Scientia Iranica, https://doi.org/10.24200/sci.2022.55619.4311

Launch Strategies for Newly Developed Short Life Cycle Products

Scientia Iranica, https://doi.org/10.24200/sci.2022.55619.4311

Prof. Fariborz Jolai[1], Ph.D.
School of Industrial Engineering, College of Engineering, University of Tehran, Iran
Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961
Email address: fjolai@ut.ac.ir (F. Jolai)

Alireza Taheri-Moghadam, Ph.D. Candidate
School of Industrial Engineering, College of Engineering, University of Tehran, Iran
Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961
Email address: taherimoghadam@ut.ac.ir (A. Taheri-Moghadam)
Phone: +989125017003 (Available everyday 03:30 to 19:30 GMT)

Prof. Jafar Razmi, Ph.D.
School of Industrial Engineering, College of Engineering, University of Tehran, Iran
Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961
Email address: jrazmi@ut.ac.ir (J. Razmi)

Associate Prof. Ata Allah Taleizadeh, Ph.D.
School of Industrial Engineering, College of Engineering, University of Tehran, Iran
Address: School of Industrial Engineering, 4th Floor, Central Building, College of Engineering, University of Tehran, North Kargar st, Tehran, Iran. Postal Code: 1439955961
Email address: taleizadeh@ut.ac.ir (A. Taleizadeh)

[1] Corresponding author

Appendix A: Calculations of optimal solutions

Derivatives, Hessian matrixes, and Calculations of the optimal solutions for all of the proposed models are provided here.

1. Monopoly model (MM)

Equation (A. 1) calculates the Hessian matrix[2] of the revenue function in order to check its concavity. As equation (A. 1) shows, the Hessian matrix is negative definite (ND) and the revenue function is concave. Please note that all of the parameters of the models are positive numbers. As the revenue function is jointly concave, the maximum value of it can be achieved by first-order derivatives, as equation (A. 2) shows. By solving the equalities of equation (A. 2), the optimum selling prices are calculated, that is presented by equations (A. 3) and (A. 4).

$$H\left(R_1\left(p_{n1},p_{o1}\right)\right)=\begin{bmatrix}-2(a_{n1}+b_{n1}) & (b_{n1}+b_{o1})\\ (b_{n1}+b_{o1}) & -2(a_{o1}+b_{o1})\end{bmatrix}$$
$$\Rightarrow\begin{cases}H\equiv\begin{bmatrix}- & +\\ + & -\end{bmatrix}\\ H_1=-2(a_{n1}+b_{n1})\overset{a_{ij},b_{ij}>0}{<}0\\ H_2=4(a_{n1}+b_{n1})(a_{o1}+b_{o1})-(b_{n1}+b_{o1})^2\overset{a_{ij},b_{ij}>0,\ a_{ij}>b_{ij}}{<}0\end{cases} \quad \text{(A. 1)}$$

$$\begin{cases}\dfrac{\partial R_1}{\partial p_{n1}}=0\\ \dfrac{\partial R_1}{\partial p_{o1}}=0\end{cases}\Rightarrow\begin{cases}-2\left(a_{n1}+b_{n1}\right)p_{n1}+\left(b_{n1}+b_{o1}\right)p_{o1}+M_{n1}+C_{n1}\left(a_{n1}+b_{n1}\right)-C_{o1}b_{o1}=0\\ \left(b_{n1}+b_{o1}\right)p_{n1}-2\left(a_{o1}+b_{o1}\right)p_{o1}+M_{o1}+C_{o1}\left(a_{o1}+b_{o1}\right)-C_{n1}b_{n1}=0\end{cases} \quad \text{(A. 2)}$$

$$p_{n1}^*=\frac{\begin{pmatrix}2M_{n1}\left(a_{o1}+b_{o1}\right)+M_{o1}\left(b_{n1}+b_{o1}\right)+C_{n1}b_{n1}\left(b_{o1}+2a_{o1}-b_{n1}\right)\\ +C_{o1}b_{o1}\left(b_{n1}-a_{o1}-b_{o1}\right)+a_{o1}b_{n1}C_{o1}+2C_{n1}a_{n1}\left(a_{o1}+b_{o1}\right)\end{pmatrix}}{2b_{n1}\left(b_{o1}+2a_{o1}\right)+4a_{n1}\left(b_{o1}+a_{o1}\right)-b_{n1}{}^2-b_{o1}{}^2} \quad \text{(A. 3)}$$

