Scientia Iranica E (2018) 25(3), 1724–1749

SCIENTIA IRANICA

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Incorporating demand, orders, lead time, and pricing decisions for reducing bullwhip effect in supply chains

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Received 3 February 2016; received in revised form 17 October 2016; accepted 1 May 2017

KEYWORDS
Supply chain;
Bullwhip effect;
Pricing;
Demand forecasting;
Ordering;
Game theory.

Abstract. The purpose of this paper is to mitigate bullwhip effect (BWE) in a Supply Chain (SC). Four main contributions are proposed. The first one is to reduce BWE through considering its multiple causes (demand, pricing, ordering, and lead time) simultaneously. The second one is to model demands, orders, and prices dynamically for reducing BWE. Demand and prices have mutual effect on each other dynamically over time. In other words, a time series model is used in a game theory method for finding the optimal prices in an SC. Moreover, the optimal prices are inserted into the time series model for forecasting price sensitive demands and orders in an SC. The third one is to use demand of each entity for forecasting its orders. This leads to drastic reduction in BWE and Mean Square Error (MSE) of the model. The fourth contribution is to use optimal prices instead of forecasted ones for demand forecasting and reducing BWE. Finally, a numerical experiment for the auto-parts SC is developed. The results show that analysing joint demand, orders, lead time, and pricing model by calculating the optimal values of prices and lead times leads to significant reduction in BWE.

1. Introduction

The competitive nature of business environment compels each company to minimize its supply, manufacturing, inventory, and distribution costs. Cost reduction techniques are more required in case of cooperating with other firms in an SC. One of the main causes of imposing extra costs to entities in an SC is demand amplification through the chain. This phenomenon has been recognized by Forrester [1], and Lee et al. [2] named it bullwhip effect (BWE) later. Such a destructive effect occurs when an end customer places an order, and its order is amplified as it moves through the chain.

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doi: 10.24200/sci.2017.4448

Dominguez et al. [3] studied the effect of Supply Chain Network (SCN) configuration and returns of goods on BWE. They showed that returning goods increased BWE in serial SCN more than divergent configuration did. Moreover, Dominguez et al. [4] investigated the impacts of important factors of SCs, including the number of nodes and echelons and the distribution of links, on BWE. In order to measure BWE, two different methods were introduced by Cannella et al. [5] including customer service level and process efficiency. Chatfield et al. [6] introduced another type of BWE in SCs, which was stock out amplification rather than demand amplification. Cannella et al. [7] demonstrated that both stock out and demand amplification were reduced in a coordinated SC.

In order to reduce demand amplification or BWE, its main causes should be investigated. Lee et al. [2,8] introduced demand forecasting, order batching, price fluctuation, rationing and shortage gaming, and none-zero lead time as the main causes of BWE. Ma et al. [9] investigated the effect of different forecasting techniques on BWE on product orders and inventory. Ma et al. [10] studied the effect of information sharing and demand forecasting on reducing BWE.

Several researchers have studied different forecasting methods for reducing that effect [11-24]. Recently, Montanari et al. [25] presented a new probabilistic demand forecasting and inventory control model for mitigating BWE. Other researchers have concentrated on order batching, such as Kelle and Milne [26], Lee and Wu [27], Potter and Disney [28], and Sodhi and Tang [29].

The other cause of BWE occurrence is pricing decisions, which are very critical in SCs profitability. For example, Wang et al. [30] investigated price forecasting impacts on BWE. Other pricing research has been performed by Özelkan and Lim [31] and Özelkan and Cakanyıldırım [32]. In spite of the fact that these papers consider pricing decisions in BWE problems, they have not studied the effect of pricing on creating BWE. Instead, the effects of supplier's selling prices on price amplifications in downstream firms, such as retailers, have been investigated. In other words, the effects of pricing decisions on demand and order amplification (BWE) have not been analysed. Zhang and Burke [33] considered pricing in BWE problems. The main drawback of that paper was that selling prices in an SC were forecasted. However, their exact values were extractives from an optimization problem, and this process is investigated in this paper.

The last causes of BWE generation, shortage gaming, and lead time were investigated by Cachon and Lariviere [34] and Agrawal et al. [35], respectively. Thus, in order to reduce BWE, its main causes have been studied in the literature, leading to production and inventory cost reduction. Although many researchers have focused on reducing BWE, there is no work in the literature to consider its multiple causes resulting in more reduction in this phenomenon. All the above papers concentrated on one of the main reasons of BWE. The only research in the literature which considered two compound causes of this effect was performed by Zhang and Burke [36]. However, this work suffered from the abovementioned drawback. Therefore, there is a huge gap in the literature on BWE, which is open to investigation. Analysing multiple causes of BWE (demand, ordering policy, pricing, and lead time) simultaneously is an important contribution to decrease BWE significantly.

In this paper, 4 main contributions are proposed. The first one is to decrease BWE through studying multiple causes of this phenomenon (demand, pricing, ordering policy, and lead time) simultaneously. This leads to more reduction in the destructive event (BWE). In a three-echelon SC consisting of a retailer, a distributor, and a manufacturer, pricing decisions are dependent and made sequentially. Therefore, optimal values of prices and lead times of the entities in an SC are obtained by modelling a sequential (Stackelberg) game theory problem. A retailer decides on prices with respect to the distributor's selling prices, and the distributor quotes prices based on the manufacturer's selling prices.

The second contribution is to model demands, orders, and prices dynamically for reducing bullwhip effect. Demand and prices have reciprocal effect on each other dynamically over time. In other words, a time series model is used in the optimization problem, which is solved by a game theory method for finding the optimal values of prices and lead times in the SC. In the time series model, demands are calculated by autoregressive functions with an exogenous variable (ARX). In addition, orders are modelled by moving average functions with an exogenous variable (MAX). Then, the optimal prices obtained from the game theory problem are inserted into the time series model for forecasting price sensitive demands and orders in the SC. This reciprocal process, in which demands are used to calculate prices and then, optimal prices are inserted into demand functions, is done dynamically over time.

The third contribution is to use demand of each entity in SCs for forecasting its ordering quantities. However, in the literature, upstream order is forecasted by using its immediate downstream order. The proposed approach in this paper, in which demands of each entity are used to forecast its ordering quantities, leads to drastic reduction in BWE and MSE of the model. The last contribution is to find optimal prices and use them for demand forecasting and reducing BWE instead of utilizing forecasted prices.

The rest of the paper is organized as follows. Problem definition and modelling are discussed in Section 2. BWE is measured and reduced in Section 3. The model proposed in Section 3 is validated and verified in Section 4. Section 5 illustrates numerical experiments. Finally, conclusions and future research are presented in Section 6.

2. Problem definition and modelling

In this paper, a three-echelon SC including a retailer, a distributor, and a manufacturer in an auto-parts SC is studied. BWE leads to demand amplification from downstream to upstream echelons. Because of this amplification, upstream firms in an SC receive inaccurate demand information, leading to excess production and inventory costs. Therefore, there is an increasing need to propose novel methods for measuring and reducing the BWE problem. Studying the main causes of BWE occurrence and trying to decrease them are significant steps for reducing BWE. Therefore, a novel model covering multiple causes of BWE (demand-pricing-ordering-lead time) is presented. The new method is an extended version of the model presented by Özelkan and Cakanyildirim [32]. However, it rectifies 3 main drawbacks of their method.

First, it investigates the effect of pricing decisions on demand and order amplification (BWE), and it works on mechanisms to reduce this effect. However, the model presented by Özelkan and Cakanyildirim [32] neither studies the effect of prices on demand amplification (BWE) nor presents mechanisms to reduce it. Instead, it tries to show price amplification in SCs. Second, in that paper, joint demand-pricing-ordering-lead time decisions are quantified for measuring and reducing BWE in SCs. However, only pricing decisions are studied and other causes of BWE have not been considered. Third, herein, due to the volatility of demands, orders, and prices, they are dynamically calculated over time. Moreover, demands and prices have mutual effect on each other. Demands are used by a time series model in the objective function of pricing problems. Then, optimal prices are inserted into the time series model for demand forecasting.

Figure 1 shows a three-echelon auto-parts SC including a retailer, a distributor, and a manufacturer. In order to solve the BWE problem, 5 main steps are implemented. First, optimal lead time and pricing values for each entity in the SC are calculated using a sequential game theory approach. Second, the optimal values are substituted in an auto-regressive exogenous input (ARX) time series for forecasting demand of each entity. Third, orders of each entity are forecasted using its demands. Then, in order to validate the model, a technique in which downstream orders have been applied for forecasting upstream orders is extracted from literature and implemented. Next, mean and variance of demands and orders are calculated for quantifying BWE. Fourth, BWE is measured by means of the two aforementioned ordering policies. The results of these methods are compared to show which method is more capable of reducing BWE (model validation).

Optimal values of selling prices and lead times are used in a time series model for forecasting demands and orders. However, autoregressive method has been used to forecast prices in an SC in literature [33]. Therefore, in the fifth step, two pricing approaches are compared with each other for validating the model proposed in this paper. MSE of order forecasting and variance of orders are calculated for both pricing approaches. Then, results are compared with each other to find which method has less forecasting error and variance of orders.

2.1. *Optimal lead time-pricing decision for retailer*

First, manufacturer quotes its selling price. Then, distributor determines its selling price based on manufacturer's quoted price. Finally, the retailer makes pricing decisions based on prices of previous echelons. The model is capable of calculating optimal values for lead times in each levels of an SC. Obtaining the optimal solutions for prices and lead times requires to design a sequential game theory model. In such a game, each player in an SC decides on its prices based on prices of other players. Table 1 indicates all parameters and variables used in the new model.

Each player in an SC tries to maximize its own profit as it is shown in Eq. (1a). Demand function is defined as a dependent time series variable; however, it was a single-valued variable in the model presented by Özelkan and Cakanyildirim [32]. The demand function depends on selling prices of each entity in SC, as well as demand of previous periods. Therefore, it is an ARX time series model, as it is shown in Eq. (1c). Eq. (1b) shows the inventory capacity constraint. The retailer's inventory level must be less than or equal to the retailer's inventory capacity. However, when the retailer receives market demand, the inventory level decreases. Thus, Eq. (1b) demonstrates that the retailer's inventory capacity minus the demand received by retailer during lead time is greater than or equal to the inventory level. Eq. (1d) indicates that the total demand for retailers' goods should be nonnegative. In addition, Eq. (1e) emphasizes the non-negativity of retailer's and distributer's selling prices (p_t and w_t):

$$\max \pi_R(p) = (p_t - w_t)(q_{t,1}(p_t, q_{t-1,1})), \tag{1a}$$

$$\text{s.t.}\ (\mu_1 - q_{t,1}(p_t, q_{t-1,1}))l_1 \geq I_1, \tag{1b}$$

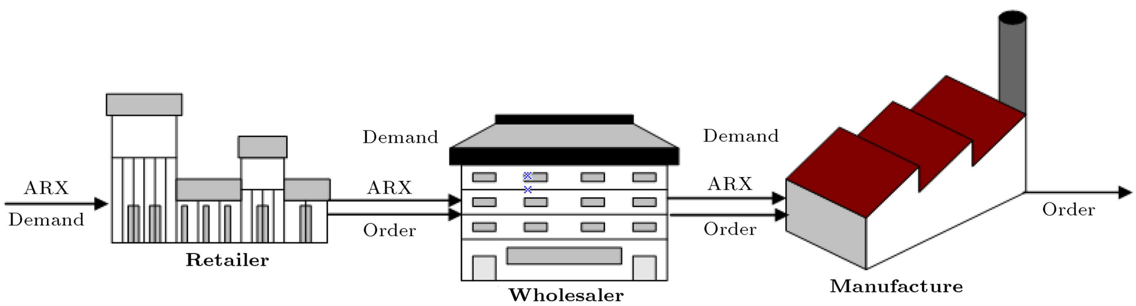

Figure 1. Structure of an auto-parts SC.

Table 1. Parameters and variables of the lead time-pricing model.

Symbol	Definition
$q_{t,1}$	Demand for retailer's goods in period t
r_1	Constant number in retailer's demand function
p_t	Price of retailer's goods
$q_{t-1,1}$	Demand for retailer's goods in period $t-1$
μ_1	Retailer's inventory capacity
l_1	Lead time of retailer
l_1^*	Optimal lead time of retailer
I_1	Retailer's inventory level
p_t^*	Optimal price of retailer's goods in period t
w_t	Price of distributor's goods
$q_{t,2}$	Demand for distributor's goods in period t
$q_{t-1,2}$	Demand for distributor's goods in period $t-1$
r_2	Constant number in distributor's demand function
w_t^*	Optimal price of distributor's goods in period t
μ_2	Distributor's inventory capacity
l_2	Lead time of distributor
l_2^*	Optimal lead time of distributor
I_2	Distributor's inventory level
z_t	Price of manufacturer's goods
$q_{t,3}$	Demand for manufacturer's products in period t
$q_{t-1,3}$	Demand for manufacturer's products in period $t-1$
r_3	Constant number in manufacturer's demand function
z_t^*	Optimal price of manufacturer's products in period t
μ_3	Manufacturer's production capacity
l_3	Lead time of manufacturer
l_3^*	Optimal lead time of manufacturer
$k=\ln\left(\frac{1}{1-s}\right)$	"s" is a desired service level; "k" is used for simplicity
m_t	Variable production cost
K	Capacity cost of manufacturer
v	A constant coefficient for calculating prices in the next period

$$q_{t,1}=r_1p_t^{-ac}q_{t-1,1}^{1-a} \quad 0<a,c<1, \tag{1c}$$

$$q_{t,1}(p_t,q_{t-1,1})\geq 0, \tag{1d}$$

$$p_t\geq w_t\geq 0, \tag{1e}$$

where a, and c are two positive numbers between zero and one. Solving the above optimization problem leads to the optimal values for retailer's lead time and selling price at period t. The optimal lead time for retailer is equal to $l_1^*=\frac{I_1}{\mu_1-r_1p_t^{-ac}q_{t-1}^{1-a}}$. Then, the retail price at period $t+1$ is calculated by Eq. (2):

$$p_{t+1}=vp_t, \quad t=1,2,...,T. \tag{2}$$

In order to find optimal values of prices, the above optimization problem is solved by extending the method presented by Özelkan and Cakanyildirim [32]. They proved that if $q'<0$ and $\frac{qq'}{(q')^2}|_{p(w)=p^*}<2$ for all p^*, where p^* denotes the critical point(s) of $\pi_R(p)$, the optimal value of p^* is equal to $p^*=\left\{p|\frac{d\pi_R(p)}{dp}=q+(p-w)q'=0\right\}$ (for more details of the proof, please refer to [32]). Using that approach and extending it to an equation including time series variable, the optimal value of retailer's selling price is calculated by Eqs. (3) and (4).

In addition, the demand function used here differs from the demand equation presented by Özelkan and Cakanyildirim [32]. In this paper, the demand function is ARX and depends on two variables (price and demand for previous period):

$$p_t^* = \left\{ p_t | \frac{d\pi_R(pt)}{dp_t} = q_{t,1}(p_t, q_{t-1,1}) + (p_t - w_t) q'_{t,1} = 0 \right\}, \tag{3}$$

$$p_t^* = \frac{acw_t}{ac-1}, \tag{4}$$

where $\frac{d\pi_R(p_t)}{dp_t}$ is a first order condition with respect to retailer's selling prices. The reaction function for the retailer is calculated by Eq. (5):

$$p_t(w_t) = q_{t,1} + (p_t - w_t) q'_{t,1} = 0, \tag{5}$$

where $q_{t,1} = r_1 p_t^{-ac} q_{t-1,1}^{1-a}$ and $q'_{t,1} = \frac{(-acr_1 q_{t-1,1}^{1-a})}{(p_t^{ac+1})}$.

