Scientia Iranica E (2018) 25(5), 2904–2911

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Hybrid cluster and data envelopment analysis with interval data

K. Kianfar[a], M. Ahadzadeh Namin[b,*], A. Alam Tabriz[c], E. Najafi[a], and F. Hosseinzadeh Lotfi[d]

a. *Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.*
b. *Department of Mathematics, Shahr-e-Qods Branch, Islamic Azad University, Tehran, Iran.*
c. *Department of Management, Shahid Beheshti University, Tehran, Iran.*
d. *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.*

Received 5 May 2017; received in revised form 24 June 2017; accepted 10 July 2017

KEYWORDS
Data envelope analysis;
Cross-efficiency matrix;
Cluster analysis.

Abstract. Data Envelope Analysis (DEA) is an approach to estimating the relative efficiency of Decision-Making Units (DMUs). Several studies have been conducted in order to prioritize efficient units, and some useful models, such as Cross-Efficiency Matrix (CEM), have been presented. Besides, a number of DEA models with interval data have been developed, and ranking DMUs with such data has been carried out. However, presenting an obtained crisp data derived from interval data is a critical problem; hence, many researches have been conducted so as to compute the weights and average of the interval data. This paper proposes a new algorithm to find highly suitable weights by applying a data mining approach of DMU's data. For this purpose, clustering and a pair-wise comparison matrix were employed to estimate the given relative efficiency of CEM. Results indicate that there is a meaningful difference between efficiency of DMUs with the lower bound and that of DMUs with the upper bound.

1. Introduction

Data Envelop Analysis (DEA) is a linear programming model and a non-parametric approach that evaluates relative technical efficiency of Decision-Making Units (DMUs) on the basis of multiple inputs and outputs by computing the ratio of weighted sum of their outputs to their inputs [1]. This technique has been used in many fields successfully with crisp values; however, in real application, there are inaccurate data similar to probabilistic, interval, ordinal, qualitative, or fuzzy data. Hence, some researchers have developed several theoretical frameworks of DEA model with data such as interval [1]. Although there are many models and techniques to solve such a problem, there is a new problem, that is, ranking the efficient DMUs with interval data. Thus, in some researches, DMUs were ranked by these ideal points [2,3]. There are several models to rank DMUs with crisp data [4]. However, in all researches, ranking DMUs with interval data has been done using ranking approaches, such as AHP or TOPSIS or hybrid algorithm, to find suitable weights in order to calculate crisp efficiency basis of interval inputs and outputs. Therefore, a new approach was conducted using data mining techniques, similar to clustering, to obtain these weights as a new model. Jahanshahloo et al. [5] focused on ranking DMUs using ideal points (ideal points are obtained by improving

*. *Corresponding author.*
E-mail addresses: kianoush.kianfar@gmail.com (K. Kianfar); mahnazahadzadehnamin@gmail.com (M. Ahadzadeh Namin); a-tabriz@sbu.ac.ir (A. Alam Tabriz); najafi1515@yahoo.com (E. Najafi); Farhad@hoseinzadeh.ir (F. Hosseinzadeh Lotfi)

doi: 10.24200/sci.2017.4482

lower bounds of DMUs) by formulating the interval DEA model in order to achieve an efficiency interval including evaluations from both the optimistic and the pessimistic viewpoints. Wu et al. [3] presented a method for ranking the performance of DMUs, named cross-efficiency method, with interval data in the DEA model to calculate the interval of cross-efficiency values based on TOPSIS method. Akbarian [6] introduced a method for ranking all extreme and non-extreme DEA-efficient DMUs based on the cross-efficiency and Analytic Hierarchy Process (AHP) methods. Jahanshahloo et al. [5] proposed a cross-efficiency model based on super-efficiency for ranking units through the TOPSIS approach. The proposed method was extended to interval data. One of the main drawbacks of the cross-efficiency method is that different optimal weights associated with the efficiency score of a given DMU may exist. In their work, a super-efficiency model was presented to overcome this problem.

