Scientia Iranica E (2018) 25(3), 1768–1781

Sharif University of Technology
Scientia Iranica
Transactions E: Industrial Engineering
http://scientiairanica.sharif.edu

Research Note

Reliability analysis of three-component mixture of distributions

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Received 3 November 2015; received in revised form 13 October 2016; accepted 28 January 2017

KEYWORDS
Three-component mixture distributions;
Reliability analysis;
Failure rate function;
Mean residual life function;
Mean waiting time function.

Abstract. This article focuses on studying three-component mixture of Exponential, Rayleigh, Pareto, and Burr Type-XII distributions in relation to reliability analysis. The main purpose of this study is to derive algebraic expressions for different functions of survival time. For these three-component mixtures of distributions, the cumulative distribution function, hazard rate function, cumulative hazard rate function, reversed hazard rate function, mean residual life function, and mean waiting time function are discussed. To study the behaviors of different reliability functions, numerical results are presented for fixed values of parameters.

1. Introduction

Reliability analysis is a tool kit of statistical procedures for analyzing time-to-failure data. Usually, reliability analysis is carried out using the classical or Bayesian statistical analysis of parametric reliability models. Reliability analysis datasets are represented with a single univariate or multivariate statistical distributions such as Exponential, Rayleigh, inverse Weibull, Pareto and Burr distributions [1-6]. Recently, some other distributions have been used to model the survival data. The use of mixture distributions has obtained popularity for modeling the heterogeneous data. Mixture of distributions is useful because it is applied to represent heterogeneous dataset where there is evidence of multimodality. Chen et al. [7] used a two-component mixture model for the analysis of cancer survival data generalizing an earlier idea in the study of Berkson and Gage [8]. In Qian's study [9], a similar model of a mixture of a Weibull component and a surviving fraction in the context of a lung cancer trial are considered. Angelis et al. [10] proposed an application of a mixture model to relative survival rates of colon cancer patients from the Finnish population-based cancer registry including major survival determinants as explicative covariates. Marin et al. [11] illustrated how Bayesian methods can be used to fit a mixture of Weibull models with an unknown number of components to heterogeneous, possibly right-censored survival data using a birth death MCMC algorithm. Abu-Taleb et al. [12] presented the Bayesian estimation of lifetime parameters of Exponential distributions when survival and censoring times are both exponentially distributed. Erişoğlu et al. [13] studied the mixture model of two different distributions to analyze the heterogeneous survival data. Krishna and Malik [14] presented the reliability estimation in Maxwell distribution using progressively type-II censored data. Ali [15] discussed

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doi: 10.24200/sci.2017.4441

the reliability properties of two-component mixture of inverse Rayleigh distributions. Also, a valuable account on the reliability properties and analysis of mixture distributions is given in [16-26], and many others alike.

Similar to two-component mixture modeling, some authors have discussed situations where data are assumed to come from a three-component mixture of distributions. For example, in order to know the proportion of failure due to a certain cause and improve the manufacturing process, Acheson and McElwee [27] divided electronic tube failures into three types of defects: gaseous defects, mechanical defects, and normal deterioration of the cathode. Further, as mentioned by Tahir et al. [28], Davis [29] reported a mixture data on lifetimes of many components used in aircraft sets. To illustrate the proposed methodology for a three-component mixture distribution, Tahir et al. [28] (2016) used the time-to-failure data on three components: transmitter tube, combination of transformers, and combination of relays. Thus, the existence of three-component mixtures of distributions is evident.

There are two approaches to statistical modeling and analyses: a model is assumed to exist in nature to generate data, and parameters of the models are estimated on the basis of these data; data are observed first and a suitable model is fitted to them. The first approach seems more logical. Following the first approach, it is assumed in this study that there exist models in nature, capable of yielding data as coming from a three-component mixture of distributions. We, specifically, assumed the existence of three-component mixture of Exponential, Rayleigh, Pareto, and Burr Type-XII distributions. For these mixtures of distributions, different reliability functions are studied both algebraically and numerically which is the main purpose of this article. This study may be useful for the practitioners dealing with the data coming from a three-component mixture of Exponential, Rayleigh, Pareto, and Burr Type-XII distributions.

A random variable, Y, is said to follow a finite mixture distribution with q components if the density function of Y can be written in the form:

$$f(y) = \sum_{m=1}^{q} p_m f_m(y),$$

where p_m $(m = 1, 2, ..., q)$ is the mth mixing proportion, such that:

$$p_q = 1 - \sum_{m=1}^{q-1} p_m,$$

and $f_m(y)$ is the mth component density function. A finite three-component mixture of distributions with mixing proportions p_1 and p_2 has the probability density function (pdf) as follows:

$$f(y;\Psi) = p_1 f_1(y;\Psi_1) + p_2 f_2(y;\Psi_2) + (1-p_1-p_2) f_3(y;\Psi_3)$$
$$p_1, p_2 \geq 0, p_1 + p_2 \leq 1, \quad (1)$$

where, $\Psi = (\lambda_1, \lambda_2, \lambda_3, p_1, p_2)$, $\Psi_m = \lambda_m$, $m = 1, 2, 3$, and $f_m(y;\Psi_m)$ is the pdf of the mth component density. The cumulative distribution function (cdf) for a finite three-component mixture distribution can be written as follows:

$$F(y;\Psi) = p_1 F_1(y;\Psi_1) + p_2 F_2(y;\Psi_2) + (1-p_1-p_2) F_3(y;\Psi_3), \quad (2)$$

where $F_m(y;\Psi_m)$ is the cdf of the mth component density.

2. Different functions of survival time

In reliability theory, classification of lifetime models is defined in terms of different functions of survival time such as reliability (survival), failure rate (hazard rate), cumulative hazard rate, reversed hazard rate, mean residual life, and mean waiting time functions. We now define these functions for a three-component mixture of distributions.

Let y be the survival time. The reliability function for a three-component mixture of distributions is defined as:

$$R(y;\Psi) = 1 - \int_0^y f(u;\Psi) du = 1 - F(y;\Psi),$$

$$R(y;\Psi) = p_1 R_1(y;\Psi_1) + p_2 R_2(y;\Psi_2) + (1-p_1-p_2) R_3(y;\Psi_3), \quad (3)$$

where $R_m(y;\Psi_m)$ is the reliability function of the mth component.

The hazard rate function for a three-component mixture of distributions is defined as ratio of lifetime model to reliability function:

$$h(y;\Psi) = \frac{f(y;\Psi)}{R(y;\Psi)}$$
$$= \frac{p_1 f_1(y;\Psi_1) + p_2 f_2(y;\Psi_2) + (1-p_1-p_2) f_3(y;\Psi_3)}{p_1 R_1(y;\Psi_1) + p_2 R_2(y;\Psi_2) + (1-p_1-p_2) R_3(y;\Psi_3)}. \quad (4)$$

The cumulative hazard rate function, $H(y;\Psi)$, and reversed hazard rate function, $r(y;\Psi)$, for a three-component mixture of distributions are given by:

$$H(y;\Psi)=\int_0^y h(u;\Psi)du=-\ln\{1-F(y;\Psi)\}=-\ln R(y;\Psi), \tag{5}$$

and:

$$r(y;\Psi)=\frac{f(y;\Psi)}{F(y;\Psi)}$$

$$=\frac{p_1f_1(y;\Psi_1)+p_2f_2(y;\Psi_2)+(1-p_1-p_2)f_3(y;\Psi_3)}{p_1F_1(y;\Psi_1)+p_2F_2(y;\Psi_2)+(1-p_1-p_2)F_3(y;\Psi_3)}. \tag{6}$$

In case $\frac{H(y;\Psi)}{y}$ is decreasing (increasing), one obtains the decreasing failure rate average (increasing failure rate average) family since the average failure rate in that case will be decreasing (increasing).