2.2. Optimal lead time-pricing decision for distributor

In order to determine the optimal lead time-pricing decisions for distributor, two steps are considered as stated by Özelkan and Cakanyildirim [32]. First, distributor calculates the retailer's reaction function presented by Eq. (5) and based on that, decides on selling prices. Then, the retailer determines its selling prices to end customers based on distributor's quoted prices. Eq. (6) shows distributor's demand function depending on retailer's pricing reaction function and demands received by the distributor at period $t-1$:

$$q_{t,2} = q_{t,2}(p_t(w_t), q_{t-1,2}) = r_2 (p_t(w_t))^{-ac} q_{t-1,2}^{1-a}. \tag{6}$$

The distributor's goal is to maximize its profit through Eq. (7a). Eq. (7b) shows capacity constraint for the distributor's inventory. The non-negativity constraint for demand function is shown by Eq. (7c). In addition, Eq. (7d) indicates that distributor's and manufacturer's selling prices are non-negative:

$$\max \pi = (w_t - z_t)(q_{t,2}(p_t(w_t), q_{t-1,2})), \tag{7a}$$

$$\text{s.t.} \ (\mu_2 - q_{t,2}(p_t(w_t), q_{t-1,2})) l_2 \geq I_2, \tag{7b}$$

$$q_{t,2}(p_t(w_t), q_{t-1,2}) \geq 0, \tag{7c}$$

$$w_t \geq z_t \geq 0. \tag{7d}$$

Lemma 1. The optimal price for distributor's goods (w_t^*) is independent of demand for previous period ($q_{t-1,2}^{1-a}$), and is given by the following equation:

$$w_t^* = \frac{acz_t}{ac-1}. \tag{8}$$

Proof. See Appendix A.

The next decision for distributor is to determine the optimal lead time between receiving retailer's orders and delivering them. The optimal lead time for distributor is obtained by solving Eq. (7b) as follows:

$$l_2^* = \frac{I_2}{\mu_2 - r_2 q_{t-1,2}^{1-a} (\frac{acw_t}{ac-1})^{-ac}}. \tag{9}$$

2.3. Optimal lead time-pricing decision for manufacturer

Manufacturer calculates the distributor's reaction function presented by Eq. (10) and decides on selling prices based on that. Eq. (11) shows manufacturer's demand function. The manufacturer's goal is to maximize its profit using Eq. (12a). The profit function for manufacturer differs from retailer's and distributor's objective functions. Manufacturer's costs include capacity costs ($K\mu_3$) as well as variable production costs (m_t). The manufacturer's inventory is subject to a capacity constraint presented by Eq. (12b). The non-negativity constraint for demand function is shown in Eq. (12c). In addition, Eq. (12d) indicates that selling prices of distributor and manufacturer as well as variable production costs should be non-negative:

$$w_t(z_t) = q_{t,2} + (w_t - z_t) q'_{t,2} = 0, \tag{10}$$

$$q_{t,3} = q_{t,3}(p_t(w_t(z_t)), q_{t-1,3}) = r_3 (p_t(w_t(z_t))^{-ac} q_{t-1,3}^{1-a}, \tag{11}$$

$$\max \pi = (z_t - m_t)(q_{t,3}(p_t(w_t(z_t)), q_{t-1,3})) - K\mu_3, \tag{12a}$$

$$\text{s.t.} \quad (\mu_3 - q_{t,3}(p_t(w_t(z_t)), q_{t-1,3})) l_3 \geq k, \tag{12b}$$

$$q_{t,3}(p_t(w_t(z_t)), q_{t-1,3} \geq 0, \tag{12c}$$

$$w_t \geq z_t \geq m_t \geq 0. \tag{12d}$$

Lemma 2. The optimal price for manufacturer's products (z_t^*) is independent of demand for previous period ($q_{t-1,3}^{1-a}$), and is given by the following equation.

$$z_t^* = \frac{(acm_t)}{(ac-1)}. \tag{13}$$

Proof. It is similar to Lemma 1 and, for brevity, is not included here.

Solving Eq. (12b) leads to finding the optimal value of lead time as follows:

$$l_3^* = \frac{k}{\mu_3 - r_3 (\frac{a^2c^2z_t}{(ac-1)^2})^{-ac} q_{t-1,3}^{1-a}}. \tag{14}$$

2.4. Demand model for retailer

Retailer's demand is forecasted by an ARX time series. In order to reach this goal, natural logarithm of retailer's demand function is taken as follows:

$$\ln(q_{t,1}) = -ac\ln(p_t^*) + (1-a)\ln(q_{t-1,1}) + \ln(r_1) + \varepsilon_t, \quad (15)$$

where ε_t is a white noise process with zero mean and variance of σ_ε^2 and p_t^* is the optimal value for retailer's price obtained from Eq. (12). The MAX process for demand forecasting is shown by the following equation:

$$\ln(q_{t,1}) = -\frac{\theta}{\varphi-1}\ln(p_t^*) - \frac{\delta}{\varphi-1} + \varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + \varphi^3\varepsilon_{t-3} + \dots, \quad (16)$$

where $\theta = -ac$, $\varphi = 1-a$, and $\delta = \ln(r_1)$.

Eqs. (17) and (18) show the expected value and variance of retailer's selling price. Using Eqs. (17) and (18), the expected value and variance of retailer's demand are calculated by Eqs. (19) and (20), respectively:

$$E[\ln(p_t^*)] = \mu_p, \quad (17)$$

$$\text{Var}[\ln(p_t^*)] = \sigma_p^2, \quad (18)$$

$$E[\ln(q_{t,1})] = \left(\frac{\theta}{1-\varphi}\right)E[\ln(p_t^*)] + \left(\frac{\delta}{1-\varphi}\right) = \left(\frac{\theta}{1-\varphi}\right)\mu_p + \left(\frac{\delta}{1-\varphi}\right), \quad (19)$$

$$\text{Var}[\ln(q_{t,1})] = \frac{\theta^2}{(1-\varphi)^2}\text{Var}[\ln(p_t^*)] + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right) = \frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right). \quad (20)$$

2.5. The retailer's ordering policy

In order to determine the retailer's ordering quantity, the extended and revised version of the Order-Up-To (OUT) level presented by Hosoda and Disney [15] is proposed here. Eqs. (21) and (22) indicate the OUT level:

$$O_{t,1} = q_{t,1} + (S_{t,1} - S_{t-1,1}), \quad (21)$$

$$S_{t,1} = \hat{q}_{t,1}^{l_1} + s\hat{\sigma}_{l_1}. \quad (22)$$

According to Hosoda and Disney [15], $\hat{\sigma}_{l_1}$ is an estimated value of the standard deviation of the forecast error considering the retailer's lead-time. $O_{t,1}$ denotes retailer's order issued at the end of period t, s is a desired service level, and $S_{t,1}$ is the OUT level at period t. Eq. (23) shows the conditional expected value of the total demand over lead time $l_1(\hat{q}_{t,1}^{l_1})$:

$$\hat{q}_{t,1}^{l_1} = E\left(\sum_{i=1}^{l_1} q_{t+i,1}\middle|\tau_t\right) = \frac{r_1p_{t+1}^\theta q_{t,1}^\varphi[1-(r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi)^{l_1}]}{1-r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi} = \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^\varphi, \quad (23)$$

where:

$$\Omega_{l_1} = \frac{1-(r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi)^{l_1}}{1-r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi}, \quad \text{and}$$

$$\tau_t = \{q_t, q_{t-1}, q_{t-2}, \dots\}$$

is the set of the demands. In order to calculate $\hat{q}_{t,1}^{l_1}$, this assumption is taken: $p_{t+i+1} = vp_{t+i}$, $i = 1, 2, \dots, l_1$. The proof for obtaining Eq. (23) is presented in Appendix B1.

Using Eqs. (21)-(23) leads to obtaining retailer's orders as follows:

$$O_{t,1} = q_{t,1} + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^\varphi - \Omega_{l_1}r_1p_t^\theta q_{t-1,1}^\varphi. \quad (24)$$

In order to measure BWE, variances of orders and demands for each stage need to be calculated. First, the equivalent value for $q_{t-1,1}^\varphi$ is obtained by Eq. (25). Then, the retailer's order is calculated by substituting Eq. (25) in Eq. (24), which is shown in Eq. (26). Next, natural logarithm of Eq. (26) is taken as it is indicated in Eq. (27). Finally, variance of retailer's order is calculated by Eqs. (28)-(30):

$$q_{t-1,1}^\varphi = \frac{q_{t,1}}{r_1p_t^\theta}, \quad (25)$$

$$O_{t,1} = q_{t,1} + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^\varphi - \Omega_{l_1}q_{t,1}, \quad (26)$$

$$\ln(O_{t,1}) = \ln(q_{t,1}) + \ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1}, \quad (27)$$

$$\text{Var}[\ln(O_{t,1})] = \text{Var}[\ln(q_{t,1})] + \text{Var}[\ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}] + 2\ \text{cov}\ (\ln(q_{t,1}), \ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}), \quad (28)$$

$$\text{Var}[\ln(O_{t,1})] = \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right] + \text{Var}[\ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})] + 2E[\ln(q_{t,1})\ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})] - 2E[\ln(q_{t,1})]E[\ln(1 + \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})], \quad (29)$$

$$\mathrm{Var}[\ln(O_{t,1})] = \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right] + \mathrm{Var}[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] + 2E[\ln(q_{t,1})\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] - 2\left[\left(\frac{\theta}{1-\varphi}\right)\mu_p + \left(\frac{\delta}{1-\varphi}\right)\right] E[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]. \quad (30)$$

Theorem 1. The retailer's order quantity at period $t+1$ is forecasted by the ARX time series $\ln(O_{t+1,1}) = \ln(r_1) + \theta\ln(v) + \theta\ln(p_t) + \ln(O_{t,1}) + \varepsilon_{t+1,1}$.

Proof. See Appendix B2.

Theorem 2. The MAX time series model of retailer's order is $\ln(O_{t+1,1}) = 2\ln(r_1) + \theta\ln(v) + 2\theta\ln(p_t) + \varphi\ln(q_{t-1,1}) + \varepsilon_t + \ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1} + \varepsilon_{t+1,1}$.

Proof. See Appendix C.

Theorem 3. The MAX time series for predicting order quantities at period $t+1$ including error terms is $\ln(O_{t+1,1}) = \ln(r_1) + \theta\ln(v) + \theta\ln(p_t) + \varepsilon_{t+1,1} + \ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1} - \frac{\theta}{\varphi-1}\ln(p_t) - \frac{\delta}{\varphi-1} + \varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + \varphi^3\varepsilon_{t-3} + \ldots$.

Proof. See Appendix D.

2.6. Demand model for distributor

In this subsection, a model for forecasting distributor's demand is proposed. Using the optimal values for distributor's selling prices from Subsection 2.2, distributor's demand function is calculated as follows:

$$q_{t,2} = r_2 w_t^{*-a_2c_2} q_{t-1,2}^{1-a_2}, \quad t = 1,2,...,T, \quad (31)$$

where w_t^* are the optimal selling prices for distributor's goods, $q_{t,2}$ indicates distributor's demand time series for the current period, and $q_{t-1,2}$ shows its demand for the previous period. r_2 is a constant coefficient. In order to forecast distributor's demand for the current period, natural logarithm of Eq. (31) is taken. Eq. (32) shows an ARX time series for distributor's demand forecasting:

$$\ln(q_{t,2}) = \theta_2\ln(w_t^*) + \varphi_2\ln(q_{t-1,2}) + \ln(r_2) + \varepsilon_{t,2}, \quad (32)$$

where $\theta_2 = -a_2c_2$, $\varphi_2 = 1-a_2$, and $\varepsilon_{t,2}$ is a white noise process of distributor's demand forecasting with zero mean and variance of $\sigma_{\varepsilon_2}^2$.

After forecasting distributor's demand, its expected value and variance should be calculated for measuring BWE in Section 3.

Lemma 3. The expected value of distributor's demand is:

$$\left(\frac{\theta_2}{1-\varphi_2}\right)\mu_w + \left(\frac{\delta_2}{1-\varphi_2}\right),$$

and its variance is:

$$\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2 + \left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right).$$

where, σ_w^2 and μ_w are variance and mean of selling prices for distributor's goods, respectively.

Proof. See Appendix E.

2.7. The distributor's ordering policy

A new method for calculating the distributor's ordering quantity is proposed. Using this method, each entity in an SC orders based on the demand it receives. However, in literature, upstream orders were calculated using downstream order. Subsection 2.7.1 describes the method proposed in this paper, whereas Subsection 2.7.2 elaborates the technique used in the literature.

2.7.1. The proposed method for forecasting the distributor's ordering quantity

In order to determine the distributor's ordering quantity, we propose the extended and revised version of the Order-Up-To (OUT) level presented by Hosoda and Disney [15]. Eqs. (33) and (34) indicate the distributor's OUT level calculated using the demand it receives:

$$O_{t,2} = q_{t,2} + (S_{t,2} - S_{t-1,2}), \quad (33)$$

$$S_{t,2} = \hat{q}_{t,2}^{l_2} + s_2\hat{\sigma}_{l_2}. \quad (34)$$

According to Hosoda and Disney [15], $\hat{\sigma}_{l_2}$ is an estimated value of the standard deviation of the forecast error considering the distributor's lead-time. $O_{t,2}$ denotes distributor's order issued at the end of period t. s_2 is a desired service level of distributor and $S_{t,2}$ is the OUT level at period t. Eq. (35) shows the conditional expected value of the total demand over lead time, l_2:

$$\hat{q}_{t,2}^{l_2} = E\left(\sum_{i=1}^{l_2} q_{t+i}\middle|\tau_{t,2}\right) = \frac{r_2(w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2}[1-(r_2^2 v'^{\theta_2} w_{t+1}^{*2\theta_2} q_{t,2}^{\varphi_2})^{l_2}]}{1-r_2^2 v'^{\theta_2} w_{t+1}^{*2\theta_2} q_{t,2}^{\varphi_2}} = \Omega_{l_2} r_2 (w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2}, \quad (35)$$

where $\Omega_{l_2} = \frac{1-(r_2^2 v'^{\theta_2} w_{t+1}^{*2\theta_2} q_{t,2}^{\varphi_2})^{l_2}}{1-r_2^2 v'^{\theta_2} w_{t+1}^{*2\theta_2} q_{t,2}^{\varphi_2}}$ and $\tau_{t,2} = \{q_{t,2}, q_{t-1,2}, q_{t-2,2}, ...\}$.

