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Research Note

A simple solution procedure for solving the multi-delivery policy into economic production lot size problem with partial rework

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Abstract. Recently, an alternative multi-delivery policy into imperfect Economic Production Quantity (EPQ) inventory model with partial rework has been proposed, which considers the number of shipments as a fixed and given value. This paper, treating the long-run average costs per unit time as a function of replenishment lot size Q and the number of shipments n, adopts the differential calculus approach to get the optimal solution of Q and n jointly. In numerical examples, it is illustrated that the solution procedure is simple and accurate.

1. Introduction

An interesting research topic in inventory theory is the development of inventory models when the manufacturing system produces defective items which can be reworked. Within this context, Chiu et al. [1] proposed a multi-delivery policy into an imperfect Economic Production Quantity (EPQ) inventory model with partial rework. Essentially, they treat their long-run average cost as a function of the replenishment lot size, Q. Consequently, the long-run average cost function of Chiu et al. [1] is a function of only one decision variable, Q. Furthermore, they consider the number of shipments, n, as a constant. Conversely, Cárdenas-Barrón et al. [2] generalized the model of Chiu et al. [1] to allow the number of shipments, n, to be a decision variable, such that the long-run average cost is a function of two decision variables, Q and n. A solution procedure to determine both Q and n is also proposed.

Following the approach of Cárdenas-Barrón et al. [2], this paper adopts the calculus approach to obtain the optimal solution of Q and n jointly. The solution procedure simplifies that of Cárdenas-Barrón et al. [2]. Research on EPQ inventory models with partial rework can be found in Cárdenas-Barrón et al. [2], Chiu et al. [1], Sana [3], Sana [4], Sana and Goyal [5], and their references.

In the following paper, Section 2 presents the mathematical formulation of the inventory model. Section 3 proposes two theorems to locate the optimal solution. Section 4 illustrates the use of the theorems

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with numerical examples. Finally, a remark conclusion is given.

2. Model formulation

We adopt all of the notation and assumptions described in Chiu et al. [1] and Cárdenas-Barrón et al. [2] to establish a new manufacturing model which allows the number of shipments n to be a decision variable. Following Chiu et al. [1] and Cárdenas-Barrón et al. [2], we can get the long-run average costs, $E[n, Q]$, per unit time for the new manufacturing model as follows:

$$E[n,Q] = \psi_3 + \psi_2(n)Q^{-1} + \psi_1(n)Q, \tag{1}$$

where:

$$\psi_3 = \frac{C\lambda}{1-\theta E[x]} + C_T\lambda + \frac{C_S E[x]\theta\lambda}{1-\theta E[x]} + \frac{C_R E[x](1-\theta)\lambda}{1-\theta E[x]}, \tag{2}$$

$$\psi_2(n) = \frac{(K+(n+1)K_1)\lambda}{1-\theta E[x]}, \tag{3}$$

$$\begin{aligned}\psi_1(n) =& \frac{h\lambda^2}{2P(1-\theta E[x])}\left[\frac{2\lambda}{P^2}E\left(\frac{1}{1-x}\right) - \frac{1}{P}\right.\\ &+ \frac{(1-\theta)}{P_1}\left[\frac{4\lambda}{P}E\left(\frac{x}{1-x}\right)\right.\\ &\left.\left.+ \frac{2\lambda(1-\theta)}{P_1}E\left(\frac{x^2}{1-x}\right) - 2E(x)\right]\right]\\ &+ \frac{h}{2(1-\theta E[x])}\left\{-\frac{\lambda[E(x)]^2(1-\theta)^2}{P_1}\left[1+\frac{\lambda}{P_1}\right]\right.\\ &\left.+ [1-\theta E(x)]^2 - \frac{\lambda[1-2\theta E(x)]}{P}\right\}\\ &- \frac{h}{2n(1-\theta E(x))}\left\{[1-\theta E(x)]^2 - \frac{\lambda}{P}\left[2-\frac{\lambda}{P}\right]\right.\\ &+ \frac{\lambda E(x)(1-\theta)}{P_1}[-2[1-\theta E(x)]\\ &\left.+ \frac{2\lambda}{P} + \frac{\lambda E(x)(1-\theta)}{P_1}\right\}\\ &+ \left\{\frac{1}{2(1-\theta E(x))}\right\}\frac{h_1(E[x])^2\lambda(1-\theta)^2}{P_1}\\ =& \frac{1}{2(1-\theta E[x])}\left\{A - \frac{B}{n} + D\right\},\end{aligned} \tag{4}$$

