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An analytical method for finding exact solitary wave solutions of the coupled (2 + 1)-dimensional Painlevé Burgers equation

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Abstract. In the present study, we obtained some new analytical solutions, such as trigonometric function, rational function, and hyperbolic function solutions by using new extension of the (G'/G)-expansion method to the coupled $(2 + 1)$-dimensional Painlevé integrable Burgers equation with the aid of the computer software Maple. This method allows one to carry out the solution process of nonlinear wave equations more thoroughly and conveniently by computer algebra systems such as the Maple and Mathematica. In addition, some figures of partial solutions are provided for direct-viewing analysis. The method can also be extended to other types of nonlinear evolution equations in mathematical physics.

1. Introduction

Exact solutions of NPDEs play an important role in the proper understanding of qualitative features of many phenomena and processes in the mentioned areas of natural science. Because exact solutions of nonlinear equations graphically and symbolically are substantiated by unscrambling the mechanisms of many complex nonlinear phenomena such as spatial localization of transfer processes, multiplicity or absence of steady states under various conditions, existence of peaking regimes, and many others. Most physical systems involve several unknown variables and unknown parameters. For example, a system of partial differential equations to describe the motion of a fluid might require density, pressure, temperature, and the particle velocity as independent variables.

Exact solutions allow researchers to design and run experiments, by creating appropriate natural conditions, to determine these parameters or functions. Therefore, investigating exact traveling wave solutions is becoming successively attractive in nonlinear sciences day by day. However, not all equations posed for these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Kudryashov method [1-3], the homotopy perturbation method [4-10], the (G'/G)-expansion method [11-15], the Exp-function method [16-18], the modified simple equation method [19-22], and Hirota's bilinear transformation method [23,24].

The objective of this article is to present new extension of the (G'/G)-expansion method [25] to construct the exact traveling wave solutions for NLEEs in mathematical physics via the coupled $(2 + 1)$-dimensional Painlevé integrable Burgers equation [25]. We assume the solution of NLEEs is of the form $u(\xi) = \sum_{i=0}^{n} \alpha_i (m + F(\xi))^i + \sum_{i=1}^{n} \beta_i (m + F(\xi))^{-i}$ where $F(\xi) = G'/G$ and $G = G(\xi)$ satisfy the ordinary differential equation $G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0$, where

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k and l are arbitrary constants. From our observation, we found that if we set $m = 0$ and leave out the portion $\sum_{i=1}^{n} \beta_i(m+F(\xi))^{-i}$ in our solution, then our solutions will coincide with the solution introduced by Wang et al. [11]. Hence, we conclude that the basic (G'/G)-expansion method established by Wang et al. [11] is the particular case of our new extension of the (G'/G)-expansion method and some useful references [26-36] that can be complementary.

The paper is organized as follows. In Section 2, the enhanced new extension of the (G'/G)-expansion method is discussed. In Section 3, we apply this method to the Painlevé integrable Burgers equation. Section 4 shows the graphical illustration of obtained solutions, and conclusions are given.

2. An analytical method

Suppose the general nonlinear partial differential equation:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, \cdots) = 0, \tag{1}$$

where $u = u(x,t)$ is an unknown function, P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of new extension of (G'/G)-expansion method combined with the algebra expansion are as follows:

- **Step 1:** The traveling wave variable ansatz:

$$\xi = x \pm \omega t, \qquad u(x,t) = u(\xi), \tag{2}$$

where $\omega \in \Re - \{0\}$ is the speed of the traveling wave, and it permits us to transform Eq. (1) into the following ODE:

$$Q(u, u', u'', \cdots) = 0, \tag{3}$$

where the superscripts stand for the ordinary derivatives with respect to ξ;

- **Step 2:** Suppose the traveling wave solution of Eq. (3) can be expressed by a polynomial in $F(\xi)$ as follows:

$$u(\xi)=\sum_{i=0}^{n} \alpha_i(m+F(\xi))^i+\sum_{i=1}^{n} \beta_i(m+F(\xi))^{-i}, \tag{4}$$

where $F(\xi) = G'/G, \alpha_n$ and β_n are not zero simultaneously. Also, $G = G(\xi)$ satisfies the ordinary differential equation:

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0, \tag{5}$$

where λ and μ are arbitrary constants to be determined later. The solutions for Eq. (5) can be written as follows:

When $\Omega = \lambda^2 - 4\mu > 0$:

$$F_1 = \frac{\sqrt{\Omega}}{2} \coth\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}, \tag{6}$$

$$F_2 = \frac{\sqrt{\Omega}}{2} \tanh\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}. \tag{7}$$

When $\Omega = \lambda^2 - 4\mu < 0$:

$$F_3 = \frac{\sqrt{\Omega}}{2} \cot\left(A + \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}, \tag{8}$$

$$F_4 = \frac{\sqrt{\Omega}}{2} \tan\left(A - \frac{\sqrt{\Omega}}{2}\xi\right) - \frac{\lambda}{2}. \tag{9}$$

When $\Omega = \lambda^2 - 4\mu = 0$:

$$F_5 = \frac{B}{A + B\xi} - \frac{\lambda}{2}. \tag{10}$$

- **Step 3:** The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (1) or Eq. (3). Moreover, precisely, we define the degree of $\mu(\xi)$ as $D(u(\xi)) = n$ which gives rise to the degree of other expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right) = n + q,$$

$$D\left(u^p \left(\frac{d^q u}{d\xi^q}\right)^s\right) = np + s(n + q). \tag{11}$$

Therefore, we can find the value of n in Eq. (4) using Eq. (11);

- **Step 4:** Substituting Eq. (4) along with Eq. (5) into Eq. (3) together with the value of n obtained in Step 3, we obtain polynomials in F^i and $F^{-i}(i = 1, 2, 3, \cdots)$, then setting each coefficient of the resulted polynomial to zero yields a system of algebraic equations for α_n, β_n, and ω;

- **Step 5:** Suppose that the values of the constants α_n, β_n, and ω can be determined by solving the system of algebraic equations obtained in Step 4. Since the general solutions of Eq. (5) are known, by substituting α_n, β_n, and ω into Eq. (4), we obtain some exact traveling wave solutions of the nonlinear evolution Eq. (1).

3. Application to the coupled (2 + 1)-dimensional Painlevé integrable Burgers equation

In the present work, we consider the following coupled $(2 + 1)$-dimensional Painlevé integrable Burgers equation [25] with parameters of the form:

$$\begin{cases} -\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial y} + \alpha v\frac{\partial u}{\partial x} + \beta\frac{\partial^2 u}{\partial y^2} + \alpha\beta\frac{\partial^2 u}{\partial x^2} = 0, \\ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0. \end{cases} \tag{12}$$

$$\alpha_0=\frac{1}{3}\frac{\alpha R+\omega}{\alpha+1}+\frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1},$$

$$\beta_1=\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right],\ \alpha_1=0,$$

$$m=\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}$$

Box I

$$a_1=2\beta,\qquad m=\frac{1}{2}\lambda,\qquad \alpha_0=-\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1},$$

$$\beta_1=-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right),\qquad R=\frac{-\omega+2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha}.$$

Box II

The traveling wave transformation equations:

$$u(\xi)=u(x,y,t),\ v(\xi)=v(x,y,t),\ \xi=x+y-\omega t, \quad (13)$$

transforms Eq. (12) to the following ordinary differential equation:

$$\omega u'+uu'+\alpha vu'+\beta u''+\alpha\beta u''=0,\quad u'-v'=0. \quad (14)$$

Integrating the second relation of Eq. (14) with respect to ξ, we get:

$$v=u+R, \quad (15)$$

where R is a constant of integration.

Substituting Eq. (15) into the first relation of Eq. (14), and then integrating it with respect to ξ, setting constant of integration to zero yields:

$$(\omega+\alpha R)u+\frac{1}{2}(\alpha+1)u^2+\beta(\alpha+1)u'=0. \quad (16)$$

By balancing the highest order derivative u' and nonlinear term $u2$ from Eq. (16), we obtain $2n=n+1$, which gives $n=1$. So:

$$u=\alpha_0+\alpha_1(m+F)+\beta_1(m+F)^{-1}. \quad (17)$$

Now, substituting Eq. (17) along with Eq. (5) into Eq. (16), we get a polynomial in $F(\xi)$. Equating the coefficient of the same power of $F^i(\xi)(i=0,\pm1,\pm2,\cdots)$, we attain the system of algebraic equations, and by solving these obtained systems of equations for α_0, α_1, β_1, m, and R as well as solving the obtained systems, we get the following values:

Set 1: Please refer to Box I.