Set:

$$w^*_{t+i+1} = v' w^*_{t+i} \begin{cases} v' \neq 1 & \text{if } i = 0, 1 \\ v' = 1 & \text{if } i > 1 \end{cases}$$

for calculating $\hat{q}^{l_2}_{t,2}$. Eq. (36) indicates distributor's order calculated by its received demand using equations (33)-(35).

$$O_{t,2} = q_{t,2} + (\Omega_{l_2} r_2 (w^*_{t+1})^{\theta_2} q^{\varphi_2}_{t,2} - \Omega_{l_2} r_2 w^{*\theta_2}_t q^{\varphi_2}_{t-1,2}). \tag{36}$$

Theorem 4. The variance of distributor's ordering quantity with the proposed method is:

$$\begin{aligned}\text{Var}[\ln(O_{t,2})] = &\left[\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2 + \left(\frac{\sigma^2_{\varepsilon_2}}{1-\varphi_2^2}\right)\right] \\ &+ \text{Var}[\ln(1 + \Omega_{l_2} r_2 (w^*_{t+1})^{\theta_2} q^{\varphi_2 - 1}_{t,2} - \Omega_{l_2})] \\ &+ 2E[\ln(q_{t,2}) \ln(1 + \Omega_{l_2} r_2 (w^*_{t+1})^{\theta_2} q^{\varphi_2 - 1}_{t,2} \\ &- \Omega_{l_2})] - 2\left[\left(\frac{\theta_2}{1-\varphi_2}\right)\mu_w + \left(\frac{\delta_2}{1-\varphi_2}\right)\right] \\ &E[\ln(1 + \Omega_{l_2} r_2 (w^*_{t+1})^{\theta_2} q^{\varphi_2 - 1}_{t,2} - \Omega_{l_2})].\end{aligned}$$

Proof. See Appendix F.

2.7.2. The distributor's ordering quantity calculated by retailer's order

In order to determine the distributor's ordering quantity, the extended and revised version of Order-Up-To (OUT) level presented by Hosoda and Disney [15] is proposed here. Eqs. (37) and (38) indicate the distributor's OUT level calculated with retailer's order.

$$O'_{t,2} = O_{t,1} + (S_{t,2} - S_{t-1,2}), \tag{37}$$

$$S_{t,2} = \hat{O}^{l_2}_{t,1} + s_2 \hat{\sigma}_{l_2}. \tag{38}$$

According to Hosoda and Disney [15], $\hat{\sigma}_{l_2}$ is an estimated value of the standard deviation of the forecast error considering the manufacturer's lead-time. $O'_{t,2}$ denotes distributor's order issued at the end of period t, which is calculated using retailer's order, and s_2 is a desired service level of distributor. Moreover, $S_{t,2}$ is the OUT level at period t and $\hat{O}^{l_2}_{t,1}$ shows the conditional expected value of the total order over lead time l_2, which is calculated by the following equation:

$$\begin{aligned}\hat{O}^{l_2}_{t,1} &= E\left(\sum_{i=1}^{l_2} O_{t+i,1} | \gamma_t\right) \\ &= \frac{r_1 v'^{\theta} p^{\theta}_t O_{t,1} (1 - r_1 v^{2\theta} p^{\theta}_t)^{l_2}}{1 - r_1 v'^{2\theta} p^{\theta}_t} = \nabla_{l_2} r_1 v'^{\theta} p^{\theta}_t O_{t,1},\end{aligned} \tag{39}$$

where $\nabla_{l_2} = \frac{1 - r_1 v'^{2\theta} p^{\theta}_t)^{l_2}}{1 - r_1 v'^{2\theta} p^{\theta}_t}$, $O_{t+1,1} = r_1 v'^{\theta} p^{\theta}_t O_{t,1}$, and $\gamma_t = \{O_{t,1}, O_{t-1,1}, O_{t-2,1}, \ldots\}$ are the set of the observed orders placed by the retailer. Now, in order to quantify BWE, variances of orders and demands for each stage should be calculated. The proof of Eq. (39) is given in Appendix G1.

Theorem 5. The variance of distributor's ordering quantity, which is calculated by orders received from retailer, is:

$$\begin{aligned}&\Big(\text{Var}[\ln(1 + \Omega_{l_1} r_1 p^{\theta}_{t+1} q^{\varphi - 1}_{t,1} - \Omega_{l_1})] \\ &+ \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right] + \text{Var}[\ln(1 + \nabla_{l_2} r_1 v'^{\theta} p^{\theta}_t - \nabla_{l_2})] \\ &+ \frac{2\theta}{1-\varphi}\text{Cov}(\ln(1 + \Omega_{l_1} r_1 p^{\theta}_{t+1} q^{\varphi-1}_{t,1} - \Omega_{l_1}), \ln(p_t)) \\ &+ 2\text{Cov}(\ln(1 + \Omega_{l_1} r_1 p^{\theta}_{t+1} q^{\varphi-1}_{t,1} - \Omega_{l_1}), \ln(1 \\ &+ \nabla_{l_2} r_1 v'^{\theta} p^{\theta}_t - \nabla_{l_2})) + \frac{2\theta}{1-\varphi}\text{Cov}(\ln(p_t), \ln(1 \\ &+ \nabla_{l_2} r_1 v'^{\theta} p^{\theta}_t - \nabla_{l_2})) + \sigma^2_{\varepsilon_2} + \sigma^2_{\varepsilon_1} + \left(\frac{\sigma^2_{\varepsilon}}{1-\varphi^2}\right)\Big).\end{aligned}$$

Proof. See Appendix G2.

Theorem 6. The distributor's order quantity calculated by retailer's order at period $t+1$ is $O'_{t+1,2} = r_2 v'^{\theta_2} w^{*\theta_2}_t O'_{t,2}$.

Proof. See Appendix H.

Lemma 4. The MAX time series for predicting order quantities at period $t+1$ is:

$$\begin{aligned}\ln(O'_{t+1,2}) = &\ln(r_2) + \theta_2 \ln(v') + \theta_2 \ln(w^*_t) \\ &+ \varepsilon_{t+1,2} + \ln(1 + \Omega_{l_2} r_2 w^{*\theta_2}_{t+1} q^{\varphi_2 - 1}_{t,2} - \Omega_{l_2}) \\ &+ \varepsilon_{t,2} - \frac{\theta_2}{\varphi_2 - 1}\ln(w^*_t) - \frac{\delta_2}{\varphi_2 - 1} + \varepsilon_t \\ &+ \varphi_2 \varepsilon_{t-1} + \varphi_2^2 \varepsilon_{t-2} + \varphi_2^3 \varepsilon_{t-3} + \ldots .\end{aligned}$$

Proof. It is similar to Theorem 3 and is not mentioned here for brevity.

2.8. Demand model for manufacturer

Eq. (40) shows an ARX time series for manufacturer's demand forecasting:

$$\ln(q_{t,3}) = \theta_3 \ln(z^*_t) + \varphi_3 \ln(q_{t-1,3}) + \ln(r_3) + \varepsilon_{t,3}, \tag{40}$$

where z^*_t is the optimal selling price for manufacturer's product. $q_{t,3}$ and $q_{t-1,3}$ indicate manufacturer's demands for periods t and $t-1$, respectively. r_3 denotes a constant coefficient and $\varepsilon_{t,3}$ is a white noise process of manufacturer's demand forecasting with zero mean and variance of $\sigma^2_{\varepsilon_3}$.

Lemma 5. The expected value of manufacturer's demand is $\left(\frac{\theta_3}{1-\varphi_3}\right)\mu_z + (\frac{\delta_3}{1-\varphi_3})$, and its variance is $\frac{\theta_3^2}{(1-\varphi_3)^2}\sigma_z^2 + \left(\frac{\sigma^2_{\varepsilon_3}}{1-\varphi_3^2}\right)$, where σ_z^2 and μ_z are variance and mean of selling prices for manufacturer's products, respectively.

Proof. It is similar to Lemma 3 and is not presented here for brevity.

2.9. The manufacturer's ordering policy

While ordering quantity is calculated using downstream order in the literature, we propose a new method, which applies demands received by each entity to calculate its orders. Subsection 2.9.1 describes the new method, and Subsection 2.9.2 elaborates the method used in the literature.

2.9.1. Manufacturer's ordering quantity calculated by its received demand

In order to determine the manufacturer's ordering quantity, the new version of OUT policy presented by Hosoda and Disney [15] is proposed in this paper. The method proposed here uses the demand received by manufacturer from distributor to place an order. However, the model presented by Hosoda and Disney [15] uses distributor's order for forecasting manufacturer's order. Eqs. (41) and (42) indicate the manufacturer's OUT level calculated by its received demand:

$$O_{t,3} = q_{t,3} + (S_{t,3} - S_{t-1,3}), \tag{41}$$

$$S_{t,3} = \hat{q}^{l_3}_{t,3} + k_3\hat{\sigma}_{l_3}, \tag{42}$$

where $O_{t,3}$ denotes manufacturer's order issued at the end of period t and s_3 is a desired service level of manufacturer. $S_{t,3}$ is the OUT level at period t and $\hat{q}^{l_3}_{t,3}$ shows the conditional expected value of the total demand over lead time l_3 as follows:

$$\hat{q}^{l_3}_{t,3} = E\left(\sum\nolimits_{i=1}^{l_3} q_{t+i}|\tau_{t,3}\right)$$

$$= \frac{r_3(z^*_{t+1})\theta_3 q^{\varphi_3}_{t,3}(1-(r_3^{\varphi_3}v''^{\theta_3}(z^*_{t+1})^{\theta_3\varphi_3}q^{\varphi_3}_{t,3})^{l_3})}{1-r_3^{\varphi_3}v''^{\theta_3}(z^*_{t+1})^{\theta_3\varphi_3}q^{\varphi_3}_{t,3}}$$

$$= \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q^{\varphi_3}_{t,3}, \tag{43}$$

where $\Omega_{l_3} = \frac{1-(r_3^{\varphi_3}v''^{\theta_3}(z^*_{t+1})^{\theta_3\varphi_3}q^{\varphi_3}_{t,3})^{l_3}}{1-r_3^{\varphi_3}v''^{\theta_3}(z^*_{t+1})^{\theta_3\varphi_3}q^{\varphi_3}_{t,3}}$ and $\tau_{t,3} = \{q_{t,3}, q_{t-1,3}, q_{t-2,3}, ...\}$ is the set of the observed demands. For calculating $\hat{q}^{l_3}_{t,3}$, it is assumed that:

$$z^*_{t+i+1} = v'' z^*_{t+i}, \begin{cases} v'' \neq 1 & \text{if } i = 0,1 \\ v'' = 1 & \text{if } i > 1 \end{cases}.$$

Eq. (44) indicates manufacturer's order, and it is obtained by using Eqs. (41)-(43).

$$O_{t,3} = q_{t,3} + (\Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q^{\varphi_3}_{t,3} - \Omega_{l_3} q_{t,3}). \tag{44}$$

Theorem 7. The variance of manufacturer's order using the proposed method is:

$$\left[\frac{\theta_3^2}{(1-\varphi_3)^2}\sigma_z^2 + \left(\frac{\sigma^2_{\varepsilon_3}}{1-\varphi_3^2}\right)\right] + \mathrm{Var}[\ln(1$$
$$+\Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q^{\varphi_3-1}_{t,3} - \Omega_{l_3})] + 2E[\ln(q_{t,3}\ln(1$$
$$+\Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q^{\theta_3-1}_{t,3} - \Omega_{l_3})] - 2\left[\left(\frac{\theta_3}{1-\varphi_3}\right)\mu_z\right.$$
$$\left.+\left(\frac{\delta_3}{1-\varphi_3}\right)\right]E[\ln(1+\Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q^{\varphi_3-1}_{t,3} - \Omega_{l_3})].$$

Proof. See Appendix I.

2.9.2. The manufacturer's ordering quantity calculated by distributor's order

In order to determine the manufacturer's ordering quantity, the revised version of OUT level presented by Hosoda and Disney [15] is used. Eqs. (45) and (46) indicate the manufacturer's OUT level calculated with distributor's order:

$$O_{t,3} = O_{t,2} + (S_{t,3} - S_{t-1,3}), \tag{45}$$

$$S_{t,3} = \hat{O}^{l_2}_{t,2} + k_3\hat{\sigma}_{l_3}, \tag{46}$$

where $O_{t,3}$ denotes manufacturer's order issued at the end of period t and s_3 is a desired service level of manufacturer. $S_{t,3}$ denotes the OUT level at period t and $\hat{O}^{l_3}_{t,1}$ shows the conditional expected value of the total order over lead time l_3 calculated by the following equation:

$$\hat{O}^{l_3}_{t,2} = E\left(\sum\nolimits_{i=1}^{l_3} O_{t+i,2}\middle|\vartheta_t\right)$$

$$= \frac{r_2 v''^{\theta} w_t^{*\theta} O_{t,2}[1-(r_2 v''^{2\theta} w_t^{*\theta})^{l_3}]}{1-r_2 v''^{2\theta} w_t^{*\theta}}$$

$$= \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} O_{t,2}, \tag{47}$$

where $\Delta_{l_3} = \frac{1-(r_2 v''^{2\theta} w_t^{*\theta})^{l_3}}{1-r_2 v''^{2\theta} w_t^{*\theta}}$, $O_{t+1,2} = r_2 v''^{\theta} w_t^{*\theta} O_{t,2}$, and $\vartheta_t = \{O_{t,2}, O_{t-1,2}, O_{t-2,2}, ...\}$ is the set of the observed orders placed by the distributor.

Theorem 8. The variance of manufacturer's order, which is calculated by distributor's order, is:

$$\begin{aligned}&\mathrm{Var}[\ln(1+\Delta_{l_3}r_2v^{''\theta}w_t^{*\theta}-\Delta_{l_3})]+\mathrm{Var}[\ln(1\\&+\Omega_{l_1}r_1(p_{t+1}^*)^\theta q_{t,1}^{\varphi-1}-\Omega_{l_1})]+\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right]\\&+\mathrm{Var}[\ln(1+\nabla_{l_2}r_1v^{*\theta}p_t^{*\theta}-\nabla_{l_2})]+2\mathrm{Cov}(\ln(1\\&+\Delta_{l_3}r_2v^{''\theta}w_t^{*\theta}-\Delta_{l_3}),\ln(1+\Omega_{l_1}r_1(p_{t+1}^*)^\theta q_{t,1}^{\varphi-1}\\&-\Omega_{l_1}))+2\mathrm{Cov}(\ln(1+\Delta_{l_3}r_2v^{''\theta}w_t^{*\theta}-\Delta_{l_3}),\ln(1\\&+\nabla_{l_2}r_1v^{''\theta}p_t^{*\theta}-\nabla_{l_2}))+2\mathrm{Cov}(\ln(1\\&+\Omega_{l_1}r_1(p_{t+1}^*)^\theta q_{t,1}^{\varphi-1}-\Omega_{l_1}),\ln(1+\nabla_{l_2}r_1v^{''\theta}p_t^{*\theta}\\&-\nabla_{l_2}))+\frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*),\ln(1+\Delta_{l_3}r_2v^{''\theta}w_t^{*\theta}\\&-\Delta_{l_3}))+\frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*),\ln(1+\Omega_{l_1}r_1(p_{t+1}^*)^\theta q_{t,1}^{\varphi-1}\\&-\Omega_{l_1}))+\frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*),\ln(1+\nabla_{l_2}r_1v^{''\theta}p_t^{*\theta}\\&-\nabla_{l_2}))+\sigma_{\varepsilon_3}^2+\sigma_{\varepsilon_2}^2+\sigma_{\varepsilon_1}^2+\left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right).\end{aligned}$$

Proof. See Appendix J.

