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Research Note

Performance evaluation of joint monitoring control charts

T. Mahmood[a,*], H.Z. Nazir[b], N. Abbas[a], M. Riaz[a] and A. Ali[b,c]

a. *Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran, KSA.*
b. *Department of Statistics, University of Sargodha, Sargodha, Pakistan.*
c. *SAA Center for Improvement of Working Conditions and Environment Labour and Human Resource Department, Government of Punjab, Lahore, Pakistan.*

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Shewhart charts.

Abstract. Shewhart-Cucconi and Shewhart-Lepage are two nonparametric control charts used for monitoring joint shifts in the process location and scale parameters. This study investigates impact of the light and heavy-tailed distributions on the performances of these charts. The effect of reference and test samples is also a part of this study.

1. Introduction

Statistical Process Control (SPC) is a set of well-known tools used to monitor the performance of a process. Control chart is a major tool of SPC that consists of a Lower Control Limit (LCL), Central Line (CL), and an Upper Control Limit (UCL). It helps us differentiate between natural and unnatural variations that refer to In-Control (IC) and Out-Of-Control (OOC) states, respectively, in a process. Normality is a typical assumption needed for parametric charts, while non-parametric charts are free from any such constraints. Reference can be made for further check out to Chakraborti et al. [1,2], Chowdhury et al. [3], and Mukherjee and Sen [4] in the literature. Moreover, a traditional approach used in SPC is to monitor each parameter separately; however, simultaneous monitoring of more than one parameter is also becoming popular in industry. Chowdhury et al. [5], McCracken and Chakraborti [6], Mukherjee and Chakraborti [7], and Mukherjee et al. [8] and the references therein may be seen in literature on simultaneous charts.

Recently, Mukherjee and Chakraborti [7] have proposed a Shewhart-type distribution-free chart for joint monitoring of the process parameters. It is based on the Lepage test, a combination of Wilcoxon rank sum test for location and Ansari Bradley test for scale (cf. Lepage [9]), and this chart is hereafter named as Shewhart-Lepage (SL) chart. On the same lines, Chowdhury et al. [5] developed a distribution-free Shewhart chart for joint monitoring that utilizes Cucconi test proposed by Cucconi [10], hereafter referred to as Shewhart Cucconi (SC) chart. Marozzi [11] provided a comparative analysis of Cucconi test versus Lepage test under some distributional setups and favored Cucconi test over Lepage test.

This study intends to investigate the impact of light and heavy-tailed distributions on the performance of SL and SC charts. In addition, the effect of reference/test samples is included in this study. The rest of the article is organized as follows: Section 2

*. *Corresponding author.*
E-mail addresses: rana.tm.19@gmail.com (T. Mahmood); hafizzafarnazir@yahoo.com (H.Z. Nazir); nasirabbas55@yahoo.com (N. Abbas); riaz76qau@yahoo.com (M. Riaz); amjidalizafar@gmail.com (A. Ali)

provides the description of SL and SC charts. Section 3 explores the properties of these charts under different distributional environments and also examines the effects of reference/tests samples. Section 4 deals with a real application related to these charts. Section 5 concludes the study with the main findings.

2. Description of SC and SL charts

Let $U_1, U_2, \cdots, U_m$ and $V_1, V_2, \cdots, V_n$ be independent random samples from their respective populations with continuous cumulative distribution functions: $F(U) = Q(\frac{U-\theta}{\delta})$ and $G(V) = Q(\frac{V-\theta}{\delta})$; $\theta \in \Re$; $\delta > 0$; this is where Q refers to some unknown continuous functions. Constants θ and δ represent unknown location and scale parameters, respectively. Let us introduce indicator variable, $I_k = 0$, or 1 depending on whether or not kth order statistic of the combined sample of $N = m + n$ observations belongs to U or V. It is to be mentioned that m is reference sample (phase I) and n is the test sample (phase II). Further, we assume that R is the linear ranks assigned to the values of the combined sample.

The popular nonparametric Wilcoxon Rank Sum (WRS) test statistic, T_1, is defined as follows:

$$T_1 = \sum_{k=1}^{N} RI_k.$$

For the equality of two scale parameters, Ansari Bradley (AB) is an efficient nonparametric test whose statistic, T_2, is defined as follows:

$$T_2 = \sum_{k=1}^{N} \left| R - \frac{1}{2}(N+1) \right| I_k.$$

Consider S_1 as the sum of the square of the ranks of V_i's in the combined sample, i.e.:

$$S_1 = \sum_{k=1}^{N} R^2 I_k.$$

Further, note that quantities $(N+1-R)I_k$, for $k = 1, 2, \cdots, N$, may be considered as the contrary ranks of V_i's. The sum of squares of the contrary ranks of V_i's in the combined sample, say S_2, is given by:

$$S_2 = \sum_{k=1}^{N} (N+1-R)^2 I_k = n(N+1)^2 - 2(N+1)T_1 + S_1.$$

Assuming that $\theta = 0$ and $\delta = 1$ refer to IC state ($F = G$), we have the following properties:

$$E(T_1|\text{IC}) = \frac{1}{2}n(N+1),$$

$$\text{Var}(T_1|\text{IC}) = \frac{1}{12}mn(N+1),$$

$$E(T_2|\text{IC}) = \begin{cases} \frac{nN}{4} & \text{when } N \text{ is even} \\ \frac{n(N^2-1)}{4N} & \text{when } N \text{ is odd} \end{cases}$$

$$\text{Var}(T_2|\text{IC}) = \begin{cases} \frac{1}{48}mn\frac{(N^2-4)}{N-1} & \text{when } N \text{ is even} \\ \frac{1}{48}\frac{mn(N+1)(N^2+3)}{N^2} & \text{when } N \text{ is odd} \end{cases}$$

$$E(S_1|\text{IC}) = E(S_2|\text{IC}) = \frac{n(N+1)(2N+1)}{6},$$

$$\text{Var}(S_1|\text{IC}) = \text{Var}(S_2|\text{IC}) = \frac{mn}{180}(N+1)(2N+1)(8N+11).$$