Set 2: Please refer to Box II.

Now, by using these sets of solutions for α_0, α_1, β_1, m, and R, and by using Eq. (17) along with Eqs. (6)-(10), we have the following solutions for coupled (2+1)-dimensional Painlevé integrable Burgers equation.

3.1. Hyperbolic function solutions

When $\Omega=\lambda^2-4\mu>0$, we get the following solutions:

Family 1: By using Set 1 and Eq. (6) along with Eq. (17), we have solutions of Eq. (12) as shown in Box III.

Related graph for this solution is displayed in Figure 1.

By using Set 1 and Eq. (7) along with Eq. (17), we have solutions of Eq. (12) as shown in Box IV, and, from Eq. (15), we have equations shown in Box V.

Related graph for this solution is displayed in Figure 2.

Family 2: By using Set 2 and Eq. (6) along with Eq. (17), we have solutions of Eq. (12) as shown in Box VI.

Related graph for this solution is displayed in

$$u_{1,1} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\alpha+1}$$

$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega - 5\alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\omega + \alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R + \beta\lambda\alpha + \beta\lambda + \frac{4}{3}\omega}{\beta(\alpha+1)} \pm \frac{1}{6}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$

$$\left. + \frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right]^{-1},$$

and:

$$v_{1,1} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\alpha+1}$$

$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega - 5\alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\omega + \alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R + \beta\lambda\alpha + \beta\lambda + \frac{4}{3}\omega}{\beta(\alpha+1)} \pm \frac{1}{6}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$

$$\left. + \frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right]^{-1} + R.$$

Box III

$$u_{2,1} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\alpha+1}$$

$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega - 5\alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\omega + \alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right]$$

$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R + \beta\lambda\alpha + \beta\lambda + \frac{4}{3}\omega}{\beta(\alpha+1)} \pm \frac{1}{6}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$

$$\left. + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right]^{-1}.$$

Box IV

$$v_{2,1} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right]^{-1}+R.$$

Box V

$$u_{3,2} = -\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A\right.\right.$$
$$\left.\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Box VI

Figure 3, and the equation shown in Box VII is obtained.

By using Set 2 and Eq. (7) along with Eq. (17), we have solutions of Eq. (12) as shown in Box VIII, and, from Eq. (15), we obtain the equation shown in Box IX.

3.2. Trigonometric function solutions

Family 3: By using Set 1 and Eq. (8) along with Eq. (17), we have solutions of Eq. (12) as shown in Box X, and, from Eq. (15), we have equation shown in Box XI.

Related graph for this solution is displayed in Figure 4.

By using Set 1 and Eq. (9) along with Eq. (17), we have solutions of Eq. (12) as shown in Box XII, and, from Eq. (15), we have equation shown in Box XIII.

Related graph for this solution is displayed in Figure 5.

Family 4: By using Set 2 and Eq. (8) along with Eq. (17), we have solutions of Eq. (12) as shown in

$$v_{3,2} = -\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A\right.\right.$$
$$\left.\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)\right.$$
$$\left.-\frac{\lambda}{2}\right)^{-1}+\frac{-\omega+2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha}.$$

Box VII

$$u_{4,2} = -\frac{2\sqrt{\beta^2\lambda^2 - 3\beta^2\mu\alpha^2 + 3\beta^2\lambda^2\alpha^2 + 2\beta^2\lambda^2\alpha - 6\beta^2\mu\alpha - 3\beta^2\mu}}{\alpha+1} + 2\beta\left(\frac{1}{2}\lambda + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right) - 2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right)^{-1}.$$

Box VIII

$$v_{4,2} = -\frac{2\sqrt{\beta^2\lambda^2 - 3\beta^2\mu\alpha^2 + 3\beta^2\lambda^2\alpha^2 + 2\beta^2\lambda^2\alpha - 6\beta^2\mu\alpha - 3\beta^2\mu}}{\alpha+1} + 2\beta\left(\frac{1}{2}\lambda + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right) - 2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda + \frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right)^{-1} + \frac{-\omega + 2\sqrt{\beta^2\lambda^2 - 3\beta^2\mu\alpha^2 + 3\beta^2\lambda^2\alpha^2 + 2\beta^2\lambda^2\alpha - 6\beta^2\mu\alpha - 3\beta^2\mu}}{\alpha}.$$

Box IX

Box XIV, and, from Eq. (15), we have the equation shown in Box XV.