The Mean Residual Life (MRL) function at a given time y measures the expected remaining life time of an individual of age y. It is denoted by $m(y;\Psi)$. The MRL function or life expectancy function for a three-component mixture of distributions is defined as follows:

$$m(y;\Psi)=\frac{1}{R(y;\Psi)}\int_y^\infty u\,f(u;\Psi)du-y,$$

$$m(y;\Psi)=\frac{1}{R(y;\Psi)}\left\{E(Y)-\int_0^y u\,f(u;\Psi)du\right\}-y. \tag{7}$$

Increasing (decreasing) mean residual life function $m(y;\Psi)$ produces the family of increasing mean residual life (decreasing mean residual life) distributions.

Another important function in reliability analysis is of Mean Waiting Time (MWT). The MWT function is known as expected inactivity time function. For a three-component mixture of distributions, the MWT function of an item failed in an interval $[0,\ y]$ is defined as follows:

$$\bar{\mu}(y;\Psi)=y-\left\{\frac{1}{F(y;\Psi)}\int_0^y uf(u;\Psi)du\right\}. \tag{8}$$

3. Reliability properties of a three-component mixture of exponential distributions

Using Eqs. (1) and (2), a finite three-Component Mixture of Exponential Distributions (3-CMED) with mixing proportions p_1 and p_2 has the pdf and cdf as follows:

$$f(y;\Psi)=p_1\lambda_1\exp(-\lambda_1y)+p_2\lambda_2\exp(-\lambda_2y)$$
$$+(1-p_1-p_2)\lambda_3\exp(-\lambda_3y), \tag{9}$$
$$0<y<\infty,\quad \lambda_m>0,\quad m=1,2,3,$$

$$F(y;\Psi)=1-p_1\exp(-\lambda_1y)-p_2\exp(-\lambda_2y)$$
$$-(1-p_1-p_2)\exp(-\lambda_3y). \tag{10}$$

3.1. Reliability and failure rate functions for a 3-CMED

The reliability function or survival function for a 3-CMED is written as:

$$R(y;\Psi)=p_1\exp(-\lambda_1y)+p_2\exp(-\lambda_2y)$$
$$+(1-p_1-p_2)\exp(-\lambda_3y) \tag{11}$$
$$0<y<\infty,\quad \lambda_m>0,\quad m=1,2,3.$$

The failure rate function or hazard rate function for a 3-CMED is defined as shown in Box I.

The behavior of hazard rate function for the 3-CMED for some fixed values of component and proportion parameters is depicted in Figures 1-3 in which the effect of parameters $\lambda_1,\lambda_2,\lambda_3,p_1$ and p_2 on hazard rate for the 3-CMED can be observed. These graphs also explain the flexibility of the hazard rate for a 3-CMED.

Using the expression in Eq. (12), the hazard rate of a 3-CMED is evaluated for the parametric values fixed in Figures 1-3. The numerical results, so obtained, are presented in Table 1.

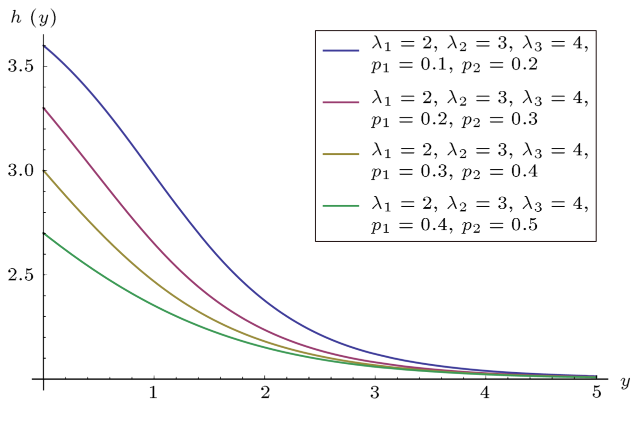

Figure 1. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1,\lambda_2,\lambda_3,p_1,p_2)=(2,\ 3,\ 4,\ 0.1,\ 0.2),(2,\ 3,\ 4,\ 0.2,\ 0.3),(2,\ 3,\ 4,\ 0.3,\ 0.4),(2,\ 3,\ 4,\ 0.4,\ 0.5)\}$.

$$h(y;\Psi)=\frac{p_1\lambda_1\exp(-\lambda_1y)+p_2\lambda_2\exp(-\lambda_2y)+(1-p_1-p_2)\lambda_3\exp(-\lambda_3y)}{p_1\exp(-\lambda_1y)+p_2\exp(-\lambda_2y)+(1-p_1-p_2)\exp(-\lambda_3y)}. \tag{12}$$

Box I

$$H(y;\Psi) = -\ln\left\{p_1\exp(-\lambda_1 y) + p_2\exp(-\lambda_2 y) + (1-p_1-p_2)\exp(-\lambda_3 y)\right\}, \tag{13}$$

and

$$r(y;\Psi) = \frac{p_1\lambda_1\exp(-\lambda_1 y) + p_2\lambda_2\exp(-\lambda_2 y) + (1-p_1-p_2)\lambda_3\exp(-\lambda_3 y)}{1 - p_1\exp(-\lambda_1 y) - p_2\exp(-\lambda_2 y) - (1-p_1-p_2)\exp(-\lambda_3 y)}. \tag{14}$$

Box II

$$m(y;\Psi) = \frac{p_1\frac{1}{\lambda_1}\exp(-\lambda_1 y) + p_2\frac{1}{\lambda_2}\exp(-\lambda_2 y) + (1-p_1-p_2)\frac{1}{\lambda_3}\exp(-\lambda_3 y)}{p_1\exp(-\lambda_1 y) + p_2\exp(-\lambda_2 y) + (1-p_1-p_2)\exp(-\lambda_3 y)}. \tag{15}$$

Box III

From Figures 1-3 and the entries in Table 1, it is obvious that, in general, failure rate for a 3-CMED follows a decreasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, failure rate decreases as proportion parameters increase (see row 1 of Table 1). On the other hand, when $\lambda_1 < \lambda_2 < \lambda_3$, failure rate increases as component parameters increase (see rows 2 and 3 of Table 1). Also, there are higher chances of failure when both the component and proportion parameters are relatively larger (see row 2 of Table 1).

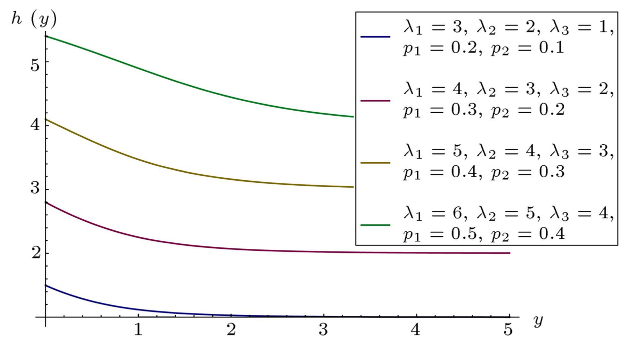

Figure 2. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (3, 2, 1, 0.2, 0.1), (4, 3, 2, 0.3, 0.2), (5, 4, 3, 0.4, 0.3), (6, 5, 4, 0.5, 0.4)\}$.