The aim of this paper is to combine clustering method with AHP (Analytic Hierarchy Process) using Sexton method, which is considered to be the novelty of this work. The rest of this paper is structured as follows. In Section 2, an overview of the research techniques includes DEA model, interval DEA model, cross-efficiency matrix, and cluster analysis, to be discussed. In Section 3, a multi-step algorithm is introduced to compute the weights of the combination of lower and upper bound efficiencies of DMUs. Therefore, crisp efficiency, instead of the interval efficiency, is obtained. In Section 4, a case study about efficiency evaluation of a commercial bank branch in Iran is implemented to illustrate and validate the proposed method. Finally, the conclusion section is given at the end of the paper.

2. Overview of the research techniques

2.1. DEA models

It is assumed that there are n DMUs to evaluate and index by $j = 1, 2, \cdots, n$; each DMU is assumed to produce different s outputs from different m inputs. Let the observed input and output vectors of DMU_j be $X_j = (x_{1j}, x_{2j}, \cdots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \cdots, y_{mj})$, respectively, where all components of vectors X_j and Y_j for all DMUs are non-negative, and each DMU has at least one strictly positive input and output. $v_i (i = 1, \cdots, m)$ is the input weight of x vectors. $y_r (r = 1, \cdots, s)$ is the output weight of y vectors. Weights of v_i and y_r are positive. In the previous section, DEA technique was defined completely. In this part, existing models are described in relation to DEA. There are three commonly orientations for DEA model, which can be formulated as follows:

1. Model 1 is an input-oriented CCR model related to the minimizing level of the inputs in order to achieve a given level of the outputs. Therefore, the further the value of θ gets from unity, the more ideal the condition of the problem will be:

$$\min \theta_p = \sum_{i=1}^{m} v_i x_{ip} \Big/ \sum_{r=1}^{s} u_r y_{rp} \,,$$

subject to:

$$\sum_{i=1}^{m} v_i x_{ij} \Big/ \sum_{r=1}^{s} u_r y_{rj} \leq 1 \qquad j = 1, \cdots, n,$$

$$u_r, v_i \geq 0. \tag{1}$$

2. Model 2 is an output-oriented CCR model concerned with the maximizing level of the outputs per given level of the inputs. Hence, the further the value of θ gets from unity, the more ideal the condition of the problem will be [7-9]:

$$\max \theta_p = \sum_{r=1}^{s} u_r y_{rp} \Big/ \sum_{i=1}^{m} v_i x_{ip} \,,$$

subject to:

$$\sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \leq 1 \qquad j = 1, \cdots, n,$$

$$u_r, v_i \geq 0. \tag{2}$$

3. Base-oriented model, unlike the others, pertains to the optimal combination of the inputs and outputs. Consequently, this model has control over inputs as well as outputs, concluding the efficiency of input utilization and efficiency of output production [8].

2.2. Interval DEA models

Entani and Tanaka [10] already proposed the interval DEA model to obtain the efficiency interval. The efficiency interval is represented by its upper and lower bounds.

Instead of exact data, models with interval data will be applied in order to rank DMUs. Input-oriented model with interval data for upper and lower bound efficiencies is formulated, respectively, as follows:

Upper bound efficiency:

$$\theta^U = \max \sum_{r=1}^{s} u_r y_{rp}^U,$$

subjected to:

$$\sum_{i=1}^{m} v_i x_{ip}^L = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^U \leq 0,$$

$$j = 1, \cdots, n, \quad j \neq p,$$

$$\sum_{r=1}^{s} u_r y_{rp}^U - \sum_{i=1}^{m} v_i x_{ip}^L \leq 0,$$

$$v_i, u_r \geq 0. \tag{3}$$

Lower bound efficiency:

$$\theta^L = \max \sum_{r=1}^{s} u_r y_{rp}^L,$$

subjected to:

$$\sum_{i=1}^{m} v_i x_{ip}^U = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj}^U - \sum_{i=1}^{m} v_i x_{ij}^L \leq 0,$$

$$j = 1, \cdots, n, \quad j \neq p,$$

$$\sum_{r=1}^{s} u_r y_{rp}^L - \sum_{i=1}^{m} v_i x_{ip}^U \leq 0,$$

$$v_i, u_r \geq 0. \tag{4}$$

The purpose of Models 3 and 4 is to raise the maximum efficiency in the interval state. Input and output vectors are individually considered as interval and Upper (U) and Lower (L) bounds. Two states are considered for each of input and output vectors: In the first state, the input value is minimum and maximum such that $X_{ij} \in [x_{ij}^L, x_{ij}^U]$. In the second state, the output value is minimum and maximum such that $Y_{rj} \in [y_{rj}^L, y_{rj}^U]$.

The purpose of Model 3 is to find the best condition of unit under assessment, DMU_p, which includes $x_{ij(p)}$ and $y_{rj(p)}$ vectors. In this model, the goal is to find an increase in outputs and a decrease in inputs for the unit under assessment. On the contrary, for other units ($j \neq p$), the purpose is to find the worst condition, implying a decrease in outputs and an increase in inputs. In Model 4 (lower bound, θ^L), the objective is finding the worst condition for DMU_p which consists of $x_{ij(p)}$ and $y_{rj(p)}$. In this model, the target is to find a decrease in outputs and an increase in inputs for the unit under assessment. Contrarily, for other units ($j \neq p$), the purpose is to find the best condition, implying a mean increase in outputs and a decrease in inputs.

2.3. Cross-efficiency matrix

Sexton et al. [11] introduced the cross-efficiency matrix in 1986. This approach aids us in evaluating efficiency of one DMU considering the optimal input and output weights of another DMU [9]. Matrix element, θ_{ij}, of the Cross-Efficiency Matrix (CEM) in the ith column and the jth row of CEM represents the efficiency of DMU_i when evaluated with the optimal weights of DMU_j, according to the following relationship:

$$\theta_{\text{DMU}_i}^{\text{DMU}_j} = \theta_{ij} = \text{eff}_i(u^j, v^j) = \sum_{r=1}^{s} u_r y_{rp}. \tag{5}$$

It is expected that 'good' DMU has several high values in its row.

2.4. Cluster analysis

Clustering is a popular data mining approach that deals with the separation of a set of objects from a useful set of mutually exclusive clusters in order that the similarity between the observations from the different clusters (i.e., subset) is low, whereas the similarity between the observations within each cluster is high [8]. Unlike decision trees which assign a class to an instance (supervised method), clustering procedures are applied when instances are divided into natural groups (unsupervised method). There are different ways of producing these clusters. The groups may be exclusive, that is, any instance belongs to only one group:

1. Probabilistic or fuzzy, i.e., an instance belongs to each group to a certain degree or probability (membership value);
2. Hierarchical group: There is a crude division of instances into groups at the top level, and each of these groups is refined further up to individual instances [12].

In other literature, the overview of the two general approaches to clustering was provided: hierarchical clustering and partitional clustering (e.g., k-means, k-median). The hierarchical clustering could make clusters by one of the two methods: agglomerative or divisive. An agglomerative method assumes that each data point is its own cluster and, with each step of the clustering process, these clusters are combined to form larger clusters, which are eventually combined to form a single cluster [13]. A divisive method of the hierarchical clustering, on the contrary, commences with the single cluster including all data points within the sample and proceeds to divide it into the smaller dissimilar clusters. Unlike hierarchical clustering, k-means clustering requires the number of resulting cluster, k, to be specified prior to analysis. Thus, k-means clustering will produce different k clusters of the greatest possible distinction [8].

3. Methodology

In this section, a multi-step algorithm is introduced to compute the weights of the combination of lower and upper bound efficiencies of DMUs so that the crisp efficiency can be obtained rather than the interval efficiency.