By using Set 2 and Eq. (9) along with Eq. (17), we have solutions of Eq. (12) as shown in Box XVI.

Related graph for this solution is displayed in Figure 6. From Eq. (15), the equation shown in Box XVII is obtained.

3.3. Rational function solutions

Family 5: By using Set 1 and Eq. (10) along with Eq. (17), we have solutions of Eq. (12) as shown in Box XVIII, and, from Eq. (15), we have the equation shown in Box XIX.

Related graph for this solution is displayed in Figure 7.

Family 6: By using Set 2 and Eq. (10) along with Eq. (17), we have solutions of Eq. (12) as shown in Box XX, and, from Eq. (15), we obtain the equation shown in Box XXI.

4. Discussion and conclusion

From obtained solutions, we observe that solutions from Family 1 to Family 2 are hyperbolic function solutions for $\lambda^2 - 4\mu > 0$, from Family 3 to Family 4 are trigonometric function solutions for $\lambda^2 - 4\mu < 0$,

$$u_{5,3} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\alpha+1} + \frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega - 5\alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right] \times\left[\omega + \alpha R + \sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}\right] \times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R + \beta\lambda\alpha + \beta\lambda + \frac{4}{3}\omega}{\beta(\alpha+1)} \pm \frac{1}{6}\frac{\sqrt{4\alpha R^2 - 6\alpha^2R^2 - 12\beta^2\mu - 4\omega\alpha R - 12\beta^2\mu\alpha^2 + 3\beta^2\lambda^2 - 2\omega^2 + 9\beta^2\lambda^2\alpha^2 - 24\beta^2\mu\alpha}}{\beta(\alpha+1)} + \frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A + \frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right) - \frac{\lambda}{2}\right]^{-1}.$$

Box X

$$v_{5,3}=\frac{1}{3}\frac{\alpha R+\omega}{\alpha+1}+\frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right]^{-1}+R.$$

Box XI

$$u_{6,3}=\frac{1}{3}\frac{\alpha R+\omega}{\alpha+1}+\frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right]^{-1}.$$

Box XII

$$v_{6,3}=\frac{1}{3}\frac{\alpha R+\omega}{\alpha+1}+\frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right]^{-1}+R.$$

Box XIII

$$u_{7,4}=-\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Box XIV

$$v_{7,4}=-\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\cot\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)^{-1}+\frac{-\omega+2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha}.$$

Box XV

and from Family 9 to Family 12 are rational function solutions for $\lambda^2-4\mu=0$. Figures 1, 3, 5, and 6 represent periodic solutions; Figures 2 and 7 represent soliton solutions, and Figure 4 represent kink solutions of Painlevé integrable Burgers equation. In this paper, we have successfully used the new extension of the (G'/G)-expansion method introduced by Islam [25] for solving the coupled (2 + 1)-dimensional Painlevé integrable Burgers equation. We have successfully obtained some exact traveling wave solutions of the coupled (2+1)-dimensional Painlevé integrable Burgers equation with parameters. When the parameters are taken as special values, the solitary wave solutions and periodic wave solutions are originated from the

$$u_{8,4}=-\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A-\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)^{-1}.$$

Box XVI

$$v_{8,4}=-\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}\tan\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{\sqrt{\lambda^2-4\mu}}{2}tan\left(A+\frac{\sqrt{\lambda^2-4\mu}}{2}(x+y-\omega t)\right)-\frac{\lambda}{2}\right)^{-1}+\frac{-\omega+2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha}.$$

Box XVII

$$u_{9,5} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right]^{-1}.$$

Box XVIII

$$v_{9,5} = \frac{1}{3}\frac{\alpha R+\omega}{\alpha+1} + \frac{1}{3}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\alpha+1}$$
$$+\frac{1}{18}\frac{1}{\beta(\alpha+1)^2}\left[-5\omega-5\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\omega+\alpha R+\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}\right]$$
$$\times\left[\frac{1}{2}\frac{\frac{4}{3}\alpha R+\beta\lambda\alpha+\beta\lambda+\frac{4}{3}\omega}{\beta(\alpha+1)}\pm\frac{1}{6}\frac{\sqrt{4\alpha R^2-6\alpha^2R^2-12\beta^2\mu-4\omega\alpha R-12\beta^2\mu\alpha^2+3\beta^2\lambda^2-2\omega^2+9\beta^2\lambda^2\alpha^2-24\beta^2\mu\alpha}}{\beta(\alpha+1)}\right.$$
$$\left.+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right]^{-1}+R.$$