The combination of AB and WRS is known as Lepage statistic (cf. [9]) and is given as follows:

$$L = \frac{(T_1 - E(T_1|\text{IC}))^2}{\text{VAR}(T_1|\text{IC})} + \frac{(T_2 - E(T_2|\text{IC}))^2}{\text{VAR}(T_2|\text{IC})}, \tag{1}$$

and Cucconi [10] statistic for testing both location and scale is defined by:

$$C = \frac{W^2 + Z^2 - 2WZ\rho}{2(1-\rho^2)}, \tag{2}$$

where W and Z are the standardized statistics given as follows:

$$W = \frac{S_1 - E(S_1|\text{IC})}{\sqrt{\text{VAR}(S_1|\text{IC})}} = \frac{6S_1 - n(N+1)(2N+1)}{\sqrt{\frac{mn}{5}(N+1)(2N+1)(8N+11)}},$$

$$Z = \frac{S_2 - E(S_2|\text{IC})}{\sqrt{\text{VAR}(S_2|\text{IC})}} = \frac{6S_2 - n(N+1)(2N+1)}{\sqrt{\frac{mn}{5}(N+1)(2N+1)(8N+11)}},$$

when $\theta > 0$ and $\delta = 1$, $E(W) > 0$ and $E(Z) < 0$; when $\theta = 0$ and $\delta > 1$, $E(W) > 0$ and $E(Z) > 0$; in general, when $\theta \neq 0$ and $\delta \neq 1$, $E(W) \neq 0$ and $E(Z) \neq 0$. Similar inequalities may be observed in other possible cases, when either θ differs from 0 or δ differs from 1 in any direction. Also, note that $E(W|\text{IC}) = E(Z|\text{IC}) = 0$ and $V(W|\text{IC}) = V(Z|IC) = 1$. Moreover, when $F = G$, the correlation coefficient between W and Z is given as (cf. [11]):

$$\rho = \text{Corr}(W, Z|\text{IC}) = \frac{2(N^2-4)}{(2N+1)(8N+11)} - 1.$$

2.1. Design of control charting constants of distribution-free charts

Construction and design of both SC and SL charts depend on the distributions of the statistics given in Eqs. (1) and (2). The lower control limits of both charts is zero as both statistics can never be negative (cf. [5,7] and the upper control limits of both charts, say H, are used to make decision. The values of H are provided in [5,7] for some selective values of n and m. We have covered more combinations of n and m to find the upper control limit, say H, for both charts, using a simulation study with 100,000 replicates (in R 3.1.1). We have taken the retrospective samples, i.e. $m = 30, 50, 100, 150, 500$, and 1000 and prospective samples, i.e. $n = 5, 8, 11, 16$, and 25, for this study, fixing $ARL_0 = 500$. The results are reported in Table 1 for SL and SC charts.

The decision procedure for the two charts is given as follows:

- **SL chart:** Statistic L is used for plotting in SL chart. If L is greater than H, then the process is declared OOC. For the follow-up analysis, we compute p-values of the WRS test for location and AB test for scale with phase I sample and ith test sample, and they are denoted as p_1 and p_2, respectively. If p_1 is very low except p_2, a shift of location is detected, or if p_2 is very low and p_1 is relatively high, a shift in scale parameter is detected. When both WRS and AB p-values are very low, a joint shift in the location and scale is considered;
- **SC chart:** Statistic C is used for plotting in SL chart. If C exceeds H, the process is declared OOC. For the follow-up analysis, we compute the p-values for Wilcoxon test (p_3) and Mood test (p_4) based on two samples (reference and test samples), (cf. [5]). The shift in location is noted when p_3 is very low except p_4; if p_3 is relatively high except p_4, then there is the indication of a shift in scale. If both p_3 and p_4 are very low, shift is noted in both location and scale. Sometimes, neither p_3 nor p_4 is very low, though the plotting statistic C is high; in this situation, the effect is due to the relation between location and scale changes or due to false alarm. So, to overcome this problem, combine ith and $(i-1)$th prospective samples and recalculate (p_3) and (p_4) for further decision.

3. Performance analyses of SL and SC charts

In this section, we will investigate the performances of SL and SC charts under different distributional environments. We will also examine the effects of reference and test samples on the performance of these charts. We will use Average Run Length (ARL) and Standard Deviation Run Length (SDRL) as performance measures. The ARL value is denoted by ARL_0 for in-control situation and ARL_1 for out-of-control situation. The distributional setups covered in this study include: Uniform: $U(\sqrt{-3}, \sqrt{3})$, Student's t: t_4, Lognormal: LN(1,1), Gamma: $G(1,1)$, and contaminated normal ($C1$: with 10% contaminations, and $C2$: with 30% contaminations). The first two are symmetric and light tailed, the next two are skewed and heavy tailed, and the last two are contaminated distributions. Abbasi and Miller [12], Alfaro and Ortega [13], Ali et al. [14], Human et al. [15], and Marozzi [11] are some useful references on the said distributional environments. The graphical displays of these distributions are given in Figure 1.

3.1. OOC performance

In order to examine the OOC performance of SL and SC charts, we have considered shifts in location and scale for these choices: $\theta = 0$, 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, and $\delta = 0.50$, 0.75, 1, 1.25, 1.5, 1.75, 2. We have chosen $m = 30$, 50, 100, 150, 500, and 1000 and $n = 5$, 8, 11, 16, and 25. It makes a total of 30 pairs (m, n). The properties of SL and SC charts, in terms of ARL and SDRL, are evaluated for different

Table 1. Constant H for SC and SL charts at $ARL_0 = 500$.

	$m = 30$	$m = 50$	$m = 100$	$m = 150$	$m = 500$	$m = 150$
			SC			
$n = 5$	4.48	5.25	5.98	6.25	6.65	6.73
$n = 8$	4.31	4.77	5.56	5.91	6.42	6.54
$n = 11$	4.45	4.8	5.34	5.67	6.29	6.42
$n = 16$	4.47	4.85	5.31	5.56	6.11	6.28
$n = 25$	4.18	4.7	5.25	5.49	6	6.16
			SL			
$n = 5$	9.4	10.32	11.25	11.5	12.02	12.14
$n = 8$	9.28	10.22	11.15	11.53	12.1	12.24
$n = 11$	9.24	10.1	11.07	11.45	12.06	12.22
$n = 16$	9.11	9.95	10.9	11.32	12.04	12.21
$n = 25$	8.4	9.5	10.74	11.17	12.02	12.2

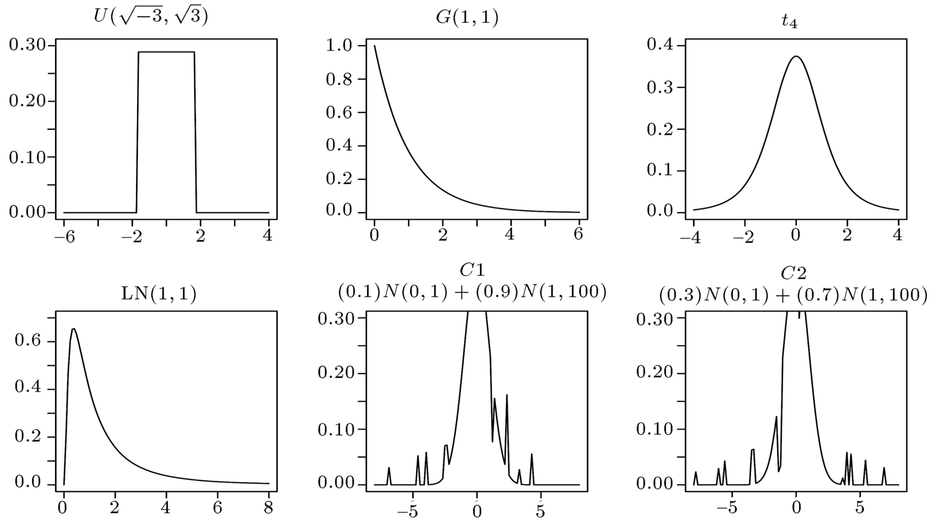

Figure 1. Probability density plots of different distributions.