In the proposed algorithm, there are five stages as follows:

1. *Evaluating efficiency of DMUs:* The DMU's performance is measured using DEA (θ^L, θ^U) based on Eqs. (3) and (4). It is underlined that input-oriented model was applied, because, in the conducted research by Samoilenko et al. (2008) [13], the most natural grouping of DMUs was provided by the results of that model using a Constant Return to Scale (CRS) criterion (i.e., CCR model);
2. *Applying the cross-efficiency matrix:* The efficient DMUs are prioritized according to CEF and Eq. (5). The matrices are implemented as in Tables 1 and 2.
3. *Cluster analysis of upper and lower bounds:* The DMUs are clustered using the k-mean approach (indicators including outputs and inputs as the attributes):
   a. Clustering of DMUs with lower efficiency is done according to the data points of Table 3;
   b. Clustering of DMUs with upper efficiency is applied by the data points of Table 4.
4. *Obtaining the score for each cluster:* The average relative efficiency of some clusters identified in the previous step is computed according to the research by semoilinko et al. (2008). It is noted that average of the clusters in the lower bound is different from that for clusters in the upper bound.
5. *Assigning the relative weight to each DMU:* A numerical scale and a derived graphic scale in AHP are approximated (analytic hierarchy process) and assigned to each cluster according to given score. A pair-wise comparison matrix is used in order to obtain relative importance for indices (e.g., clusters) and rank them. In fact, results indicate relative importance of DMU within each cluster considering the obtained relative importance of its cluster.

Table 1. CEM for the lower efficiency of the DMU.

	1	**2**	**3**	$\cdots$	n	**Avg.**
1	θ^L_{11}	θ^L_{12}	θ^L_{13}	$\cdots$	θ^L_{1n}	$\left(\sum_{j=1}^n \theta^L_{1j}\right)/n$
2	θ^L_{21}	θ^L_{22}	θ^L_{23}	$\cdots$	θ^L_{2n}	$\left(\sum_{j=1}^n \theta^L_{2j}\right)/n$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	θ^L_{n1}	θ^L_{n2}	θ^L_{n3}	$\cdots$	θ^L_{nn}	$\left(\sum_{j=1}^n \theta^L_{nj}\right)/n$

Table 2. CEM for the upper efficiency of the DMU.

No.	**1**	**2**	**3**	$\cdots$	n	**Avg.**
1	θ^U_{11}	θ^U_{12}	θ^U_{13}	$\cdots$	θ^U_{1n}	$\left(\sum_{j=1}^n \theta^U_{1j}\right)/n$
2	θ^U_{21}	θ^U_{22}	θ^U_{23}	$\cdots$	θ^U_{2n}	$\left(\sum_{j=1}^n \theta^U_{2j}\right)/n$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	θ^U_{n1}	θ^U_{n2}	θ^U_{n3}	$\cdots$	θ^U_{nn}	$\left(\sum_{j=1}^n \theta^U_{nj}\right)/n$

Table 3. Clustering with lower efficiency.

	Indicators					
Attributes	x^L_{11}	x^L_{i1}	x^L_{m1}	y^L_{11}	y^L_{r1}	y^L_{s1}
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
		x^L_{ij}			y^L_{rj}	
	x^L_{1n}	x^L_{in}	x^L_{mn}	y^L_{1n}	y^L_{rj}	y^L_{sn}

Table 4. Clustering with upper efficiency.

	Indicators						**Cluster no.**
Attributes	x^U_{11}	x^U_{i1}	x^U_{m1}	y^U_{11}	y^U_{r1}	y^U_{s1}	
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	
		x^U_{ij}			y^U_{rj}		
	x^U_{1n}	x^U_{in}	x^U_{mn}	y^U_{1n}	y^U_{rj}	y^U_{sn}	

As a result, the final crisp efficiency of each DMU is calculated by computing Weighted Average (W.A.) of $(\sum_{j=1}^n \theta^L_{1j})/n$ and $(\sum_{j=1}^n \theta^U_{1j})/n$. Therefore, more W.A. indicates more ranks for each DMU. The proposed algorithm is shown in Figure 1 [14].