$$p_{o1}^*=\frac{\begin{pmatrix}2M_{o1}\left(a_{n1}+b_{n1}\right)+M_{n1}\left(b_{n1}+b_{o1}\right)+2C_{o1}a_{n1}\left(a_{o1}+b_{o1}\right)\\ +\left(C_{o1}b_{o1}-C_{n1}b_{n1}-C_{n1}a_{n1}\right)\left(b_{n1}-b_{o1}\right)+2C_{o1}b_{n1}a_{o1}\end{pmatrix}}{2b_{n1}\left(b_{o1}+2a_{o1}\right)+4a_{n1}\left(b_{o1}+a_{o1}\right)-b_{n1}{}^2-b_{o1}{}^2} \quad \text{(A. 4)}$$

2. Duopoly model (DM)

2.1. Nash equilibrium

As it is shown by equations (A. 5) and (A. 6), R_1 and R_2 are concave functions according to their variables (please note that R_2 is a single-variable function and there is no need for calculating its Hessian matrix). Hence, the Nash equilibrium can be calculated by first-order derivatives of the R_1 and R_2. Equations (A. 7) to (A. 9) calculate the first-order derivatives of the profit functions.

$$H\left(R_1\left(p_{n1},p_{o1}\right)\right)=\begin{bmatrix}-2(a_{n1}+2b_{n1}) & (b_{n1}+b_{o1})\\ (b_{n1}+b_{o1}) & -2\left(a_{o1}+b_{o1}\left(\beta_o+1\right)\right)\end{bmatrix}\overset{a,b,\beta>0,\ a>b}{\Rightarrow} H \text{ is ND.} \quad \text{(A. 5)}$$

$$\frac{\partial^2 R_2\left(p_{o2}\right)}{\partial p_{o2}{}^2}=-2\left(a_{o2}+b_{o2}\left(\beta_o+1\right)\right)\overset{a,b,\beta>0}{\leq}0 \quad \text{(A. 6)}$$

$$\frac{\partial R_1\left(p_{n1},p_{o1}\right)}{\partial p_{n1}}=-2\left(a_{n1}+2b_{n1}\right)p_{n1}+\left(b_{n1}+b_{o1}\right)p_{o1}+b_{n1}p_{o2}+\underbrace{\left(M_{n1}+C_{n1}\left(a_{n1}+2b_{n1}\right)-C_{o1}b_{o1}\right)}_{K_1} \quad \text{(A. 7)}$$

$$\frac{\partial R_1\left(p_{n1},p_{o1}\right)}{\partial p_{o1}}=\left(b_{n1}+b_{o1}\right)p_{n1}-2\left(a_{o1}+b_{o1}\left(\beta_o+1\right)\right)p_{o1}+b_{o1}\beta_o p_{o2}+\underbrace{\left(M_{o1}-C_{n1}b_{n1}+C_{o1}\left(a_{o1}+b_{o1}\left(\beta_o+1\right)\right)\right)}_{K_2} \quad \text{(A. 8)}$$

---

[2] Urruty, H., Baptiste, J., Strodiot, Jacques, J., Nguyen, and Hien, V., “Generalized Hessian matrix and second-order optimality conditions for problems withC 1, 1 data,” Appl. Math. Optim., vol. 11, no. 1, pp. 43–56, 1984.

$$\frac{\partial R_2(p_{o2})}{\partial p_{o2}} = b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)p_{o2} + \underbrace{\left(M_{o2} + C_{o2}\left(a_{o2} + b_{o2}(\beta_o + 1)\right)\right)}_{K_3} \quad \text{(A. 9)}$$

There are three equalities and three variables (p_{n1}, p_{o1}, p_{o2}) in order to achieve the Nash equilibrium as equation (A. 10) shows. Please note that K_1, K_2, and K_3 are defined to simplify the equations, and their formulations are presented by equations (A. 7), (A. 8), and (A. 9) respectively.

$$\begin{cases} -2(a_{n1} + 2b_{n1})p_{n1} + (b_{n1} + b_{o1})p_{o1} + b_{n1}p_{o2} + K_1 = 0 \\ (b_{n1} + b_{o1})p_{n1} - 2\left(a_{o1} + b_{o1}(\beta_o + 1)\right)p_{o1} + b_{o1}\beta_o p_{o2} + K_2 = 0 \\ b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)p_{o2} + K_3 = 0 \end{cases} \Rightarrow \begin{cases} p_{n1}^* \\ p_{o1}^* \\ p_{o2}^* \end{cases} \quad \text{(A. 10)}$$

