | | | 0.865 | 4.105 | 7.297 | 14.125 | 47.327 | 207.021 | 512.749 | 788.186 | 683.896 |
| ARL | 2.7 | | 1.262 | 3.709 | 6.346 | 12.267 | 43.333 | 199.239 | 443.726 | 527.443 | 457.272 |
| CV | | | 0.456 | 0.855 | 0.918 | 0.958 | 0.988 | 0.998 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.183 | 0.916 | 1.678 | 3.383 | 12.322 | 57.174 | 127.508 | 151.592 | 131.405 |
| Q2 | | | 0.442 | 2.206 | 4.042 | 8.151 | 29.689 | 137.756 | 307.221 | 365.249 | 316.610 |
| Q3 | | | 0.883 | 4.413 | 8.0841 | 16.302 | 59.377 | 275.511 | 614.442 | 730.498 | 633.221 |
| ARL | 3 | | 1.277 | 4.047 | 7.259 | 14.952 | 59.746 | 282.346 | 505.086 | 474.878 | 411.578 |
| CV | | | 0.466 | 0.868 | 0.929 | 0.966 | 0.992 | 0.998 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 0.188 | 1.014 | 1.941 | 4.156 | 17.044 | 81.082 | 145.160 | 136.469 | 118.259 |
| Q2 | | | 0.454 | 2.442 | 4.677 | 10.014 | 41.065 | 195.361 | 349.752 | 328.814 | 284.938 |
| Q3 | | | 0.907 | 4.885 | 9.353 | 20.027 | 82.131 | 390.721 | 699.504 | 657.627 | 569.875 |

TABLE S.9. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability for the lower-sided chart with $\alpha_0=1.50$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|-------|-------|-------|-------|--------|--------|---------|----------|----------|
| ARL | 0.1 | | 1.010 | 1.042 | 1.053 | 1.064 | 1.081 | 1.099 | 1.111 | 1.149 | 1.176 |
| CV | | 0.1 | 0.199 | 0.224 | 0.224 | 0.245 | 0.274 | 0.299 | 0.316 | 0.360 | 0.387 |
| Q1 | | 0.062 | 0.089 | 0.096 | 0.102 | 0.102 | 0.111 | 0.119 | 0.125 | 0.141 | 0.151 |
| Q2 | | 0.151 | 0.215 | 0.231 | 0.246 | 0.268 | 0.288 | 0.301 | 0.339 | 0.365 | |
| Q3 | | 0.301 | 0.431 | 0.463 | 0.493 | 0.535 | 0.575 | 0.602 | 0.679 | 0.729 | |
| ARL | 0.4 | | 1.036 | 1.163 | 1.212 | 1.264 | 1.352 | 1.450 | 1.523 | 1.784 | 2.001 |
| CV | | 0.187 | 0.374 | 0.418 | 0.457 | 0.510 | 0.557 | 0.586 | 0.663 | 0.707 | |
| Q1 | | 0.086 | 0.146 | 0.165 | 0.184 | 0.214 | 0.246 | 0.269 | 0.349 | 0.415 | |
| Q2 | | 0.207 | 0.353 | 0.397 | 0.443 | 0.515 | 0.593 | 0.649 | 0.843 | 1.000 | |
| Q3 | | 0.414 | 0.705 | 0.794 | 0.886 | 1.029 | 1.185 | 1.297 | 1.685 | 2.001 | |
| ARL | 0.5 | | 1.044 | 1.202 | 1.265 | 1.335 | 1.452 | 1.588 | 1.692 | 2.079 | 2.423 |
| CV | | 0.206 | 0.410 | 0.458 | 0.501 | 0.558 | 0.609 | 0.639 | 0.721 | 0.766 | |
| Q1 | | 0.091 | 0.161 | 0.184 | 0.208 | 0.246 | 0.289 | 0.322 | 0.439 | 0.540 | |
| Q2 | | 0.219 | 0.389 | 0.444 | 0.501 | 0.594 | 0.698 | 0.775 | 1.057 | 1.302 | |
| Q3 | | 0.438 | 0.778 | 0.887 | 1.002 | 1.188 | 1.396 | 1.550 | 2.114 | 2.604 | |