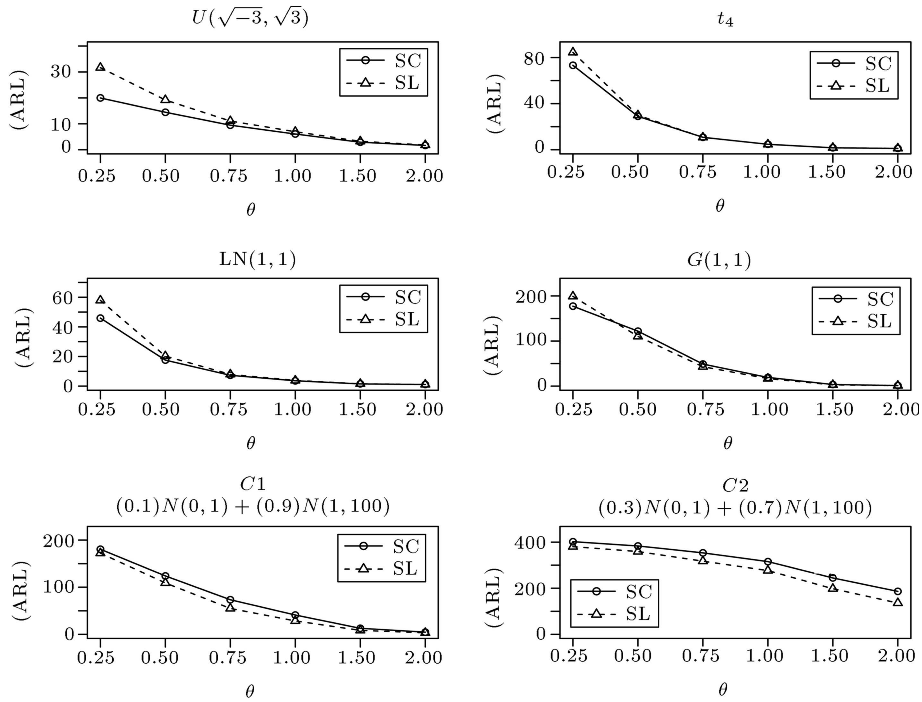

Figure 2. ARL_1 curve with varying location shifts θ and fixed $\delta = 1.25$.

combinations, θ and δ. These results are provided in Tables 2 and 3 under different distributions. For the sake of brevity, we only discuss the results of the pair (100, 5). Moreover, some useful ARL curves are also produced and provided in Figures 2 and 3.

The useful findings about the two charts are listed as follows:

1. In *general*, the run length follows right skewed distribution; the run length distributions of both charts decrease with the increase in the location and scale shifts; shift in the scale parameter is detected faster than the shift in the location parameter; both charts are sensitive to shifts in location and scale, but both charts react more quickly to detect a shift in standard deviation rather than mean;

2. For the case of *uniform* distribution, SC chart performs slightly better than SL chart. For instance, when $\theta = 0.25$ and $\delta = 1.25$, ARL_1 values of SC and SL charts are 19.97 and 31, respectively; when $\theta = 0.0$ and $\delta = 1.25$, ARL_1 values of SC and SL charts are 22.81 and 39.65, respectively; and when $\theta = 0.25$ and $\delta = 1.00$, ARL_1 values of SC and SL charts are 111.64 and 133.09;

3. SC chart performs slightly better than SL chart

Table 2. ARLs of SC and SL charts under different distributions using $m = 100$ and $n = 5$.