2.2. Stackelberg

We should determine the best response of the second manufacturer, before calculating the Stackelberg optimum solution. As it is explained before, the profit functions are concave, hence the best response of the second manufacturer ($p_{o2}^*(p_{n1},p_{o1})$) can be determined by equation (A. 11).

$$\frac{\partial R_2(p_{o2})}{\partial p_{o2}} = 0 \Rightarrow$$
$$b_{o2}p_{n1} + b_{o2}\beta_o p_{o1} - 2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)p_{o2} + \left(M_{o2}C_{o2}\left(a_{o2} + b_{o2}(\beta_o + 1)\right)\right) = 0 \quad \text{(A. 11)}$$
$$\Rightarrow p_{o2}^*(p_{n1}, p_{o1}) = \frac{M_{o2} + b_{o2}(\beta_o p_{o1} + p_{n1})}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)} + \frac{C_{o2}}{2}$$

If we replace p_{o2} in equation (9), with the best response function ($p_{o2}^*(p_{n1},p_{o1})$) which is calculated by equation (A. 11), the profit function of the first manufacturer will be changed as equation (A. 18), in which, k_4 to k_9 are defined by equations (A. 12) to (A. 17).

$$k_4 = b_{n1}\left(\frac{b_{o2}}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)} - 2\right) - a_{n1} \quad \text{(A. 12)}$$

$$k_5 = b_{n1}\left(1 + \frac{b_{o2}\beta_o}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)}\right) \quad \text{(A. 13)}$$

$$k_6 = M_{n1} + b_{n1}\left(\frac{M_{o2}}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)} + \frac{C_{o2}}{2}\right) \quad \text{(A. 14)}$$

$$k_7 = b_{o1}\left(1 + \frac{\beta_o}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)}\right) \quad \text{(A. 15)}$$

$$k_8 = b_{o1}\left(\frac{b_{o2}\beta_o^2}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)} - (\beta_o + 1)\right) - a_{o1} \quad \text{(A. 16)}$$

$$k_9 = M_{o1} + b_{o1}\left(\frac{\beta_o M_{o2}}{2\left(a_{o2} + b_{o2}(\beta_o + 1)\right)} + \frac{C_{o2}}{2}\right) \quad \text{(A. 17)}$$

$$R_1(p_{n1}, p_{o1}) = (k_4p_{n1} + k_5p_{o1} + k_6)(p_{n1} - C_{n1}) + (k_7p_{n1} + k_8p_{o1} + k_9)(p_{o1} - C_{o1}) \quad \text{(A. 18)}$$

Optimum decisions of the first manufacturer are calculated by first-order derivatives of his profit function as equations (A. 19) to (A. 21) show.

$$\begin{cases} \frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{n1}} = 0 \\ \frac{\partial R_1(p_{n1}, p_{o1})}{\partial p_{o1}} = 0 \end{cases} \quad \text{(A. 19)}$$

$$p_{n1}^* = \frac{C_{n1}k_5^2 - k_5k_9 + 2k_6k_8 - k_7k_9 - 2C_{n1}k_4k_8 + C_{n1}k_5k_7 + C_{o1}k_5k_8 - C_{o1}k_7k_8}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad \text{(A. 20)}$$

$$p_{o1}^* = \frac{C_{o1}k_7^2 - k_5k_6 + 2k_4k_9 - k_6k_7 - C_{n1}k_4k_5 + C_{n1}k_4k_7 - 2C_{o1}k_4k_8 + C_{o1}k_5k_7}{k_5^2 + 2k_5k_7 + k_7^2 - 4k_4k_8} \quad \text{(A. 21)}$$

3. Duopoly model II (DM-II)

3.1. Nash equilibrium

As the Hessian matrixes of the profit functions show (equations (A. 22) and (A. 23)), the profit functions (R_1 and R_2) are concave according to their own variables. Hence, the Nash equilibrium can be calculated by first-order derivatives of the profit functions. Equations (A. 24) to (A. 27) show the first-order derivatives of the profit functions.