| ARL | 0.6 | | 1.051 | 1.241 | 1.319 | 1.407 | 1.558 | 1.739 | 1.879 | 2.433 | 2.954 |
| CV | | 0.221 | 0.441 | 0.492 | 0.538 | 0.599 | 0.652 | 0.684 | 0.768 | 0.813 | |
| Q1 | | 0.095 | 0.176 | 0.203 | 0.232 | 0.280 | 0.336 | 0.379 | 0.543 | 0.696 | |
| Q2 | | 0.229 | 0.423 | 0.489 | 0.558 | 0.675 | 0.809 | 0.913 | 1.309 | 1.677 | |
| Q3 | | 0.459 | 0.846 | 0.977 | 1.117 | 1.349 | 1.619 | 1.826 | 2.618 | 3.354 | |
| ARL | 0.9 | | 1.071 | 1.357 | 1.485 | 1.634 | 1.912 | 2.273 | 2.577 | 3.964 | 5.562 |
| CV | | 0.258 | 0.513 | 0.571 | 0.623 | 0.691 | 0.748 | 0.782 | 0.865 | 0.906 | |
| Q1 | | 0.106 | 0.215 | 0.257 | 0.304 | 0.389 | 0.496 | 0.586 | 0.989 | 1.451 | |
| Q2 | | 0.256 | 0.519 | 0.619 | 0.732 | 0.936 | 1.196 | 1.411 | 2.384 | 3.497 | |
| Q3 | | 0.512 | 1.038 | 1.238 | 1.465 | 1.872 | 2.392 | 2.823 | 4.768 | 6.994 | |
| ARL | 1 | | 1.077 | 1.395 | 1.541 | 1.715 | 2.043 | 2.483 | 2.863 | 4.690 | 6.952 |
| CV | | 0.268 | 0.532 | 0.592 | 0.646 | 0.714 | 0.773 | 0.807 | 0.887 | 0.925 | |
| Q1 | | 0.109 | 0.228 | 0.275 | 0.329 | 0.428 | 0.558 | 0.669 | 1.199 | 1.852 | |
| Q2 | | 0.263 | 0.549 | 0.662 | 0.792 | 1.031 | 1.345 | 1.613 | 2.891 | 4.463 | |
| Q3 | | 0.526 | 1.099 | 1.324 | 1.584 | 2.062 | 2.689 | 3.226 | 5.781 | 8.926 | |
| ARL | 1.3 | | 1.094 | 1.509 | 1.714 | 1.968 | 2.481 | 3.228 | 3.925 | 7.891 | 14.046 |
| CV | | 0.293 | 0.581 | 0.645 | 0.701 | 0.773 | 0.831 | 0.863 | 0.934 | 0.964 | |
| Q1 | | 0.117 | 0.265 | 0.328 | 0.405 | 0.557 | 0.776 | 0.978 | 2.123 | 3.895 | |
| Q2 | | 0.283 | 0.639 | 0.791 | 0.977 | 1.343 | 1.869 | 2.357 | 5.115 | 9.385 | |
| Q3 | | 0.565 | 1.276 | 1.583 | 1.954 | 2.686 | 3.739 | 4.714 | 10.230 | 18.771 | |
| ARL | 1.5 | | 1.104 | 1.585 | 1.832 | 2.149 | 2.813 | 3.837 | 4.844 | 11.296 | 23.069 |
| CV | | 0.307 | 0.607 | 0.674 | 0.731 | 0.803 | 0.859 | 0.891 | 0.955 | 0.978 | |
| Q1 | | 0.122 | 0.289 | 0.365 | 0.459 | 0.655 | 0.953 | 1.244 | 3.104 | 6.492 | |
| Q2 | | 0.294 | 0.695 | 0.878 | 1.107 | 1.578 | 2.296 | 2.997 | 7.478 | 15.641 | |
| Q3 | | 0.588 | 1.391 | 1.757 | 2.214 | 3.156 | 4.592 | 5.995 | 14.956 | 31.282 | |
| ARL | 2 | | 1.127 | 1.774 | 2.142 | 2.646 | 3.812 | 5.878 | 8.195 | 28.769 | 87.139 |
| CV | | 0.336 | 0.660 | 0.730 | 0.789 | 0.859 | 0.911 | 0.937 | 0.982 | 0.994 | |
| Q1 | | 0.132 | 0.347 | 0.457 | 0.606 | 0.946 | 1.543 | 2.211 | 8.132 | 24.924 | |
| Q2 | | 0.318 | 0.835 | 1.102 | 1.459 | 2.278 | 3.717 | 5.326 | 19.593 | 60.054 | |
| Q3 | | 0.636 | 1.671 | 2.204 | 2.919 | 4.557 | 7.434 | 10.653 | 39.186 | 120.107 | |