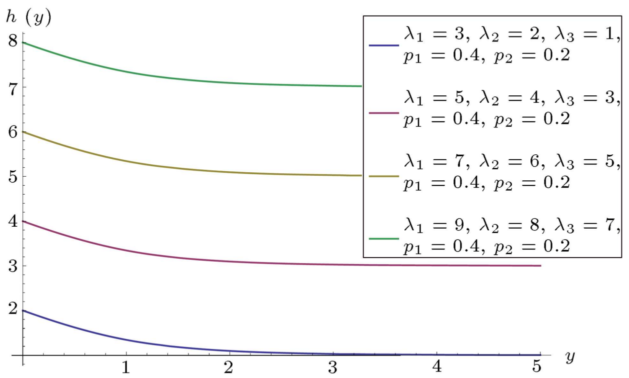

Figure 3. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2), (9, 8, 7, 0.4, 0.2)\}$.

3.2. Cumulative hazard rate and reversed hazard rate functions for a 3-CMED

The cumulative hazard rate function, $H(y;\Psi)$, and reversed hazard rate function, $r(y;\Psi)$, for a 3-CMED using Eqs. (5) and (6) are obtained by Eqs. (13) and (14), as shown in Box II.

3.3. Mean residual life and mean waiting time functions for a 3-CMED

The MRL function or life expectancy function for a 3-CMED is obtained by Eq. (15) as shown in Box III. Using the above expression Eq. (15), the MRL of a 3-CMED is evaluated for the parametric values fixed in

Table 1. Hazard rate of a 3-CMED.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
2, 3, 4, 0.1, 0.2	2.980380	2.013920	2.000090
2, 3, 4, 0.2, 0.3	2.649940	2.010230	2.000070
2, 3, 4, 0.3, 0.4	2.468170	2.008990	2.000060
2, 3, 4, 0.4, 0.5	2.353170	2.008370	2.000050
3, 2, 1, 0.2, 0.1	1.119030	1.000990	1.000010
4, 3, 2, 0.3, 0.2	2.252010	2.002740	2.000020
5, 4, 3, 0.4, 0.3	3.470680	3.006810	3.000050
6, 5, 4, 0.5, 0.4	4.897300	4.026680	4.000180
3, 2, 1, 0.4, 0.2	1.344590	1.003450	1.000020
5, 4, 3, 0.4, 0.2	3.344590	3.003450	3.000020
7, 6, 5, 0.4, 0.2	5.344590	5.003450	5.000020
9, 8, 7, 0.4, 0.2	7.344590	7.003450	7.000020

$$\bar{\mu}(y;\Psi) = \frac{y - \frac{p_1}{\lambda_1}\Big\{\exp(-\lambda_1 y) - 1\Big\} - \frac{p_2}{\lambda_2}\Big\{\exp(-\lambda_2 y) - 1\Big\} - \frac{(1-p_1-p_2)}{\lambda_3}\Big\{\exp(-\lambda_3 y) - 1\Big\}}{1 - p_1\exp(-\lambda_1 y) - p_2\exp(-\lambda_2 y) - (1-p_1-p_2)\exp(-\lambda_3 y)}. \quad (16)$$

Box IV

Figures 1-3. The numerical results, so obtained, are showcased in Table 2.

From the entries in Table 2, it is seen that, in general, MRL for a 3-CMED follows an increasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, MRL increases as proportion parameters increase (see row 1 of Table 2). On the other hand, when $\lambda_1 < \lambda_2 < \lambda_3$, MRL decreases as component parameters increase (see rows 2 and 3 of Table 2). Using Eq. (8), the MWT function for a 3-CMED is obtained by Eq. (16) as shown in Box IV.

Using Eq. (16), the MWT of a 3-CMED is evaluated for the parametric values fixed in Figures 1-3. The numerical results, so obtained, are given in Table 3.

From the entries in Table 3, it is observed that, in general, MWT for a 3-CMED follows an increasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, MWT increases as proportion parameters increase (see row 1 of Table 2). On the other hand, when $\lambda_1 < \lambda_2 < \lambda_3$, MWT decreases as component parameters increase (see rows 2 and 3 of Table 2).

4. Reliability properties of a three-component mixture of Rayleigh distributions

Using Eqs. (1) and (2), a finite three-component mixture of Rayleigh distributions (3-CMRD) with mixing proportions, p_1 and p_2, has the pdf and cdf as:

$$f(y;\Psi) = p_1\frac{y}{\lambda_1^2}\exp\left(-\frac{y^2}{2\lambda_1^2}\right) + p_2\frac{y}{\lambda_2^2}\exp\left(-\frac{y^2}{2\lambda_2^2}\right) + (1-p_1-p_2)\frac{y}{\lambda_3^2}\exp\left(-\frac{y^2}{2\lambda_3^2}\right),$$

$$0 < y < \infty, \quad \lambda_m > 0. \quad m = 1,2,3, \quad (17)$$

$$F(y;\Psi) = 1 - p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right) - p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right) - \left(1 - p_1 - p_2\right)\exp\left(-\frac{y^2}{2\lambda_3^2}\right). \quad (18)$$

4.1. Reliability and failure rate functions for a 3-CMRD

The reliability function or survival function for a 3-CMRD is written as follows:

$$R(y;\Psi) = p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right) + p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right) + \left(1 - p_1 - p_2\right)\exp\left(-\frac{y^2}{2\lambda_3^2}\right),$$

$$0 < y < \infty,\ \lambda_m > 0, \qquad m = 1,2,3. \quad (19)$$

Table 2. MRL of a 3-CMED.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
2, 3, 4, 0.1, 0.2	0.366027	0.497706	0.499985
2, 3, 4, 0.2, 0.3	0.406593	0.498304	0.499989
2, 3, 4, 0.3, 0.4	0.428908	0.498505	0.499990
2, 3, 4, 0.4, 0.5	0.443027	0.498605	0.499991
3, 2, 1, 0.2, 0.1	0.952296	0.999511	0.999997
4, 3, 2, 0.3, 0.2	0.463508	0.499545	0.499997
5, 4, 3, 0.4, 0.3	0.297994	0.332768	0.333330
6, 5, 4, 0.5, 0.4	0.208717	0.248670	0.249991
3, 2, 1, 0.4, 0.2	0.861899	0.998291	0.999989
5, 4, 3, 0.4, 0.2	0.308037	0.333048	0.333331
7, 6, 5, 0.4, 0.2	0.189491	0.199885	0.199999
9, 8, 7, 0.4, 0.2	0.137111	0.142796	0.142857

Table 3. MWT of a 3-CMED.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
2, 3, 4, 0.1, 0.2	1.326540	5.291690	10.29170
2, 3, 4, 0.2, 0.3	1.374520	5.325040	10.32500
2, 3, 4, 0.3, 0.4	1.424020	5.358400	10.35830
2, 3, 4, 0.4, 0.5	1.475120	5.391760	10.39170
3, 2, 1, 0.2, 0.1	2.154490	5.839520	10.81700
4, 3, 2, 0.3, 0.2	1.475810	5.391780	10.39170
5, 4, 3, 0.4, 0.3	1.277660	5.255000	10.25500
6, 5, 4, 0.5, 0.4	1.194010	5.188330	10.18830
3, 2, 1, 0.4, 0.2	1.819170	5.645900	10.63350
5, 4, 3, 0.4, 0.2	1.289110	5.263330	10.26330
7, 6, 5, 0.4, 0.2	1.173980	5.170480	10.17050
9, 8, 7, 0.4, 0.2	1.127060	5.126590	10.12660

$$h(y;\Psi)=\frac{p_1\frac{y}{\lambda_1^2}\exp\left(-\frac{y^2}{2\lambda_1^2}\right)+p_2\frac{y}{\lambda_2^2}\exp\left(-\frac{y^2}{2\lambda_2^2}\right)+(1-p_1-p_2)\frac{y}{\lambda_3^2}\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}{p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right)+p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right)+(1-p_1-p_2)\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}. \tag{20}$$

Box V

Table 4. Hazard rate of a 3-CMRD.