$$H\left(R_1\left(p_{n1},p_{o1}\right)\right)=\begin{bmatrix}-2(a_{n1}+b_{n1}\left(\beta_n+2\right)) & (b_{n1}+b_{o1}) \\ (b_{n1}+b_{o1}) & -2\left(a_{o1}+b_{o1}\left(\beta_o+2\right)\right)\end{bmatrix}\overset{a,b,\beta>0}{\underset{a>b}{\Rightarrow}} H \text{ is ND} \tag{A. 22}$$

$$H\left(R_2\left(p_{n2},p_{o2}\right)\right)=\begin{bmatrix}-2(a_{n2}+b_{n2}\left(\beta_n+2\right)) & (b_{n2}+b_{o2}) \\ (b_{n2}+b_{o2}) & -2\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)\end{bmatrix}\overset{a,b,\beta>0}{\underset{a>b}{\Rightarrow}} H \text{ is ND} \tag{A. 23}$$

$$\begin{aligned}\frac{\partial R_1\left(p_{n1},p_{o1}\right)}{\partial p_{n1}}=&-2\left(a_{n1}+b_{n1}\left(\beta_n+2\right)\right)p_{n1}+\left(b_{n1}+b_{o1}\right)p_{o1}+b_{n1}\beta_n p_{n2}+b_{n1}p_{o2}\\&+\underbrace{\left(M_{n1}+C_{n1}\left(a_{n1}+b_{n1}\left(\beta_n+2\right)\right)-C_{o1}b_{o1}\right)}_{K_{10}}\end{aligned} \tag{A. 24}$$

$$\begin{aligned}\frac{\partial R_1\left(p_{n1},p_{o1}\right)}{\partial p_{o1}}=&\left(b_{n1}+b_{o1}\right)p_{n1}-2\left(a_{o1}+b_{o1}\left(\beta_o+2\right)\right)p_{o1}+b_{o1}p_{n2}+b_{o1}\beta_o p_{o2}\\&+\underbrace{\left(M_{o1}-C_{n1}b_{n1}+C_{o1}\left(a_{o1}+b_{o1}\left(\beta_o+2\right)\right)\right)}_{K_{11}}\end{aligned} \tag{A. 25}$$

$$\begin{aligned}\frac{\partial R_2\left(p_{o2}\right)}{\partial p_{n2}}=&b_{n2}\beta_n p_{n1}+b_{n2}p_{o1}-2\left(a_{n2}+b_{n2}\left(\beta_n+2\right)\right)p_{n2}+\left(b_{n2}+b_{o2}\right)p_{o2}\\&+\underbrace{\left(M_{n2}+C_{n2}\left(a_{n2}+b_{n2}\left(\beta_n+2\right)\right)-C_{o2}b_{o2}\right)}_{K_{12}}\end{aligned} \tag{A. 26}$$

$$\begin{aligned}\frac{\partial R_2\left(p_{o2}\right)}{\partial p_{o2}}=&b_{o2}p_{n1}+b_{o2}\beta_o p_{o1}+\left(b_{n2}+b_{o2}\right)p_{n2}-2\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)p_{o2}\\&+\underbrace{\left(M_{o2}-C_{n2}b_{n2}+C_{o2}\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)\right)}_{K_{13}}\end{aligned} \tag{A. 27}$$

In order to determine the Nash equilibrium, four equalities have to be solved, that is shown by equation (A. 28). Please note that K_{10}, K_{11}, K_{12}, and K_{13} are defined to simplify the equations, and the formulations of them are presented by equations (A. 24), (A. 25), (A. 26), and (A. 27) respectively.

$$\begin{cases}-2\left(a_{n1}+b_{n1}\left(\beta_n+2\right)\right)p_{n1}+\left(b_{n1}+b_{o1}\right)p_{o1}+b_{n1}\beta_n p_{n2}+b_{n1}p_{o2}+K_{10}=0\\\left(b_{n1}+b_{o1}\right)p_{n1}-2\left(a_{o1}+b_{o1}\left(\beta_o+2\right)\right)p_{o1}+b_{o1}p_{n2}+b_{o1}\beta_o p_{o2}+K_{11}=0\\b_{n2}\beta_n p_{n1}+b_{n2}p_{o1}-2\left(a_{n2}+b_{n2}\left(\beta_n+2\right)\right)p_{n2}+\left(b_{n2}+b_{o2}\right)p_{o2}+K_{12}=0\\b_{o2}p_{n1}+b_{o2}\beta_o p_{o1}+\left(b_{n2}+b_{o2}\right)p_{n2}-2\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)p_{o2}+K_{13}=0\end{cases}\Rightarrow\begin{cases}p_{n1}^*\\p_{o1}^*\\p_{n2}^*\\p_{o2}^*\end{cases} \tag{A. 28}$$