4. Numeric example and results

4.1. Data

The numerical example is taken into account, and this approach is applied to the selected commercial bank branch in Iran, used as interval data in the conducted research by Jahanshahloo and Hosseinzadeh Lotfi in 2009 [15] (reference of data). Each branch utilizes three inputs to generate five outputs, and there are 20 branches. The three inputs are payable interest, personnel, and non-performing loans. The five outputs of the bank include the total sum of four main deposits, other deposits, granted loans, received interest, and fee.

4.2. Implementation, results, and discussion

First, the cluster analysis is applied to the whole data including 20 bank branches. For this purpose, the k-means approach is employed according to the introduced approach of the conducted research by Samoilenko et al. in 2008 [13]; hence, they defined $K_{\max}$ and τ_{Outlier} as parameters, and then $K_{\max}$ clusters was generated. Following the examination of the segmentation with K clusters, they were able to show that the current segmentation with K clusters does not provide the natural grouping of DMUs; if $K > 1$, there is at least one cluster that includes less than τ_{Outlier} percentage of DMUs. Therefore, by decreasing

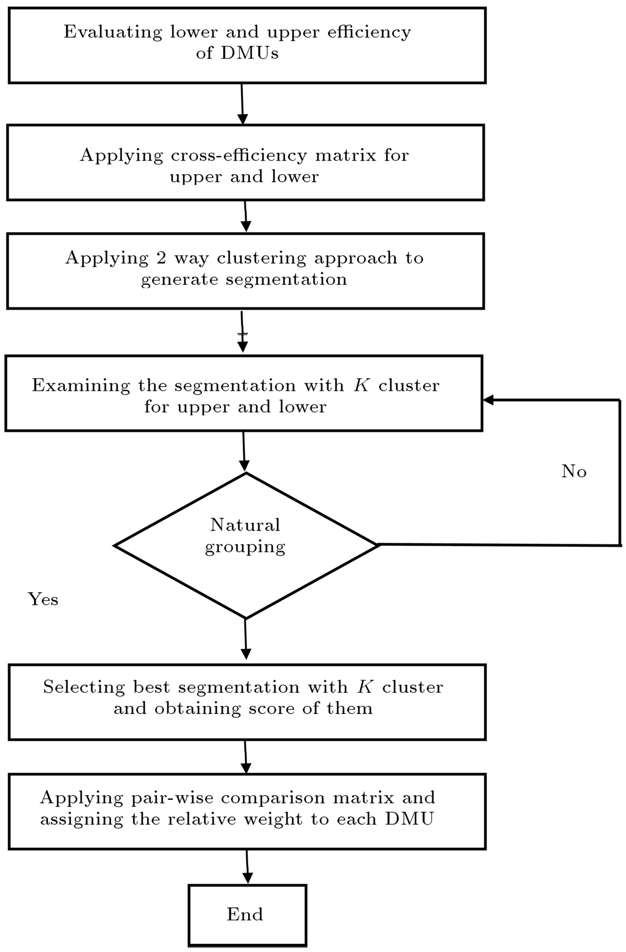

Figure 1. Proposed algorithm for evaluating DMUs.

the number of clusters, the previous examination is repeated in order to evaluate a new segmentation. Otherwise, they could access the best segmentation with K clusters.

It should be noted that τ_{Outlier} is an index to identify natural clustering. Moreover, a silhouette index introduced by Peter J. Rousseeuw in 1987 is considered so as to find natural and qualified clustering. Average silhouette width is inside the interval $[-1, 1]$ so that the value close to 1 indicates natural grouping and that close to -1 indicates incorrect clustering.