		$U(\sqrt{-3}, \sqrt{3})$		$t_{(4)}$		LN(1,1)		$G(1,1)$		$C1$		$C2$	
δ	θ	SC	SL	SC	SL	SC	SL	SC	SL	SC	SL	SC	SL
	0.00	133.60	1171.98	15598.55	4741.50	1958.81	1351.42	14130.31	332.10	4244.22	771.56	753.65	374.45
	0.25	153.60	1184.99	15053.16	4721.17	4446.38	1405.95	12805.36	2288.34	3179.54	652.55	717.98	342.99
	0.50	4596.41	990.82	3651.45	6725.76	9842.98	1701.95	5629.85	1595.17	1644.35	479.31	655.09	314.40
0.5	**0.75**	2185.38	182.76	271.02	3925.29	12483.72	1703.05	1238.17	296.32	625.48	226.86	560.32	281.93
	1.00	16.18	11.66	13.11	2888.23	9251.59	1183.41	218.56	35.65	215.57	71.55	453.31	238.19
	1.50	1.59	1.39	1.14	1607.68	1044.82	164.16	3.66	1.32	22.81	5.46	288.71	133.39
	2.00	1.00	1.00	1.01	795.95	47.68	11.86	1.05	1.00	3.79	1.49	185.64	68.50
	0.00	3639.70	5250.33	5776.59	9287.89	12139.71	1784.44	10500.39	1406.33	1497.88	692.08	631.99	360.24
	0.25	10453.54	1930.49	2348.37	8441.62	6892.77	1146.05	3579.71	290.75	1198.41	535.60	612.85	345.37
	0.50	201.87	122.63	285.72	6249.00	1657.41	405.53	866.22	150.12	645.93	270.15	552.64	314.30
0.75	**0.75**	25.75	21.42	34.85	4173.70	269.82	104.72	183.01	55.13	287.55	107.38	483.19	257.58
	1.00	8.05	7.39	6.00	3025.55	55.96	28.35	38.16	11.77	105.60	36.96	412.44	207.27
	1.50	2.07	1.96	1.32	1435.98	5.97	4.12	2.86	1.43	17.23	5.48	273.54	118.28
	2.00	1.11	1.07	1.03	589.88	1.80	1.57	1.08	1.01	3.91	1.71	185.71	71.33
	0.00	506.23	499.45	511.47	503.09	503.04	505.16	503.96	506.55	511.82	500.85	506.78	507.24
	0.25	111.64	133.09	267.71	245.33	261.97	259.18	782.65	534.14	423.98	404.52	494.57	490.28
	0.50	31.67	37.89	66.51	55.74	71.47	69.17	240.77	162.23	266.94	216.13	455.29	450.29
1	**0.75**	13.19	15.55	16.09	13.54	20.79	20.23	72.60	53.58	133.49	96.20	411.23	387.42
	1.00	6.75	7.89	5.17	4.64	7.64	7.64	22.60	16.62	63.85	39.75	359.26	321.20
	1.50	2.52	2.85	1.51	1.49	2.10	2.16	3.02	2.23	13.94	8.47	260.57	207.84
	2.00	1.36	1.41	1.07	1.09	1.20	1.23	1.14	1.06	4.40	2.86	190.03	139.47
	0.00	22.81	39.65	112.22	136.30	76.10	102.55	18.35	28.47	203.03	198.85	410.70	389.44
	0.25	19.97	31.65	73.29	84.49	45.91	57.89	177.22	199.10	180.53	172.20	401.98	380.56
	0.50	14.46	19.18	28.97	29.93	17.54	20.25	121.88	110.31	123.96	109.05	383.55	359.37
1.25	**0.75**	9.47	11.07	10.80	10.74	7.31	7.97	48.95	43.15	73.42	54.95	353.64	317.61
	1.00	6.06	6.96	4.71	4.68	3.62	3.85	19.29	16.75	41.02	28.50	315.83	276.75
	1.50	2.90	3.25	1.68	1.72	1.50	1.57	3.62	2.85	12.83	8.47	244.72	197.99
	2.00	1.66	1.81	1.14	1.17	1.09	1.11	1.26	1.14	4.65	3.24	186.30	135.86
	0.00	8.09	14.67	40.09	56.25	24.25	37.13	7.06	11.07	100.81	97.56	340.70	307.79
	0.25	7.83	13.64	30.78	40.54	17.07	23.69	18.87	31.48	88.99	89.52	342.21	301.97
	0.50	7.01	11.14	16.53	19.25	8.28	10.24	61.19	69.09	68.48	63.88	324.87	286.24
1.5	**0.75**	6.17	8.41	8.05	9.00	4.20	4.81	38.18	37.91	46.47	38.58	301.81	268.04
	1.00	5.18	6.29	4.32	4.63	2.45	2.71	18.40	17.78	29.86	22.70	281.01	238.85
	1.50	3.14	3.49	1.82	1.91	1.31	1.36	4.38	3.83	11.48	8.05	229.81	181.89
	2.00	1.97	2.14	1.23	1.27	1.05	1.07	1.49	1.32	5.08	3.72	186.46	135.55
	0.00	4.74	8.46	19.39	28.97	11.72	18.94	4.31	6.69	59.37	55.99	288.24	250.96
	0.25	4.64	8.15	16.38	23.38	9.06	13.34	7.47	12.63	50.90	54.32	283.87	246.25
	0.50	4.42	7.33	10.88	13.94	5.19	6.62	15.58	25.51	43.14	42.68	271.38	237.93
1.75	**0.75**	4.20	6.29	6.43	7.71	3.03	3.51	27.12	30.59	32.22	29.04	264.18	227.34
	1.00	3.80	5.29	3.96	4.50	1.97	2.18	17.83	18.30	22.62	19.19	247.84	208.32
	1.50	3.12	3.60	1.92	2.08	1.21	1.26	5.24	5.07	10.84	8.22	214.13	168.13
	2.00	2.20	2.39	1.31	1.37	1.03	1.04	1.84	1.65	5.39	4.10	180.64	132.26
	0.00	3.38	5.81	11.54	18.11	7.12	11.84	3.20	4.87	40.46	35.55	248.38	209.49
	0.25	3.35	5.68	10.24	15.29	5.86	8.79	4.55	7.44	33.66	37.23	250.30	207.66
	0.50	3.24	5.41	7.75	10.65	3.77	4.89	7.11	12.18	29.54	31.10	242.79	200.31
2	**0.75**	3.17	4.97	5.25	6.69	2.42	2.88	11.64	17.69	24.08	23.29	234.72	193.50
	1.00	2.99	4.40	3.63	4.32	1.70	1.89	14.87	16.88	18.16	16.44	225.19	184.62
	1.50	2.68	3.41	1.98	2.20	1.16	1.20	6.14	6.36	9.79	8.20	195.52	153.90
	2.00	2.32	2.55	1.38	1.47	1.03	1.03	2.29	2.15	5.54	4.51	170.17	132.57

Table 3. SDRLs of SC and SL charts under different distributions using $m = 100$ and $n = 5$.

