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Robust Holt-Winter based control chart for monitoring autocorrelated simple linear profiles with contaminated data

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Abstract. Profile monitoring is a useful technique in statistical process control used when quality of the product or process is represented by a function over a time period. This function represents the relationship between a response variable and one or more explanatory variables. Most existing control charts for monitoring profiles are based on the assumption that the observations within each profile are independent of each other, which is often violated in practice. Sometimes there are one or more outliers in each profile which lead to poor statistical performance of the control chart. This paper focuses on Phase II monitoring of a simple linear profile with autocorrelation within profile data in the presence of outliers. In this paper, we propose a new combined control chart based on the robust Holt-Winter model to decrease the effect of outliers. We first evaluate the effect of outliers on the performance of the proposed combined control chart. Then, we apply robust Holt-Winter and design a robust combined control chart to overcome the effect of outliers. The performance of the proposed robust Holt-Winter control chart is evaluated through extensive simulation studies. The results show that the proposed robust control chart performs well.

1. Introduction

The quality of some industrial processes is described using a relationship between a response variable and one or more explanatory variables known as profile [1]. Different types of profile based on the type of response variable, including linear profiles, nonlinear profiles, generalized linear profiles, multivariate profiles, and so on, are considered by researchers. Profile monitoring is done in Phases I and II. In Phase I, a control chart is designed based on an existing data set to check the stability of the process. In this phase, the process stability is evaluated and the process parameters under in-control status are estimated. In Phase II, the estimated parameters in Phase I are used to design the control chart for monitoring the process over time and detecting any shifts in the process [2,3]. Woodall et al. [4] presented an introduction as well as a literature review on profile monitoring. Several applications of profiles monitoring have been presented by Kang and Albin [5], Mahmuod and Woodall [6], Wang and Tsung [7], Montgomery [8], Zou et al. [9], and Williams et al. [10]. Simple linear profiles for Phases I and II were reported by Mestek et al. [11], Stover and Brill [12], Kang and Albin [5], Kim et al. [13], Noorossana et al. [14], Wang and Tsung [7], Zou et al. [9], Zhang et al. [15], Saghaei et al. [16], Soleimani and Noorossana [17], Narvand et al. [18], Soleimani et al. [19], and Soleimani and Noorossana [20]. The

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main assumption in many control charts used for profile monitoring is that the observations within or between profiles are independent, which is often violated in practice. Some researchers show the spatial or serial correlation within or between profiles [21]. Jensen et al. [21] proposed a Linear Mixed Model (LMM) to explain the autocorrelation within a linear profile in Phase I. Jensen and Birch [22] showed that using mixed models has considerable advantages when there is autocorrelation within nonlinear regression profiles. Soleimani et al. [23] proposed a transformation to eliminate the AR(1) structure between observations within simple linear profiles and then used the traditional control procedures in the literature to monitor the profiles in Phase II. Soleimani and Noorossana [17] evaluated the effect of autocorrelation in monitoring multivariate simple linear profiles in Phase II. They assessed the effect of three main models namely AR(1), MA(1), and ARMA(1,1) on the monitoring schemes of multivariate simple linear profiles. Narvand et al. [18] used the linear mixed model for monitoring linear profiles within autocorrelation structure while there are simultaneous random and fixed effects in the structure of autocorrelated linear profiles. In this paper, they used T^2 Hotelling, Multivariate Exponential Weighted Moving Average (MEWMA), and multivariate cumulative sum (MCUSUM) control charts to monitor the process. Another research in this scope is the paper of Soleimani et al. [19] in which they monitored the multivariate simple linear profiles in Phase II when there was autocorrelation between observations within each profile. They proposed an efficient technique based on a transformation method to eliminate the autocorrelation structure within the profiles. Zhang et al. [24] proposed a new approach to monitor a linear profile when there is autocorrelation within each profile. Their approach is Gaussian process model which describes the Within Profile Correlation (WPC). Also, they presented two types of Shewhart multivariate control charts in Phase II to monitor the linear trend and the WPC, separately. Also, Soleimani and Noorossana [20] focused on the monitoring of multivariate simple linear profiles with autocorrelation between profiles. They presented three methods based on time series models to eliminate the effects of autocorrelation. Also, they evaluated another case with the presence of outliers in the autocorrelated observations within each profile. Finally, they showed that the presence of outliers has a detrimental effect on the parameter estimation and the control chart performance.