| ARL | 2.5 | | 1.148 | 1.963 | 2.472 | 3.213 | 5.102 | 8.939 | 13.866 | 76.945 | 370.370 |
| CV | | 0.359 | 0.700 | 0.772 | 0.829 | 0.897 | 0.942 | 0.963 | 0.993 | 0.999 | |
| Q1 | | 0.140 | 0.404 | 0.555 | 0.772 | 1.319 | 2.425 | 3.843 | 21.992 | 106.405 | |
| Q2 | | 0.338 | 0.974 | 1.337 | 1.859 | 3.177 | 5.843 | 9.260 | 52.987 | 256.374 | |
| Q3 | | 0.676 | 1.947 | 2.674 | 3.718 | 6.354 | 11.686 | 18.521 | 105.974 | 512.749 | |
| ARL | 2.7 | | 1.155 | 2.039 | 2.611 | 3.462 | 5.715 | 10.553 | 17.112 | 115.483 | 681.474 |
| CV | | 0.366 | 0.714 | 0.785 | 0.843 | 0.908 | 0.951 | 0.970 | 0.996 | 0.999 | |
| Q1 | | 0.143 | 0.427 | 0.596 | 0.844 | 1.496 | 2.889 | 4.778 | 33.078 | 195.904 | |
| Q2 | | 0.345 | 1.029 | 1.435 | 2.033 | 3.603 | 6.963 | 11.511 | 79.699 | 472.015 | |
| Q3 | | 0.690 | 2.057 | 2.870 | 4.066 | 7.207 | 13.925 | 23.023 | 159.399 | 944.030 | |
| ARL | 3 | | 1.165 | 2.155 | 2.825 | 3.859 | 6.756 | 13.513 | 23.461 | 215.017 | 1755.504 |
| CV | | 0.377 | 0.732 | 0.804 | 0.861 | 0.923 | 0.962 | 0.978 | 0.998 | 0.999 | |
| Q1 | | 0.147 | 0.461 | 0.658 | 0.959 | 1.796 | 3.742 | 6.604 | 61.712 | 504.883 | |
| Q2 | | 0.355 | 1.111 | 1.586 | 2.311 | 4.327 | 9.016 | 15.913 | 148.692 | 1216.476 | |
| Q3 | | 0.710 | 2.223 | 3.173 | 4.623 | 8.654 | 18.031 | 31.826 | 297.383 | 2432.951 | |

TABLE S.10. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability for the upper-sided chart with $\alpha_0=1.50$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| ARL | 0.1 | | 138768.8 | 34692.19 | 27753.75 | 23128.13 | 18502.5 | 15418.75 | 13876.88 | 10674.52 | 9251.251 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 39921.14 | 9980.177 | 7984.113 | 6653.403 | 5322.694 | 4435.554 | 3991.985 | 3070.724 | 2661.275 |
| Q2 | | | 96186.83 | 24046.45 | 19237.09 | 16030.85 | 12824.61 | 10687.12 | 9618.371 | 7398.667 | 6412.132 |
| Q3 | | | 192373.7 | 48092.89 | 38474.18 | 32061.7 | 25649.22 | 21374.23 | 19236.74 | 14797.33 | 12824.26 |
| ARL | 0.4 | | 34695.94 | 8673.985 | 6939.188 | 5782.657 | 4626.125 | 3855.104 | 3469.594 | 2668.918 | 2313.063 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 9981.256 | 2495.206 | 1996.136 | 1663.423 | 1330.709 | 1108.9 | 997.996 | 767.656 | 665.283 |
| Q2 | | | 24049.05 | 6012.002 | 4809.532 | 4007.886 | 3206.239 | 2671.808 | 2404.593 | 1849.606 | 1602.946 |
| Q3 | | | 48098.09 | 12024 | 9619.064 | 8015.772 | 6412.478 | 5343.616 | 4809.185 | 3699.213 | 3205.893 |