		$U(\sqrt{-3},\sqrt{3})$		$t_{(4)}$		LN(1,1)		$G(1,1)$		$C1$		$C2$	
δ	θ	SC	SL	SC	SL	SC	SL	SC	SL	SC	SL	SC	SL
0.5	0.00	1523.32	3438.77	26154.47	15223.50	10820.01	3922.19	24104.39	1212.42	5762.79	1152.54	1165.85	462.47
	0.25	3023.32	3506.40	22552.20	14967.63	15555.80	3377.95	20772.47	7520.43	4863.72	973.78	1135.92	419.56
	0.50	14639.87	2769.44	10064.97	14773.53	21235.22	4046.90	13380.19	5877.11	3319.82	767.23	1063.13	417.88
	0.75	7716.73	499.28	2384.35	13315.89	21560.04	3839.40	5596.81	2573.27	1796.22	496.24	1013.83	394.96
	1.00	22.29	15.20	106.34	11305.25	17742.19	2831.89	1910.50	475.51	818.40	281.56	823.35	410.21
	1.50	1.07	0.83	0.71	8864.32	4978.54	777.76	35.83	2.47	149.53	22.06	655.10	319.56
	2.00	0.07	0.05	0.07	6302.91	408.96	41.36	0.64	0.08	30.15	2.89	494.31	185.81
0.75	0.00	13916.04	9698.16	9485.65	19606.95	17472.98	2883.92	16745.08	2724.92	2298.94	865.22	1005.81	461.84
	0.25	18810.79	4380.18	5963.97	19060.68	12867.26	1896.86	8592.19	523.63	2021.06	744.40	989.55	432.95
	0.50	327.14	185.02	1039.44	16231.99	4985.29	859.17	3075.42	329.09	1381.87	467.73	918.59	438.71
	0.75	30.13	24.54	221.31	13388.02	942.35	262.07	908.14	165.33	907.08	363.98	859.89	395.57
	1.00	8.37	7.76	17.96	11673.86	136.19	54.03	186.15	34.21	427.66	123.80	785.62	370.67
	1.50	1.58	1.49	0.79	8030.86	8.67	4.79	9.81	1.87	134.33	17.72	615.42	264.77
	2.00	0.36	0.29	0.17	4801.92	1.43	1.10	0.60	0.14	27.68	2.10	452.18	203.34
1	0.00	851.89	702.26	853.95	712.30	818.90	720.95	836.01	711.49	723.14	804.96	827.01	729.03
	0.25	134.50	162.99	535.68	417.24	461.83	424.46	1915.06	1037.74	760.01	646.67	871.64	711.38
	0.50	34.76	42.01	146.08	130.01	124.03	111.20	662.72	313.50	617.50	437.40	765.56	703.94
	0.75	13.56	16.23	33.98	21.43	31.48	27.53	188.94	137.30	386.99	268.98	738.65	669.15
	1.00	6.61	7.78	8.02	5.66	9.10	8.94	52.61	36.38	256.72	147.65	723.87	629.51
	1.50	2.04	2.42	1.04	0.95	1.70	1.76	5.51	3.76	99.17	54.36	590.70	493.13
	2.00	0.73	0.79	0.29	0.32	0.51	0.55	0.69	0.39	35.49	27.53	495.19	424.78
1.25	0.00	23.81	42.05	163.10	174.78	97.38	126.44	18.82	29.69	286.95	335.96	728.38	570.31
	0.25	20.59	33.27	109.80	123.10	59.77	72.91	281.84	283.74	327.52	271.34	733.74	587.17
	0.50	14.58	19.67	44.99	44.47	21.03	24.47	222.75	182.89	271.87	214.53	701.67	579.38
	0.75	9.29	11.07	15.30	14.19	7.96	8.59	101.57	69.38	200.18	126.31	674.40	563.25
	1.00	5.77	6.69	5.48	5.09	3.41	3.63	33.07	27.11	157.51	85.21	611.01	529.24
	1.50	2.42	2.78	1.21	1.20	0.91	0.99	5.50	3.74	60.55	43.06	548.87	476.71
	2.00	1.08	1.25	0.42	0.46	0.31	0.35	0.85	0.57	24.00	15.72	470.69	373.64
1.5	0.00	7.77	14.68	51.18	67.60	27.76	41.45	6.73	10.82	138.08	168.01	686.31	478.92
	0.25	7.45	13.42	41.65	51.14	19.44	26.69	20.10	33.57	160.81	137.43	601.55	485.66
	0.50	6.64	10.96	21.10	23.69	8.71	10.88	84.64	96.83	167.35	113.32	668.38	463.00
	0.75	5.79	8.12	9.63	10.33	4.00	4.58	55.69	53.46	111.48	75.83	576.60	472.61
	1.00	4.76	5.90	4.59	4.73	1.99	2.29	26.45	25.01	97.94	53.64	547.22	470.79
	1.50	2.62	3.00	1.35	1.43	0.65	0.72	5.75	4.69	55.60	21.25	511.73	434.71
	2.00	1.39	1.59	0.55	0.60	0.23	0.27	1.19	0.91	27.12	8.49	493.54	391.42
1.75	0.00	4.26	8.03	22.75	32.97	12.28	19.95	3.85	6.29	80.30	97.01	568.63	394.23
	0.25	4.16	7.76	19.15	26.84	9.33	13.94	7.27	12.66	95.33	76.66	506.45	404.74
	0.50	3.90	6.98	12.77	16.22	5.03	6.52	16.98	28.03	93.15	69.49	487.01	398.63
	0.75	3.71	5.85	7.11	8.44	2.59	3.11	34.93	38.86	75.73	48.14	481.79	418.78
	1.00	3.31	4.83	3.93	4.47	1.43	1.65	22.75	22.82	62.09	38.74	472.90	405.47
	1.50	2.61	3.10	1.42	1.60	0.51	0.57	6.26	5.95	41.53	19.02	442.68	385.83
	2.00	1.64	1.82	0.65	0.74	0.19	0.21	1.62	1.36	26.37	6.29	443.42	341.43
2	0.00	2.87	5.39	12.63	19.82	7.01	12.18	2.70	4.41	58.03	64.65	436.40	347.44
	0.25	2.82	5.22	11.26	16.66	5.68	8.62	4.14	7.13	57.65	51.27	478.36	342.76
	0.50	2.70	4.92	8.42	11.68	3.37	4.58	6.94	12.39	53.91	46.07	449.94	332.65
	0.75	2.65	4.52	5.36	6.95	1.91	2.42	12.64	19.66	50.35	39.54	457.56	345.47
	1.00	2.46	3.88	3.42	4.17	1.11	1.33	17.57	19.59	45.03	26.37	439.34	356.90
	1.50	2.13	2.89	1.47	1.72	0.43	0.49	6.84	7.10	27.69	15.50	430.07	348.88
	2.00	1.76	2.02	0.73	0.85	0.16	0.18	2.11	1.96	31.29	7.96	387.64	351.30

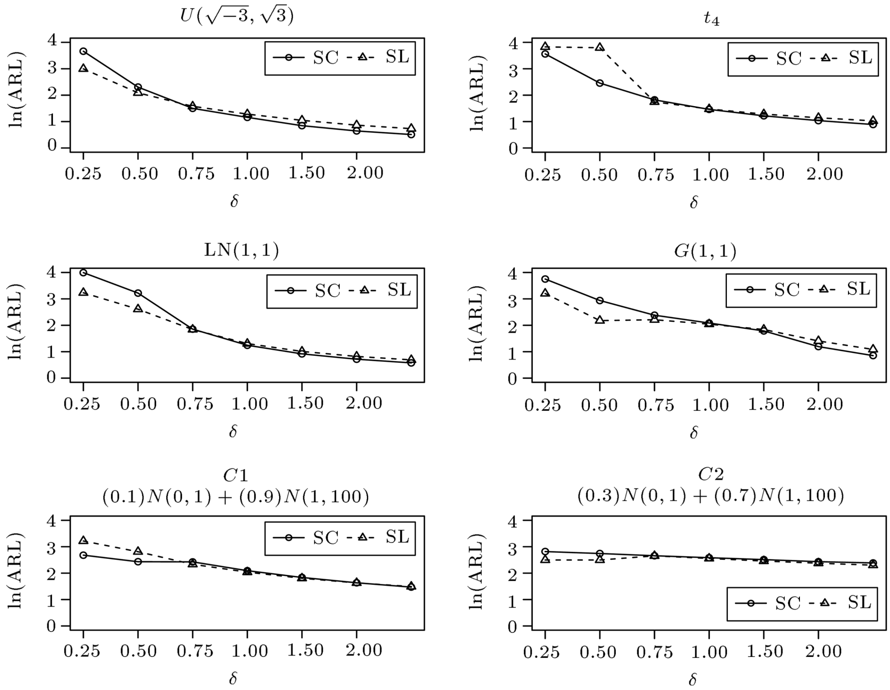

Figure 3. ($\ln \text{ARL}_1$) profile with respect to scale shift (δ) on fixed $\theta = 0.5$.

under t_4. For example, when $\theta = 0.25$ and $\delta = 1.25$, ARL_1 values of SC and SL charts are 73.29 and 84.49, respectively; when $\theta = 0.0$ and $\delta = 1.25$, ARL_1 values of SC and SL charts are 112.36 and 136.30, respectively, while when $\theta = 0.25$ and $\delta = 1.00$, ARL_1 values of SC and SL charts are 267.71 and 245.33;

4. For the case of *lognormal* distribution, SC chart performs slightly better than SL chart. Due to an upward shift in $\theta = 0.25$ and $\delta = 1.25$, ARL_1 decreasing status concerning 45.91 of SC and 57.89 of SL charts is clear. When $\theta = 0.0$ and $\delta = 1.25$, ARL_1 values of SC and SL charts are 76.10 and 102.55, respectively, while when $\theta = 0.25$ and $\delta = 1.00$, ARL_1 values of both SC and SL charts decrease as approximately 48%;