One of the efficient methods to solve this problem is a robust approach. Robust is a useful technique to decrease or eliminate the effects of outlier on the control charts proposed for profiles monitoring in both Phases I and II. Some researchers have used this technique in their papers, such as Ebadi and Shahriari [25] who proposed a new approach in Phase I monitoring of simple linear profiles based on robust approach. They applied two weighted functions, i.e. Huber and Bisquare, in the parameter estimation and proposed a robust method to estimate the error variance. Jearkpaporn et al. [26] proposed a new model based on control and response variables and evaluated the effect of outliers in multistage processes. They introduced a robust approach to modify the effect of outliers on monitoring multistage processes characterized by the Generalized Linear Model. Results showed that the average run length performance of the proposed model was appropriate to detect the small shifts in the presence of outliers. Asadzadeh and Aghaie [27] proposed a robust monitoring approach based on Huber's M-estimator to reduce outlier's effect. Then, they investigated the performance of the robust and non-robust Cause Selecting Control charts (CSCs) using the average run length criterion. The results showed that the Huber based CSC had better performance than the traditional CSC to detect the out-of-control conditions of the process. Shahriari et al. [28] proposed a new approach for parameter estimation in the simple linear profile instead of ordinary least square method. They used two weighted functions in the proposed robust algorithm and then presented a new method to estimate the process variance. Then, the results of their research showed that the robust estimators, like classic estimators, had a suitable performance in the absence of outliers, while in the presence of outliers, the robust estimators had better performance than the classic estimators. Also, Asadzadeh et al. [29] developed a robust approach for monitoring the multistage processes. They assumed that the process consists of two stages and there are outliers in historical quality data. Then, they presented a robust method based on compound-estimator to build the relationship between the quality characteristics.

In this paper, we specifically concentrate on Phase II monitoring of within-profile autocorrelation in simple linear profiles with the presence of outliers. We propose a combined EWMA/χ^2 control chart based on the Holt-Winter model and investigate the effect of outliers on the statistical performance of the proposed control chart. In addition, the performance of the proposed robust control chart is evaluated by extensive simulation studies in terms of some criteria such as Average Run Length (ARL), Median Absolute Deviance (MAD), and Interquantile Range (IQR).

This paper is organized as follows: In the next section, we first present the autocorrelated simple linear profile model. Then, we develop Holt-Winter and robust Holt-Winter, first proposed by Croux et al. [30], to use them in monitoring the autocorrelated simple linear profiles without and with presence of outliers, respectively. In Section 3, a combined

EWMA/χ^2 control chart based on the ordinary and robust Holt-Winter models is proposed to monitor the autocorrelated simple linear profiles with and without outliers, respectively. In Section 4, extensive simulation studies are performed to evaluate the performance of the robust Holt-Winter based control chart in terms of ARL, MAD, and IQR criteria. Our concluding remarks and some future research are presented in Section 5.

2. Problem definition and the ordinary and robust Holt-Winter models

In this section, we first define and model the problem and define the corresponding assumptions. Then, we develop Ordinary Holt-Winter model (O-HW) and Robust Holt-Winter model (R-HW) corresponding to the problem. In this section, we describe the autocorrelated simple linear profile model for the jth sample profile as we have observations (x_i, y_{ij}), $i = 1, 2, ..., n$. It is assumed that when the process is in-control, the autocorrelation within simple linear profile can be modeled using the following equation:

$$y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij},$$
$$\varepsilon_{ij} = \phi\varepsilon_{(i-1)j} + a_{ij}, \tag{1}$$

where y_{ij} and x_i are the ith response variable in the jth sample profile and x_i is the ith value of explanatory variable. Also, A_0 and A_1 are the intercept and slope parameters, respectively; ε_{ij}'s are the autocorrelated error terms; and a_{ij}'s are independent identically distributed normal random variables with mean 0 and variance (i.e. $a_{ij} \sim N(0, \sigma^2)$). In this paper, we focus on Phase II monitoring of simple linear profile; therefore, we assume that the parameters A_0, A_1, and σ^2 are known. We assume that there is autocorrelation within a simple linear profile and the autocorrelation structure is a first-order autoregressive (AR(1)) model. In the next subsection, we present the ordinary and robust Holt-Winter models developed for our problem and calculate the residuals based on these two models. Then, the performance of the proposed robust control chart in the presence of outliers is tested through simulation studies.