3. Measuring and reducing BWE

In this section, BWE is quantified using orders and demands of each entity in the SC calculated in the previous sections. Two methods are utilized for measuring BWE. In the first method, orders of downstream echelons are used to forecast upstream orders as shown in Eqs. (48)-(50). In contrast to the first method, the second one utilizes demand of each echelon for forecasting its own ordering quantity through Eqs. (51) and (52). For example, distributor's demand is used to forecast its relevant ordering quantity. Comparing Eq. (49) with Eq. (52) shows that BWE is significantly reduced by the second method, which uses distributor's demand for forecasting distributor's order. Moreover, comparing Eq. (50) with Eq. (53) demonstrates that BWE is mitigated in manufacturer echelon if the second method is used. Therefore, if order quantity of each entity in an SC is forecasted by its demand, BWE will be reduced significantly in comparison with the cases in which downstream orders are used for forecasting upstream orders.

4. Validation and verification

In order to validate the model, MSE of demand forecasting and variance of orders calculated by the proposed method are compared with those calculated by the technique presented by Zhang and Burke [33].

Theorem 9. If the optimal values of prices are calculated using the proposed method, the forecasting error (MSE) will be less than that in the case in which prices are forecasted as studied by Zhang and Burke [33].

Proof. See Appendix K.

Theorem 10. The proposed method in this paper, in which optimal prices are used for forecasting demands and orders in SCs, reduces BWE significantly.

Proof. See Appendix L. Eqs. (48)-(53) represent the bullwhip effect metrics in each echelon of the SC (Eq. (48) is shown in Box I):

$$\begin{aligned}B_2=&\frac{\mathrm{Var}[\ln(O_{t,2})]}{\mathrm{Var}[\ln(q_{t,1})]}=[\mathrm{Var}[\ln(1+\Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1}\\&-\Omega_{l_1})]+\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right]+\mathrm{Var}[\ln(1+\nabla_{l_2}r_1v^\theta p_t^\theta\\&-\nabla_{l_2})]+\frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(1+\Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1}\end{aligned}$$

$$B_1=\frac{\mathrm{Var}[\ln(O_{t,1})]}{\mathrm{Var}[\ln(q_{t,1})]}$$

$$=\frac{\begin{gathered}\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2+\left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right]+\mathrm{Var}[\ln(1+\Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1}-\Omega_{l_1})]+2E[\ln(q_{t,1})\ln(1+\Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1}-\Omega_{l_1})]\\-2\left[\left(\frac{\theta}{1-\varphi}\right)\mu_p+\left(\frac{\delta}{1-\varphi}\right)\right]E[\ln(1+\Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^{\varphi-1}-\Omega_{l_1})]\end{gathered}}{\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2+\left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)}. \quad (48)$$

Box I

$$- \Omega_{l_1}), \ln(p_t)) + 2\text{Cov}(\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}), \ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})) + \frac{2\theta}{1-\varphi}\text{Cov}(\ln(p_t), \ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})) + \sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_1}^2 + \left(\frac{\sigma_{\varepsilon}^2}{1-\varphi^2}\right)\Bigg] \Big/ \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_{\varepsilon}^2}{1-\varphi^2}\right)\right], \tag{49}$$

$$B_3 = \frac{\text{Var}[\ln(O_{t,3})]}{\text{Var}[\ln(q_{t,1})]} = [\text{Var}[\ln(1 + \Delta_{l_3} r_2 v^{\theta} w_t^{\theta} - \Delta_{l_3})] + \text{Var}[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] + \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right] + \text{Var}[\ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})] + 2\text{Cov}(\ln(1 + \Delta_{l_3} r_2 v^{\theta} w_t^{\theta} - \Delta_{l_3}), \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})) + 2\text{Cov}(\ln(1 + \Delta_{l_3} r_2 v^{\theta} w_t^{\theta} - \Delta_{l_3}), \ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})) + 2\text{Cov}(\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}), \ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})) + \frac{2\theta}{1-\varphi}\text{Cov}(\ln(p_t), \ln(1 + \Delta_{l_3} r_2 v^{\theta} w_t^{\theta} - \Delta_{l_3})) + \frac{2\theta}{1-\varphi}\text{Cov}(\ln(p_t), \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})) + \frac{2\theta}{1-\varphi}\text{Cov}(\ln(p_t), \ln(1 + \nabla_{l_2} r_1 v^{\theta} p_t^{\theta} - \nabla_{l_2})) + \sigma_{\varepsilon_3}^2 + \sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_1}^2 + \left(\frac{\sigma_{\varepsilon}^2}{1-\varphi^2}\right)\Bigg] \Big/ \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_{\varepsilon}^2}{1-\varphi^2}\right)\right], \tag{50}$$

Eqs. (51) to (53) are shown in Box II.

5. Numerical experiments

In order to validate the proposed methods, a data set from auto-parts industry is used to analyse the contributions of this paper, namely, (I) using optimal prices instead of forecasted ones for demand and order forecasting; (II) investigating the effect of joint demand-order-pricing-lead time decisions on reducing BWE; (III) calculating order quantities for each echelon in an SC through its relevant demand instead of using downstream orders for measuring upstream orders.

This section is organized as follows. In Subsection 5.1, the effect of joint demand-order-lead time and optimal prices on reducing BWE is investigated using data set of an auto-parts SC. After calculating BWE metric with forecasted prices, the results are compared with the case in which BWE is calculated with the optimal prices. Subsection 5.2 compares the proposed method, in which demand of each entity is used to forecast its order quantity, with the method in which upstream orders are predicted by downstream orders.

5.1. Joint demand-pricing-lead time model for reducing BWE in auto-parts industry

In this subsection, the proposed joint demand-pricing-lead time method is used to reduce BWE. Then, the method is compared with the model in which prices are forecasted. In order to show the effect of joint demand-pricing-lead time decisions on reducing BWE, we use a data set of an auto-parts manufacturing company. Figure 2 shows the demand functions of retailer, distributor, and manufacturer.

For calculating joint demand-pricing-lead time model, retailer's optimal selling price is used to forecast its demand using Eq. (15). A statistical test called coefficient test is applied by EViews software to determine ARX structure of Eq. (15). Table 2 shows the coefficient test for retailer's demand. In Table 2, AR(10) shows 10th order auto-regressive variable of natural logarithm of retailer's demand, ln ($q_{t-10,1}$). The first to 9th order AR variables (AR(1), AR(2),..., AR(9)) have been examined with coefficient test. Since the corresponding p-values for the first to 9th order AR variables are higher than 0.025, variables are rejected and AR(10), whose p-value is lower than 0.025, is accepted. P-value is the probability of obtaining a result equal to or more than what is observed. The coefficient of AR(10) is extracted from Table 2. Moreover, the expected value and variance of retailer's demand are calculated through Eqs. (19) and (20). In the next step, retailer's order quantity is calculated through Eq. (27). Table 3 shows coefficient test for retailer's order quantity. In Table 3, OMEGA is a representative of $[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]$ in Eq. (35).

After identifying ARX coefficients, demands of retailer, distributor, and manufacturer are forecasted and compared with the actual ones for period t. Figure 3(a) shows the actual and forecasted demands for retailer, distributor, and manufacturer. The figure illustrates that there is a trivial difference between actual and forecasted demands. This fact shows that the method proposed in this paper has high capability of demand forecasting with low error. Figure 3(b) depicts demands of entities calculated with optimal prices (the proposed method in this paper) as well

$$B_1 = \frac{\text{Var}[\ln(O_{t,1})]}{\text{Var}[\ln(q_{t,1})]}$$

$$= \frac{\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right] + \text{Var}[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] + 2E[\ln(q_{t,1})\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] - 2\left[\left(\frac{\theta}{1-\varphi}\right)\mu_p + \left(\frac{\delta}{1-\varphi}\right)\right] E[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]}{\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)}. \quad (51)$$

$$B_2 = \frac{\text{Var}[\ln(O_{t,2})]}{\text{Var}[\ln(q_{t,1})]}$$

$$= \frac{\left[\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2 + \left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right)\right] + \text{Var}[\ln(1+\Omega_{l_2} r_2 w_{t+1}^{\theta_2} q_{t,2}^{\varphi_2-1} - \Omega_{l_2})] + 2E[\ln(q_{t,2})\ln(1+\Omega_{l_2} r_2 w_{t+1}^{\theta_2} q_{t,2}^{\varphi_2-1} - \Omega_{l_2})] - 2\left[\left(\frac{\theta_2}{1-\varphi_2}\right)\mu_w + \left(\frac{\delta_2}{1-\varphi_2}\right)\right] E[\ln(1+\Omega_{l_2} r_2 w_{t+1}^{\theta_2} q_{t,2}^{\varphi_2-1} - \Omega_{l_2})]}{\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)}, \quad (52)$$

$$B_3 = \frac{\text{Var}[\ln(O_{t,3})]}{\text{Var}[\ln(q_{t,1})]}$$

$$= \frac{\left[\frac{\theta_3^2}{(1-\varphi_3)^2}\sigma_z^2 + \left(\frac{\sigma_{\varepsilon_3}^2}{1-\varphi_3^2}\right)\right] + \text{Var}[\ln(1+\Omega_{l_3} r_3 z_{t+1}^{\theta_3} q_{t,3}^{\varphi_3-1} - \Omega_{l_3})] + 2E[\ln(q_{t,3})\ln(1+\Omega_{l_3} r_3 z_{t+1}^{\theta_3} q_{t,3}^{\varphi_3-1} - \Omega_{l_3})] - 2\left[\left(\frac{\theta_3}{1-\varphi_3}\right)\mu_z + \left(\frac{\delta_3}{1-\varphi_3}\right)\right] E[\ln(1+\Omega_{l_3} r_3 z_{t+1}^{\theta_3} q_{t,3}^{\varphi_3-1} - \Omega_{l_3})]}{\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)}. \quad (53)$$

Box II

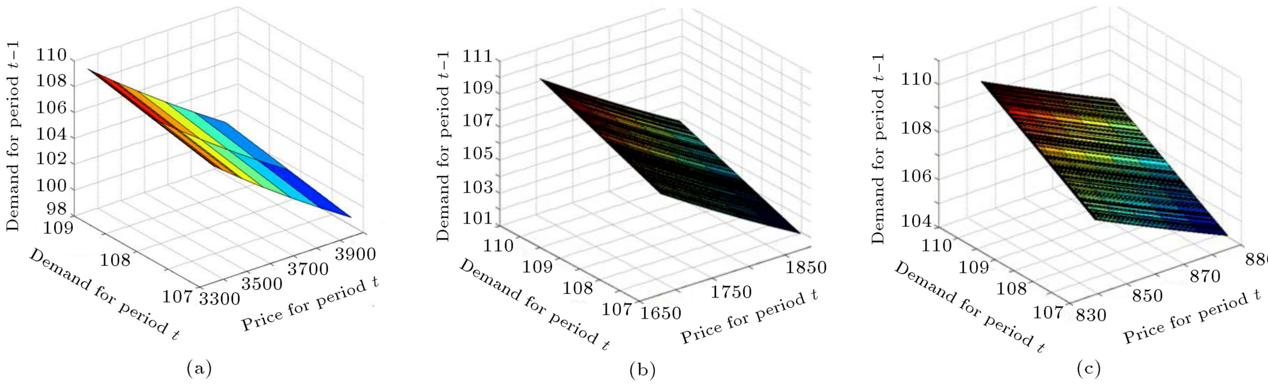

Figure 2. (a) Retailer's demand calculated using Eq. (1c). (b) Distributor's demand calculated using Eq. (6). (c) Manufacturer's demand calculated using Eq. (11).

as the forecasted ones (the method presented in the literature) in an auto-parts SC. Figure 3(b) demonstrates that demand amplification from retailer to manufacturer is significantly reduced by applying optimal prices rather than forecasted ones. As it is illustrated in the figure, demands of retailer, distributor, and manufacturer calculated with optimal prices are very close to each other in comparison with the demands obtained by the forecasted prices. Figure 3(a) and (b) show that demands forecasted using optimal prices are better estimations of actual demands than the demands obtained by forecasted prices are.

Figure 3(c) shows demands and orders of entities in the SC calculated by the optimal prices. Figure 3(d) indicates that BWE exists in the SC because order of each entity is amplified as it moves through the

Table 2. Coefficient test for retailer's demand.

Variable	Coefficient	Std. error	t-statistic	p-values
$\ln(r_1)$	4.546337	0.102659	44.28589	0.0000
p_t^*	–0.005083	0.002148	–2.365807	0.0187
$AR(10)$	1.005144	0.002769	363.0504	0.0000
R-squared	0.997943	Mean dependent var		4.686812
Adjusted R-squared	0.997928	S.D. dependent var		0.007760
S.E. of regression	0.000353	Akaike info criterion		–13.04874
Sum of squared residual	3.57E-05	Schwarz criterion		–13.01068
Log likelihood	1888.542	Hannan-Quinn criterion		–13.03348
Durbin-Watson statistics	0.161957			

Table 3. Coefficient test for retailer's order.

Variable	Coefficient	Std. error	t-statistic	p-values
$\ln(q_{t,1})$	1.000002	3.10E-06	322602.8	0.0000
OMEGA	0.998380	0.002814	354.7612	0.0000
R-squared	1.000000	Mean dependent var		4.691569
Adjusted R-squared	1.000000	S.D. dependent var		0.008035
S.E. of regression	3.71E-07	Akaike info criterion		–26.76835
Sum of squared residual	4.11E-11	Schwarz criterion		–26.74366
Log likelihood	4017.252	Hannan-Quinn criterion		–26.75847
Durbin-Watson statistics	3.003381			

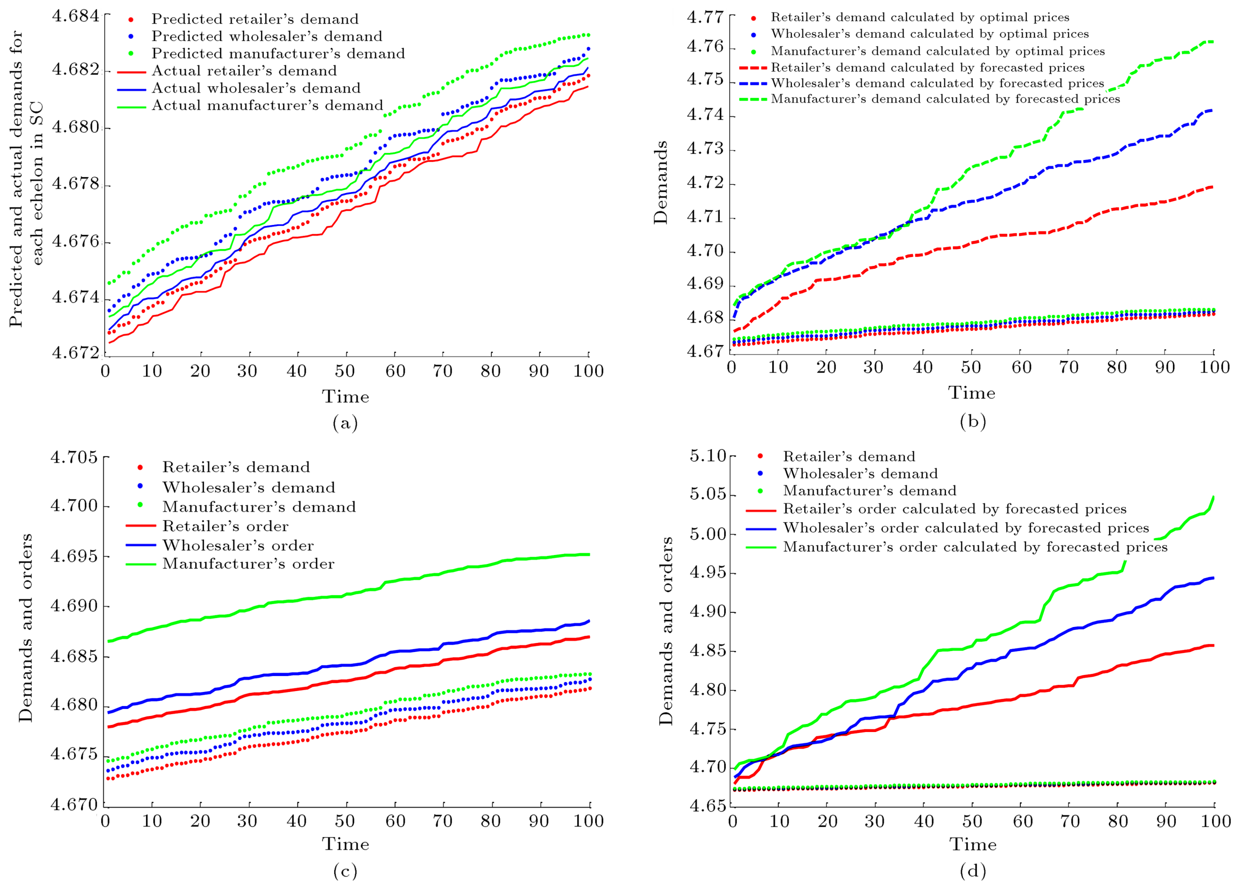

Figure 3. (a) Actual and forecasted demands of retailer, distributor, and manufacturer. (b) Demands of retailer, distributor, and manufacturer calculated by optimal and forecasted prices. (c) Demands and orders of retailer, distributor, and manufacturer calculated by the optimal prices. (d) Demands and orders of retailer, distributor, and manufacturer calculated by the forecasted prices.