$$\begin{aligned}A =& h\left\{\frac{\lambda^2}{P}\left[\frac{2\lambda}{P^2}E\left(\frac{1}{1-x}\right) - \frac{1}{P}\right.\right.\\ &+ \frac{(1-\theta)}{P_1}\left[\frac{4\lambda}{P}E\left(\frac{x}{1-x}\right)\right.\\ &\left.\left.\left.+ \frac{2\lambda(1-\theta)}{P_1}E\left(\frac{x^2}{1-x}\right) - 2E(x)\right]\right]\right\},\end{aligned} \tag{5}$$

$$\begin{aligned}B =& h\left[[1-\theta E(x)]^2 - \frac{\lambda}{P}\left(2-\frac{\lambda}{P}\right)\right.\\ &+ \frac{\lambda E(x)(1-\theta)}{P_1}\left[-2[1-\theta E(x)] + \frac{2\lambda}{P}\right.\\ &\left.\left.+ \frac{\lambda E(x)(1-\theta)}{P_1}\right]\right],\end{aligned} \tag{6}$$

$$\begin{aligned}D =& h\left([1-\theta E(x)]^2 - \frac{\lambda[1-2\theta E(x)]}{P}\right.\\ &\left.- \frac{\lambda(E(x))^2(1-\theta)^2}{P_1}\left[1+\frac{\lambda}{P_1}\right]\right)\\ &+ \frac{h_1(E[x])^2\lambda(1-\theta)^2}{P_1}.\end{aligned} \tag{7}$$

3. The optimal solution (n^*, Q^*) of $E[n, Q]$

When n is given, Chiu et al. [1] obtain the optimal replenishment lot size $Q^*(n)$ as:

$$Q^*(n) = \sqrt{\frac{2(K+(n+1)K_1)\lambda}{A-\frac{B}{n}+D}}. \tag{8}$$

So, the optimal solution (n^*, Q^*) of $E[n, Q]$ can be obtained by:

$$E[n^*, Q^*] = \underset{n\geq 1}{\text{minimum}}\left\{E[n, Q^*(n)]\right\}. \tag{9}$$

If we treat n as a continuous variable, Eqs. (8) and (9) yield:

$$\frac{dQ^*(n)}{dn} = \frac{(K_1\lambda)\frac{\sqrt{A-\frac{B}{n}+D}}{\sqrt{2(K+(n+1)K_1)\lambda}} - \frac{B\sqrt{2(K+(n+1)K_1)\lambda}}{(2n^2)\sqrt{A-\frac{B}{n}+D}}}{A-\frac{B}{n}+D}, \tag{10}$$

$$\frac{dE[n,Q^*(n)]}{dn} = \frac{K_1\lambda}{(1-\theta E[x])Q^*(n)} - \frac{\psi_2(n)}{(Q^*(n))^2}\frac{dQ^*(n)}{dn} + \frac{BQ^*(n)}{2n^2(1-\theta E[x])} + \psi_1(n)\frac{dQ^*(n)}{dn}. \quad (11)$$

Incorporating Eqs. (10) and (11), we have:

$$\frac{dE[n,Q^*(n)]}{dn} = \frac{K_1\lambda\sqrt{A-\frac{B}{n}+D}}{(1-\theta E[x])\sqrt{2(K+(n+1)K_1)\lambda}} + \frac{B\sqrt{2(K+(n+1)K_1)\lambda}}{2n^2(1-\theta E[x])\sqrt{A-\frac{B}{n}+D}}$$

$$= \frac{2K_1\lambda n^2\left(A-\frac{B}{n}+D\right)+2B(K+(n+1)K_1)\lambda}{2n^2(1-\theta E[x])\sqrt{2(K+(n+1)K_1)\lambda}\sqrt{A-\frac{B}{n}+D}}$$

$$= \frac{2\lambda[K_1n^2(A+D)+B(K+K_1)]}{2n^2(1-\theta E[x])\sqrt{2(K+(n+1)K_1)\lambda}\sqrt{A-\frac{B}{n}+D}}. \quad (12)$$