$\lambda_1,\lambda_2,\lambda_3,p_1,p_2$	$Y=1$	$Y=5$	$Y=10$
12, 14, 16, 0.1, 0.2	0.004449	0.022190	0.044054
12, 14, 16, 0.2, 0.3	0.004872	0.024280	0.048072
12, 14, 16, 0.3, 0.4	0.005295	0.026392	0.052263
12, 14, 16, 0.4, 0.5	0.005719	0.028527	0.056635
12, 10, 8, 0.2, 0.1	0.013320	0.065804	0.126239
14, 12, 10, 0.3, 0.2	0.007917	0.039301	0.076806
16, 14, 12, 0.4, 0.3	0.005176	0.025784	0.050983
18, 16, 14, 0.5, 0.4	0.003616	0.018055	0.035965
10, 8, 6, 0.5, 0.3	0.015220	0.073516	0.133827
12, 10, 8, 0.5, 0.3	0.009592	0.047322	0.090994
14, 12, 10, 0.5, 0.3	0.006633	0.032957	0.064684
16, 14, 12, 0.5, 0.3	0.004872	0.024280	0.048072

The failure rate function or hazard rate function for a 3-CMRD is defined as shown in Box V.

The trend of the hazard rate function (Eq. (20)) for some fixed values of component and proportion parameters is depicted in Figures 4-6. From Figures 4-6, the effects of parameters $\lambda_1,\lambda_2,\lambda_3,p_1$ and p_2 on hazard rate for the 3-CMRD can be observed. Flexibility of hazard rate function can be seen in Figures 4-6.

Using Eq. (20), the hazard rate of a 3-CMRD is evaluated for the parametric values fixed in Figures 4-6. The numerical results, so obtained, are showcased in Table 4.

From Figures 4-6 and the entries in Table 4, it is noticed that, in general, failure rate for a 3-CMRD follows an increasing trend over time. When $\lambda_1,\lambda_2,\lambda_3$ failure rate increases as proportion parameters increase (see row 1 of Table 4). On the other hand, when $\lambda_1,\lambda_2,\lambda_3$ failure rate decreases as component parameters increase (see rows 2 and 3 of Table 4). Also, there are higher chances of failure when both the component and proportion parameters are relatively smaller (see row 2 of Table 4).

4.2. Cumulative hazard rate and reversed hazard rate functions for a 3-CMRD

The cumulative hazard rate function, $H(y;\Psi)$, and reversed hazard rate function, $r(y;\Psi)$, for a 3-CMRD

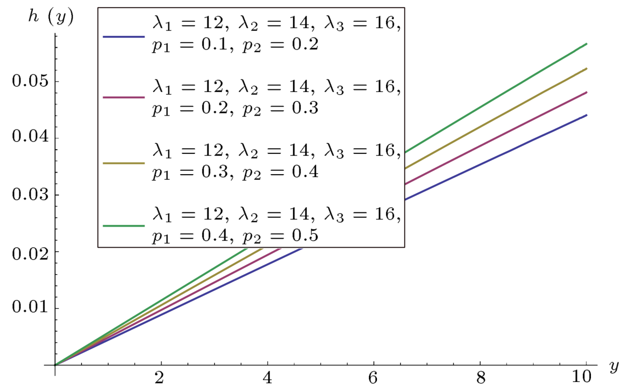

Figure 4. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1,\lambda_2,\lambda_3,p_1,p_2)=(12, 14, 16, 0.1, 0.2), (12, 14, 16, 0.2, 0.3), (12, 14, 16, 0.3, 0.4), (12, 14, 16, 0.4, 0.5)\}$.

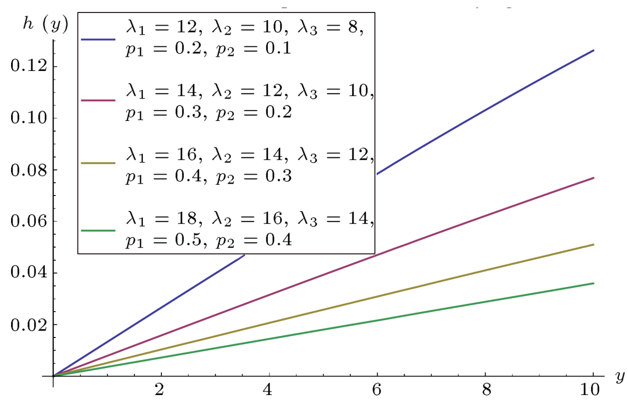

Figure 5. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1,\lambda_2,\lambda_3,p_1,p_2)=(12, 10, 8, 0.2, 0.1), (14, 12, 10, 0.3, 0.2), (16, 14, 12, 0.4, 0.3), (18, 16, 14, 0.5, 0.4)\}$.

using Eqs. (5) and (6) are obtained by Eqs. (21) and (22), respectively, as shown in Box VI.

4.3. Mean residual life and mean waiting time functions for a 3-CMRD

The MRL function or life expectancy function for a 3-CMRD is obtained by Eq. (23) as shown in Box VII. Using Eq. (23), the MRL of a 3-CMRD is evaluated for the parametric values fixed in Figures 4-6. The numerical results, so obtained, are given in Table 5.

From the entries in Table 5, it is seen that, in general, MRL for a 3-CMRD follows a decreasing trend

$$H(y;\Psi) = -\ln\left\{p_1 \exp\left(-\frac{y^2}{2\lambda_1^2}\right) + p_2 \exp\left(-\frac{y^2}{2\lambda_2^2}\right) + \left(1-p_1-p_2\right)\exp\left(-\frac{y^2}{2\lambda_3^2}\right)\right\}, \tag{21}$$

and

$$r(y;\Psi) = \frac{p_1 \frac{y}{\lambda_1^2}\exp\left(-\frac{y^2}{2\lambda_1^2}\right) + p_2 \frac{y}{\lambda_2^2}\exp\left(-\frac{y^2}{2\lambda_2^2}\right) + (1-p_1-p_2)\frac{y}{\lambda_3^2}\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}{1 - p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right) - p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right) - (1-p_1-p_2)\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}. \tag{22}$$

Box VI

$$m(y;\Psi) = \frac{p_1\lambda_1\sqrt{\frac{\pi}{2}}\left\{1-Erf\left(\frac{y}{\sqrt{2}\lambda_1}\right)\right\} + p_2\lambda_2\sqrt{\frac{\pi}{2}}\left\{1-Erf\left(\frac{y}{\sqrt{2}\lambda_2}\right)\right\} + (1-p_1-p_2)\lambda_3\sqrt{\frac{\pi}{2}}\left\{1-Erf\left(\frac{y}{\sqrt{2}\lambda_3}\right)\right\}}{p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right) + p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right) + \left(1-p_1-p_2\right)\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}. \tag{23}$$

Box VII

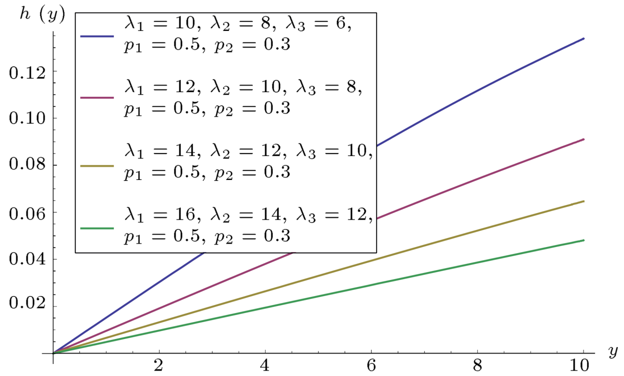

Figure 6. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (10, 8, 6, 0.5, 0.3), (12, 10, 8, 0.5, 0.3), (14, 12, 10, 0.5, 0.3), (16, 14, 12, 0.5, 0.3)\}$.