| ARL | 0.5 | | 27757.75 | 6939.438 | 5551.55 | 4626.292 | 3701.034 | 3084.195 | 2775.775 | 2135.212 | 1850.517 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 7985.263 | 1996.208 | 1596.938 | 1330.757 | 1064.577 | 887.124 | 798.397 | 614.118 | 532.217 |
| Q2 | | | 19239.86 | 4809.705 | 3847.695 | 3206.355 | 2565.015 | 2137.454 | 1923.674 | 1479.67 | 1282.334 |
| Q3 | | | 38479.72 | 9619.411 | 7695.389 | 6412.709 | 5130.029 | 4274.909 | 3847.348 | 2959.339 | 2564.668 |
| ARL | 0.6 | | 23132.29 | 5783.073 | 4626.459 | 3855.382 | 3084.306 | 2570.255 | 2313.229 | 1779.407 | 1542.153 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 6654.602 | 1663.543 | 1330.805 | 1108.98 | 887.156 | 739.272 | 665.331 | 511.759 | 443.506 |
| Q2 | | | 16033.74 | 4008.174 | 3206.47 | 2672.001 | 2137.531 | 1781.218 | 1603.062 | 1233.044 | 1068.592 |
| Q3 | | | 32067.47 | 8016.348 | 6412.941 | 5344.001 | 4275.063 | 3562.437 | 3206.123 | 2466.089 | 2137.185 |
| ARL | 0.9 | | 15423.2 | 3855.799 | 3084.639 | 2570.533 | 2056.426 | 1713.688 | 1542.32 | 1186.4 | 1028.213 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 4436.833 | 1109.1 | 887.252 | 739.352 | 591.453 | 492.854 | 443.554 | 341.162 | 295.655 |
| Q2 | | | 10690.2 | 2672.29 | 2137.762 | 1781.411 | 1425.059 | 1187.491 | 1068.708 | 822.003 | 712.356 |
| Q3 | | | 21380.4 | 5344.579 | 4275.524 | 3562.822 | 2850.119 | 2374.983 | 2137.416 | 1644.006 | 1424.713 |
| ARL | 1 | | 13881.38 | 3470.344 | 2776.275 | 2313.563 | 1850.85 | 1542.375 | 1388.138 | 1067.798 | 925.425 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Q1 | | | 3993.279 | 998.212 | 798.541 | 665.427 | 532.313 | 443.569 | 399.198 | 307.043 | 266.084 |
| Q2 | | | 9621.49 | 2405.112 | 1924.021 | 1603.293 | 1282.565 | 1068.746 | 961.837 | 739.795 | 641.109 |
| Q3 | | | 19242.98 | 4810.225 | 3848.041 | 3206.586 | 2565.13 | 2137.493 | 1923.674 | 1479.589 | 1282.218 |
| ARL | 1.3 | | 10679.13 | 2669.784 | 2135.827 | 1779.856 | 1423.885 | 1186.571 | 1067.914 | 821.472 | 711.943 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 3072.052 | 767.905 | 614.295 | 511.889 | 409.482 | 341.211 | 307.076 | 236.179 | 204.669 |
| Q2 | | | 7401.865 | 1850.206 | 1480.096 | 1233.355 | 986.615 | 822.121 | 739.875 | 569.054 | 493.134 |
| Q3 | | | 14803.73 | 3700.413 | 2960.192 | 2466.711 | 1973.231 | 1644.243 | 1479.749 | 1138.109 | 986.269 |
| ARL | 1.5 | | 9255.916 | 2313.979 | 1851.183 | 1542.653 | 1234.122 | 1028.435 | 925.592 | 711.994 | 617.061 |
| CV | | | 1 | 1 | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 2662.617 | 665.547 | 532.408 | 443.649 | 354.891 | 295.718 | 266.132 | 204.684 | 177.374 |