Box XIX

$$u_{10,6} = -\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right)$$
$$-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right)^{-1}.$$

Box XX

$$v_{10,6} = -\frac{2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha+1}+2\beta\left(\frac{1}{2}\lambda+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right)$$
$$-2\beta\left(-\frac{1}{4}\lambda^2+\mu\right)\left(\frac{1}{2}\lambda+\frac{B}{A+B(x+y-\omega t)}-\frac{\lambda}{2}\right)^{-1}$$
$$+\frac{-\omega+2\sqrt{\beta^2\lambda^2-3\beta^2\mu\alpha^2+3\beta^2\lambda^2\alpha^2+2\beta^2\lambda^2\alpha-6\beta^2\mu\alpha-3\beta^2\mu}}{\alpha}.$$

Box XXI

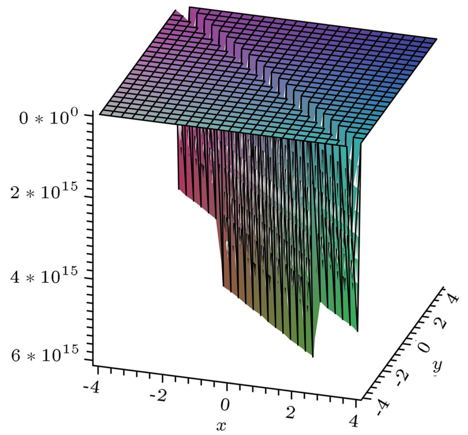

Figure 1. Kink profile of solutions $u_{1,1}$ and $v_{1,1}$ of Painlevé integrable Burgers equation for $\mu = -1$, $R = 0$, $\omega = 0$, $\beta = 1$, $\alpha = 1$, $\lambda = 1$ and $A = 0$ within the intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

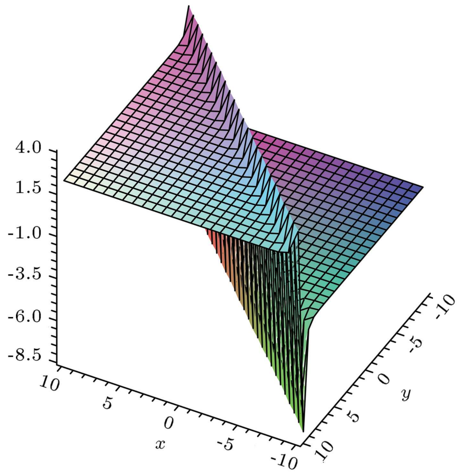

Figure 2. Soliton profile of solutions $u_{2,1}$, and $v_{2,1}$ of Painlevé integrable Burgers equation for $\mu = -1$, $R = 0$, $\omega = 0$, $\beta = 1$, $\alpha = 1$, $\lambda = 1$, and $A = 0$ within the intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

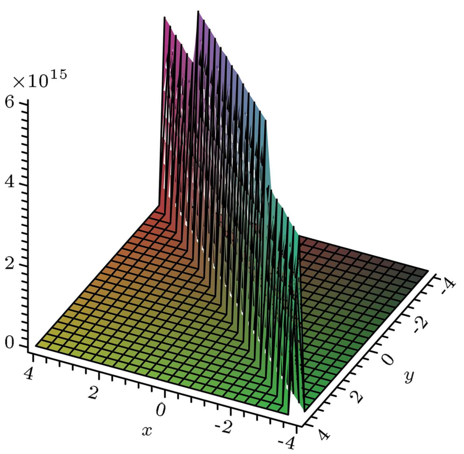

Figure 3. Periodic solutions $u_{3,2}$ and $u_{7,4}$ of Painlevé integrable Burgers equation for $\beta = 2$, $\mu = -1$, $R = 0$, $\omega = 0$, $\alpha = 1$, $\lambda = 1$, and $A = 0$ within the intervals $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$.