Table 4. BWE metrics, variance of orders, and demands for each entity in SC.

Row	Criteria	Retailer	Distributor	Manufacturer
1	BWE metric with optimal prices	1.000404	1.000517	1.000454
2	BWE metric with forecasted prices	1.05297769	3.55964187	6.335687873
3	Variance of demands with optimal prices	6.45431×10^{-5}	6.50565×10^{-5}	6.64263×10^{-5}
4	Variance of demands with forecasted prices	0.000875031	0.001846765	0.00329832
5	Variance of orders with optimal prices	6.45692×10^{-5}	6.50901×10^{-5}	6.64565×10^{-5}
6	Variance of orders with forecasted prices	0.000921388	0.006573823	0.020897128

Table 5. BWE metrics, variance of orders calculated by both methods, and variance of demands.

	Retailer	Distributor	Manufacturer
BWE metric using orders calculated by downstream orders	1.000404	1.079685662	1.05694084
BWE metric using orders calculated by demands	1.000404	1.000517073	1.00045357
Variance of demands	6.45431×10^{-5}	6.50565×10^{-5}	6.64263×10^{-5}
Variance of orders calculated by downstream orders	6.45692×10^{-5}	7.02406×10^{-5}	7.0209×10^{-5}
Variance of orders calculated by demands	6.45692×10^{-5}	6.50901×10^{-5}	6.64565×10^{-5}

chain. The difference between demand and order of each entity represents the existence of BWE in the SC. Figure 3(d) depicts demands and orders measured by the forecasted prices. In Figure 3(d), the differences between demands and orders of entities in the SC illustrate the existence of BWE. However, comparing Figure 3(c) with Figure 3(d) indicates that orders are more amplified when calculated with the forecasted prices than when calculated with the optimal prices. Therefore, BWE is significantly reduced by using optimal prices in demand and order calculation rather than by using forecasted ones.

Table 4 shows BWE metric and variances of demands and orders for each entity measured by optimal prices as well as by forecasted ones. Table 4 indicates that the BWE metric calculated with the proposed method, using the optimal prices, is lower than the BWE metric measured by the forecasted prices. The first row of Table 4 shows that BWE metrics for retailer, distributor, and manufacturer are close to each other and approximately equal to 1. Thus, BWE is significantly reduced and it can be claimed that BWE is almost eliminated by the method presented in this paper.

The second row of Table 4 shows that the BWE metric measured by forecasted prices is very high. Comparing row 3 with row 4 of Table 4 shows that variances of demands are significantly reduced by applying the proposed method using optimal prices. Moreover, when demand is calculated by optimal prices, demand amplification is lower than when it is measured by forecasted prices. Comparing rows 5 and 6 of Table 4 shows that variances of orders are significantly reduced and orders are not amplified significantly by applying the proposed method using the optimal prices.

5.2. The effect of ordering policies on BWE

As it was discussed in Subsection 5.1, statistical tests are applied to find the best time series for forecasting orders. Those tests are not included here for brevity. Figure 4(a) shows orders of each entity in the SC calculated by two methods: (I) Orders of each entity are calculated using its received demands; (II) Orders are measured using downstream orders. Retailer's orders for both methods are the same, because retailer is the first echelon, and there is no downstream echelon after it. Therefore, its order is calculated by its own demand. Comparing solid lines with diamonded-solid ones shows that orders of each echelon quantified by its received demand are amplified less than orders calculated with downstream orders. In other words, the proposed method, in which order of each entity in the SC is calculated through its received demand, significantly reduces BWE.

Figure 4(b) depicts demands and orders of each echelon in the SC calculated by downstream orders. Figure 4(b) shows that orders are significantly amplified and BWE is a large value when orders are calculated by downstream orders. Figure 4(c) illustrates demands of each echelon in the SC and orders calculated by the received demands. Figure 4(c) shows that orders that have been calculated by the received demands are not amplified significantly. Thus, BWE is reduced significantly when orders are calculated by demands.

Table 5 shows that the BWE metric quantified

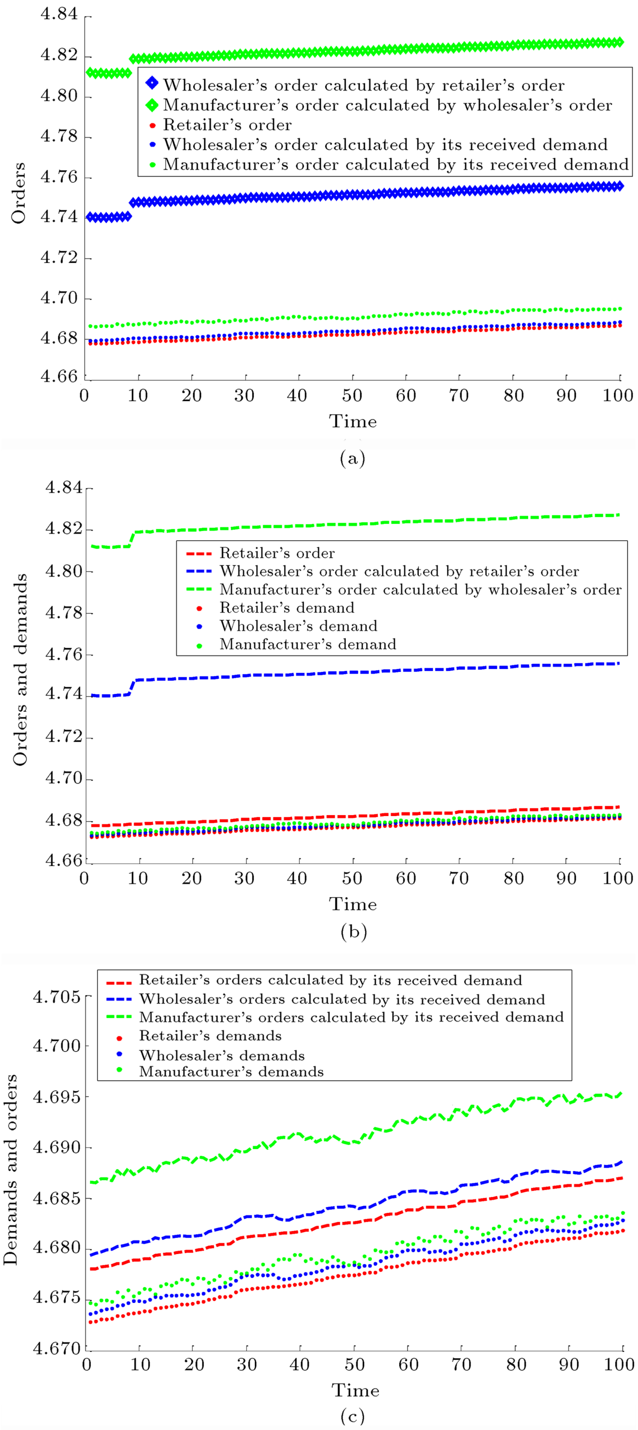

Figure 4. (a) Orders for each echelon in the SC calculated by its received demands as well as its downstream orders. (b) Demands and orders for each echelon in the SC calculated by downstream orders. (c) Demands and orders for each echelon in SC calculated by the received demands.

with the method proposed in this paper is lower than the metric measured using the orders that are calculated by demands. Comparing the 5th row of Table 5 with the 6th one indicates that the variance of orders calculated by downstream orders is more than the variance of orders measured by demands.

6. Implications

This works demonstrates that 3 factors can significantly reduce BWE in SCs. The first one is joint demand, pricing, ordering, and lead time decisions. This occurs due to the fact that eliminating the causes of BWE generation will lead to its reduction. If multiple causes of BWE are analysed simultaneously, it decreases more significantly. Demand forecasting is one of those causes. From downstream to upstream echelons of the SC, demand forecasting errors are accumulated and added to the next echelon of the chain, leading to demand amplification (BWE) and inaccurate demand information. These inaccuracies will increase the variance of orders through the SC. If variance of orders increases in the SC, fluctuations occur in production system, which lead to either generating huge inventories or shortage of products and loss of customers. Both of them impose extravagant costs to the entities in the SC.

Thus, providing more accurate demand forecasting helps production managers to provide smoother production plan with the least fluctuations, leading to reduction in inventory and shortage costs. In this paper, the results of demand forecasting with ARX model showed that variance of orders and BWE were reduced significantly, which would lead to further cost reductions in an SC and production planning without high fluctuations. Inaccurate or improper ordering policies, pricing, and lead time decisions also lead to more ordering variance through the SC, which consequently increase costs of each entity. The results of presenting the new methods for ordering policy, lead time, and pricing decisions demonstrated that variance of orders and BWE were reduced using the proposed methods.

The second factor is to use optimal prices instead of the forecasted ones. As it was proved mathematically and shown by numerical experiments, optimal prices reduce MSE of demand forecasting and, consequently, reduce BWE. The third factor is adopting an appropriate ordering policy. In this paper, it was mathematically and numerically proved that using demand of each entity for calculating its order quantities would significantly reduce BWE in comparison with the method in which downstream orders were used. It is worthwhile to mention that there is a difference between the demand received by the distributor (or the manufacturer) and its downstream order in practice.

Practically, in an SC, the manufacturer requires to have distributor's demand in advance in order to be able to produce adequate products. Assume that the manufacturer decides to provide the raw material to produce next week's products. In the current week, the manufacturer does not have market demand for the next week. Thus, the manufacturer uses the forecasted demand of the distributor, which is predicted by demand planning department. Manufacturer will place its order for providing the required raw material based on the forecasted demand of the distributor. Then, at the end of forecasting period, the distributor places its actual order and manufacturer will receive actual demand of the distributor. Thus, the distributor's order differs from the demand that the manufacturer receives from demand forecasting department. This occurs due to the fact that actual demand of distributor is not available in the planning period (current week); hence, its forecasted demand is used. This paper showed that using demand of each entity for calculating its order quantities would reduce BWE significantly in comparison with the method in which downstream orders were used. This is due to the fact that downstream orders accumulate forecasting errors; however, using demand of each entity only includes forecast errors of one stage.

Production managers can use the proposed techniques for reducing costs of SC and making production plan smoother with less fluctuations in inventory and ordering. In addition to managers and practitioners, academic communications also benefit from this study. They can use the proposed method accompanied by investigating the effect of shortage gaming on BWE.

7. Conclusions and future research

This paper investigated the impact of joint demand, orders, lead time, and pricing decisions on reducing BWE. In order to mitigate it, 4 major contributions were proposed. The first contribution was considering multiple causes of BWE (demand, orders, lead time, and pricing) simultaneously for reducing it. The second one was to model demands, orders, and prices dynamically. Demand and prices have mutual effect on each other over time. Therefore, a time series model was applied in a game theory method for finding the optimal values of prices in an SC. Then, optimal prices were inserted in the time series model for demand forecasting. The third contribution was proposing a new policy to find order quantities for each entity in the SC. The new method used demand of each entity to calculate its order quantities. In order to validate the new ordering policy, it was compared with the method in the literature, which used downstream orders for forecasting upstream orders.

It was proved that using demand of each entity for calculating its order quantities would reduce BWE significantly in comparison with the method in which downstream orders were used. The fourth contribution was to find optimal prices and use them for demand forecasting and reducing BWE. It was proved that the proposed method, which used optimal prices to forecast demands, had low forecasting error in comparison with the technique that forecasted prices. Furthermore, it was proved that using optimal prices for forecasting demands and orders in SCs would reduce BWE significantly.

Then, the proposed model was validated using a data set of an auto-parts SC. First, the effect of the proposed joint demand, orders, lead-time, and pricing model on BWE was investigated. In order to reach that goal, the effect of optimal prices on BWE was compared with the impact of the forecasted prices on BWE. Statistical tests were used to find the most appropriate time series for demand and order forecasting. The results showed that BWE and variance of orders/demands were significantly reduced when optimal prices were used. Second, the proposed ordering policy, which used the received demands of each entity to find its order quantities, was examined by a data set of an auto-parts SC. The results were compared with the method in which orders of each entity were obtained by downstream orders. This comparison indicated that BWE and variance of orders were significantly reduced when orders of each entity were calculated by its received demands. It can be claimed that BWE was almost removed from the SC using the proposed method. In addition, this paper provides a fundamental structure for future research. That is, analysing the impact of compound causes of BWE, including shortage gaming, on reducing it.