2.1. *The proposed ordinary Holt-Winter model for the explained problem*

Consider a time series observed up to time point t-1. The Holt-Winter model for AR(1) is a one-step-ahead model to make a prediction of the time series at time t, denoted by $\hat{y}_{t|t-1}$ [30]. If y_{ij} is the ith observation in the jth profile, the one-step-ahead for autocorrelated simple linear profile is as follows:

$$e_{ij} = y_{ij} - \hat{y}_{ij|i-1}. \tag{2}$$

The Holt-Winter model has some parameters including local level and local trend [30] denoted by α_{ij} and β_{ij} in our problem, respectively, and calculated using Eq. (3):

$$\hat{\alpha}_{ij} = \lambda_1 y_{ij} + (1-\lambda_1)\left(\hat{\alpha}_{(i-1)j} + \hat{\beta}_{(i-1)j}\right),$$
$$\hat{\beta}_{ij} = \lambda_2\left(\hat{\alpha}_{ij} - \hat{\alpha}_{(i-1)j}\right) + (1-\lambda_2)\hat{\beta}_{(i-1)j}, \tag{3}$$

and the forecast value of the ith observation in the jth profile is equal to $\hat{y}_{ij|i-1} = \hat{\alpha}_{(i-1)j} + \hat{\beta}_{(i-1)j}$ [30].

The parameters λ_1 and λ_2 in Eq. (3) are smoothing parameters with values between zero and one. The smoothing parameters λ_1 and λ_2 are selected by minimizing the sum of squares forecast errors. In this paper, we assume that the values of $\hat{\alpha}_{0j}$ and $\hat{\beta}_{0j}$ in the Holt-Winter model are equal to the values of A_0 and A_1 in the autocorrelated simple linear profile given in Eq. (1).

2.2. *The proposed robust Holt-Winter model for the explained problem*

When there are outliers in the observations, the smoothing parameters in the Holt-Winter model are biased. Also, the prediction error has turgid scale [30]. In addition, we illustrate through simulation studies that the outliers affect the statistical performance of the developed control chart. Hence, we represent the robust Holt-Winter of Croux et al. [30] in the case of AR(1) autocorrelated simple linear profile.

A cleaned version, y^*_{ij}, of the ith observation in the jth profile (y_{ij}) is obtained by Eq. (4):

$$y^*_{ij} = \psi_k\left(\frac{y_{ij} - \hat{\alpha}_{(i-1)j} - \hat{\beta}_{(i-1)j}}{\hat{\sigma}_{ij}}\right)\hat{\sigma}_{ij} + \hat{\alpha}_{(i-1)j} + \hat{\beta}_{(i-1)j}, \tag{4}$$

where $\psi_k(k) = \max(-k, \min(y, k))$ is the Huber ψ-function with boundary value k, and $\hat{\sigma}_{ij}$ is a local-scale estimate of the one-step-ahead forecast errors [30]. In this paper, we assume that the boundary value k is equal to 2 to limit the effect of forecast errors more than two times the estimate of local scale. The local scale estimate is computed, recursively, as [30]:

$$\hat{\sigma}^2_{ij} = \lambda_\sigma \rho_k\left(\frac{y_{ij} - \hat{\alpha}_{(i-1)j} - \hat{\beta}_{(i-1)j}}{\hat{\sigma}_{(i-1)j}}\right)\hat{\sigma}^2_{(i-1)j} + (1-\lambda_\sigma)\hat{\sigma}^2_{(i-1)j}, \tag{5}$$

where $\rho_k(y) = \min(k^2, y^2)$ is a bounded loss function with boundary value $k = 2$ and $\lambda_\sigma = 0.3$ [30]. Then, the new recursive equation of robust Holt-Winter algorithm is as [30]:

$$\hat{\alpha}_{ij} = \lambda_1 y^*_{ij} + (1-\lambda_1)\left(\hat{\alpha}_{(i-1)j} + \hat{\beta}_{(i-1)j}\right),$$
$$\hat{\beta}_{ij} = \lambda_2\left(\hat{\alpha}_{ij} - \hat{\alpha}_{(i-1)j}\right) + (1-\lambda_2)\hat{\beta}_{(i-1)j}. \tag{6}$$