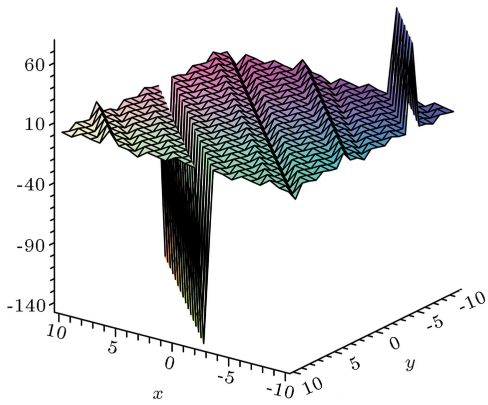

Figure 4. Periodic profile of solutions $u_{5,3}$ and $v_{5,3}$ of Painlevé integrable Burgers equation for $\mu = -1$, $R = 0$, $\omega = 0$, $\beta = 1$, $\alpha = 1$, $\lambda = 1$, and $A = 0$ within the intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

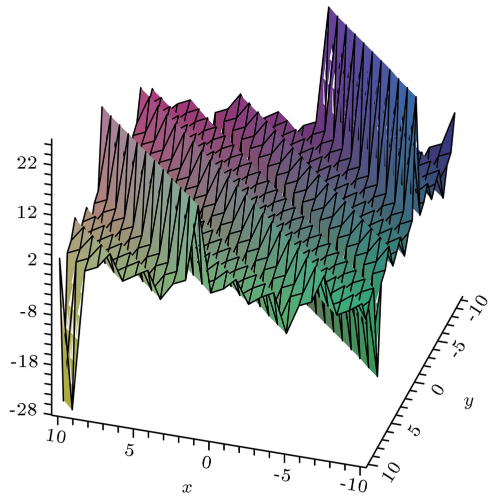

Figure 5. Periodic profile of solutions $u_{6,3}$ and $v_{6,3}$ of Painlevé integrable Burgers equation for $\mu = -1$, $R = 0$, $\omega = 0$, $\beta = 1$, $\alpha = 1$, $\lambda = 1$, and $A = 0$ within the intervals $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

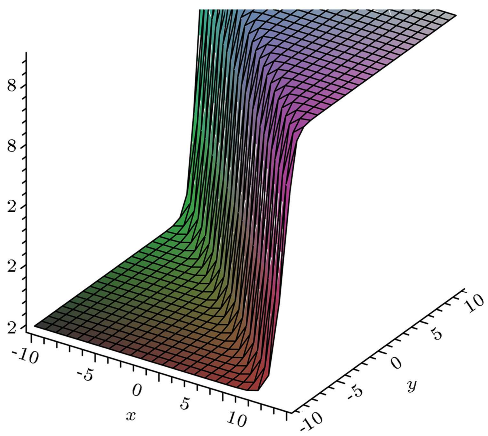

Figure 6. Periodic solutions $u_{4,2}$ and $u_{8,4}$ of Painlevé integrable Burgers equation for $\beta = 2$, $\mu = -1$, $R = 0$, $\omega = 0$, $\alpha = 1$, $\lambda = 1$, and $A = 0$ within the intervals $-4 \leq x \leq 4$ and $-4 \leq y \leq 4$

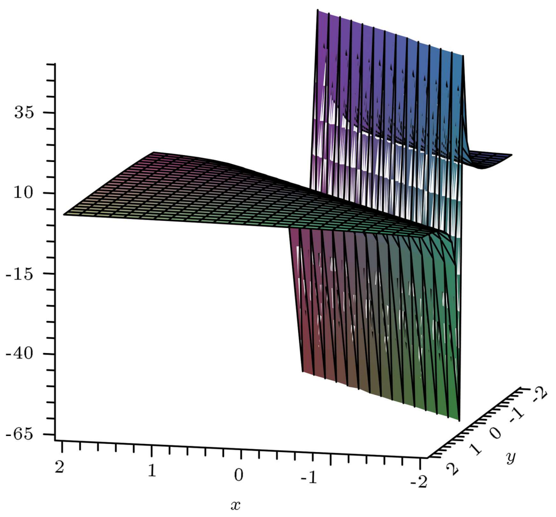

Figure 7. Soliton profile of solution $v_{9,5}$ of Painlevé integrable Burgers equation for $\mu = -1$, $R = 0$, $\omega = 0$, $\beta = 1$, $\alpha = 1$, $\lambda = 1$, $A = 0$, and $B = 1$ within the intervals $-2 \le x \le 2$, $-2 \le y \le 2$.

exact solutions. The merit of the method is that it is independent of the integrability of the coupled NLPDEs; therefore, it can be used to solve both integrable and nonintegrable coupled NLPDEs. This work shows that the new extension of the (G'/G)-expansion method is sufficient, effective, and suitable for solving other nonlinear evolution equations; it deserves further applying and studying as well. To our knowledge, the solutions obtained in this paper have not been reported in the literature so far.