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Appendix A

The method presented by Özelkan and Cakanyildirim [32] is extended here. In order to find the optimal values of prices, concavity of the objective function should be investigated. Thus, the second order condition should be negative:

$$\frac{d^2(\pi(p_t(w_t)))}{d^2(p_t(w_t))} = 2q'^2_{t,1}(p_t(w_t), q_{t-1,2})$$

$$-q_{t,1}(p_t(w_t), q_{t-1,2})q''_{t,1}(p_t(w_t), q_{t-1,2}) < 0$$

$$\rightarrow \frac{q_{t,1}(p_t(w_t), q_{t-1,2})q''_{t,1}(p_t(w_t), q_{t-1,2})}{q'^2_{t,1}(p_t(w_t), q_{t-1,2})} < 2$$

$$\rightarrow \frac{1}{a^2c^2(acw_t)^{ac}} < 2.$$

Therefore, the optimal value for w_t must satisfy the inequality $\frac{1}{a^2c^2(acw_t)^{ac}} < 2$ leading to the concave profit function. Assume that the second order condition is satisfied; thus, the first order condition should be investigated to find the optimal values of w_t. The optimal price for distributor's goods is obtained by solving:

$$w_t^* = \{w_t | q_{t,1}(p_t(w_t), q_{t-1,2})$$

$$+ (w_t - z_t)\frac{d(q_{t,1}(p_t(w_t), q_{t-1,2})}{dw_t} = 0\},$$

where:

$$\frac{d(q_{t,1}(p_t(w_t), q_{t-1,2})}{dw_t} = \frac{-(a^2c^2r_2q_{t-1,2}^{1-a})}{(ac-1)(acw_t)/(ac-1)^{ac+1}},$$

and:

$$q_{t,2} = q_{t,1}(p_t(w_t), q_{t-1,2}) = r_2q_{t-1,2}^{1-a}(p_t^*(w_t))^{-ac}$$

$$= r_2q_{t-1,2}^{1-a}(\frac{acw_t}{ac-1})^{-ac}.$$

After solving the above equations, w_t is obtained as:

$$w_t^* = \{r_2q_{t-1,2}^{1-a}(\frac{acw_t}{ac-1})^{-ac}$$

$$+ (w_t - z_t)\frac{-(a^2c^2r_2q_{t-1,2}^{1-a})}{(ac-1) * \frac{acw_t}{(ac-1)^{ac+1}}} = 0\},$$

and:

$$w_t = \frac{acz_t}{ac-1}.$$

Selling price is a positive number ($w_t > 0$); therefore ($ac-1$) should be positive. This shows that ac is greater than one. Hence, the inequality $\frac{1}{a^2c^2(acw_t)^{ac}} < 2$ and the second order condition are satisfied. As a result, the optimal price for distributor's goods (w_t^*) is $w_t^* = \frac{acz_t}{ac-1}$.

Appendix B1

AS in the method presented by Hosoda and Disney [15], $E(\sum_{i=1}^{l_1} q_{t+i,1}|\tau_t)$ is equal to the sum of the first l_1 terms of a geometric progression. In that geometric progression, terms are demands over lead time. Thus, using the formulation for calculating sum of the first l_1 terms of a geometric progression, having $q_{t+1,1}$ as the first term, its progression ratio is $r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi$. The proof is given as follows:

$$q_{t+1,1} = r_1p_{t+1}^{\theta_1}q_{t,1}^{\varphi_1}, \tag{B1.1}$$

$$q_{t+2,1} = r_1p_{t+2}^{\theta_2}q_{t+1,1}^{\varphi_2} = r_1v^{\theta_2}p_{t+1}^{\theta_2}q_{t+1,1}^{\varphi_2}$$

$$= r_1v^{\theta_2}p_{t+1}^{\theta_2}r_1^{\varphi_2}p_{t+1}^{\theta_1\varphi_2}q_{t,1}^{\varphi_1\varphi_2}, \tag{B1.2}$$

$$q_{t+3,1} = r_1p_{t+3}^{\theta_3}q_{t+2,1}^{\varphi_3} = r_1v^{\theta_3}v^{\theta_3}p_{t+1}^{\theta_3}r_1^{\varphi_3}v^{\theta_2\varphi_3}$$

$$p_{t+1}^{\theta_2\varphi_3}r_1^{\varphi_2\varphi_3}p_{t+1}^{\theta_1\varphi_2\varphi_3}q_{t,1}^{\varphi_1\varphi_2\varphi_3}, \tag{B1.3}$$

$$\vdots$$

Let $\theta_1 = \theta_2 = \theta_3 = \theta_4 = ... = \theta$ and $\varphi_1 = \varphi$, $\varphi_2 = 2$, and $\varphi_1\varphi_2 = 2\varphi$, $\varphi_1\varphi_2\varphi_3 = 3\varphi$, $\varphi_1\varphi_2\varphi_3\varphi_4 = 4\varphi, ...$. Let $p_{t+i+1} = vp_{t+i}$, $i = 1, 2, ..., l_1$, then we have:

$$q_{t+1,1} = r_1p_{t+1}^\theta q_{t,1}^\varphi, \tag{B1.4}$$

$$q_{t+2,1} = r_1v^\theta p_{t+1}^\theta r_1^2p_{t+1}^{2\theta}q_{t,1}^{2\varphi} = r_1^3v^\theta p_{t+1}^{3\theta}q_{t,1}^{2\varphi}, \tag{B1.5}$$

$$q_{t+3,1} = r_1v^\theta p_{t+1}^\theta r_1v^\theta p_{t+1}^{2\theta}r_1^3p_{t+1}^{2\theta}q_{t,1}^{3\varphi} = r_1^5v^{2\theta}p_{t+1}^{5\theta}q_{t,1}^{3\varphi}, \tag{B1.6}$$

$$\hat{q}_{t,1}^{l_1} = E\left(\sum_{i=1}^{l_1} q_{t+i,1}|\tau_t\right)$$

$$= \frac{r_1p_{t+1}^\theta q_{t,1}^\varphi[1-(r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi)^{l_1}]}{1-r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi}$$

$$= \Omega_{l_1}r_1p_{t+1}^\theta q_{t,1}^\varphi, \tag{B1.7}$$

where $\Omega_{l_1} = \frac{1-(r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi)^{l_1}}{1-r_1^2v^\theta p_{t+1}^{2\theta}q_{t,1}^\varphi}$, and $\tau_t = \{q_t, q_{t-1}, q_{t-2}, ...\}$ is the set of the demands.

Appendix B2

By applying Eq. (26) and setting the equivalent value for $q_{t,1}$, the following equation is obtained:

$$O_{t,1} = r_1p_t^\theta q_{t-1,1}^\varphi + \Omega_{l_1}(r_1p_{t+1}^\theta q_{t,1}^\varphi - r_1p_t^\theta q_{t-1,1}^\varphi). \tag{B2.1}$$

Then, in order to find retailer's order at period $t+1$, we extend Eq. (B2.1) to $t+1$ as follows:

$$O_{t+1,1} = r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi} + \Omega_{l_1} r_1 p_{t+2}^{\theta} q_{t+1,1}^{\varphi} - \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi}. \quad \text{(B2.2)}$$

Then, the equivalent value for $q_{t,1}$ is substituted in Eq. (B2.2), resulting in the following equation:

$$O_{t+1,1} = r_1 p_{t+1}^{\theta} (r_1 p_t^{\theta} q_{t-1,1}^{\varphi})^{\varphi} + \Omega_{l_1} r_1 p_{t+2}^{\theta} (r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi})^{\varphi} - \Omega_{l_1} r_1 p_{t+1}^{\theta} (r_1 p_t^{\theta} q_{t-1,1}^{\varphi})^{\varphi}. \quad \text{(B2.3)}$$

Eq. (B2.4) is obtained by substituting $p_t^{\theta} = v^{\theta} p_{t-1}^{\theta}$ and $q_{t,1} = r_1 p_t^{\theta} q_{t-1,1}^{\varphi}$ in Eq. (B2.3):

$$O_{t+1,1} = r_1 v^{\theta} p_t^{\theta} (r_1 p_t^{\theta} q_{t-1,1}^{\varphi})^{\varphi} + \Omega_{l_1} (r_1 v^{2\theta} p_t^{\theta} (r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi})^{\varphi} - r_1 v^{\theta} p_t^{\theta} (r_1 p_t^{\theta} q_{t-1,1}^{\varphi})^{\varphi}), \quad \text{(B2.4)}$$

where v is a constant number indicating how much information about price is transferred from the present period to the next period. Having $\varphi \approx 1$, Eq. (B2.4) is converted to the following equation.

$$O_{t+1,1} = r_1 v^{\theta} p_t^{\theta} r_1 p_t^{\theta} q_{t-1,1}^{\varphi} + \Omega_{l_1} (r_1 v^{2\theta} p_t^{\theta} r_1 p_{t+1}^{\theta} q"_{t,1}^{\varphi} - r_1 v^{\theta} p_t^{\theta} r_1 p_t^{\theta} q_{t-1,1}^{\varphi}). \quad \text{(B2.5)}$$

Because θ is a very small quantity, we suppose that $v^{2\theta} \approx v^{\theta}$; thus, Eq. (B2.6) is obtained. Finally, (Eq. B2.7) shows the retailer's ordering quantity at period $t+1$:

$$O_{t+1,1} = r_1 v^{\theta} p_t^{\theta} [r_1 p_t^{\theta} q_{t-1,1}^{\varphi} + \Omega_{l_1} (r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi} - r_1 p_t^{\theta} q_{t-1,1}^{\varphi})], \quad \text{(B2.6)}$$

$$O_{t+1,1} = r_1 v^{\theta} p_t^{\theta} O_{t,1}. \quad \text{(B2.7)}$$

The ARX time series model is used to forecast retailer's order at period $t+1$. This process is necessary for measuring and reducing BWE. Eq. (B2.8) shows ARX model for forecasting retailer's order at period $t+1$. This equation is obtained by taking natural logarithm of Eq. (B2.7):

$$\ln(O_{t+1,1}) = \ln(r_1) + \theta \ln(v) + \theta \ln(p_t) + \ln(O_{t,1}) + \varepsilon_{t+1,1}, \quad \text{(B2.8)}$$

where $\varepsilon_{t+1,1}$ is a white noise process at period $t+1$ with zero mean and variance of $\sigma_{\varepsilon_1}^2$.

Appendix C

Eq. (C.1) shows the time series equation for retailer's ordering quantity at period t, and Eq. (C.3) is obtained by substituting Eq. (C.2) in Eq. (C.1):

$$\ln(O_{t,1}) = \ln(q_{t,1}) + \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1}, \quad \text{(C.1)}$$

$$\ln(q_{t,1}) = \theta \ln(p_t) + \varphi \ln(q_{t-1,1}) + \ln(r_1) + \varepsilon_t, \quad \text{(C.2)}$$

$$\ln(O_{t,1}) = \theta \ln(p_t) + \varphi \ln(q_{t-1,1}) + \ln(r_1) + \varepsilon_t + \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1}. \quad \text{(C.3)}$$

Eq. (C.2) is used to extract the equivalent time series for retailer's order at period $t+1$. This time series is substituted in Eq. (C.1), and Eq. (C.3) is generated. Eq. (C.3) is substituted in Eq. (B2.8) and Eq. (C.4) is created, which shows the MAX time series model for retailer's order at period $t+1$. The proof is now completed:

$$\ln(O_{t+1,1}) = 2\ln(r_1) + \theta \ln(v) + 2\theta \ln(p_t) + \varphi \ln(q_{t-1,1}) + \varepsilon_t + \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1} + \varepsilon_{t+1,1}. \quad \text{(C.4)}$$

Appendix D

Using Eq. (16), the MAX time series equation for $\ln(q_{t,1})$ is extracted. Substituting MAX model of $\ln(q_{t,1})$ in Eq. (C.1), the following equation is generated:

$$\ln(O_{t,1}) = \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1} - \frac{\theta}{\varphi - 1} \ln(p_t) - \frac{\delta}{\varphi - 1} + \varepsilon_t + \varphi \varepsilon_{t-1} + \varphi^2 \varepsilon_{t-2} + \varphi^3 \varepsilon_{t-3} + \dots . \quad \text{(D.1)}$$

Finally, the right-hand side of Eq. (D.1) is substituted in Eq. (B2.8), and the MAX of retailer's order at period $t+1$, including previous error terms, is obtained as follows:

$$\ln(O_{t+1,1}) = \ln(r_1) + \theta \ln(v) + \theta \ln(p_t) + \varepsilon_{t+1,1} + \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) + \varepsilon_{t,1} - \frac{\theta}{\varphi - 1} \ln(p_t) - \frac{\delta}{\varphi - 1} + \varepsilon_t + \varphi \varepsilon_{t-1} + \varphi^2 \varepsilon_{t-2} + \varphi^3 \varepsilon_{t-3} + \dots . \quad \text{(D.2)}$$

Appendix E

In order to calculate mean and variance of distributor's demand, Eq. (32) is converted to its equivalent MAX time series by Eqs. (E.1)-(E.4):

$$\ln(q_{t,2}) = \theta_2 \ln(w_t^*) + \varphi_2 \ln(q_{t-1,2}) + \ln(r_2) + \varepsilon_{t,2}, \ln(r_2) = \delta_2, \quad \text{(E.1)}$$

$$q_{t-1,2}=\left(\frac{q_{t,2}}{r_2 w_t^{*\theta_2}}\right)^{1/\varphi_2}, \tag{E.2}$$

$$\ln(q_{t-1,2})=\frac{\ln(q_{t,2})}{\varphi_2}-\frac{\theta_2}{\varphi_2}\ln(w_t^*)-\frac{\varepsilon_{t,2}}{\varphi_2}-\frac{\delta_2}{\varphi_2}, \tag{E.3}$$

$$\ln(q_{t,2})=-\frac{\theta_2}{\varphi_2-1}\ln(w_t^*)-\frac{\delta_2}{\varphi_2-1}+\varepsilon_{t,2}+\varphi_2\varepsilon_{t-1,2}+\varphi_2^2\varepsilon_{t-2,2}+\varphi_2^3\varepsilon_{t-3,2}+\dots . \tag{E.4}$$

Then, the expected value and variance of Eq. (E.4) are taken. Afterwards, $E[\ln(w_t^*)]=\mu_w$ and $\text{Var}[\ln(w_t^*)]=\sigma_w^2$ are substituted in Eqs. (E.5) and (E.6). The proof is now completed:

$$E[\ln(q_{t,2})]=\left(\frac{\theta_2}{1-\varphi_2}\right)E[\ln(w_t^*)]+\left(\frac{\delta_2}{1-\varphi_2}\right)=\left(\frac{\theta_2}{1-\varphi_2}\right)\mu_w+\left(\frac{\delta_2}{1-\varphi_2}\right), \tag{E.5}$$

$$\text{Var}[\ln(q_{t,2})]=\frac{\theta_2^2}{(1-\varphi_2)^2}\text{Var}[\ln(w_t^*)]+\left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right)=\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2+\left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right), \tag{E.6}$$

where σ_w^2 is variance of selling prices for distributor's goods, and μ_w is mean of selling prices for distributor's goods.