| Q2 | | | 6415.366 | 1603.581 | 1282.796 | 1068.939 | 855.082 | 712.510 | 641.225 | 493.169 | 427.368 |
| Q3 | | | 12830.73 | 3207.163 | 2565.592 | 2137.878 | 1710.163 | 1425.021 | 1282.449 | 986.339 | 854.735 |
| ARL | 2 | | 6943.187 | 1735.797 | 1388.638 | 1157.198 | 925.758 | 771.465 | 694.319 | 534.092 | 462.879 |
| CV | | | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 1997.286 | 499.214 | 399.342 | 332.761 | 266.180 | 221.793 | 199.599 | 153.505 | 133.018 |
| Q2 | | | 4812.304 | 1202.816 | 962.184 | 801.762 | 641.340 | 534.392 | 480.919 | 369.857 | 320.497 |
| Q3 | | | 9624.607 | 2405.632 | 1924.367 | 1603.524 | 1282.68 | 1068.785 | 961.837 | 739.715 | 640.994 |
| ARL | 2.5 | | 5555.549 | 1388.887 | 1111.11 | 925.925 | 740.740 | 617.284 | 555.555 | 427.350 | 370.370 |
| CV | | | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 1598.088 | 399.414 | 319.503 | 266.228 | 212.954 | 177.438 | 159.679 | 122.797 | 106.405 |
| Q2 | | | 3850.466 | 962.357 | 769.816 | 641.456 | 513.095 | 427.522 | 384.735 | 295.869 | 256.375 |
| Q3 | | | 7700.933 | 1924.713 | 1539.632 | 1282.911 | 1026.191 | 855.043 | 769.469 | 591.739 | 512.749 |
| ARL | 2.7 | | 5144.397 | 1286.099 | 1028.88 | 857.399 | 685.919 | 571.599 | 514.44 | 395.723 | 342.960 |
| CV | | | 1 | 1 | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 |
| Q1 | | | 1479.807 | 369.844 | 295.846 | 246.515 | 197.183 | 164.295 | 147.851 | 113.699 | 98.519 |
| Q2 | | | 3565.478 | 891.109 | 712.818 | 593.958 | 475.097 | 395.856 | 356.236 | 273.948 | 237.375 |
| Q3 | | | 7130.955 | 1782.219 | 1425.637 | 1187.915 | 950.193 | 791.712 | 712.472 | 547.896 | 474.750 |
| ARL | 3 | | 4630.457 | 1157.614 | 926.092 | 771.743 | 617.395 | 514.495 | 463.046 | 356.189 | 308.698 |
| CV | | | 1 | 1 | 0.999 | 0.999 | 0.999 | 0.999 | 0.999 | 0.998 | 0.998 |
| Q1 | | | 1331.956 | 332.881 | 266.276 | 221.873 | 177.469 | 147.867 | 133.066 | 102.325 | 88.663 |
| Q2 | | | 3209.241 | 802.051 | 641.571 | 534.585 | 427.599 | 356.274 | 320.612 | 246.545 | 213.626 |
| Q3 | | | 6418.483 | 1604.101 | 1283.142 | 1069.17 | 855.197 | 712.549 | 641.225 | 493.09 | 427.252 |

TABLE S.11. ARL, CV and quartiles of the run length distribution assuming 0.0027 as the false alarm probability for the two-sided chart with $\alpha_0=1.50$, $\lambda_0=2.5$, $\lambda_1 \in (0.1,0.4,0.5,0.6,0.9,1,1.3,1.5,2,2.5,2.7,3)$ and $\alpha_1 \in (0.1,0.4,0.5,0.6,0.75,0.9,1,1.3,1.5)$

| | λ | α | 0.1 | 0.4 | 0.5 | 0.6 | 0.75 | 0.9 | 1 | 1.3 | 1.5 |