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References

1. Neirameh, A. "Exact analytical solutions for 3D-Gross-Pitaevskii equation with periodic potential by using the Kudryashov method", *Journal of the Egyptian Mathematical Society*, **24**(1), pp. 49-53 (2016). DOI: 10.1016/j.joems.2014.11.004.
2. Eslami, M. and Neirameh, A. "New solitary and double periodic wave solutions for a generalized sinh-Gordon equation", *Eur. Phys. J. Plus*, **129**, p. 54 (2014).
3. Kim, H., Bae, J.H. and Sakthivel, R. "Exact travelling wave solutions of two important nonlinear partial differential equations", *Z. Naturforsch A. Phys. Sci.*, **69a**, pp. 155-62 (2014).
4. Mohiud-Din, S.T. and Muhammad, A.N. "Homotopy perturbation method for solving fourth-order boundary value problems", *Math. Prob. Eng.*, **2007**, pp. 1-15 (2007).
5. Mohyud-Din, S.T. and Noor, M.A. "Homotopy perturbation method for solving partial differential equations", *Z. Naturforsch A: Phys. Sci.*, **64a**, pp. 157-70 (2009).
6. Mohyud-Din, S.T., Yildirim, A. and Sariaydin, S. "Numerical soliton solutions of the improved Boussinesq equation", *Int. J. Numer. Methods Heat Fluid Flow*, **21**(7), pp. 822-7 (2011).
7. Mohyud-Din, S.T., Yildirim, A. and Demirli, G. "Analytical solution of wave system in Rn with coupling controllers", *Int. J. Numer. Methods Heat Fluid Flow*, **21**(2), pp. 198-205 (2011).
8. Mohyud-Din, S.T., Yildirim, A. and Sariaydin, S. "Numerical soliton solution of the Kaup-Kupershmidt equation", *Int. J. Numer. Methods Heat Fluid Flow*, **21**(3), pp. 272-81 (2011).
9. Sakthivel, R., Chun, C. and Lee, J. "New travelling wave solutions of Burgers equation with finite transport memory", *Z. Naturforsch. A: Phys. Sci.*, **65a**, p. 40 (2010).
10. Chun, C. and Sakthivel, R. "Homotopy perturbation technique for solving two point boundary value problems - comparison with other methods", *Comput. Phys. Commun.*, **181**, pp. 1021-4 (2010).
11. Wang, M., Li, X. and Zhang, J. "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics", *Phys. Lett. A*, pp. 417-23 (2008).
12. Zayed, E.M.E. and Gepreel, K.A. "The (G'/G)-expansion method for finding the traveling wave solutions of nonlinear partial differential equations in mathematical physics", *J. Math. Phys.*, **50**, 013502-14 (2009).
13. Guo, S. and Zhou, Y. "The extended (G'/G)-expansion method and its applications tothe Whitham-Broer-Kaup-like equations and coupled Hirota-Satsuma KdV equations", *Appl. Math. Comput.*, **215**, pp. 3214-3221 (2010).
14. Kim, H. and Sakthivel, R. "New exact travelling wave solutions of some nonlinear higher dimensional physical models", *Z. Naturforsch A: Phys. Sci.*, **65a**, pp. 197-202 (2010).
15. Kim, H. and Sakthivel, R. "New exact traveling wave solutions of some nonlinear higher-dimensional physical models", *Rep. Math. Phys.*, **70**(1), pp. 39-50 (2012).
16. Bekir, A. and Boz, A. "Exact solutions for nonlinear evolution equations using Exp function method", *Phys. Lett. A*, **372**, pp. 16-19-25 (2008).
17. Akbar, M.A. and Ali, N.H.M. "Exp-function method for Duffing equation and new solutions of (2+1) dimensional dispersive long wave equations", *Prog. Appl. Math.*, **1**(2), pp. 30-42 (2011).
18. Naher, H., Abdullah, A.F. and Akbar, M.A. "The Exp-function method for new exact solutions of the nonlinear partial differential equations", *Int. J. Phys. Sci.*, **6**(29), pp. 6706-16 (2011).

