Electronic supplementary information (ESI) for the manuscript entitled:

"Designing and analyzing two non-invasive current sensors using Ampere's Force Law"

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Force applied on moving/piezoelectric microbeams

The both microbeams are carrying currents $i_1=c_1 sin(\omega_1 t)$ and $i_2=c_2 sin(\omega_2 t)$, respectively. Figure ESI-1 shows the microbeams cross-view, which assumed to be in the y-z plane along the y-axis. The cross-sections of the wires considered rectangular shapes to be consistent with the MEMS/NEMS fabrication processes.

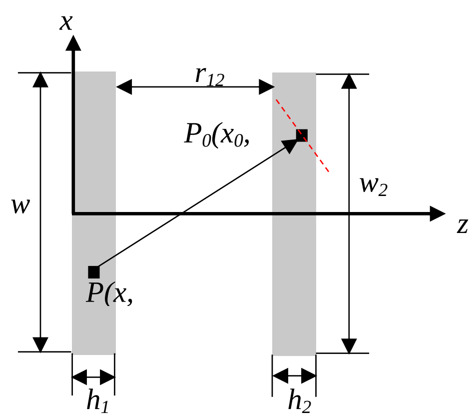

Figure ESI-1: Cross-section of the microbeams.

Since the lengths of the wires are much larger than the microbeams cross-sections and the distance between them, the microbeams are assumed to be infinite. Therefore, the magnetic field of each element of the moving microbeam, like point $P(x,y)$, at point $P_0(x_0,z_0)$ of the stationary microbeam can be written using Biot-Savart-Laplace law as follows

$$dH_x(x_0,z_0)=\frac{2i_1}{cA_1}\frac{z_0-z}{(x_0-x)^2+(z_0-z)^2}dxdz$$
$$dH_z(x_0,z_0)=-\frac{2i_1}{cA_1}\frac{x_0-x}{(x_0-x)^2+(z_0-z)^2}dxdz \quad (1)$$

where, A_1 is the cross-section of the moving microbeam; and c is the light velocity. Since the lengths of the wires were assumed infinitive, the magnetic field in y-direction is zero. Taking integral over the moving microbeam cross-section, the components of the magnetic fields are achieved as follows

$$H_x(x_0,z_0)=\frac{i_1}{cA_1}\left\{\begin{bmatrix}\left(x_0+\frac{w_1}{2}\right)\ln\left(1+\frac{h_1+2z_0}{\left(x_0+\frac{w_1}{2}\right)^2+z_0^2}h_1\right)\\ +2(z_0+h_1)\arctan\left(\frac{x_0+\frac{w_1}{2}}{z_0+h_1}\right)-2z_0\arctan\left(\frac{x_0+\frac{w_1}{2}}{z_0}\right)\end{bmatrix}-\begin{bmatrix}\left(x_0-\frac{w_1}{2}\right)\ln\left(1+\frac{h_1+2z_0}{\left(x_0-\frac{w_1}{2}\right)^2+z_0^2}h_1\right)\\ +2(z_0+h_1)\arctan\left(\frac{x_0-\frac{w_1}{2}}{z_0+h_1}\right)-2z_0\arctan\left(\frac{x_0-\frac{w_1}{2}}{z_0}\right)\end{bmatrix}\right\} \tag{2}$$

$$H_z(x_0,z_0)=\frac{i_1}{cA_1}\left\{\begin{bmatrix}z_0\ln\left(1+\frac{\left(x_0+\frac{w_1}{2}\right)^2}{z_0^2}\right)-(z_0+h_1)\ln\left(1+\frac{\left(x_0+\frac{w_1}{2}\right)^2}{(z_0+h_1)^2}\right)\\ -2\left(x_0+\frac{w_1}{2}\right)\left(\arctan\left(\frac{z_0+h_1}{x_0+\frac{w_1}{2}}\right)-\arctan\left(\frac{z_0}{x_0+\frac{w_1}{2}}\right)\right)\end{bmatrix}-\begin{bmatrix}z_0\ln\left(1+\frac{\left(x_0-\frac{w_1}{2}\right)^2}{z_0^2}\right)-(z_0+h_1)\ln\left(1+\frac{\left(x_0-\frac{w_1}{2}\right)^2}{(z_0+h_1)^2}\right)\\ -2\left(x_0-\frac{w_1}{2}\right)\left(\arctan\left(\frac{z_0+h_1}{x_0-\frac{w_1}{2}}\right)-\arctan\left(\frac{z_0}{x_0-\frac{w_1}{2}}\right)\right)\end{bmatrix}\right\} \tag{3}$$

where, w_1 is the width of the moving microbeam; h_1 is the height of the moving microbeam. The direction of the magnetic field vector is not shown in the Figure ESI-1 because it changes by the currents' direction; however, it is always perpendicular to the line connecting P to P_0, parallel to the shown red-dashed line in Figure ESI-1. Using the magnetic field components determined in Eqs. (2) and (3), the force applied on the stationary microbeam by the moving microbeam can be calculated using Ampere Law as follows

$$\vec{F}_{12} = \mu_0 \frac{i_1 i_2}{A_2} \left\{ \hat{x} \left[\int_{r_{12}+h_1}^{r_{12}+h_1+h_2} \int_{-w_2/2}^{w_2/2} H_z(x_0, z_0) dx_0 dz_0 \right] - \hat{z} \left[\int_{r_{12}+h_1}^{r_{12}+h_1+h_2} \int_{-w_2/2}^{w_2/2} H_x(x_0, z_0) dx_0 dz_0 \right] \right\} \quad (4)$$

Here, r_{12} is the distance between the wires 1 and 2 shown in Fig. 2. According to Eq. (4), the force per unit length applied on the stationary microbeam has components in *x*- and *z*- directions. It can be readily shown that the force in *x*-direction is zero due to the symmetric geometry of the microbeams. Furthermore, the same force applied to the moving microbeam is applied to the stationary microbeam as well. To avoid the vibration of the stationary microbeam, the stationary microbeam is assumed to be thicker than the moving microbeam. By substituting Eqs. (2) and (3) into Eq. (4), the total force applied to the moving microbeam can be calculated as presented in the manuscript, Eq. (3) of manuscript.