Appendix F

First, the equivalent value for $q_{t-1,2}^{\varphi_2}$ is obtained by Eq. (F.1). Then, by substituting Eq. (F.1) in Eq. (36), the distributor's order and its natural logarithm are obtained using Eqs. (F.2) and (F.3), respectively. Revising Eq. (F.3) with time lagged error terms leads to a MAX time series as it is demonstrated in Eq. (F.4). Finally, variance of retailer's order is calculated by Eqs. (F.5)-(F.7):

$$q_{t-1,2}^{\varphi_2}=\frac{q_{t,2}}{r_2 w_t^{*\theta_2}}, \tag{F.1}$$

$$O_{t,2}=q_{t,2}(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2}), \tag{F.2}$$

$$\ln(O_{t,2})=\ln(q_{t,2})+\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2}), \tag{F.3}$$

$$\ln(O_{t,2})=\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})-\frac{\theta_2}{\varphi_2-1}\ln(w_t^*)-\frac{\delta_2}{\varphi_2-1}+\varepsilon_{t,2}+\varphi_2\varepsilon_{t-1,2}+\varphi_2^2\varepsilon_{t-2,2}+\varphi_2^3\varepsilon_{t-3,2}+\dots, \tag{F.4}$$

$$\text{Var}[\ln(O_{t,2})]=\text{Var}[\ln(q_{t,2})]+\text{Var}[\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2}]+2\text{cov}(\ln(q_{t,2}),\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2}), \tag{F.5}$$

$$\text{Var}[\ln(O_{t,2})]=\left[\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2+\left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right)\right]+\text{Var}[\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})]+2E[\ln(q_{t,2})\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})]-2E[\ln(q_{t,2})]E[\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})], \tag{F.6}$$

$$\text{Var}[\ln(O_{t,2})]=\left[\frac{\theta_2^2}{(1-\varphi_2)^2}\sigma_w^2+\left(\frac{\sigma_{\varepsilon_2}^2}{1-\varphi_2^2}\right)\right]+\text{Var}[\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})]+2E[\ln(q_{t,2})\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})]-2\left[\left(\frac{\theta_2}{1-\varphi_2}\right)\mu_w+(\frac{\delta_2}{1-\varphi_2})\right]E[\ln(1+\Omega_{l_2}r_2(w_{t+1}^*)^{\theta_2}q_{t,2}^{\varphi_2-1}-\Omega_{l_2})]. \tag{F.7}$$

Appendix G1

As in the method presented by Hosoda and Disney [15], $E(\sum_{i=1}^{l_2}O_{t+i,1}|\gamma_t)$ is equal to the sum of the first l_1 terms of a geometric progression. In the geometric progression, terms are orders over lead time. Thus, using the formulation for calculating sum of the first l_1 terms of a geometric progression having $O_{t+1,1}$ as the first term, its progression ratio is $r_1v^{2\theta}p_t^\theta$. The proof is given as follows:

$$O_{t+1,1}=r_1p_{t+1}^\theta O_{t,1}=r_1v^\theta p_t^\theta O_{t,1}, \tag{G1.1}$$

$$O_{t+2,1}=r_1p_{t+2}^\theta O_{t+1,1}=r_1v^{2\theta}p_t^\theta O_{t+1,1}, \tag{G1.2}$$

$$O_{t+3,1}=r_1p_{t+3}^\theta O_{t+2,1}=r_1v^{2\theta}p_t^\theta O_{t+2,1}, \tag{G1.3}$$

$$\vdots$$

Eq. (G1.4) shows the sum of a geometric progression of orders over lead time:

$$\hat{O}_{t,1}^{l_2} = E\left(\sum_{i=1}^{l_2} O_{t+i,1}|\gamma_t\right) = \frac{r_1 v^\theta p_t^\theta O_{t,1}[1-(r_1 v^{2\theta} p_t^\theta)^{l_2}]}{1-r_1 v^{2\theta} p_t^\theta} = \nabla_{l_2} r_1 v^\theta p_t^\theta O_{t,1}, \quad \text{(G1.4)}$$

where:

$$\nabla_{l_2} = \frac{1-(r_1 v^{2\theta} p_t^\theta)^{l_2}}{1-r_1 v^{2\theta} p_t^\theta},$$

$$O_{t+1,1} = r_1 v^\theta p_t^\theta O_{t,1},$$

$$\gamma_t = \{O_{t,1}, O_{t-1,1}, O_{t-2,1}, ...\}, \quad \text{(G1.5)}$$

and:

$$p_{t+i+1} = v p_{t+i} \begin{cases} v \neq 1 & \text{if } i = 0, 1 \\ v = 1 & \text{if } i > 1 \end{cases}.$$

Appendix G2

First, Eq. (37) is used for calculating distributor's ordering quantity. The corresponding values for $S_{t,2}$ and $S_{t-1,2}$ are obtained by Eqs. (G2.1) and (G2.2). Then, these values are substituted in Eqs. (37); as the result of this substitution, Eq. (G2.3) is obtained:

$$S_{t,2} = \nabla_{l_2} r_1 v^\theta p_t^\theta O_{t,1} + k_2 \hat{\sigma}_{l_2}, \quad \text{(G2.1)}$$

$$S_{t-1,2} = \nabla_{l_2} r_1 v^\theta p_{t-1}^\theta O_{t-1,1} + k_2 \hat{\sigma}_{l_2}, \quad \text{(G2.2)}$$

$$O_{t,2} = O_{t,1} + \nabla_{l_2} r_1 v^\theta p_t^\theta O_{t,1} - \nabla_{l_2} r_1 v^\theta p_{t-1}^\theta O_{t-1,1}. \quad \text{(G2.3)}$$

The goal of this subsection is to calculate distributor's order at period $t(O_{t,2})$ by the use of retailer's order at period $t(O_{t,1})$. Therefore, retailer's order in the previous period $(O_{t-1,1})$ should be converted to $O_{t,1}$. For achieving this goal, the equivalent value of $O_{t,1}$ from Eq. (G2.4) is substituted in Eq. (G2.3) as it is indicated in Eq. (G2.5):

$$O_{t+1,1} = r_1 v^\theta p_t^\theta O_{t,1} \rightarrow O_{t,1} = r_1 v^\theta p_{t-1}^\theta O_{t-1,1}, \quad \text{(G2.4)}$$

$$O_{t,2} = O_{t,1} + \nabla_{l_2} r_1 v^\theta p_t^\theta O_{t,1} - \nabla_{l_2} O_{t,1}. \quad \text{(G2.5)}$$

Then, natural logarithm of Eq. (G2.5) is taken as follows:

$$\ln(O_{t,2}) = \ln(O_{t,1}) + \ln(1 + \nabla_{l_2} r_1 v^\theta p_t^\theta - \nabla_{l_2}) + \varepsilon_{t,2}. \quad \text{(G2.6)}$$

By substituting Eq. (D.1) in Eq. (G2.6), distributor's order is estimated by the following equation:

$$\ln(O_{t,2}) = \ln(1 + \Omega_{l_1} r_1 p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}) - \frac{\theta}{\varphi-1}\ln(p_t) + \ln(1 + \nabla_{l_2} r_1 v^\theta p_t^\theta - \nabla_{l_2}) - \frac{\delta}{\varphi-1} + \varepsilon_{t,2} + \varepsilon_{t,1} + \varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} + \varphi^3\varepsilon_{t-3} + \ldots . \quad \text{(G2.7)}$$

Finally, variance of distributor's order is calculated by Eq. (G2.8), and the proof is completed:

$$\begin{aligned}\mathrm{Var}[\ln(O_{t,2})] =& \mathrm{Var}[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})] + \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right] + \mathrm{Var}[\ln(1 + \nabla_{l_2} r_1 v^\theta p_t^\theta - \nabla_{l_2})] + \frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(1 + \Omega_{l_1} r_1 p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}), \ln(p_t)) \\ &+ 2\mathrm{Cov}(\ln(1 + \Omega_{l_1} r_1 p_{t+1}^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1}), \ln(1 + \nabla_{l_2} r_1 v^\theta p_t^\theta - \nabla_{l_2})) + \frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t), \ln(1 + \nabla_{l_2} r_1 v^\theta p_t^\theta - \nabla_{l_2})) + \sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_1}^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right). \quad \text{(G2.8)}\end{aligned}$$

Appendix H

Eq. (H.1) shows distributor's order quantity at period $t+1$:

$$O'_{t+1,2} = q_{t+1,2} + \Omega_{l_2}(q_{t+2,2} - q_{t+1,2}). \quad \text{(H.1)}$$

The corresponding values for $q_{t+1,2}$ and $q_{t+2,2}$ are substituted in Eq. (H.1), and Eq. (H.2) is generated. Then, the equivalent values for $q_{t,2}$ and $q_{t+1,2}$ are substituted in Eq. (H.2) and Eq. (H.3) is produced:

$$O'_{t+1,2} = r_2(w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2} + \Omega_{l_2} r_2 (w_{t+2}^*)^{\theta_2} q_{t+1,2}^{\varphi_2} - \Omega_{l_1} r_2 (w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2}, \quad \text{(H.2)}$$

$$O'_{t+1,2} = r_2(w_{t+1}^*)^{\theta_2}(r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2})^{\varphi_2} + \Omega_{l_2} r_2 (w_{t+2}^*)^{\theta_2}(r_2(w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2})^{\varphi_2} - \Omega_{l_1} r_2 (w_{t+1}^*)^{\theta_2}(r_2 w_t{}^{*\theta_2} q_{t-1,2}^{\varphi_2})^{\varphi_2}, \quad \text{(H.3)}$$

where $w_{t+i+1}^* = v' w_{t+i}^*$ and $(w_{t+i+1}^*)^{\theta_2} = v'^{\theta_2}(w_{t+i}^*)^{\theta_2}$, $i = 0, 1, 2, ..., l_2$. Setting $(w_{t+1}^*)^{\theta_2} = v'^{\theta_2}(w_t^*)^{\theta_2}$ and having $\varphi_2 \approx 1$, Eq. (H.3) is rewritten as follows.

$$\begin{aligned}O'_{t+1,2} =& r_2 v'^{\theta_2} w_t^{*\theta_2} r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2} \\ &+ \Omega_{l_2} r_2 v'^{2\theta_2} w_t^{*\theta_2} r_2 w_{t+1}^{*\theta_2} q_{t,2}^{\varphi_2} \\ &- \Omega_{l_1} r_2 v'^{\theta_2} w_t^{*\theta_2} r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2},\end{aligned} \tag{H.4}$$

where v is a constant number indicating how much price information is transferred from the present period to the next period. Because θ_2 is a very small quantity, it can be inferred that $v'^{2\theta_2} \approx v'^{\theta_2}$, which leads to Eq. (H.5). Finally, Eq. (H.6) shows the distributor's ordering quantity at period $t+1$. The proof is now completed:

$$\begin{aligned}O'_{t+1,2} =& r_2 v'^{\theta_2} w_t^{*\theta_2} [r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2} + \Omega_{l_2} r_2 (w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2} \\ &- \Omega_{l_1} r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2}],\end{aligned} \tag{H.5}$$

$$O'_{t+1,2} = r_2 v'^{\theta_2} w_t^{*\theta_2} O_{t,2}. \tag{H.6}$$

After obtaining distributor's order at period $t+1$, ARX time series should be calculated to forecast distributor's order at period $t+1$. The order forecasting process is necessary for measuring and reducing BWE. Eq. (H.7) shows ARX model for predicting distributor's order at period $t+1$. This equation is obtained by taking natural logarithm of Eq. (H.6):

$$\begin{aligned}\ln(O'_{t+1,2}) =& \ln(r_2) + \theta_2 \ln(v') + \theta_2 \ln(w_t^*) \\ &+ \ln(O_{t,2}) + \varepsilon_{t+1,2},\end{aligned} \tag{H.7}$$

where $\varepsilon_{t+1,2}$ is a white noise process for distributor's order forecasting at period $t+1$ with zero mean and variance of $\sigma^2_{\varepsilon_2}$.

Appendix I

First, the equivalent value for $q^{\varphi_3}_{t-1,3}$ is obtained by Eq. (I.1). Then, by substituting Eq. (I.1) in Eq. (44), the manufacturer's order is obtained as it is indicated in Eq. (I.2). Natural logarithm of Eq. (I.2) is calculated by Eq. (I.3), and its MAX time series is shown by Eq. (I.4). Finally, variance of retailer's order is calculated through Eqs. (I.5)-(I-7):

$$q^{\varphi_3}_{t-1,3} = \frac{q_{t,3}}{r_3 z_t^{*\theta_3}}, \tag{I.1}$$

$$O_{t,3} = q_{t,3}(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3}), \tag{I.2}$$

$$\ln(O_{t,3}) = \ln(q_{t,3}) + \ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3}), \tag{I.3}$$

$$\begin{aligned}\ln(O_{t,3}) =& \ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3}) \\ &- \frac{\theta_3}{\varphi_3 - 1}\ln(z_t^*) - \frac{\delta_3}{\varphi_3 - 1} + \varepsilon_{t,3} + \varphi_3 \varepsilon_{t-1,3} \\ &+ \varphi_3^2 \varepsilon_{t-2,3} + \varphi_3^3 \varepsilon_{t-3,3} + ...,\end{aligned} \tag{I.4}$$

$$\begin{aligned}\mathrm{Var}[\ln(O_{t,3})] =& \mathrm{Var}[\ln(q_{t,3})] + \mathrm{Var}[\ln(1 + \Omega l_3 r_3 \\ &(z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3}] \\ &+ 2\mathrm{cov}(\ln(q_{t,3}), \ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} \\ &q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3}),\end{aligned} \tag{I.5}$$

$$\begin{aligned}\mathrm{Var}[\ln(O_{t,3})] =& \left[\frac{\theta_3^2}{(1-\varphi_3)^2}\sigma_z^2 + \left(\frac{\sigma^2_{\varepsilon_3}}{1-\varphi_3^2}\right)\right] + \mathrm{Var}[\ln(1 \\ &+ \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})] \\ &+ 2E[\ln(q_{t,3}) \ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} \\ &q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})] - 2E[\ln(q_{t,3})]E[\ln(1 \\ &+ \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})],\end{aligned} \tag{I.6}$$

$$\begin{aligned}\mathrm{Var}[\ln(O_{t,3})] =& \left[\frac{\theta_3^2}{(1-\varphi_3)^2}\sigma_z^2 + \left(\frac{\sigma^2_{\varepsilon_3}}{1-\varphi_3^2}\right)\right] + \mathrm{Var}[\ln(1 \\ &+ \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})] \\ &+ 2E[\ln(q_{t,3}) \ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} \\ &q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})] - 2\left[\left(\frac{\theta_3}{1-\varphi_3}\right)\mu_z \right. \\ &\left. + \left(\frac{\delta_3}{1-\varphi_3}\right)\right] E[\ln(1 + \Omega_{l_3} r_3 (z^*_{t+1})^{\theta_3} \\ &q_{t,3}^{\varphi_3 - 1} - \Omega_{l_3})].\end{aligned} \tag{I.7}$$

Appendix J

The corresponding values of OUT policy for manufacturer in periods t and $t-1$ are obtained by Eqs. (J.1) and (J.2). Then, these values are substituted in Eq. (45), and Eq. (J.3) is obtained:

$$S_{t,3} = \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} O_{t,2} + k_3 \hat{\sigma}_{l_3}, \tag{J.1}$$

$$S_{t-1,3} = \Delta_{l_3} r_2 v''^{\theta} (w^*_{t-1})^{\theta} O_{t-1,2} + k_3 \hat{\sigma}_{l_3}, \tag{J.2}$$

$$\begin{aligned}O_{t,3} =& O_{t,2} + \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} O_{t,2} \\ &- \Delta_{l_3} r_2 v''^{\theta} (w^*_{t-1})^{\theta} O_{t-1,2}.\end{aligned} \tag{J.3}$$

The goal of this subsection is to calculate manufacturer's order at period $t(O_{t,3})$ using distributor's order at period $t(O_{t,2})$ as follows:

$$O_{t,3} = O_{t,2} + \nabla_{l_3} r_2 v''^{\theta} w_t^{*\theta} O_{t,2} - \nabla_{l_3} O_{t,2}. \qquad \text{(J.4)}$$

Eq. (J.5) shows the natural logarithm of Eq. (J.4):

$$\ln(O_{t,3}) = \ln(O_{t,2}) + \ln(1+\Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} - \Delta_{l_3}) + \varepsilon_{t,3}. \qquad \text{(J.5)}$$