5. *Gamma (1,1)* provides substantial results when $\theta = 0.0$ and $\delta = 1.0$. When $\theta = 0.25$ and $\delta = 1.00$, ARL_1 values of both charts are greater than the intended ARL_0 values, making both charts less effective and ARL biased for such a shift. By varying δ, we observe the same effect on the results of the said charts. Moreover, having $\theta = 1.5$ and 2 with $\delta = 1.25$ shows an increasing trend as compared to the results when δ remains IC. Similar type of the finding of the exponential distribution was also noted by Riaz and Does [16];

6. In contaminated environment ($C1$ and $C2$), effectiveness of detecting the shift in location and scale is affected for both SC and SL charts as compared to other environments. SL chart performs slightly better than SC chart. In $C1$, reduction in ARL_1 values of SC and SL charts is reported as: 64% and 66% on $\theta = 0.25$ and $\delta = 1.25$; 59% and 60% on $\theta = 0.0$ and $\delta = 1.25$; and approximately 15% and 19% on $\theta = 0.25$ and $\delta = 1.00$. On the other hand, in $C2$, reduction in ARL_1 values of SC and SL charts is as: 20% and 24% on $\theta = 0.25$ and $\delta = 1.25$; 18% and 22% on $\theta = 0.0$ and $\delta = 1.25$; and approximately 1.08% and 1.9% on $\theta = 0.25$ and $\delta = 1.00$;

7. Consider the effect of specific shift in $\delta = 1.25$ on the charts with respect to different environments. The shifts in θ (on horizontal axis) and ARL_1 (on vertical axis) are portrayed in Figure 2. The results revealed better performances of SC and SL charts with the increase of θ. Further, results from Figure 3 show better performances of SC and SL charts with an increase in δ on fixed $\theta = 0.5$. Moreover, in light-tailed distributions, SC chart performs well, while SL chart is superior in heavy-tailed environments; both chartslose their performance in the case of $C2$.

3.2. Effect of reference sample and test sample on the performance of charts

Control limits of nonparametric charts are estimated from reference sample (m), and this may have significant effect on the performance of phase-II chart, which is reported in Table 4. In general, increasing m produces a decreasing trend in ARL_1 of both charts under all environments. Specifically, at fixed $\delta = 1.5$, ARL_1 of SC chart under $G(1, 1)$ decreases about 44.5%

Table 4. Profile of ARL and SDRL using $n = 11$ and $\delta = 1.5$.

Dist.	chart	m	30			50			100			150			500			1000		
		θ	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00	0.25	0.75	2.00
$G(1,1)$	SC	ARL	14.69	37.63	1.03	10.22	20.90	1.01	8.66	13.51	1.00	8.71	11.90	1.00	9.06	11.01	1.00	8.88	10.58	1.00
		SDRL	21.98	112.93	0.32	12.77	53.01	0.11	9.44	23.26	0.03	8.97	16.00	0.02	8.83	11.69	0.00	8.53	10.58	0.00
	SL	ARL	27.32	41.05	1.04	20.89	24.80	1.01	18.32	15.71	1.00	17.41	13.58	1.00	16.36	10.78	1.00	16.20	10.49	1.00
		SDRL	42.83	137.68	0.36	27.62	57.12	0.17	21.20	24.12	0.03	18.94	18.23	0.02	16.28	11.28	0.00	15.97	10.59	0.00
$U(\sqrt{-3},\sqrt{3})$	SC	ARL	15.85	5.90	1.10	11.16	4.54	1.06	9.71	4.04	1.05	9.74	3.99	1.04	3.60	2.73	1.12	3.61	2.70	1.11
		SDRL	21.68	6.55	0.34	12.84	4.45	0.26	10.32	3.73	0.22	9.91	3.62	0.21	3.10	2.18	0.36	3.10	2.15	0.36
	SL	ARL	28.72	7.48	1.13	22.31	6.15	1.10	19.19	5.49	1.08	18.40	5.27	1.07	6.36	3.76	1.17	6.35	3.75	1.16
		SDRL	42.41	8.67	0.42	28.36	6.52	0.35	21.62	5.34	0.30	19.79	4.99	0.28	5.89	3.26	0.44	5.86	3.21	0.44
$t_{(4)}$	SC	ARL	38.16	6.44	1.02	18.66	3.52	1.01	12.88	2.73	1.00	12.06	2.60	1.00	11.49	2.52	1.00	11.39	2.50	1.00
		SDRL	136.96	32.75	0.16	41.36	5.34	0.09	18.81	2.72	0.07	15.26	2.34	0.06	11.86	2.04	0.06	11.26	1.99	0.05
	SL	ARL	50.84	7.63	1.03	18.66	3.52	1.01	20.40	3.31	1.01	18.26	3.07	1.01	15.91	2.78	1.01	15.43	2.77	1.01
		SDRL	164.23	27.94	0.18	41.36	5.34	0.09	29.15	3.47	0.09	22.99	2.88	0.08	16.36	2.29	0.08	15.41	2.25	0.07
LN(1,1)	SC	ARL	14.56	2.44	1.00	8.81	1.91	1.00	6.84	1.73	1.00	6.73	1.71	1.00	6.72	1.69	1.00	6.67	1.69	1.00
		SDRL	34.97	3.00	0.02	13.34	1.55	0.01	7.84	1.21	0.01	7.18	1.15	0.01	6.40	1.11	0.01	6.23	1.09	0.01
	SL	ARL	21.92	2.81	1.00	14.54	2.28	1.00	11.46	2.06	1.00	10.92	1.99	1.00	9.72	1.89	1.00	9.57	1.87	1.00
		SDRL	52.03	3.30	0.03	21.84	2.08	0.02	13.95	1.62	0.01	12.20	1.48	0.01	9.55	1.32	0.01	9.27	1.29	0.01
$C1$	SC	ARL	152.50	58.14	2.17	74.16	28.52	1.58	40.24	12.77	1.30	35.42	10.52	1.25	30.63	8.64	1.21	29.52	8.21	1.20
		SDRL	531.70	235.39	4.01	266.60	128.20	2.33	92.66	31.75	0.81	58.63	17.81	0.63	36.22	9.37	0.52	31.30	8.15	0.50
	SL	ARL	157.89	75.30	3.23	88.63	34.24	1.95	51.95	14.30	1.34	43.38	11.37	1.29	35.06	8.83	1.23	33.30	8.38	1.23
		SDRL	550.94	334.08	31.30	293.00	202.80	51.15	102.50	30.53	0.81	64.53	19.93	0.67	37.99	9.03	0.53	35.21	8.33	0.54
$C2$	SC	ARL	435.72	241.76	19.27	350.80	240.10	27.12	233.60	159.90	24.68	215.70	133.61	17.44	198.37	102.18	9.73	192.37	92.92	8.73
		SDRL	1018.67	609.59	40.09	950.00	663.20	82.12	528.30	444.70	143.30	393.70	315.30	67.56	250.32	139.36	13.56	216.89	109.03	9.89
	SL	ARL	480.23	331.12	53.29	385.60	299.00	92.69	269.80	178.50	26.97	224.90	132.70	13.54	174.28	78.53	6.53	171.01	69.12	5.76
		SDRL	1175.49	846.16	171.27	923.10	806.40	573.60	518.30	428.20	143.90	370.10	287.60	49.11	208.62	106.99	7.89	194.29	79.72	6.44