Note that in the robust model, to compute the smoothing parameters in Eq. (6), instead of minimizing the sum of squares of the one-step-ahead forecast errors over the training sample, we use the measure in Eq. (7) [30]. Hence, the smoothing parameters of the robust Holt-Winter model are different from the ones in the ordinary Holt-Winter model:

$$\left(\lambda_1^{\text{opt}}, \lambda_2^{\text{opt}}\right) = \arg \min_{(\lambda_1,\lambda_2)} \sum_{i=1}^{n} \rho_k \left(\frac{y_{ij} - \hat{y}_{ij|i-1}}{s_0}\right), \quad (7)$$

where $s_0 = \text{med}_{1\leq i\leq n}|e_{ij}|$ [30].

3. The proposed control scheme

As mentioned in the previous sections, this paper focuses on Phase II monitoring of AR(1) autocorrelated simple linear profile. In this section, we propose a combined control scheme, including Exponentially Weighted Moving Average (EWMA) and chi-square, based on the residuals obtained from ordinary and robust Holt-Winter models. EWMA control chart is used to monitor the mean of residuals, while the chi-square control chart is designed to monitor the standard deviation of the residuals. These two control charts are simultaneously used to monitor the mean and standard deviation of the residuals. In addition, the chi-square control chart helps the EWMA control chart to detect large shifts in the parameters of a simple linear profile.

3.1. The EWMA statistic

The EWMA statistic can be calculated using residuals obtained from the ordinary and robust Holt-Winter models. The average of residuals is $\bar{e}_j = \frac{\sum_{i=1}^{n} e_{ij}}{n}$. Hence, the EWMA control chart, based on Holt-Winter model for the jth profile, is given by:

$$z_j = \theta\bar{e}_j + (1-\theta)z_{j-1}, \quad (8)$$

where θ is the smoothing parameter of the EWMA control chart ($0 < \theta < 1$). We assumed the value of 0.2 for θ and $z_0 = 0$. The lower and upper control limits for the Holt-Winter based EWMA control chart are computed by Eq. (9):

$$\text{UCL} = \mu_{\bar{e}} + L\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\theta}{2-\theta}},$$

$$\text{CL} = \mu_{\bar{e}},$$

$$\text{LCL} = \mu_{\bar{e}} - L\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\theta}{2-\theta}}, \quad (9)$$

where $\mu_{\bar{e}}$ is equal to 0, n is the number of observations in each profile, and σ is the standard deviation of residuals. The coefficient of control limits (L) is obtained using simulation studies to give a specified in-control ARL.

3.2. The chi-square statistic

The second statistic used in the proposed scheme is the chi-square statistic. The chi-square statistic and the upper control limit based on the residuals obtained from the Holt-Winter model are given in Eq. (10):

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{e_{ij}-0}{\sigma}\right)^2 \sim \chi_n^2,$$

$$\text{UCL} = \chi_{n,\alpha}^2, \quad (10)$$

where $\chi_{n,\alpha}^2$ is the 100 $(1-\alpha)$ percentile of the chi-square distribution with n degrees of freedom and α is the probability of type I error.

In the next section, we first evaluate the effect of outliers on the performance of the proposed control chart scheme which is designed with residuals of the ordinary Holt-Winter model. Then, we apply the control scheme, designed based on the residual of robust Holt-Winter, under the contaminated data. Through simulation studies, we show that the control scheme based on the robust Holt-Winter is superior to the control scheme based on the ordinary Holt-Winter.