Table 5. MRL of a 3-CMRD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
12, 14, 16, 0.1, 0.2	18.09130	14.94920	12.15690
12, 14, 16, 0.2, 0.3	17.34140	14.23790	11.52970
12, 14, 16, 0.3, 0.4	16.59120	13.51910	10.87580
12, 14, 16, 0.4, 0.5	15.84060	12.79260	10.19340
12, 10, 8, 0.2, 0.1	10.35080	7.721280	5.929370
14, 12, 10, 0.3, 0.2	13.59350	10.70350	8.428180
16, 14, 12, 0.4, 0.3	16.84150	13.76350	11.11020
18, 16, 14, 0.5, 0.4	20.09260	16.87520	13.92330
10, 8, 6, 0.5, 0.3	9.855810	7.325830	5.642930
12, 10, 8, 0.5, 0.3	12.34580	9.549830	7.433610
14, 12, 10, 0.5, 0.3	14.84200	11.86790	9.419580
16, 14, 12, 0.5, 0.3	17.34140	14.23790	11.52970

over time. When $\lambda_1 < \lambda_2 < \lambda_3$, MRL decreases as proportion parameters increase (see row 1 of Table 5). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, MRL increases as component parameters increase (see rows 2 and 3 of Table 5). Using Eq. (8), the MWT function for a 3-CMRD is obtained by Eq. (24) as shown in Box VIII.

Using Eq. (24), the MWT of a 3-CMRD is evaluated for the parametric values fixed in Figures 4-6. The numerical results, so obtained, are presented in Table 6.

From the entries in Table 6, it is obvious that, in general, MWT for a 3-CMRD follows an increasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, MWT increases as

$$\mu(y;\Psi) = \frac{y - p_1\lambda_1\sqrt{\frac{\pi}{2}}Erf\left(\frac{y}{\sqrt{2}\lambda_1}\right) - p_2\lambda_2\sqrt{\frac{\pi}{2}}Erf\left(\frac{y}{\sqrt{2}\lambda_2}\right) - (1-p_1-p_2)\lambda_3\sqrt{\frac{\pi}{2}}Erf\left(\frac{y}{\sqrt{2}\lambda_3}\right)}{1 - p_1\exp\left(-\frac{y^2}{2\lambda_1^2}\right) - p_2\exp\left(-\frac{y^2}{2\lambda_2^2}\right) - \left(1-p_1-p_2\right)\exp\left(-\frac{y^2}{2\lambda_3^2}\right)}. \tag{24}$$

Box VIII

Table 6. MWT of a 3-CMRD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
12, 14, 16, 0.1, 0.2	0.333488	1.686060	3.488280
12, 14, 16, 0.2, 0.3	0.333505	1.688110	3.504600
12, 14, 16, 0.3, 0.4	0.333519	1.689840	3.518610
12, 14, 16, 0.4, 0.5	0.333531	1.691330	3.530760
12, 10, 8, 0.2, 0.1	0.333810	1.726250	3.807210
14, 12, 10, 0.3, 0.2	0.333617	1.702140	3.616060
16, 14, 12, 0.4, 0.3	0.333516	1.689510	3.515760
18, 16, 14, 0.5, 0.4	0.333458	1.682190	3.457630
10, 8, 6, 0.5, 0.3	0.333940	1.741480	3.904260
12, 10, 8, 0.5, 0.3	0.333691	1.711170	3.683860
14, 12, 10, 0.5, 0.3	0.333572	1.696450	3.570310
16, 14, 12, 0.5, 0.3	0.333505	1.688110	3.504600

proportion parameters increase (see row 1 of Table 6). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, MWT decreases as component parameters increase (see rows 2 and 3 of Table 6).

5. Reliability properties of a three-component mixture of Pareto distributions

Using Eqs. (1) and (2), a finite three-Component Mixture of Pareto Distributions (3-CMPD) with mixing proportions p_1 and p_2 has the pdf and cdf as follows:

$$f(y;\Psi) = p_1\lambda_1 y^{-(\lambda_1+1)} + p_2\lambda_2 y^{-(\lambda_2+1)} + (1-p_1-p_2)\lambda_3 y^{-(\lambda_3+1)}, \quad (25)$$

$$1<y<\infty, \quad \lambda_m>0, \quad m=1,2,3$$

$$F(y;\Psi) = 1-p_1 y^{-\lambda_1} - p_2 y^{-\lambda_2} - (1-p_1-p_2)y^{-\lambda_3}. \quad (26)$$

5.1. Reliability and failure rate functions for a 3-CMPD

The reliability function or survival function for a 3-CMPD is written as follows:

$$R(y;\Psi) = p_1 y^{-\lambda_1} + p_2 y^{-\lambda_2} + (1-p_1-p_2)y^{-\lambda_3},$$

$$1<y<\infty, \quad \lambda_m>0, \quad m=1,2,3. \quad (27)$$

The failure rate function or hazard rate function for a 3-CMPD is defined as follows:

$$h(y;\Psi) = \frac{p_1\lambda_1 y^{-(\lambda_1+1)} + p_2\lambda_2 y^{-(\lambda_2+1)} + (1-p_1-p_2)\lambda_3 y^{-(\lambda_3+1)}}{p_1 y^{-\lambda_1} + p_2 y^{-\lambda_2} + (1-p_1-p_2)y^{-\lambda_3}}. \quad (28)$$

The trend of the hazard rate Function (Eq. (28)) for some values of component and proportion parameters is shown in Figures 7-9. From Figures 7-9, the effects of parameters $\lambda_1, \lambda_2, \lambda_3$ and p_2 on hazard rate for the 3-CMPD can be observed. These graphs also explain the flexibility of the hazard rate for a 3-CMPD.

Using Eq. (28), the hazard rate of a 3-CMPD is evaluated for the parametric values fixed in Figures 7-

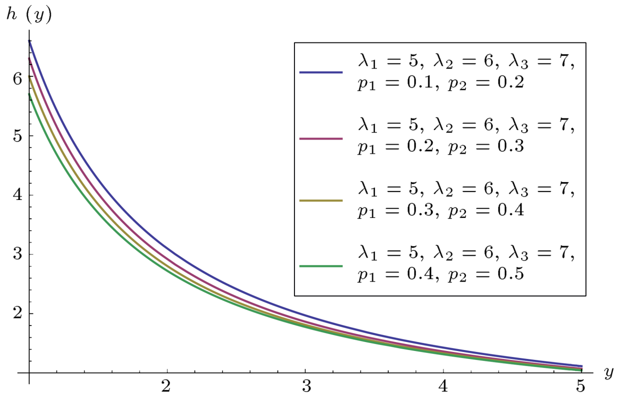

Figure 7. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (5, 6, 7, 0.1, 0.2), (5, 6, 7, 0.2, 0.3), (5, 6, 7, 0.3, 0.4), (5, 6, 7, 0.4, 0.5)\}$.

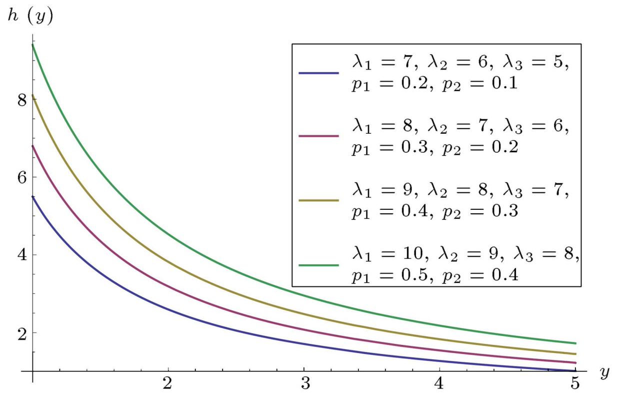

Figure 8. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (7, 6, 5, 0.2, 0.1), (8, 7, 6, 0.3, 0.2), (9, 8, 7, 0.4, 0.3), (10, 9, 8, 0.5, 0.4)\}$.