due to an increase in m from 30 to 50, while at fixed $\theta = 0.75$, it decreases about 64.1%, 68.4%, 70.7%, and 71.8% from 30 to 100, 150, 500, and 1000 samples, respectively. On the other hand, in SL chart, 23.5%, 32.9%, 36.3%, 40.1%, and 40.7% fall out is reported in ARL_1 from $m = 30$ to 1000, respectively, on the fixed location parameter $\theta = 0.25$. Moreover, the same findings are examined for different δ at fixed $\theta = 1$.

The test sample (n) also exhibits significant effects on the performance of the phase-II chart and its profile study is given in Table 5. At fixed $\delta = 1.25$, ARL_1 of SC chart under t_4 environment decreases about 53.9% due to increase in n from 5 to 8 at fixed $\theta = 0.75$, while it decreases 66.5%, 76.6%, and 84.7% from 5 to 11, 16, and 25 samples, respectively. On the other hand, a decrease of 39.6%, 66.4%, 74.5%, and 84.7% in ARL_1 of SL chart is reported with $n = 5$ to 25, respectively, on the fixed location parameter $\theta = 0.75$. The same findings are also observed at fixed $\theta = 1$ and varying values of δ. In general, increasing the test sample size produces decreasing trend in ARL_1 of both charts under all environments.

4. Illustrative example

In this section, we apply our SC and SL charts to a dataset containing duration of contract strikes in US manufacturing industries (cf. [17]). A strike is a refusal of employees to perform their required work as a form of protest. In industries, strikes may cause losses in manufacturing and production departments. So, administration and human resource management always try to avoid it. In case of a strike, they monitor the strike duration to minimize loss. From the said data, we have considered the data from January 1968 to October 1976. Further (following Mukherjee and Sen [4]), we have considered 100 observations between January 1968 and February 1969 as a reference sample and remaining 460 data points as test samples (each of size 10). The control limits for SC and SL charts are obtained by the same simulation procedure, as mentioned in Table 1, and they are given as: 5.37 for SC chart and 11.1 for SL chart at $\text{ARL}_0 = 500$. The values of the plotting statistics for SC and SL charts, along with test samples, are reported in Table 6 and their corresponding control charts are given in Figure 4.

Table 5. Profile of ARL and SDRL using $m = 50$ and $\delta = 1.25$.

Distributions	Chart		5		8		11		16		25	
			0.75	**2**	**0.75**	**2**	**0.75**	**2**	**0.75**	**2**	**0.75**	**2**
$G(1,1)$	SC	ARL	66.23	1.53	30.14	1.03	15.81	1.00	6.20	1.00	1.88	1.00
		SDRL	206.14	2.53	111.94	0.30	37.51	0.09	17.84	0.00	7.67	0.00
	SL	ARL	59.59	1.35	30.20	1.03	16.91	1.00	5.94	1.00	1.86	1.00
		SDRL	171.20	1.68	63.90	0.31	36.08	0.10	17.68	0.00	5.49	0.00
$U(\sqrt{-3},\sqrt{3})$	SC	ARL	5.81	1.94	5.45	1.17	4.52	1.14	3.65	1.01	1.68	1.00
		SDRL	5.55	1.39	5.37	0.46	4.46	0.41	3.60	0.10	1.18	0.06
	SL	ARL	8.28	2.16	8.12	1.29	6.19	1.22	4.42	1.01	2.10	1.00
		SDRL	8.18	1.64	8.05	0.66	6.53	0.54	4.68	0.13	1.72	0.07
$t_{(4)}$	SC	ARL	13.29	1.17	6.12	1.02	4.46	1.00	3.11	1.00	2.03	1.00
		SDRL	33.41	0.49	13.75	0.14	9.94	0.05	5.49	0.02	4.60	0.00
	SL	ARL	13.29	1.17	8.03	1.03	4.46	1.00	3.39	1.00	2.03	1.00
		SDRL	33.41	0.49	23.69	0.18	9.94	0.05	6.81	0.02	3.15	0.00
LN(1,1)	SC	ARL	7.57	1.09	4.06	1.01	3.14	1.00	2.33	1.00	1.61	1.00
		SDRL	9.77	0.32	5.19	0.09	3.86	0.02	2.79	0.00	1.62	0.00
	SL	ARL	8.72	1.12	5.38	1.01	3.82	1.00	2.58	1.00	1.69	1.00
		SDRL	11.01	0.38	7.40	0.12	4.93	0.04	3.15	0.01	1.68	0.00
$C1$	SC	ARL	121.24	21.19	61.43	2.81	43.57	1.38	27.96	1.13	16.04	1.03
		SDRL	541.81	317.18	369.53	46.72	197.72	1.59	110.00	0.54	62.73	0.21
	SL	ARL	101.05	13.09	71.54	3.88	53.65	1.66	28.68	1.14	17.18	1.03
		SDRL	370.64	225.37	347.72	28.89	235.46	22.57	121.34	0.59	77.65	0.22
$C2$	SC	*ARL*	371.56	246.12	305.37	137.23	257.64	21.62	145.77	6.97	105.54	3.54
		SDRL	1008.69	934.20	1021.67	962.15	662.90	65.37	311.41	11.49	269.41	5.15
	SL	ARL	388.05	242.54	374.93	182.64	325.04	75.86	178.23	8.41	121.87	3.79
		SDRL	943.40	854.49	994.99	976.19	786.37	409.76	444.82	15.83	318.78	6.16

Table 6. Contract strikes, test samples, and corresponding SC and SL statistics.