|-----|------------|------------|-------|-------|-------|-------|--------|--------|--------|---------|---------|
| ARL | 0.1 | | 1.011 | 1.045 | 1.057 | 1.069 | 1.089 | 1.108 | 1.122 | 1.164 | 1.193 |
| CV | | | 0.104 | 0.208 | 0.233 | 0.255 | 0.285 | 0.312 | 0.329 | 0.375 | 0.403 |
| Q1 | | | 0.064 | 0.092 | 0.099 | 0.105 | 0.115 | 0.124 | 0.129 | 0.147 | 0.158 |
| Q2 | | | 0.153 | 0.221 | 0.238 | 0.254 | 0.276 | 0.298 | 0.312 | 0.353 | 0.381 |
| Q3 | | | 0.307 | 0.442 | 0.476 | 0.507 | 0.555 | 0.596 | 0.624 | 0.707 | 0.762 |
| ARL | 0.4 | | 1.039 | 1.177 | 1.231 | 1.289 | 1.387 | 1.499 | 1.582 | 1.884 | 2.142 |
| CV | | | 0.194 | 0.388 | 0.433 | 0.474 | 0.528 | 0.577 | 0.607 | 0.685 | 0.730 |
| Q1 | | | 0.088 | 0.152 | 0.172 | 0.193 | 0.225 | 0.261 | 0.288 | 0.380 | 0.457 |
| Q2 | | | 0.212 | 0.366 | 0.414 | 0.464 | 0.543 | 0.629 | 0.693 | 0.916 | 1.102 |
| Q3 | | | 0.423 | 0.732 | 0.828 | 0.928 | 1.086 | 1.259 | 1.386 | 1.833 | 2.204 |
| ARL | 0.5 | | 1.047 | 1.219 | 1.289 | 1.367 | 1.499 | 1.655 | 1.774 | 2.231 | 2.647 |
| CV | | | 0.213 | 0.425 | 0.474 | 0.518 | 0.577 | 0.629 | 0.661 | 0.743 | 0.789 |
| Q1 | | | 0.093 | 0.168 | 0.193 | 0.219 | 0.262 | 0.310 | 0.347 | 0.484 | 0.606 |
| Q2 | | | 0.224 | 0.405 | 0.464 | 0.527 | 0.630 | 0.748 | 0.836 | 1.166 | 1.461 |
| Q3 | | | 0.448 | 0.809 | 0.928 | 1.054 | 1.260 | 1.495 | 1.672 | 2.331 | 2.921 |
| ARL | 0.6 | | 1.055 | 1.262 | 1.349 | 1.446 | 1.617 | 1.825 | 1.989 | 2.651 | 3.296 |
| CV | | | 0.229 | 0.456 | 0.509 | 0.556 | 0.618 | 0.673 | 0.705 | 0.789 | 0.835 |
| Q1 | | | 0.098 | 0.183 | 0.213 | 0.245 | 0.299 | 0.362 | 0.412 | 0.607 | 0.796 |
| Q2 | | | 0.235 | 0.441 | 0.512 | 0.589 | 0.719 | 0.873 | 0.992 | 1.464 | 1.917 |
| ARL | | 0.9 | | 1.076 | 1.389 | 1.530 | 1.699 | 2.017 | 2.442 | 2.806 | 4.538 |
| CV | | | 0.266 | 0.529 | 0.589 | 0.641 | 0.710 | 0.768 | 0.802 | 0.883 | 0.922 |
| Q1 | | | 0.109 | 0.226 | 0.271 | 0.324 | 0.420 | 0.546 | 0.653 | 1.156 | 1.765 |
| Q2 | | | 0.262 | 0.544 | 0.654 | 0.781 | 1.012 | 1.316 | 1.573 | 2.785 | 4.253 |
| Q3 | | | 0.524 | 1.088 | 1.308 | 1.561 | 2.025 | 2.631 | 3.145 | 5.569 | 8.505 |
| ARL | 1 | | 1.083 | 1.429 | 1.592 | 1.788 | 2.167 | 2.687 | 3.146 | 5.463 | 8.515 |
| CV | | | 0.276 | 0.548 | 0.609 | 0.664 | 0.734 | 0.792 | 0.826 | 0.904 | 0.939 |
| Q1 | | | 0.112 | 0.239 | 0.291 | 0.351 | 0.465 | 0.618 | 0.752 | 1.423 | 2.303 |
| Q2 | | | 0.269 | 0.576 | 0.701 | 0.846 | 1.119 | 1.489 | 1.812 | 3.428 | 5.549 |
| Q3 | | | 0.539 | 1.153 | 1.402 | 1.693 | 2.239 | 2.978 | 3.624 | 6.857 | 11.097 |
| ARL | 1.3 | | 1.100 | 1.553 | 1.782 | 2.072 | 2.670 | 3.569 | 4.434 | 9.683 | 18.531 |