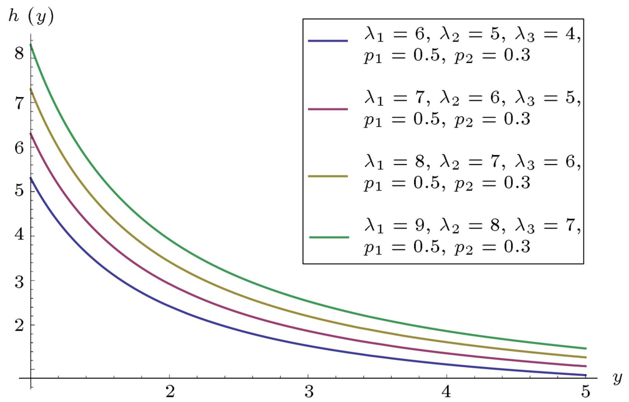

Figure 9. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (6, 5, 4, 0.5, 0.3), (7, 6, 5, 0.5, 0.3), (8, 7, 6, 0.5, 0.3), (9, 8, 7, 0.5, 0.3)\}$.

Table 7. Hazard rate of a 3-CMPD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y=1$	$Y=5$	$Y=10$
5, 6, 7, 0.1, 0.2	6.600000	1.114290	0.526772
5, 6, 7, 0.2, 0.3	6.300000	1.071430	0.517021
5, 6, 7, 0.3, 0.4	6.000000	1.053060	0.513411
5, 6, 7, 0.4, 0.5	5.700000	1.042860	0.168017
7, 6, 5, 0.2, 0.1	5.500000	1.009890	0.501966
8, 7, 6, 0.3, 0.2	6.800000	1.223190	0.604971
9, 8, 7, 0.4, 0.3	8.100000	1.448940	0.711377
10, 9, 8, 0.5, 0.4	9.400000	1.720000	0.834483
6, 5, 4, 0.5, 0.3	5.300000	0.871429	0.417021
7, 6, 5, 0.5, 0.3	6.300000	1.071430	0.517021
8, 7, 6, 0.5, 0.3	7.300000	1.271430	0.617021
9, 8, 7, 0.5, 0.3	8.300000	1.471430	0.717021

Table 8. MRL of a 3-CMPD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y=1$	$Y=5$	$Y=10$
5, 6, 7, 0.1, 0.2	0.181667	1.121030	2.375330
5, 6, 7, 0.2, 0.3	0.193333	1.166670	2.418440
5, 6, 7, 0.3, 0.4	0.205000	1.186220	2.434400
5, 6, 7, 0.4, 0.5	0.216667	1.197090	2.442720
7, 6, 5, 0.2, 0.1	0.228333	1.238550	2.490640
8, 7, 6, 0.3, 0.2	0.176190	0.981712	1.983980
9, 8, 7, 0.4, 0.3	0.142857	0.805471	1.640290
10, 9, 8, 0.5, 0.4	0.119841	0.662698	1.368360
6, 5, 4, 0.5, 0.3	0.241667	1.529760	3.198580
7, 6, 5, 0.5, 0.3	0.193333	1.166670	2.418440
8, 7, 6, 0.5, 0.3	0.161429	0.943878	1.945290
9, 8, 7, 0.5, 0.3	0.138690	0.792942	1.627410

9. The numerical results, so obtained, are showcased in Table 7.

From Figures 7-9 and the entries in Table 7, it is observed that, in general, failure rate for a 3-CMPD follows a decreasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, failure rate decreases as proportion parameters increase (see row 1 of Table 7). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, failure rate increases as component parameters increase (see rows 2 and 3 of Table 7). Also, there are higher chances of failure when both the component and proportion parameters are relatively larger (see row 2 of Table 7).

5.2. Cumulative hazard rate and reversed hazard rate functions for a 3-CMPD

The cumulative hazard rate function, $H(y;\Psi)$, and reversed hazard rate function, $r(y;\Psi)$, for a 3-CMPD using Eqs. (5) and (6) are obtained by Eqs. (29) and (30), respectively, as shown in Box IX.

5.3. Mean residual life and mean waiting time functions for a 3-CMPD

The MRL function or life expectancy function for a 3-CMPD is:

$$m(y;\Psi)=$$

$$\frac{\frac{p_1}{(\lambda_1-1)}y^{1-\lambda_1}+\frac{p_2}{(\lambda_2-1)}y^{1-\lambda_2}+\frac{(1-p_1-p_2)}{(\lambda_3-1)}y^{1-\lambda_3}}{p_1y^{-\lambda_1}+p_2y^{-\lambda_2}+(1-p_1-p_2)y^{-\lambda_3}}. \quad (31)$$

Using Eq. (31), the MRL of a 3-CMPD is evaluated for the parametric values fixed in Figures 7-9. The numerical results, so obtained, are presented in Table 8.

From the entries available in Table 8, it is obvious that, in general, MRL for a 3-CMPD follows an increasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, MRL increases as proportion parameters increase (see row 1 of Table 8). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, MRL decreases as component parameters increase (see rows 2 and 3 of Table 8). Using Eq. (8), the MWT function for a 3-CMPD is obtained by Eq. (32) as shown in Box X.

Using the above Expression (32), the MWT of a 3-CMPD is evaluated for the parametric values fixed in Figures 7-9. The numerical results, so obtained, are given in Table 9.

From the entries in Table 9, it is seen that, in general, MWT for a 3-CMPD follows an increasing trend over time. When $\lambda_1<\lambda_2<\lambda_3$, MWT increases as

$$H(y;\Psi)=-\ln\left\{p_1y^{-\lambda_1}+p_2y^{-\lambda_2}+(1-p_1-p_2)y^{-\lambda_3}\right\}, \quad (29)$$

and

$$r(y;\Psi)=\frac{p_1\lambda_1y^{-(\lambda_1+1)}+p_2\lambda_2y^{-(\lambda_2+1)}+(1-p_1-p_2)\lambda_3y^{-(\lambda_3+1)}}{1-p_1y^{-\lambda_1}-p_2y^{-\lambda_2}-(1-p_1-p_2)y^{-\lambda_3}}. \quad (30)$$

Box IX

$$\bar{\mu}(y;\Psi)=\frac{y-\frac{p_1}{(1-\lambda_1)}\left\{y^{1-\lambda_1}-\lambda_1\right\}-\frac{p_2}{(1-\lambda_2)}\left\{y^{1-\lambda_2}-\lambda_2\right\}-\frac{(1-p_1-p_2)}{(1-\lambda_3)}\left\{y^{1-\lambda_3}-\lambda_3\right\}}{1-p_1y^{-\lambda_1}-p_2y^{-\lambda_2}-(1-p_1-p_2)y^{-\lambda_3}}. \quad (32)$$

Box X

Table 9. MWT of a 3-CMPD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y=1$	$Y=5$	$Y=10$
5, 6, 7, 0.1, 0.2	2.866750	6.181940	11.18170
5, 6, 7, 0.2, 0.3	2.906390	6.193780	11.19340
5, 6, 7, 0.3, 0.4	2.946910	6.205630	11.20500
5, 6, 7, 0.4, 0.5	2.988360	6.217480	11.21670
7, 6, 5, 0.2, 0.1	3.029530	6.229500	11.22840
8, 7, 6, 0.3, 0.2	2.849370	6.176370	11.17620
9, 8, 7, 0.4, 0.3	2.742810	6.142880	11.14290
10, 9, 8, 0.5, 0.4	2.677120	6.119840	11.11980
6, 5, 4, 0.5, 0.3	3.071430	6.243780	11.24190
7, 6, 5, 0.5, 0.3	2.906390	6.193780	11.19340
8, 7, 6, 0.5, 0.3	2.800570	6.161520	11.16140
9, 8, 7, 0.5, 0.3	2.730250	6.138710	11.13870

proportion parameters increase (see row 1 of Table 9). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, MWT decreases as component parameters increase (see rows 2 and 3 of Table 9).