Serial no.	Test samples (n)										SC	SL
1	5	18	44	44	59	60	7	14	31	32	0.942	1.557
2	77	1	2	7	10	18	23	25	36	42	0.466	1.361
3	46	47	50	77	9	37	41	49	52	119	2.47	4.58
4	2	13	25	31	31	35	44	45	53	111	0.784	2.149
5	3	4	5	6	7	9	14	23	26	37	2.407	4.86
6	46	47	77	2	11	16	147	2	2	4	1.745	3.181
7	6	16	18	31	42	6	7	32	44	70	0.92	1.628
8	32	71	7	27	14	26	4	4	43	60	0.186	0.639
9	62	64	68	82	3	13	30	154	3	17	1.502	2.632
10	**19**	**28**	**72**	**99**	**104**	**114**	**152**	**153**	**216**	**15**	**6.95**	**12.23**
11	21	52	109	3	5	9	26	52	61	148	0.674	1.258
12	168	2	11	19	26	30	36	47	50	87	0.416	1.124
13	3	5	7	17	23	30	104	108	192	18	0.326	0.129
14	40	47	57	1	5	10	15	19	28	42	0.594	1.346
15	64	148	4	6	12	12	28	105	112	163	2.12	3.774
16	11	12	29	50	235	10	19	41	52	100	0.769	1.534
17	3	4	10	12	34	88	101	102	104	124	2.629	5.101
18	15	61	98	22	24	38	64	84	5	6	0.445	0.886
19	70	70	1	11	18	19	50	90	9	15	0.084	0.149
20	20	24	84	117	1	23	25	59	63	179	1.582	2.399
21	92	153	17	226	13	23	2	38	3	3	1.417	1.311
22	6	139	2	25	85	13	125	4	54	91	2.193	3.807
23	38	2	6	61	18	64	122	11	16	31	0.046	0.089
24	39	41	2	4	5	7	9	13	38	3	1.911	3.883
25	10	4	5	22	27	28	36	39	85	191	0.119	0.7
26	5	44	56	6	21	33	109	125	127	8	0.762	1.574
27	9	13	14	15	28	50	60	135	5	7	0.184	0.253
28	16	21	37	41	2	2	20	24	57	8	0.651	2.062
29	16	24	59	115	123	141	146	146	3	15	3.952	6.63
30	15	18	20	26	34	84	122	174	4	14	0.347	1.274
31	15	17	22	24	39	53	107	5	9	10	0.782	1.978
32	16	22	24	31	31	34	38	42	65	74	2.706	6.862
33	101	130	1	2	2	3	4	8	11	22	4.704	7.843
34	23	27	32	33	35	43	43	44	100	2	1.378	4.147
35	19	20	20	20	23	24	33	33	63	67	2.719	8.275
36	94	116	1	8	15	15	22	23	26	27	0.176	1.844
37	55	160	5	8	13	20	42	53	59	83	0.782	1.755
38	101	8	11	15	22	58	60	108	31	42	0.832	1.656
39	45	50	61	106	142	36	52	99	38	47	4.435	8.895
40	62	38	51	98	133	9	86	141	9	5	2.534	5.763
41	49	8	13	2	6	37	28	36	48	136	0.07	0.118
42	139	2	14	15	33	143	42	8	122	56	0.886	1.503
43	14	14	106	127	131	140	141	163	22	23	4.387	7.672
44	29	99	118	2	12	12	21	21	27	38	0.247	1.458
45	42	117	2	12	19	22	75	126	8	36	0.216	0.36
46	107	5	5	29	151	9	16	29	35	65	0.137	0.274

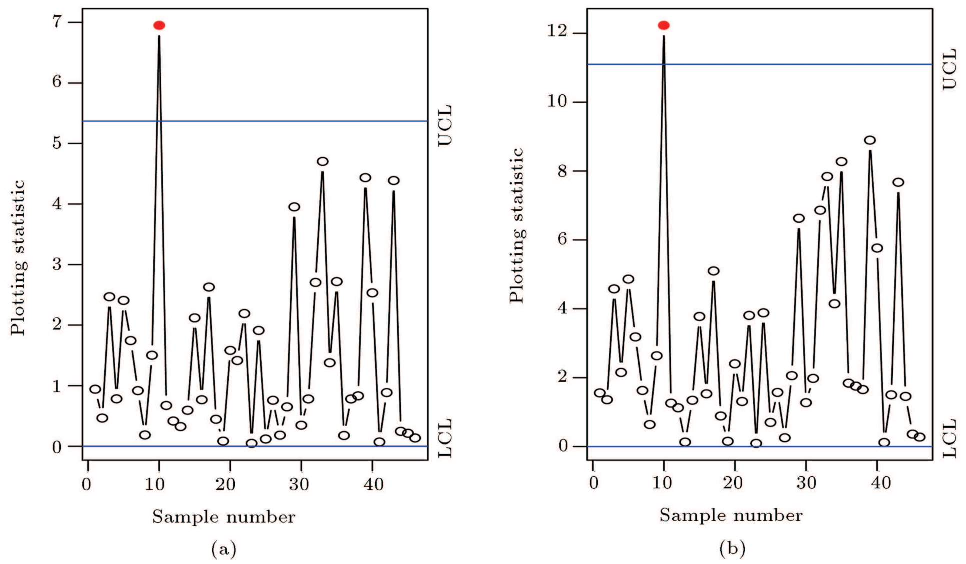

Figure 4. Control chart displays: (a) SC chart, and (b) SL chart.

It is evident that both SC and SL charts indicate an OOC signal at 10th point. For the follow-up diagnosis of shift by SL chart, we have computed the p-values for Wilcoxon test ($p_1 = 0.001684$) and Ansari Bradley test ($p_2 = 0.1267$), indicating a shift in location parameter. Similarly, for SC chart, we got the p-values for Wilcoxon test ($p_3 = 0.001684$) and mood test ($p_4 = 0.04445$), referring to locational shift. The results of this example are also in line with those of Mukherjee and Sen [4] which concluded that there is no scale shift in the process.

5. Concluding remarks

This study investigated two nonparametric, SC and SL, charts for the joint monitoring of location and scale parameters. The performance analysis revealed that SC takes an edge over SL under light-tailed distributions, while SL is a good alternative under heavy-tailed distributions. Moreover, a reasonably larger reference and test samples produce better ARL performance of these charts. Some interesting future research directions might include studying the performance of these charts: EWMA and CUSUM setups; under multiple structural breaks; when a shift occurs at steady-state.

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Biographies

Tahir Mahmood is a student of MS in Applied Statistics at the Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran, Saudi Arabia. His current research interests include statistical process control, nonparametric tech- niques, and linear profile monitoring.

Hafiz Zafar Nazir obtained his PhD in Statistics from the Institute for Business and Industrial Statistics, University of Amsterdam, the Netherlands. He is serving as an Assistant Professor in the Department of Statistics, University of Sargodha, Pakistan. His current research interests include statistical process control, nonparametric techniques, and robust methods.

Nasir Abbas is serving as an Assistant Professor at Department of Mathematics and Statistics, KFUPM. He obtained his PhD in Industrial Statistics from the Institute for Business and Industrial Statistics, University of Amsterdam, the Netherlands in 2012. His current research interests include mathematical statistics and quality control, particularly control charting methodologies under parametric and nonparametric environments.

Muhammad Riaz obtained PhD degree in statistics from the Institute for Business and Industrial Statistics, University of Amsterdam, the Netherlands in 2008. He holds the position of Associate Professor in the Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. His current research interests include statistical process control, non-parametric techniques, mathematical statistics, and experimental design.

Amjid Ali is a student of MPhil Statistics at the Department of Statistics, University of Sargodha, Sargodha, Pakistan. His current research interests include statistical process control, multivariate techniques, biostatistics, and development of new robust methods.