By substituting Eq. (G.7) in Eq. (J.5), manufacturer's order is estimated by the following equation:

$$\begin{aligned}\ln(O_{t,3}) =& \ln(1 + \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} - \Delta_{l_3}) + \ln(1 \\ &+ \Omega_{l_1} r_1 (p_{t+1}^*)^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1}) - \frac{\theta}{\varphi-1}\ln(p_t^*) \\ &+ \ln(1+\nabla_{l_2} r_1 v''^{\theta} p_t^{*\theta} - \nabla_{l_2}) - \frac{\delta}{\varphi-1} + \varepsilon_{t,3} \\ &+ \varepsilon_{t,2} + \varepsilon_{t,1} + \varepsilon_t + \varphi\varepsilon_{t-1} + \varphi^2\varepsilon_{t-2} \\ &+ \varphi^3\varepsilon_{t-3} + \dots . \qquad \text{(J.6)}\end{aligned}$$

Finally, variance of manufacturer's order is calculated by Eq. (J.7). The proof is now completed:

$$\begin{aligned}\mathrm{Var}[\ln(O_{t,3})] =& \mathrm{Var}[\ln(1 + \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} - \Delta_{l_3})] \\ &+ \mathrm{Var}[\ln(1+ \Omega_{l_1} r_1 (p_{t+1}^*)^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] \\ &+ \left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2\right] + \mathrm{Var}[\ln(1 \\ &+ \nabla_{l_2} r_1 v''^{\theta} p_t^{*\theta} - \nabla_{l_2})] + 2\mathrm{Cov}(\ln(1 \\ &+ \Delta_{l_3} r_2 v^{*\theta} w_t^{*\theta} - \Delta_{l_3}), \ln(1 \\ &+ \Omega_{l_1} r_1 (p_{t+1}^*)^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})) \\ &+ 2\mathrm{Cov}(\ln(1 + \Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} - \Delta_{l_3}), \\ &\ln(1 + \nabla_{l_2} r_1 v''^{\theta} p_t^{*\theta} - \nabla_{l_2})) \\ &+ 2\mathrm{Cov}(\ln(1 + \Omega_{l_1} r_1 (p_{t+1}^*)^{\theta} q_{t,1}^{\varphi-1} \\ &- \Omega_{l_1}), \ln(1 + \nabla_{l_2} r_1 v''^{\theta} p_t^{*\theta} - \nabla_{l_2})) \\ &+ \frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*), \ln(1+\Delta_{l_3} r_2 v''^{\theta} w_t^{*\theta} \\ &- \Delta_{l_3})) + \frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*), \ln(1 \\ &+ \Omega_{l_1} r_1 (p_{t+1}^*)^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})) \\ &+ \frac{2\theta}{1-\varphi}\mathrm{Cov}(\ln(p_t^*), ln(1+\nabla_{l_2} r_1 v''^{\theta} p_t^{*\theta} \\ &- \nabla_{l_2})) + \sigma_{\varepsilon_3}^2 + \sigma_{\varepsilon_2}^2 + \sigma_{\varepsilon_1}^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right). \qquad \text{(J.7)}\end{aligned}$$

Appendix K

Eqs. (K.1) and (K.2) are used to forecast retailer's prices. In order to calculate MSE of retailer's demand, the actual values of demands are subtracted from the forecasted ones. The MSE of retailer's demand is shown in Eq. (K.3). Eq. (K.4) is obtained by substituting Eq. (K.2) in Eq. (K.3). The MSE of retailer's demand is calculated through Eq. (K.5) for the case in which the optimal values of prices are used, Eqs. (K.1) to (K.5) are shown in Box K.I.

Since some part of the price information is lost in each period of time and it is not transferred to the next period, price inequality $(p_t^*)^{ac} \leq (\lambda p_{t-1}^{a_p})^{ac}$ exists, where a_p is a declining exponent. $acln(p_t^*) \leq ac[\ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{p,t}]$ is the natural logarithm of the price inequality. Two positive terms $(1-a)\ln(q_{t-1,1})$ and $\ln(r_1)$ are subtracted from both sides of the above inequality, and the positive term $\ln(q_{t,1})$ is added to both sides as shown in Box K.II.

The following operations prove that $\mathrm{MSE}_{p2} \leq \mathrm{MSE}_{p1}$:

$$0 \leq S_{1t} \leq S_{2t} \rightarrow S_{1t}^2 \leq S_{2t}^2 \rightarrow \sum_{t=1}^{n} S_{1t}^2 \leq \sum_{t=1}^{n} S_{2t}^2$$

$$\rightarrow \mathrm{MSE}_{p2} \leq \mathrm{MSE}_{p1}.$$

The proposed method, which uses optimal prices to forecast future demands, has low forecasting error in comparison with the technique presented by Zhang and Burke [28], which forecasts prices. Therefore, the proof for Theorem 9 is completed.

Appendix L

BWE is calculated using two methods. First, BWE is quantified through the proposed method, in which optimal prices are calculated and used for forecasting demands and orders. Second, BWE is measured through the method in the literature using forecasted prices for predicting demands and orders.

B_1^* is the BWE in retailer's level, where p_t^* includes the optimal values for retailer price at period t, $t = \{1, 2, ..., n\}$. Let $\ln(q_{t,1})$ and $\ln(1+\Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})$ be two independent variables; therefore, their covariance is equal to zero. Table L.1 shows that these variables are independent.

As it is shown in Table L.1, $E[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] \times E[\ln(q_{t,1})] = E[\ln(q_{t,1})\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]$, which are equal to 0.024191.

$$p_t = \lambda p_{t-1}^{a_p}, \tag{K.1}$$

$$\ln(p_t) = \ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{p,t}, \tag{K.2}$$

$$\mathrm{MSE}_{p1} = \frac{\sum_{t=1}^{n} [\ln(q_{t,1}) - (-acln(p_t) + (1-a)\ln(q_{t-1,1}) + \ln(r_1))]^2}{n}, \quad \forall\ n > 0, \tag{K.3}$$

$$\mathrm{MSE}_{p1} = \frac{\sum_{t=1}^{n} [\ln(q_{t,1}) - (-ac[\ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{p,t}] + (1-a)\ln(q_{t-1,1}) + \ln(r_1))]^2}{n}$$

$$= \frac{\sum_{t=1}^{n} [\ln(q_{t,1}) + ac[\ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{p,t}] - (1-a)\ln(q_{t-1,1}) - \ln(r_1)]^2}{n}, \tag{K.4}$$

$$\mathrm{MSE}_{p2} = \frac{\sum_{t=1}^{n} [\ln(q_{t,1}) - (-acln(p_t^*) + (1-a)\ln(q_{t-1,1}) + \ln(r_1))]^2}{n}$$

$$= \frac{\sum_{t=1}^{n} [\ln(q_{t,1}) + (acln(p_t^*) - (1-a)\ln(q_{t-1,1}) - \ln(r_1))]^2}{n}. \tag{K.5}$$

Box K.I

$$\overbrace{\ln(q_{t,1}) + (acln(p_t^*) - (1-a)\ln(q_{t-1,1}) - \ln(r_1))}^{S_{1t}} \leq$$

$$\overbrace{\ln(q_{t,1}) + (ac[\ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{p,t}] - (1-a)\ln(q_{t-1,1}) - \ln(r_1))}^{S_{2t}}.$$

Box K.II

Table L.1. Independency of two variables.

$E[\ln(q_{t,1})]$	4.686407
$E[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]$	0.005162
$E[\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})] \times E[\ln(q_{t,1})]$	0.024191
$E[\ln(q_{t,1}) \ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]$	0.024191

Thus, $\ln(q_{t,1})$ and $\ln(1 + \Omega_{l_1} r_1 p_{t+1}^{\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})$ are two independent variables.

Eq. (L.1) shows BWE in retailer's level calculated through optimal prices as shown in Box L.I.

For the case in which forecasted prices are used, variances of demands and orders are calculated. First, variance of price is calculated using Eqs. (L.2) and (L.3). Eq. (L.2) is an Auto-Regressive (AR) model for price forecasting. Then, variance of prices is used for calculating variance of demands. Eq. (L.4) shows variance of demands, which is used as a denominator of BWE equation presented by Eq. (L.5).

$$\ln(p_t) = ln(\lambda) + a_p \ln(p_{t-1}) + \varepsilon_{t,p}, \tag{L.2}$$

$$\mathrm{Var}[\ln(p_{t-1})] = \sigma_p^2,$$

$$\mathrm{Var}[\varepsilon_{t,p}] = \sigma_{\varepsilon_p}^2 \rightarrow \mathrm{Var}[\ln(p_t)] = a_p^2 \sigma_p^2 + \sigma_{\varepsilon_p}^2, \tag{L.3}$$

$$\mathrm{Var}[\ln(q_{t,1})] = \frac{\theta^2}{(1-\varphi)^2} \mathrm{Var}[\ln(p_t)] + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)$$

$$= \frac{\theta^2}{(1-\varphi)^2} (a_p^2 \sigma_p^2 + \sigma_{\varepsilon_p}^2) + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right). \tag{L.4}$$

After calculating variance of demands, variance of orders is calculated and used as a numerator of BWE equation. Eq. (L.5) as shown in Box L.II, shows BWE

$$B_1^* = \frac{\mathrm{Var}[\ln(O_{t,1})]}{\mathrm{Var}[\ln(q_{t,1})]} = \frac{\left[\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right] + \mathrm{Var}[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{*\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]\right]}{\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right]}$$

$$= 1 + \frac{\overbrace{\mathrm{Var}[\ln(1+\Omega_{l_1} r_1 p_{t+1}^{*\theta} q_{t,1}^{\varphi-1} - \Omega_{l_1})]}^{F_1}}{\underbrace{\left[\frac{\theta^2}{(1-\varphi)^2}\sigma_p^2 + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right]}_{F_2}}. \tag{L.1}$$

Box L.I

$$B_1 = \frac{\mathrm{Var}[\ln(O_{t,1})]}{\mathrm{Var}[\ln(q_{t,1})]} = \frac{\left[\left[\frac{\theta^2}{(1-\varphi)^2}(a_p^2\sigma_p^2 + \sigma_{\varepsilon_p}^2) + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right] + \mathrm{Var}[\ln(1+\Omega_{l_1} r_1 (\lambda p_t^{a_p})^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})]\right]}{\left[\frac{\theta^2}{(1-\varphi)^2}(a_p^2\sigma_p^2 + \sigma_{\varepsilon_p}^2) + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right]}$$

$$= 1 + \frac{\mathrm{Var}\,\overbrace{[\ln(1+\Omega_{l_1} r_1 (\lambda p_t^{a_p})^\theta q_{t,1}^{\varphi-1} - \Omega_{l_1})]}^{G_1}}{\underbrace{\left[\frac{\theta^2}{(1-\varphi)^2}(a_p^2\sigma_p^2 + \sigma_{\varepsilon_p}^2) + \left(\frac{\sigma_\varepsilon^2}{1-\varphi^2}\right)\right]}_{G_2}}. \tag{L.5}$$

Box L.II

in retailer echelon when prices are forecasted instead of using the optimal values of prices.

Since some part of the price information is lost in each period of time and it is not transferred to the next period, price inequality $p_{t+1}^{*\theta} < (\lambda p_t^{a_p})^\theta$ exists; therefore, $F_1 < G_1$.

The following inequalities show that BWE is significantly reduced by utilizing the method proposed in this paper in comparison with the method used in literature. The model proposed here finds the optimal values for prices. The optimal prices are substituted in demand and order forecasting models. Finally, variances of demands and orders are calculated and BWE is quantified. However, the method in the literature uses forecasted prices, leading to higher demand amplification and more BWE value:

$$0 < a_p < 1 \rightarrow 0 < a_p^2 < 1 \rightarrow a_p^2\sigma_p^2 < \sigma_p^2$$

$$\text{and } \sigma_{\varepsilon_p}^2 \approx 0 \quad (\sigma_{\varepsilon_p}^2 \text{ is a very small value}),$$

$$a_p^2\sigma_p^2 + \sigma_{\varepsilon_p}^2 < \sigma_p^2 \rightarrow G_2 < F_2, \tag{L.6}$$

$$\begin{cases} F_1 < G_1 \\ F_2 > G_2 \end{cases} \rightarrow B_1^* < B_1. \tag{L.7}$$

Similarly, it can be proved that BWE in distributor's and manufacturer's echelons is minimal at optimal price and lead time; however, the proof is not included here for briefness.

Appendix M

In this part of the paper, the theoretical and practical distinctions between the demand received by the distributor (or the manufacturer) and its downstream order are described.

M.1. In theory

The demand received by the manufacturer differs from its downstream order as follows [31,32]:

$$q_{t,3} = q_{t,2} + (O_{t,2} - O_{t-1,2}),$$

$$O_{t,2} = m_{t,2} + z_2\sqrt{v_{t,2}},$$

where $m_{t,2} = E(\sum_{i=1}^{l_3} q_{2,t+i}|q_{2,t})$ is the mean of demands and $v_{t,2} = \text{var}(\sum_{i=1}^{l_3} q_{2,t+i}|q_{2,t})$ is the variance of demands.

As it is observable from the above equations, the demand received by the manufacturer ($q_{t,3}$) is not equal to its downstream order ($O_{t,2}$). Instead, the demand received by the distributor is equal to the demand of retailer plus the difference between retailer's orders at two consecutive periods of time. In this paper, the theoretical distinction between the demand received by the distributor and its downstream order is modelled as follows:

$$\begin{aligned} q_{t,3} =& r_3 w_t^{*-a_3c_3} q_{t-1,3}^{1-a_3} \neq O_{t,2} = q_{t,2} \\ &+ (\Omega_{l_2} r_2 (w_{t+1}^*)^{\theta_2} q_{t,2}^{\varphi_2} - \Omega_{l_2} r_2 w_t^{*\theta_2} q_{t-1,2}^{\varphi_2}) \\ \ln(q_{t,3}) =& \theta_3 \ln(z_t^*) + \varphi_3 \ln(q_{t-1,3}) + \ln(r_3) + \varepsilon_{t,3} \\ &\neq \ln(O_{t,2}) = \ln(q_{t,2}) + \ln(1 + \Omega_{l_2} r_2 w_{t+1}^{\theta_2} q_{t,2}^{\varphi_2 - 1} \\ &- \Omega_{l_2}) + \varepsilon_{t,2}. \end{aligned}$$

Both the demand received by the manufacturer ($q_{t,3}$) and the logarithm of the demand received by the manufacturer ($\ln(q_{t,3})$) differ from distributor's order ($O_{t,2}$) and logarithm of distributor's order ($\ln(O_{t,2})$).

M.2. In practice

In this paper, a three-echelon auto-parts supply chain has been practically investigated. In a supply chain, the manufacturer requires to have distributor's demand in advance in order to be able to produce adequate products. Assume that the manufacturer decides to provide the raw material to produce the next week's products. In the current week, the manufacturer does not have market demand for the next week. Thus, the manufacturer uses the forecasted demand of the distributor, which is predicted by demand planning department. Manufacturer will place its order for providing the required raw material based on the forecasted demand of the distributor. Then, at the end of forecasting period, the distributor places its actual order and manufacturer will receive actual demand of the distributor. Thus, the distributor's order differs from demand that the manufacturer receives from demand forecasting department. This occurs due to the fact that actual demand of distributor is not available in the planning period (current week); hence, its forecasted demand is used. This paper showed that using demand of each entity for calculating its order quantities would reduce BWE significantly in comparison with the method in which downstream orders were used. This is due to the fact that downstream orders accumulate forecasting errors; however, using demand of each entity only includes forecast errors of one stage.

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