3.2. Stackelberg

As it is explained before, the profit functions are concave and the rational reaction functions of the second manufacturer ($p_{n2}^*(p_{n1},p_{o1})$, $p_{o2}^*(p_{n1},p_{o1})$) can be calculated by first-order derivatives, as equation (A. 29) shows. By solving the equalities of equation (A. 29), $p_{n2}^*(p_{n1},p_{o1})$, $p_{o2}^*(p_{n1},p_{o1})$ are determined as equations (A. 30) and (A. 31), in which, k_{14} to k_{20} are defined by equation (A. 29).

$$\begin{cases} \frac{\partial R_2(p_{n2},p_{o2})}{\partial p_{n2}} = 0 \\ \frac{\partial R_2(p_{n2},p_{o2})}{\partial p_{o2}} = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \overbrace{b_{n2}\beta_n}^{k_{14}} p_{n1} + b_{n2}p_{o1} \overbrace{-2\left(a_{n2}+b_{n2}\left(\beta_n+2\right)\right)}^{k_{15}} p_{n2} + \overbrace{\left(b_{n2}+b_{o2}\right)}^{k_{16}} p_{o2} \\ \qquad + \underbrace{\left(M_{n2}+C_{n2}\left(a_{n2}+b_{n2}\left(\beta_n+2\right)\right)-C_{o2}b_{o2}\right)}_{k_{17}} = 0 \\ b_{o2}p_{n1} + \overbrace{b_{o2}\beta_o}^{k_{18}} p_{o1} + \overbrace{\left(b_{n2}+b_{o2}\right)}^{k_{16}} p_{n2} \overbrace{-2\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)}^{k_{19}} p_{o2} \\ \qquad + \underbrace{\left(M_{o2}-C_{n2}b_{n2}+C_{o2}\left(a_{o2}+b_{o2}\left(\beta_o+2\right)\right)\right)}_{k_{20}} = 0 \end{cases} \tag{A. 29}$$

$$p^*_{n2}(p_{n1},p_{o1}) = \frac{\left(k_{16}b_{o2}-k_{14}k_{19}\right)p_{n1} + \left(k_{16}k_{18}-k_{19}b_{n2}\right)p_{o1} + \left(k_{16}k_{20}-k_{17}k_{19}\right)}{k_{15}k_{19}-k_{16}^{\ 2}} \tag{A. 30}$$

$$p^*_{o2}(p_{n1},p_{o1}) = \frac{\left(k_{14}k_{16}-k_{15}b_{o2}\right)p_{n1} + \left(k_{16}b_{n2}-k_{15}k_{18}\right)p_{o1} + \left(k_{16}k_{17}-k_{15}k_{20}\right)}{k_{15}k_{19}-k_{16}^{\ 2}} \tag{A. 31}$$

The best response of the second manufacturer ($p_{n2}^*(p_{n1},p_{o1})$, $p_{o2}^*(p_{n1},p_{o1})$) should be replaced in equation (19) in order to determine the profit of the first manufacturer. Then the optimal decision of the first manufacturer (p_{n1}^*, p_{o1}^*) according to the best rational response of the second manufacturer is determined. Equations (A. 32) and (A. 33) show equalities which determine p_{n1}^* and p_{o1}^*.

$$p^*_{n1} = \frac{\left(\begin{array}{l} p^*_{n2}\left(b_{o1}\left(b_{n1}\left(2\beta_n\left(\beta_o+2\right)+1\right)\right)+2\beta_n b_{n1}a_{o1}\right) \\ +p^*_{o2}\left(b_{o1}\left(\beta_o\left(3b_{n1}+b_{o1}\right)+4b_{n1}\right)+2a_{o1}b_{n1}\right) \\ +C_{n1}\left(\begin{array}{l} 2a_{o1}b_{n1}\left(2+\beta_n\right)+b_{n1}b_{o1}\left(4\left(\beta_n+\beta_o\right)+2\beta_n\beta_o+7\right) \\ +2a_{n1}\left(b_{o1}\left(\beta_o+2\right)+a_{o1}\right)-b_{n1}^{\ 2} \end{array}\right) \\ +C_{o1}\left(b_{o1}\left(b_{n1}-b_{o1}\right)\left(\beta_o+2\right)+a_{o1}\left(b_{n1}-b_{o1}\right)\right) \\ +2M_{n1}\left(b_{o1}\left(\beta_o+2\right)+a_{o1}\right)+M_{o1}\left(b_{n1}+b_{o1}\right)+4\beta_n b_{n1}b_{o1} \end{array}\right)}{\left(\begin{array}{l} 4\left(a_{o1}+2b_{o1}\right)\left(a_{n1}+\beta_n b_{n1}\right)+4\beta_o b_{o1}\left(a_{n1}+b_{n1}\left(\beta_o+2\right)\right) \\ +2b_{n1}\left(4a_{o1}+7b_{o1}\right)-b_{n1}^{\ 2}-b_{o1}^{\ 2} \end{array}\right)} \tag{A. 32}$$