4. Simulation studies and performance evaluation

In this paper, the following autocorrelated model is considered to evaluate the performance of the proposed methods:

$$y_{ij} = 3 + 2x_i + \varepsilon_{ij},$$

$$\varepsilon_{ij} = 0.7\varepsilon_{(i-1)j} + a_{ij}, \quad (11)$$

where a_{ij} is the residual of AR(1) autoregressive model, which follows a normal distribution with the mean equal to zero and variance equal to one ($a_{ij} \sim N(0,1)$). Also, the x_i values are equal to 2, 4, 6, and 8. The in-control ARL for the proposed combined control scheme is assumed to be 200. Hence, each control chart is designed such that the in-control ARL of 400 is achieved. Therefore, L in the EWMA control chart is set equal to 3.2137. Also, the smoothing parameter in the EWMA control chart is assumed equal to 0.2. The probability of Type I error in the chi-square control chart is set equal to 0.0025 to obtain the in-control ARL of 400. Also, the smoothing parameters of the ordinary Holt-Winter model for the obtained in-control ARL are calculated as $\lambda_1 = 0.3324$ and $\lambda_2 = 0.1958$:

$$\text{ARL}_{0 \text{ overall}} = 200, \ \alpha_{0 \text{ overall}} = \frac{1}{200} = 0.005$$

$$\Rightarrow \alpha_{\text{Each control chart}} = 1$$

$$-\sqrt{(1-0.005)} = 0.0025,$$

$$\mathrm{ARL}_{0\ \text{Each control chart}} = \frac{1}{\alpha_{\text{Each control chart}}} = 400. \quad (12)$$

4.1. Performance evaluation of the combined control chart under both clean and outlier data

In this subsection, we first evaluate the performance of the combined EWMA/χ^2 control chart when the data are clean. Then, we provide p percent contaminated data and evaluate the performance of the combined control chart. Finally, we use robust Holt-Winter model to construct the combined control chart and compare the results of these three situations with each other.

The ARL_0 values of the combined EWMA/χ^2 control chart under the explained situations are computed using 10,000 simulation runs and the results are summarized in Table 1. In this table, p shows the percent of outliers in the data and d shows the difference between the values of parameter A_0 in the clean and contaminated data. Note that all of the ARL_0 and ARL_1 values for clean data are calculated based on the ordinary Holt-Winter model in our simulation studies.

The results of Table 1 show that the presence of outliers in the clean observations leads to decrease in ARL_0 value. Moreover, however the p and d values increase, the ARL_0 values will be smaller. Finally, the control chart based on the robust Holt-Winter model is robust against the outliers, because with using this method, the ARL_0 value is close to ARL_0 under clean data.

Also, the ARL_1 values of the combined EWMA /χ^2 control chart are summarized in Table 2. Note that the magnitude of shift in the parameter A_0 is considered equal to 0.1.

The results of Table 2 show that increase in the p and d values leads to decrease in ARL_1 for the ordinary Holt-Winter. However, the robust Holt-Winter model can increase the ARL_1 values of the combined control chart and close them to the ARL_1 values of the control chart under clean data. These results show that the robust model is resistant against the outliers in the observations.

Now, we compare the performances of the combined EWMA/χ control chart in the explained situations. Hence, we determine the out-of-control ARL values for the shifts from 0.1 to 0.9 in parameter A_0. Also, we set the percent of outliers equal to 0.05 and the difference between the values of parameter A_0 in clean and contaminated data is 0.1. Table 3 shows the out-of-control ARL values of the combined control chart for clean and contaminated data. In addition, the in-control ARL values of the proposed control chart with clean and contaminated data (O-HW and R-HW) are reported in this table.

Figure 1 shows the ARL curves of the combined control chart for clean and contaminated data based on two ordinary and robust Holt-Winter models.

As shown in Figure 1, the ARL_0 value of con-

Table 1. Comparison of the values of ARL_0 in the presence of outlier (p) and the difference between A_0 parameters under clean and contaminated data.

p	Clean data	d = 0.1 O-HW	0.1 R-HW	0.2 O-HW	0.2 R-HW	0.3 O-HW	0.3 R-HW	0.4 O-HW	0.4 R-HW
0.00	200.69								
0.05		98.83	195.95	93.48	191.13	80.21	188.44	71.75	168.87
0.10		66.88	190.25	62.40	187.65	57.69	175.33	56.88	167.05
0.15		61.07	189.33	58.57	188.31	53.39	171.45	52.28	159.76
0.20		48.32	186.23	43.41	186.65	42.63	170.43	33.7	159.18

Table 2. ARL_1 values in the presence of (p) percent of outliers and the difference between the values of (d) in the intercept for outlier data.

p	Clean data	d = 0.1 O-HW	0.1 R-HW	0.2 O-HW	0.2 R-HW	0.3 O-HW	0.3 R-HW	0.4 O-HW	0.4 R-HW
0.00	164.78								
0.05		69.85	131.63	66.60	125.69	63.65	119.92	65.61	108.80
0.10		67.24	123.43	65.39	121.91	63.94	110.63	63.72	108.74
0.15		66.38	121.14	61.69	119.18	56.24	117.58	55.52	103.14
0.20		63.76	117.52	57.67	106.20	53.07	99.18	49.92	100.69

Table 3. The ARL values of the clean and contaminated data under shift in the intercept.