Parameters $K_{\max}$ and τ_{Outlier} are set to 5 and 10%; thus, with the average silhouette more than 0.5, we could come up with two solutions that disaggregate upper data and lower data into two and three clusters, respectively. Results of clustering are shown in Table 5.

As discussed in the steps of algorithm, CCR (constant return to scale) and input-oriented models are applied in order to measure the relative efficiency of DMUs with interval data. Details are shown in Table 6.

In addition, the average relative efficiencies of the two and three identified clusters of upper and lower data are calculated separately, and the results are shown in Table 7.

Table 5. Results of clustering.

	Number of clusters	Number of DMUs in each cluster	Average Silhouette width
Upper	Five clusters	1, 7, 1, 6, 5	0.2481
	Four clusters	11, 5, 3, 1	0.4734
	Three clusters	7, 1, 12	0.5098
	Two clusters	12, 8	0.6037
Lower	Five clusters	3, 5, 8, 3, 1	0.2335
	Four clusters	7, 1, 5, 7	0.2039
	Three clusters	3, 14, 3	0.5735

Table 6. Interval efficiency of DMU.

DMU	(θ^L, θ^U)	DMU	(θ^L, θ^U)
1	(1.0, 0.29)	11	(1.0, 1.0)
2	(0.21, 0.77)	12	(0.32, 0.49)
3	(0.52, 1.0)	13	(0.44, 0.70)
4	(1.0, 1.0)	14	(0.25, 0.72)
5	(0.63, 0.38)	15	(0.41, 1.0)
6	(0.90, 1.0)	16	(0.22, 1.0)
7	(0.73, 1.0)	17	(1.0, 1.0)
8	(1.0, 1.0)	18	(0.26, 0.95)
9	(1.0, 1.0)	19	(0.99, 1.0)
10	(1.0, 1.0)	20	(0.18, 0.97)

Table 7. Score of clusters.

	Cluster no.	Score
Upper	C1	0.9113
	C2	0.8317
Lower	C1	0.9967
	C2	0.6433
	C3	0.5814

Table 8. Numerical scale of lower data.

Index	C1	C2	C3
C1	1	2	7
C2	0.5	1	6
C3	0.143	0.167	1
Sum	1.643	3.147	14

In this stage, a numerical scale is approximated considering the given score of each cluster (Tables 8 and 9). Therefore, the relative importance or the weight of each DMU for upper and lower efficiencies is obtained (Table 10).

Finally, the crisp efficiency of each DMU is com-

Table 9. Numerical scale of upper data.

Index	C1	C2
C1	1	2
C2	0.5	1
Sum	1.5	3

Table 10. Relative importance of clusters.

	Cluster	Weight
Upper	C1	0.67
	C2	0.33
Lower	C1	0.58
	C2	0.35
	C3	0.07

puted using the obtained relative weight and score concerning to upper and lower data in interval data (Table 11). The ranking process is implemented for the following parts separately:

1. DMU with the interval efficiency ($\theta_L = 1$, $\theta_U = 1$);
2. DMU with the interval semi-efficiency ($\theta_L < 1$, $\theta_U = 1$) or ($\theta_L = 1$, $\theta_U < 1$);
3. DMU with the interval inefficiency ($\theta_L < 1$, $\theta_U < 1$).

It is noted that the efficient value of DMUs is set according to results of the cross-efficiency matrix.

5. Conclusion

This paper studied the ranking methodology of DMUs with the interval data. There are several approaches to prioritizing DMUs using the combination of DEA and ranking techniques such as AHP or TOPSIS. In contrast, Data Mining (DM) techniques were applied, similar to cluster analysis, in order to investigate partitional data (DMUs) based on their attributes. Assigning the relative weights to DMUs with interval data (lower and upper) helped compute the weighted average of lower and upper data; however, the approximation of weights and suitable methodology to obtain these is an important problem. Therefore, clustering as a DM approach has the ability to explore appropriate relative importance for all DMUs that are similar to each other. On the other hand, the efficiency of DMUs was evaluated by applying DEA and was ranked using the CEM approach. The proposed

Table 11. Ranking DMU with interval data.