$$p^*_{o1} = \frac{\left(\begin{array}{l} p^*_{o2}\left(b_{n1}\left(b_{o1}\left(2\beta_o\left(\beta_n+2\right)+1\right)\right)+2\beta_o b_{o1}a_{n1}\right) \\ +p^*_{n2}\left(b_{n1}\left(\beta_n\left(3b_{o1}+b_{n1}\right)+4b_{o1}\right)+2a_{n1}b_{o1}\right) \\ +C_{o1}\left(\begin{array}{l} 2a_{n1}b_{o1}\left(2+\beta_o\right)+b_{o1}b_{n1}\left(4\left(\beta_o+\beta_n\right)+2\beta_o\beta_n+7\right) \\ +2a_{o1}\left(b_{n1}\left(\beta_n+2\right)+a_{n1}\right)-b_{o1}^{\ 2} \end{array}\right) \\ +C_{n1}\left(b_{n1}\left(b_{o1}-b_{n1}\right)\left(\beta_n+2\right)+a_{n1}\left(b_{o1}-b_{n1}\right)\right) \\ +2M_{o1}\left(b_{n1}\left(\beta_n+2\right)+a_{n1}\right)+M_{n1}\left(b_{o1}+b_{n1}\right)+4\beta_o b_{o1}b_{n1} \end{array}\right)}{\left(\begin{array}{l} 4\left(a_{n1}+2b_{n1}\right)\left(a_{o1}+\beta_o b_{o1}\right)+4\beta_n b_{n1}\left(a_{o1}+b_{o1}\left(\beta_n+2\right)\right) \\ +2b_{o1}\left(4a_{n1}+7b_{n1}\right)-b_{o1}^{\ 2}-b_{n1}^{\ 2} \end{array}\right)} \tag{A. 33}$$

Appendix B: Parameter estimation of the case study

In this section calculation of the case study for eliminating inflation rate, and seasonal factor is explained as well as the heuristic method which is used for parameter estimation.

The inflation rate affects selling prices and manufacturing costs. Besides, as it is mentioned before, the demand for the textile products alters continuously as season changes. Hence, the impacts of the inflation rate and seasons should be eliminated before parameter estimation. The inflation rate of Iran between 2012 to 2017 is presented by Table B1[3].

---

[3] CBI, "CPI and Inflation," Central Bank of the Islamic republic of Iran, 2018. https://www.cbi.ir/Inflation/Inflation_en.aspx (accessed Feb. 15, 2018).

The seasonal factor is implemented in order to eliminate the impact of seasons. One month is determined as an origin month, and the demands of other months are compared with the origin month, in order to show how the demands change during different months. The seasonal factor is determined as a ratio of the average demand of each month over the average demand of April (as the origin month), equation (B. 1) represents the calculation of the seasonal factor, and Table B2 presents the seasonal factors determined for each month.

$$\text{Seasonal factor}_m = \frac{\text{Average demand of month } (m)}{\text{Average demand of "April"}} \quad \text{(B. 1)}$$

*****Please insert Table B1 about here.*****

*****Please insert Table B2 about here.*****

In order to modify the datasets by using seasonal factor (to modify demand) and inflation rate (to modify costs and prices), equations (B. 2) and (B. 3) are employed. Equation (B. 3) is a common formula that is being used for calculating future value (*FV*) of a present payment (*PV*), in which the inflation rate of period t is equal to i_t and n is the number of periods [44].

$$\text{modified } (D^s_{ij}) = \frac{D^s_{ij}}{\text{related seasonal factor}} \quad \text{(B. 2)}$$

$$FV = PV \times \prod_{t=1}^{n} (1+i_t)^n \quad \text{(B. 3)}$$

There are two basic methods for parameters estimation. The first method is market research, which requires a long time process. The second method is estimating the parameters of the demand functions by the collected dataset. Although the second method is less accurate than the first method, it requires fewer resources and time. We prefer to use the second method for parameters estimation because we do not intend to focus on the market research methods here.