Eq. (12) illustrates that if $B \geq 0$, then $E[n,Q^*(n)]$ is increasing with respect to $n \geq 1$. So, based on Eq. (9), we have:

$$E[n^*,Q^*] = E[1,Q^*(1)] = \underset{n\geq 1}{\text{minimize}}\{E[n,Q^*(n)]\} \text{ if } B \geq 0. \quad (13)$$

Consequently, if $B \geq 0$, we have $n^* = 1$ and $Q^* = Q^*(1) = \sqrt{\frac{2(K+2K_1)\lambda}{A-B+D}}$. Therefore, we conclude that the optimal number of shipments is $n^* = 1$ if $B \geq 0$. Hence, we obtain the following results.

Theorem 1: Suppose that $B \geq 0$. Then:

(i) $E[n,Q^*(n)]$ is increasing with respect to $n \geq 1$;

(ii) The optimal solution, (n^*,Q^*), of $E[n,Q]$ can be expressed by Eq. (13), that is $(n^*,Q^*) = (1,Q^*(1)) = (1,\sqrt{\frac{2(K+2K_1)\lambda}{A-B+D}})$;

(iii) The optimal number of shipments of the proposed model is $n^* = 1$.

Conversely, if $B < 0$, there exist two cases:

Case (I): If $A+D \leq 0$, Eq. (12) implies $\frac{dE(n,Q^*(n))}{dn} < 0$ for all $n \geq 1$. So, $E[n,Q^*(n)]$ is decreasing on $n \geq 1$. Therefore, $(n^*,Q^*) = (\infty,\infty)$;

Case (II): If $A+D > 0$, Eq. (12) implies:

$$\frac{dE[n,Q^*(n)]}{dn}\begin{cases} < 0 & \text{if } 0 < n < \Omega, & (14a)\\ = 0 & \text{if } n = \Omega, & (14b)\\ > 0 & \text{if } n > \Omega, & (14c)\end{cases}$$

where:

$$\Omega = \sqrt{\frac{-B(K+K_1)}{K_1(A+D)}}. \quad (15)$$

Eqs. (14a), (14b), and (14c) reveal that $\frac{dE[n,Q^*(n)]}{dn}$ is decreasing on $(0,\Omega]$ and increasing on $[\Omega,\infty)$. Let $n_0 = \lfloor\Omega\rfloor$ = the greatest integer $\leq \Omega$. Then, we have the following results.

Theorem 2: Suppose that $B < 0$. Then:

Case (I): If $A+D \leq 0$, then $(n^*,Q^*) = (\infty,\infty)$;

Case (II): If $A+D > 0$, then $E[n^*,Q^*] = \text{minimum}\{E[n_0,Q^*(n_0)],E[n_0+1,Q^*(n_0+1)]\}$. That is, $(n^*,Q^*) = (n_0,Q^*(n_0))$ or $(n_0+1,Q^*(n_0+1))$ associated with the less cost.

4. Numerical examples

In this paper, all instances in Cárdenas-Barrón et al. [2] are used for comparison. The data are shown in Table 1. Table 2 reveals that all optimal solutions are consistent with those in Cárdenas-Barrón et al. [2].

5. Conclusion

Cárdenas-Barrón et al. [2] treated the long-run average costs per unit time for the model of Chiu et al. [1] as a function of the replenishment lot size, Q, and the number of shipments n. Since n is restricted to being an integer number, they did not use the differential calculus approach to find the optimal solution (n^*,Q^*). Basically, they adopted the approach of García-Laguna et al. [6] and used Eqs. (13) and (14) in Cárdenas-Barrón et al. [2] to develop the algorithm for locating the optimal solution (n^*,Q^*). However, this paper considered n as a continuous variable and used the differential calculus approach to obtain Theorems 1 and 2 to locate the optimal solution (n^*,Q^*). There exist two cases:

Case (I): When $B \geq 0$, then $(n^*,Q^*) = (1,Q^*(1))$;

Case (II): When $B < 0$ and $A+D > 0$, then:

$$(n^*,Q^*) = \text{minimize}\{(n_0,Q^*(n_0)),(n_0+1,Q^*(n_0+1))\},$$

Table 1. Data of the instances in Cárdenas-Barrón et al. (2012) [2].