19. Jawad, A.J.M., Petkovic, M.D. and Biswas, A. "Modified simple equation method for nonlinear evolution equations", *Appl. Math. Comput.*, **217**, pp. 869-77 (2010).

20. Zayed, E.M.E. "A note on the modified simple equation method applied to Sharma-Tasso-Olver equation", *Appl. Math. Comput.*, **218**, pp. 3962-4 (2011).

21. Khan, K. and Akbar, M.A. "Exact and solitary wave solutions for the Tzitzeica-Dodd-Bullough and the modified KdV-Zakharov-Kuznetsov equations using the modified simple equation method", *Ain Shams Eng. J.*, **4**(4), pp. 903-909 (2013). DOI:10.1016/j.asej.2013.01.010.

22. Khan, K. and Akbar, M.A. "Exact solutions of the (2+1)-dimensional cubic Klein-Gordon equation and the (3+1)-dimensional Zakharov-Kuznetsov equation using the modified simple equation method", *J. Assoc. Arab. Univ. Basic Appl. Sci.*, **15**, pp. 74-81 (2014).

23. Hirota, R. "Exact envelope soliton solutions of a nonlinear wave equation", *J. Math. Phys.*, **14**, pp. 805-810 (1973).

24. Hirota, R. and Satsuma, J. "Soliton solutions of a coupled KDV equation", *Phys. Lett. A*, **85**, pp. 404-8 (1981).

25. Islam, M.S., Kamruzzaman, K. and Ali Akbar, M. "An analytical method for finding exact solutions of modified Korteweg-de Vries equation", *Results Phys.* **5**, pp. 131-135 (2015).

26. Abdel-Gawad, H.I. "Towards a unified method for exact solutions of evolution equations. An application to reaction diffusion equations with finite memory transport", *J. of Statistical Phys.*, **147**, pp. 506-518 (2012).

27. Hamdy, I., Gawad, A. and Tantawy, M. "Exact solutions of the Shamel-Korteweg-de Vries equation with time dependent coefficients", *Inf. Sci. Lett.*, **3**(3), pp. 103-109 (2014).

28. Wazwaz, A.M., *Partial Differential Equations and Solitary Wave Solutions*, Springer (2009).

29. Muhammad, Y., Syed Tahir, R.R. and Safdar, A. "Analytical and soliton solutions: nonlinear model of nanobioelectronics transmission lines", *Applied Mathematics and Computation*, **265**, pp. 994-1002 (2015).

30. Muhammad, Y. and Safdar, A. "Bright, dark and singular solitons in magneto-electro-elastic circular rod", *Waves in Random and Complex Media*, **25**(4), pp. 549-555 (2015).

31. Muhammad, Y. and Safdar, A. "Solitary wave and shock wave solutions to the transmission line model for nano-ionic currents along microtubules", *Applied Mathematics and Computation*, **246**, pp. 460-463 (2014).

32. Safdar, A., Syed Tahir, R.R. and Muhammad, Y. "Traveling wave solutions for nonlinear dispersive water wave systems with time dependent coefficients", *Nonlinear Dynamics*, **82**(4), pp. 1755-1762 (2015).

33. Muhammad, Y. and Syed Tahir, R.R. "Dispersive dark optical soliton in (2+1)-dimensions by G'/G-expansion with dual-power law nonlinearity", *Optik-International Journal for Light and Electron Optics*, **126**(24), pp. 5812-5814 (2015).

34. Muhammad, Y., Safdar, A. and Syed Amer, M. "Solitons for compound KdV-Burgers' equation with variable coefficients and power law nonlinearity", *Nonlinear Dynamics*, **81**(3), pp. 1191-1196 (2015).

35. Muhammad, Y., Hamoodur, R. and Muzamil. I. "Travelling wave solutions to some nonlinear evolution equations", *Applied Mathematics and Computation*, **249**, pp. 81-88 (2014).

36. Nadia, C. and Muhammad, Y. "New and more general traveling wave solutions for nonlinear Schrodinger equation", *Waves in Random and Complex Media*, **26**(1), pp. 84-91 (2016).

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