| CV | | | 0.302 | 0.597 | 0.663 | 0.719 | 0.791 | 0.848 | 0.880 | 0.947 | 0.973 |
| Q1 | | | 0.120 | 0.279 | 0.349 | 0.437 | 0.613 | 0.875 | 1.126 | 2.639 | 5.186 |
| Q2 | | | 0.289 | 0.672 | 0.842 | 1.052 | 1.477 | 2.109 | 2.712 | 6.359 | 12.495 |
| Q3 | | | 0.579 | 1.343 | 1.684 | 2.104 | 2.954 | 4.218 | 5.424 | 12.718 | 24.989 |
| ARL | 1.5 | | 1.111 | 1.636 | 1.914 | 2.276 | 3.057 | 4.304 | 5.573 | 14.349 | 31.879 |
| CV | | | 0.316 | 0.623 | 0.691 | 0.749 | 0.820 | 0.876 | 0.906 | 0.965 | 0.984 |
| Q1 | | | 0.125 | 0.304 | 0.389 | 0.497 | 0.726 | 1.0885 | 1.455 | 3.983 | 9.026 |
| Q2 | | | 0.301 | 0.733 | 0.938 | 1.198 | 1.749 | 2.622 | 3.505 | 9.596 | 21.748 |
| Q3 | | | 0.602 | 1.467 | 1.875 | 2.396 | 3.498 | 5.243 | 7.009 | 19.191 | 43.496 |
| ARL | 2 | | 1.135 | 1.841 | 2.258 | 2.839 | 4.234 | 6.819 | 9.849 | 39.392 | 125.695 |
| CV | | | 0.345 | 0.676 | 0.746 | 0.805 | 0.874 | 0.924 | 0.948 | 0.987 | 0.996 |
| Q1 | | | 0.135 | 0.367 | 0.492 | 0.663 | 1.068 | 1.814 | 2.687 | 11.188 | 36.016 |
| Q2 | | | 0.325 | 0.885 | 1.185 | 1.597 | 2.572 | 4.371 | 6.474 | 26.957 | 86.778 |
| Q3 | | | 0.651 | 1.770 | 2.369 | 3.193 | 5.145 | 8.742 | 12.948 | 53.913 | 173.556 |
| ARL | 2.5 | | 1.156 | 2.049 | 2.627 | 3.489 | 5.779 | 10.697 | 17.329 | 106.788 | 370.370 |
| CV | | | 0.367 | 0.715 | 0.787 | 0.845 | 0.909 | 0.952 | 0.971 | 0.995 | 0.999 |
| Q1 | | | 0.144 | 0.429 | 0.600 | 0.852 | 1.514 | 2.931 | 4.840 | 30.577 | 106.405 |
| Q2 | | | 0.346 | 1.035 | 1.446 | 2.053 | 3.649 | 7.063 | 11.662 | 73.673 | 256.374 |
| Q3 | | | 0.692 | 2.070 | 2.893 | 4.105 | 7.297 | 14.125 | 23.324 | 147.345 | 512.749 |
| ARL | 2.7 | | 1.163 | 2.132 | 2.781 | 3.776 | 6.522 | 12.773 | 21.678 | 154.307 | 467.899 |
| CV | | | 0.375 | 0.727 | 0.800 | 0.857 | 0.920 | 0.960 | 0.977 | 0.997 | 0.999 |
| Q1 | | | 0.147 | 0.454 | 0.646 | 0.935 | 1.729 | 3.529 | 6.091 | 44.247 | 134.462 |
| Q2 | | | 0.353 | 1.095 | 1.556 | 2.253 | 4.165 | 8.502 | 14.676 | 106.611 | 323.976 |
| Q3 | | | 0.706 | 2.190 | 3.111 | 4.506 | 8.329 | 17.004 | 29.353 | 213.221 | 647.953 |
| ARL | 3 | | 1.174 | 2.258 | 3.022 | 4.236 | 7.793 | 16.613 | 30.222 | 248.968 | 539.485 |
| CV | | | 0.385 | 0.746 | 0.818 | 0.874 | 0.934 | 0.969 | 0.983 | 0.998 | 0.999 |
| Q1 | | | 0.151 | 0.492 | 0.716 | 1.068 | 2.095 | 4.634 | 8.549 | 71.479 | 155.056 |
| Q2 | | | 0.363 | 1.185 | 1.725 | 2.574 | 5.047 | 11.165 | 20.599 | 172.224 | 373.596 |
| Q3 | | | 0.727 | 2.370 | 3.449 | 5.148 | 10.095 | 22.330 | 41.199 | 344.449 | 747.191 |