ARL	Shift									
	0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Clean data	199.98	148.79	99.988	71.65	59.10	48.77	35.27	32.40	29.16	15.89
O-HW	99.98	69.26	57.11	40.89	38.67	33.88	24.03	19.75	18.22	12.40
R-HW	192.70	130.07	84.14	64.70	54.22	40.81	33.43	32.18	27.33	21.79

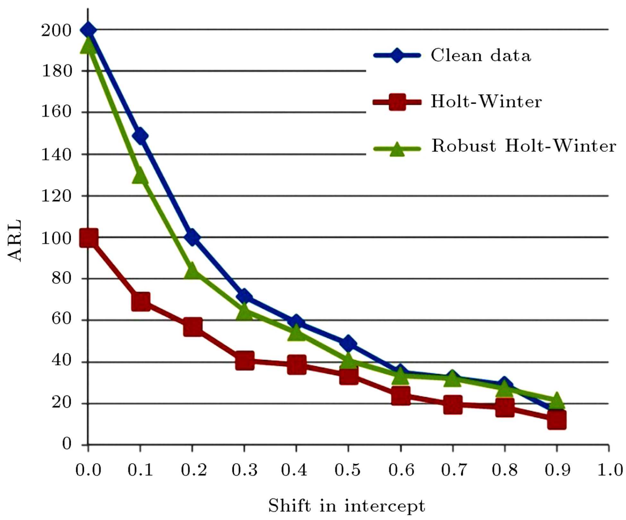

Figure 1. Comparison among the robust and non-robust ARLs in detecting different shifts in the intercept parameter.

taminated data based on ordinary Holt-Winter model is much reduced. However, the ARL_0 value based on the robust Holt-Winter model is close to that of the clean data. Also, the ARL_1 values of the combined control chart based on robust Holt-Winter model with shift from 0.1 to 0.9 in the intercept parameter are close to the clean data situation. These results represent the satisfactory performance of the proposed combined control chart based on robust Holt-Winter model against the outlier's effects.

The out-of-control ARL values of the combined control chart for clean and contaminated data under shift in slope parameter (A_1) are computed through simulation studies and the results are summarized in Table 4. In the simulation studies of this table, the percent of outliers is set equal to 0.05 and the difference between the values of parameter A_1 in clean and contaminated data is 0.025.

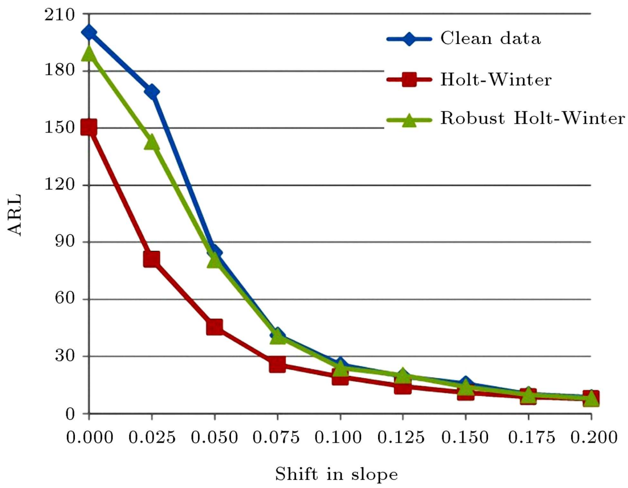

Figure 2. Comparison among the robust and non-robust ARLs in detecting different shifts in the slope parameter.

Figure 2 shows the ARL curves of the combined control chart for clean and contaminated data under shift in the slope parameter (A_1) based on two ordinary and robust Holt-Winter models.