6. Reliability properties of a three-component mixture of Burr type-XII distributions

Using Eqs. (1) and (2), a finite three-component mixture of Burr Type-XII distributions (3-CMBD) with mixing proportions p_1 and p_2 has the pdf and cdf as follows:

$$f(y;\Psi)=p_1\lambda_1(1+y)^{-(\lambda_1+1)}+p_2\lambda_2(1+y)^{-(\lambda_2+1)}+(1-p_1-p_2)\lambda_3(1+y)^{-(\lambda_3+1)},$$

$$0<y<\infty,\ \ \lambda_m>0,\ \ m=1,2,3, \quad (33)$$

$$F(y;\Psi)=1-p_1(1+y)^{-\lambda_1}-p_2(1+y)^{-\lambda_2}-(1-p_1-p_2)(1+y)^{-\lambda_3}. \quad (34)$$

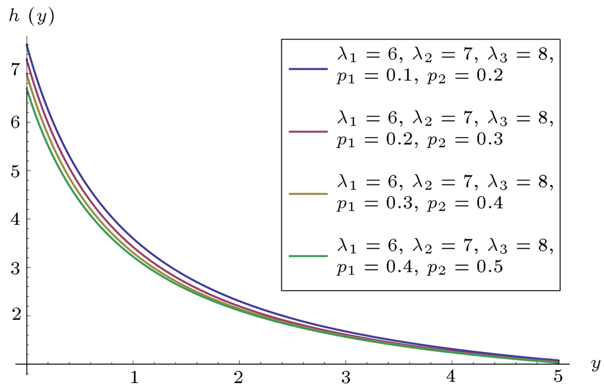

Figure 10. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (6,\ 7,\ 8,\ 0.1,\ 0.2),\ (6,\ 7,\ 8,\ 0.2,\ 0.3),\ (6,\ 7,\ 8,\ 0.3,\ 0.4),\ (6,\ 7,\ 8,\ 0.4,\ 0.5)\}$.

6.1. Reliability and failure rate functions for a 3-CMBD

The reliability function or survival function for a 3-CMBD is written as:

$$R(y;\Psi)=p_1(1+y)^{-\lambda_1}+p_2(1+y)^{-\lambda_2}+(1-p_1-p_2)(1+y)^{-\lambda_3}$$

$$0<y<\infty,\ \ \lambda_m>0,\ \ m=1,2,3. \quad (35)$$

The failure rate function or hazard rate function for a 3-CMBD is defined by Eq. (36) as shown in Box XI.

The trend of the hazard rate Function (Eq. (36)) for some fixed values of component and proportion parameters is illustrated in Figures 10-12. From Figures 10-12, the effects of parameters $\lambda_1, \lambda_2, \lambda_3, p_1$ and p_2 on hazard rate for the 3-CMBD can be observed.

Using Eq. (36), the hazard rate of a 3-CMBD is evaluated for the parametric values fixed in Figures 10-12. The numerical results, so obtained, are presented in Table 10.

From Figures 10-12 and the entries in Table 10, it is seen that, in general, failure rate for a 3-CMED

$$h(y;\Psi)=\frac{p_1\lambda_1(1+y)^{-(\lambda_1+1)}+p_2\lambda_2(1+y)^{-(\lambda_2+1)}+(1-p_1-p_2)\lambda_3(1+y)^{-(\lambda_3+1)}}{p_1(1+y)^{-\lambda_1}+p_2(1+y)^{-\lambda_2}+(1-p_1-p_2)(1+y)^{-\lambda_3}}. \quad (36)$$

Box XI

$$H(y;\Psi) = -\ln\left\{p_1(1+y)^{-\lambda_1} + p_2(1+y)^{-\lambda_2} + (1-p_1-p_2)(1+y)^{-\lambda_3}\right\}, \quad (37)$$

$$r(y;\Psi) = \frac{p_1\lambda_1(1+y)^{-(\lambda_1+1)} + p_2\lambda_2(1+y)^{-(\lambda_2+1)} + (1-p_1-p_2)\lambda_3(1+y)^{-(\lambda_3+1)}}{1 - p_1(1+y)^{-\lambda_1} - p_2(1+y)^{-\lambda_2} - (1-p_1-p_2)(1+y)^{-\lambda_3}}. \quad (38)$$

Box XII

$$m(y;\Psi) = \frac{\frac{p_1}{(\lambda_1-1)}(1+y)^{1-\lambda_1} + \frac{p_2}{(\lambda_2-1)}(1+y)^{1-\lambda_2} + \frac{(1-p_1-p_2)}{(\lambda_3-1)}(1+y)^{1-\lambda_3}}{p_1(1+y)^{-\lambda_1} + p_2(1+y)^{-\lambda_2} + (1-p_1-p_2)(1+y)^{-\lambda_3}}. \quad (39)$$

Box XIII

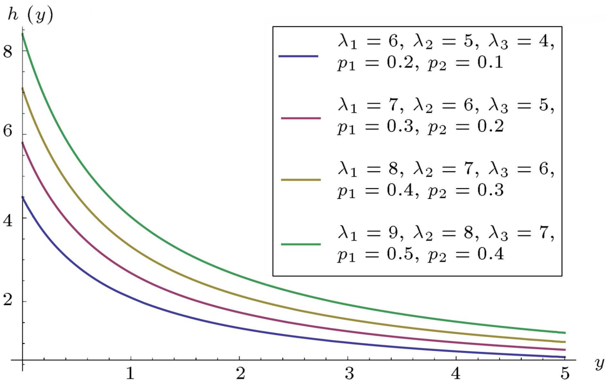

Figure 11. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (6, 5, 4, 0.2, 0.1), (7, 6, 5, 0.3, 0.2), (8, 7, 6, 0.4, 0.3), (9, 8, 7, 0.5, 0.4)\}$.

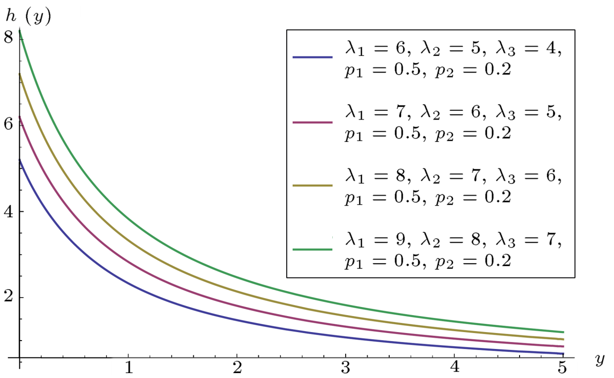

Figure 12. Graphs of hazard rate function for a 3-CMED for parameters $\{(\lambda_1, \lambda_2, \lambda_3, p_1, p_2) = (6, 5, 4, 0.5, 0.2), (7, 6, 5, 0.5, 0.2), (8, 7, 6, 0.5, 0.2), (9, 8, 7, 0.5, 0.2)\}$.

follows a decreasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$, failure rate decreases as proportion parameters increase (see row 1 of Table 10). On the other hand, when $\lambda_1 > \lambda_2 > \lambda_3$, failure rate increases as component parameters increase (see rows 2 and 3 of Table 10). Also, there are higher chances of failure when both the component and proportion parameters are relatively larger (see row 2 of Table 10).