After modification of the raw data, in order to eliminate the impact of the inflation rate and seasonal effect, we should estimate parameters of the proposed model. The demand functions are multivariable functions. Besides, they are not regular polynomial functions and some of the parameters are common between the demand functions and they should be estimated simultaneously. Hence, regular regression methods are not appropriate for this case and we developed a heuristic method in order to estimate the parameters. The proposed heuristic approach is explained step by step as follows:

Step 1: Determine lower bound and upper bound for each parameter (usually a_{ij}&$b_{ij}\in[0,0.5]$, $\beta_j\in[0,2]$, and the ranges of production cost (C_{ij}) and market size (M_{ij}) can be easily estimated by previous data).

Step 2: Divide the determined window of each parameter into 1000 equal sections.

Step 3: Calculate the demand functions for each set of predetermined parameters (middle of the mentioned sections).

Step 4: Calculate mean square errors (as equation (B. 4) shows) for each set of predetermined parameters. Determine the minimum mean square error (MMSE).

Step 5: Compare the MMSE with MSE that is evaluated by the previous iteration. If it was improved less than 0.01%, save the related parameters and **stop**. Else, narrow lower bound and upper bound of parameters and go to **step 2** in order to increase the accuracy of parameter estimation.

Please note that we can increase the accuracy of the heuristic method if it is needed, but calculation time increases too. As it is mentioned, equation (B. 4) determines the mean square error. In which, S is the number of datasets, D^s_{ij} is actual demand of product i produced by manufacturer j achieved by dataset s, and D'^k_{ij} is estimated demand of product i produced by manufacturer j estimated by k^{th} predetermined parameters set.

$$MSE_k = \frac{\sum_{s=1}^{S}\left(\left(D^s_{n1} - D'^k_{n1}\right)^2 + \left(D^s_{o1} - D'^k_{o1}\right)^2 + \left(D^s_{n2} - D'^k_{n2}\right)^2 + \left(D^s_{o2} - D'^k_{o2}\right)^2\right)}{4\times S} \quad \text{(B. 4)}$$

Usually, these brands introduce their products simultaneously (DM-II condition). Hence, the data set of the first and second conditions are less than the DM-II condition. That is why the accuracy of the estimation of the parameters for the third condition is higher than the other conditions.

Fig. B1, Fig. B2, and Fig. B3 indicates the accuracy of parameters' estimation for the first, second, and third conditions respectively.

The raw dataset, modified dataset, modification codes, parameter estimation code, and solving codes are presented by supplementary data.

*****Please insert Fig. B1 about here.*****

*****Please insert Fig. B2 about here.*****
*****Please insert Fig. B3 about here.*****

List of Tables

Table B1, Inflation rate of Iran.

Year-Month	2012-12	2013-12	2014-12	2015-12	2016-12	2017-12
Point to point inflation rate	25.7%	39.3%	17.2%	13.7%	8.6%	10.0%

Table B2, Seasonal factors of each month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Seasonal factor	0.33	0.24	1.05	1.00	0.94	0.84	0.81	0.89	0.89	0.76	0.43	0.31

List of Figures

Fig. B1. Evaluation of the estimation of the parameters for the first condition.