Parameter	Instance 1 from Chiu et al. (2012) [1]	Instance 2	Instance 3	Instance 4	Instance 5	Instance 6
C	100	190	21	100	120	300
C_R	60	170	5	50	110	100
C_s	20	100	4	25	80	20
C_T	0.1	40	3	40	60	5
K	20000	4000	400	65690	4000	89000
K_1	4350	1	0.1	0.5	1	9.8
h	20	30	10	10	90	150
h_1	40	20	30	3	80	50
λ	3400	560	210	300	400	600
P	60000	590	260	300	800	1200
P_1	2200	360	130	200	200	300
x	$U(0, 0.3)$	$U(0, 0.21)$	$U(0, 0.11)$	$U(0, 0.4)$	$U(0, 0.69)$	$U(0, 0.35)$
$E(x)$	0.15	0.105	0.055	0.2	0.345	0.175
θ	0.1	0.3	0.26	0.4	0.65	0.2
$E(1/(1-x))$	0.18889164	1.12248730	1.0593983	1.27706405	1.6973666	1.23080833
$E(x/(1-x))$	0.1889164	0.12248730	0.0593983	0.27706405	0.6973666	0.23080833
$E(x^2/(1-x))$	0.0389164	0.01748730	0.00493983	0.07706405	0.3523666	0.0550833

Table 2. The optimal solution of $E[n, Q]$.

Parameters	Instance					
	1	2	3	4	5	6
B	10.325275	–1.523229	–0.104973	–0.92400	–20.07692	–0.11625
A	–0.29032	39.65082	5.532814	23.16403	53.42922	43.47971
D	17.99790	0.98289	1.88041	–0.41120	23.80281	54.25375
Ω	—	12.24683	7.52697	73.04401	32.25033	3.286856
n_0	1	12	7	73	32	3
ψ_3	374512.58883	141306.76304	5159.51912	47217.39130	101974.86304	198668.39378
$\psi_2(n_0)$	99065989.84772	2320371.70883	85389.06361	21432717.39130	2079535.93297	55361160.62176
$\psi_2(n_0+1)$	—	2320949.92256	85410.36827	21432880.43478	2080051.56300	55367253.88601
$\psi_1(n_0)$	3.74736	21.04318	3.76799	12.37254	50.18333	50.65918
$\psi_1(n_0+1)$	—	21.03814	3.76704	12.37245	50.17107	50.65416
$Q^*(n_0)$	5141.61287	332.06497	150.53795	1316.16125	203.56518	1045.37842
$Q^*(n_0+1)$	—	332.14613	150.57573	1316.17120	203.61527	1045.48774
$E[n_0, Q^*(n_0)]$	413047.57481	155282.17086	6293.97144	79785.91957	122406.01888	304584.41390
$E[n_0+1, Q^*(n_0+1)]$	—	155282.23779	6293.96978	79785.92110	122406.05676	304584.99488
(n^*, Q^*)	$(1, Q^*(1))$	$(12, Q^*(12))$	$(8, Q^*(8))$	$(73, Q^*(73))$	$(32, Q^*(32))$	$(3, Q^*(3))$

where:

$$n_0 = \left\lfloor \sqrt{\frac{-B(K+K_1)}{K_1(A+D)}} \right\rfloor .$$

Compared with the solution procedure for Cases 1-1 and 1-2 described in Cárdenas-Barrón et al. [2], the one shown by Theorems 1 and 2 in this paper is simpler. In addition, numerical examples illustrate that our solution procedure is rather accurate. A relevant managerial insight is that we consider the number of shipments in the paper as a decision variable and also as an integer value. Future research directions are in line for applying our proposed solution procedure to other inventory models and also extending it to more realistic situations such as considering different types of variable demand, e.g. stock-dependent demand, credit-linked demand, and advertisement-dependent demand. These are some interesting research directions on which studies can be done in the near future.

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