$(\theta_L = 1, \theta_U = 1)$						
DMU	**Score**	**Weight**	**Score**	**Weight**	**Crisp**	**Rank**
4	0.466	0.07	0.578	0.33	3.520857	4
8	0.7	0.58	1.195	0.67	2.750431	6
9	0.525	0.07	1.432	0.67	14.90129	2
10	0.959	0.07	4.097	0.33	20.60343	1
11	0.376	0.07	0.599	0.67	6.779286	3
17	0.892	0.58	1.257	0.67	3.014052	5
$(\theta_L < 1, \theta_U = 1)$ or $(\theta_L = 1, \theta_U < 1)$						
DMU	**Score**	**Weight**	**Score**	**Weight**	**Crisp**	**Rank**
1	1.166	0.35	0.29	0.67	2.391143	13
3	0.52	0.35	1.256	0.67	3.594343	9
6	0.9	0.07	0.659	0.33	4.336714	8
7	0.73	0.07	0.436	0.33	3.115429	11
15	0.41	0.35	1.136	0.67	3.254629	10
16	0.22	0.07	0.951	0.33	5.033286	7
19	0.99	0.58	1.128	0.67	2.963034	12
$(\theta_L < 1, \theta_U < 1)$						
DMU	**Score**	**Weight**	**Score**	**Weight**	**Crisp**	**Rank**
2	0.21	0.07	0.77	0.33	4.17	16
5	0.63	0.07	0.38	0.33	2.751429	20
12	0.32	0.07	0.49	0.33	2.96	19
13	0.44	0.07	0.7	0.33	4.07	17
14	0.25	0.07	0.72	0.33	3.974286	18
18	0.26	0.07	0.95	0.33	5.068571	15
20	0.18	0.07	0.97	0.33	5.082857	14

algorithm was employed for 20 bank branches. Results show the crisp efficiency obtained for each DMU by using the combined cluster analysis, and CEM was computed based upon the correct weighting. Because this weighting originates from natural similarity of DMUs to each other, their inputs and outputs have been considered as attributes. Future works could focus on the DEA and cluster analysis with fuzzy data. In addition, the multi-stage DEA with fuzzy data can be considered as our next research in the future.

Nomenclature

y_r^L	Lower limit of an output vector
y_r^u	Upper limit of an output vector
x_i^L	Lower limit of an input vector
x_i^u	Upper limit of an input vector
θ^u	Efficiency of upper limit
θ_p	Efficiency of "P" point
v_i	Input weight
u_r	Output weight
x_i	Input vector
y_r	Output vector
θ^L	Efficiency of lower limit