Table 10. Hazard rate of a 3-CMBD.

$\lambda_1, \lambda_2, \lambda_3, p_1, p_2$	$Y = 1$	$Y = 5$	$Y = 10$
6, 7, 8, 0.1, 0.2	3.600000	1.078790	0.567273
6, 7, 8, 0.2, 0.3	3.421050	1.049120	0.559416
6, 7, 8, 0.3, 0.4	3.304350	1.037040	0.556541
6, 7, 8, 0.4, 0.5	3.222220	1.030480	0.555051
6, 5, 4, 0.2, 0.1	2.093750	0.673077	0.365222
7, 6, 5, 0.3, 0.2	2.685190	0.848718	0.458586
8, 7, 6, 0.4, 0.3	3.318180	1.033330	0.554773
9, 8, 7, 0.5, 0.4	4.029410	1.253850	0.665241
6, 5, 4, 0.5, 0.2	2.333330	0.696000	0.371096
7, 6, 5, 0.5, 0.2	2.833330	0.862667	0.462005
8, 7, 6, 0.5, 0.2	3.333330	1.029330	0.552914
9, 8, 7, 0.5, 0.2	3.833330	1.196000	0.643823

6.2. *Cumulative hazard rate and reversed hazard rate functions for a 3-CMBD*

The cumulative hazard rate function, $H(y;\Psi)$, and reversed hazard rate function, $r(y;\Psi)$, for a 3-CMBD using Eq. (5) and (6) are obtained by Eqs. (37) and (38), respectively, as shown in Box XII.

6.3. *Mean residual life and mean waiting time functions for a 3-CMBD*

The MRL function or life expectancy function for a 3-CMBD is obtained by Eq. (39) as shown in Box XIII. Using Eq. (39), the MRL of a 3-CMBD is evaluated for the parametric values fixed in Figures 10-12. The numerical results, so obtained, are given in Table 11.

From the entries available in Table 11, it is observed that, in general, MRL for a 3-CMBD follows an increasing trend over time. When $\lambda_1 < \lambda_2 < \lambda_3$,

$$\bar{\mu}(y;\Psi)=\frac{y-\frac{p_1}{(1-\lambda_1)}\left\{(1+y)^{1-\lambda_1}-1\right\}-\frac{p_2}{(1-\lambda_2)}\left\{(1+y)^{1-\lambda_2}-1\right\}-\frac{(1-p_1-p_2)}{(1-\lambda_3)}\left\{(1+y)^{1-\lambda_3}-1\right\}}{1-p_1(1+y)^{-\lambda_1}-p_2(1+y)^{-\lambda_2}-(1-p_1-p_2)(1+y)^{-\lambda_3}}. \quad (40)$$

Box XIV

Table 11. MRL of a 3-CMBD.

$\lambda_1,\lambda_2,\lambda_3,p_1,p_2$	$Y=1$	$Y=5$	$Y=10$
6, 7, 8, 0.1, 0.2	0.328889	1.112730	2.116890
6, 7, 8, 0.2, 0.3	0.348872	1.144060	2.145560
6, 7, 8, 0.3, 0.4	0.361905	1.156830	2.156050
6, 7, 8, 0.4, 0.5	0.371076	1.163760	2.161490
6, 5, 4, 0.2, 0.1	0.639583	1.982310	3.651530
7, 6, 5, 0.3, 0.2	0.466667	1.473850	2.726430
8, 7, 6, 0.4, 0.3	0.361039	1.161760	2.163460
9, 8, 7, 0.5, 0.4	0.286415	0.928022	1.752070
6, 5, 4, 0.5, 0.2	0.571429	1.920000	3.596150
7, 6, 5, 0.5, 0.2	0.441270	1.451200	2.707220
8, 7, 6, 0.5, 0.2	0.360091	1.167090	2.171260
9, 8, 7, 0.5, 0.2	0.304422	0.976286	1.812680

MRL increases as proportion parameters increase (see row 1 of Table 11). On the other hand, when $\lambda_1>\lambda_2>\lambda_3$, MRL decreases as component parameters increase (see rows 2 and 3 of Table 11).

Using Eq. (8), the MWT function for a 3-CMBD is obtained by Eq. (40) as shown in Box XIV.

Using the above expression (40), the MWT of a 3-CMBD is evaluated for the parametric values fixed in Figures 10-12. The numerical results, so obtained, are showcased in Table 12.

From the entries in Table 12, it is obvious that, in general, MWT for a 3-CMBD follows an increasing trend over time. When $\lambda_1<\lambda_2<\lambda_3$, MWT increases as proportion parameters increase (see row 1 of Table 12). On the other hand, when $\lambda_1>\lambda_2>\lambda_3$, MWT decreases as component parameters increase (see rows 2 and 3 of Table 12).

Table 12. MWT of a 3-CMBD.

$\lambda_1,\lambda_2,\lambda_3,p_1,p_2$	$Y=1$	$Y=5$	$Y=10$
6, 7, 8, 0.1, 0.2	1.158190	5.153350	10.15330
6, 7, 8, 0.2, 0.3	1.167500	5.161450	10.16140
6, 7, 8, 0.3, 0.4	1.176850	5.169560	10.16950
6, 7, 8, 0.4, 0.5	1.186220	5.177660	10.17760
6, 5, 4, 0.2, 0.1	1.333000	5.300180	10.29870
7, 6, 5, 0.3, 0.2	1.231130	5.215260	10.21500
8, 7, 6, 0.4, 0.3	1.174130	5.167170	10.16710
9, 8, 7, 0.5, 0.4	1.139140	5.136310	10.13630
6, 5, 4, 0.5, 0.2	1.273020	5.250890	10.25010
7, 6, 5, 0.5, 0.2	1.210960	5.198500	10.19830
8, 7, 6, 0.5, 0.2	1.171420	5.164790	10.16480
9, 8, 7, 0.5, 0.2	1.144520	5.141080	10.14110

7. Concluding remarks

The reliability analyses of three-component mixture of Exponential, Rayleigh, Pareto, and Burr Type-XII distributions are performed. It is observed that, in general, failure rate functions of 3-CMED, 3-CMPD, and 3-CMBD (3-CMRD) follow a decreasing trend (increasing) over time. Moreover, when $\lambda_1<\lambda_2<\lambda_3$, hazard rates of 3-CMED, 3-CMPD, and 3-CMBD (3-CMRD) decrease (increase) as proportion parameters increase. On the other hand, when $\lambda_1>\lambda_2>\lambda_3$, hazard rates of 3-CMED, 3-CMPD, and 3-CMBD (3-CMRD) increase (decrease) as component parameters increase. When $\lambda_1<\lambda_2<\lambda_3$, MRL of 3-CMED, 3-CMPD, and 3-CMBD (3-CMRD) increase (decrease) as proportion parameters increase, whereas when $\lambda_1>\lambda_2>\lambda_3$, the MRL of 3-CMED, 3-CMPD, and 3-CMBD (3-CMRD) decrease (increase) as component parameters increase. The behavior of MWT is observed to be increasing when $\lambda_1<\lambda_2<\lambda_3$, and proportion parameters increase for 3-CMED, 3-CMPD, and 3-CMBD. On the other hand, when $\lambda_1>\lambda_2>\lambda_3$, MWTs of 3-CMED, 3-CMPD, and 3-CMBD decrease as component parameters increase.

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