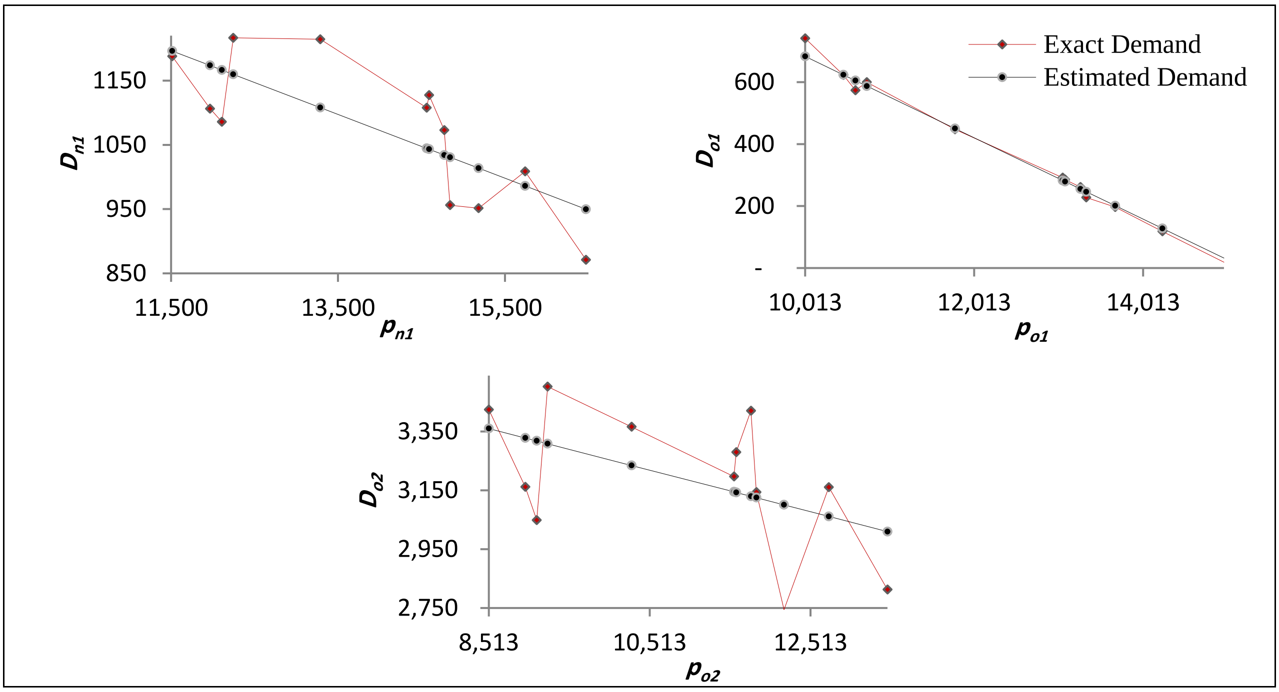

Fig. B2. Evaluation of the estimation of the parameters for the second condition.

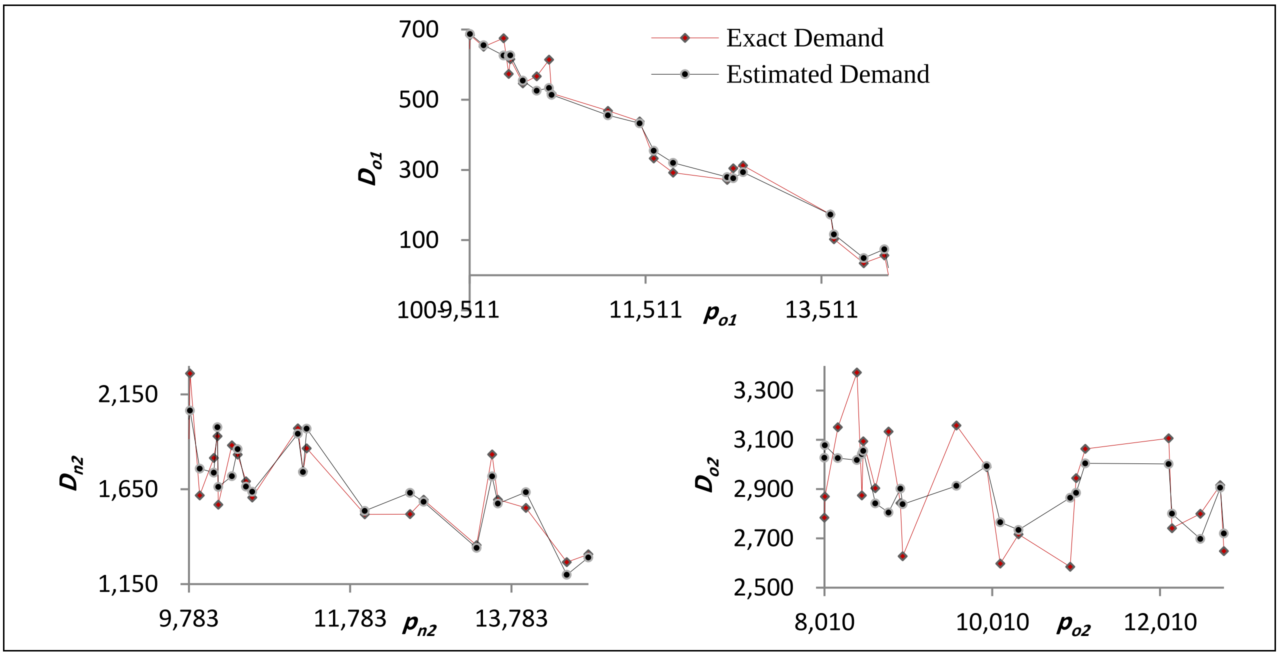

Fig. B3. Evaluation of the estimation of the parameters for the third condition.