References

1. Lu, T. and Liu, S.T. "Ranking DMUs by comparing DEA cross-efficiency intervals using entropy measures", *Entropy*, **18**(12), p. 452 (2016).
2. Sadeghi Gavgani, S. and Zohrehbandian, M. "A Cross-efficiency based ranking method for finding the most efficient DMU", *Math. Probl. Eng.*, **2014**, pp. 1-6 (2014).
3. Wu, J., Sun, J., Song, M., and Liang, L. "A ranking method for DMUs with interval data based on DEA cross-efficiency evaluation and TOPSIS", *J. Syst. Sci. and Syst. Eng.*, **22**(2), pp. 191-201 (2013).
4. Mirhashemi, A. and Izadikhah, M. "Ranking DMUs in the presence of undesirable data", *J. Basic. Appl. Sci. Res.*, **3**(4), pp. 910-919 (2013).
5. Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Shoja, N., Tohidi, G., and Razavyan, S. "Ranking using l1-norm in data envelopment analysis", *Appl. Math. Comput.*, **153**(1), pp. 215-224 (2004).
6. Akbarian, D. "Ranking all DEA-efficient DMUs based on cross efficiency and analytic hierarchy process methods", *J. Opt.*, **2015**, pp. 1-10 (2015).
7. Zohrehbandian, M. and Sadeghi Gavgani, S. "Cross-efficiency evaluation under the principle of rank priority of DMUs", *World. Appl. Sci. J.*, **21**, pp. 46-49 (2013).
8. Yang, F., Ang, S., Xia, Q., and Yang, C. "Ranking DMUs by using interval DEA cross efficiency matrix with acceptability analysis", *Eur. J. Oper. Res.*, **223**(2), pp. 483-488 (2012).
9. Caklovic, L. and Hunjak, T. "Measuring DMU-efficiency by modified cross-efficiency approach", *Math. Communication*, **17**, pp. 559-573 (2012).
10. Entani, T., Maeda, Y., and Tanaka, H. "Dual models of interval DEA and its extension to interval data", *Eur. J. Oper. Res.*, **136**(1), pp. 32-45 (2002).
11. Sexton, T.R., Silkman, R.H., and Hogan, A.J. "Data envelopment analysis: critique and extensions" In: Silkman, R.H. (Ed.), *Measuring Efficiency: An Assessment of Data Envelopment Analysis*, Jossey-Bass, San Francisco, CA (1986).
12. Dai, X. and Kuosmanen, T. "Best-practice benchmarking using clustering methods: Application to energy regulation", *Omega*, **42**(1), pp. 179-188 (2014).
13. Samoilenko, S. and Osei-Bryson, K.M. "Increasing the discriminatory power of DEA in the presence of the sample heterogeneity with cluster analysis and decision trees", *Exp. Syst. .Appl.*, **34**(2), pp. 1568-1581 (2008).
14. Amin, G.R., Emrouznejad, A., and Rezaei, S. "Some clarifications on the DEA clustering approach", *Eur. J. Oper. Res.*, **215**(2), pp. 498-501 (2011).
15. Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Rostamy Malkhalifeh, M., and Ahadzadeh Namin, M. "A generalized model for data envelopment analysis with interval data", *Appl. Math. Model.*, **33**(7), pp. 3237-3244 (2009).

Biographies

Kianoosh Kianfar is a PhD Student of Industrial Engineering, Operations Research, and Systems Engineering in Science and Research Branch, Islamic Azad University, Tehran, Iran. His research interests are industrial engineering, data envelopment analysis, operations research, and decision theory. He has published two papers in national and international journals.

Mahnaz Ahadzadeh received his PhD degree in Applied Mathematics from the Science and Research Branch, Islamic Azad University, Tehran, Iran in 2009. She is currently an Assistant Professor in Islamic Azad University, Shahr-e-Qods Branch. Her research interests are data envelopment analysis, efficiency analysis, applied mathematics, productivity analysis, and applied econometrics. Some of her publications are available in https://www.researchgate.net/profile/Mahnaz_Namin

Akbar Alam Tabriz received his PhD degree in Management from Turkey in 1989. He is currently an Associated Professor in Shahid Beheshti University. His research interests are performance evaluation in production industries, implementation of Total Quality Management (TQM), and efficiency and productivity measurement in Industry. He has published several papers in national and international journals.

Esmaeil Najafi received his PhD in Industrial Engineering from Science and Research Branch, Islamic Azad University, Tehran, Iran in 2009. He is currently an Associated Professor in Islamic Azad University, Science & Research Branch. His research interests are industrial engineering, applied and computational Mathematics, operations management, and optimization. Some of his publications are available in the following link: https://www.researchgate.net/profile/Esmaeil_Najafi3

Farhad Hosseinzadeh Lotfi received his PhD degree in Applied Mathematics from Science & Research Branch, Islamic Azad University, Tehran, Iran in 2000. He is currently a Professor in Islamic Azad University, Science & Research Branch. His research interests are mathematics, operation research, data envelopment analysis, and efficiency. Some of his publications are available in the following link: http://scholar.google.com/citations?user=gc_qn8gAAAAJ&hl=en