Similar to the previous case (shift in the intercept parameter), the ARL_0 value of contaminated data based on ordinary Holt-Winter model is much reduced. However, the ARL_0 value based on the robust Holt-Winter model is close to that of the clean data. Furthermore, the ARL_1 values of the combined control chart based on robust Holt-Winter model with shifts in the slope ranges from 0.025 to 0.2 are close to the clean data situation. These results represent the appropriate performance of the combined EWMA/χ control chart based on the robust Holt-Winter model against the outlier's effects under the shift in the slope parameter. Also, in this subsection, we evaluate the performance of the proposed robust Holt-Winter based control charts under simultaneous shifts in the intercept and slope parameters. Note that we set the

Table 4. The ARL values of the clean and contaminated data under shift in the slope.

ARL	Shift								
	0	**0.025**	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.2**
Clean data	200.34	169.25	84.52	41.13	25.70	19.74	15.76	10.32	8.64
O-HW	150.35	81.10	45.53	25.76	19.31	14.51	11.09	8.92	7.75
R-HW	189.29	142.90	80.66	40.78	24.03	20.21	14.06	9.95	8.05

Table 5. The ARL values of the clean and contaminated data under simultaneous shifts in the intercept (d_1) and slope (d_2) parameters.

ARL	d_1	d_2							
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**
Clean data	0.1	104.05	52.76	28.90	21.22	14.07	11.17	9.84	7.76
	0.2	62.29	37.14	23.18	16.14	12.30	9.97	8.19	6.95
	0.3	44.52	26.71	17.78	13.66	10.72	8.79	7.46	6.33
	0.4	33.22	20.72	14.86	11.59	9.37	7.72	7.01	5.90
	0.5	24.25	16.21	12.77	10.18	8.69	7.28	6.20	5.70
O-HW	0.1	52.68	33.00	21.51	16.41	12.22	9.53	8.82	6.92
	0.2	37.80	24.34	17.54	13.49	10.31	8.67	7.46	6.48
	0.3	29.71	19.22	14.34	11.32	9.48	7.68	6.96	5.92
	0.4	21.58	16.18	13.11	9.99	8.43	7.18	6.27	5.63
	0.5	18.42	13.37	10.93	8.86	7.45	6.55	5.84	5.27
R-HW	0.1	100.27	48.93	30.84	16.61	12.95	10.68	9.63	7.65
	0.2	60.65	40.70	19.57	15.21	11.52	9.43	7.77	6.62
	0.3	33.41	24.20	17.05	10.70	13.13	8.98	7.70	6.43
	0.4	28.94	20.60	14.57	11.31	9.91	7.30	6.99	6.06
	0.5	23.82	17.84	13.30	11.02	9.12	8.03	6.58	5.78

percent of outliers equal to 0.05 and the magnitude of shifts in the parameters A_0 and A_1 under both clean and contaminated data is set equal to 0.1 and 0.025, respectively. The results are presented in Table 5.

As shown in Table 5, the ARL_0 value of contaminated data based on ordinary Holt-Winter model is much reduced. But, the ARL_0 value based on robust Holt-Winter model is close to that of the clean data. Also, the ARL_1 values of the proposed combined control chart based on robust Holt-Winter model are close to the ARL_1 values under clean data. These results show the suitable performance of the proposed EWMA/χ control chart based on robust Holt-Winter model against the outlier's effects under different simultaneous shifts in the intercept and slope parameters of a simple linear profile.

In this paper, in addition to the comparison of the ARL values, we propose the most commonly used robust scale estimates, namely the Median Absolute Deviation (MAD) and the Inter-Quartile Range (IQR), to show the better performance of the robust Holt-Winter model than that of the ordinary Holt-Winter model for monitoring the autocorrelated simple linear profile. The MAD and the IQR robust scale estimates are calculated as follows [27]:

$$\mathrm{MAD} = \mathrm{Mediane}|e_i - \mathrm{Median}(e_i)|/0.6745,$$

$$i = 1, 2, ..., N = 4 \times RL_0, \tag{13}$$

$$\mathrm{IQR} = 0.64 \times (75\text{th} - 25\text{th percentile}), \tag{14}$$

where RL_0 is the run length of autocorrelated profiles and gets a signal from the control chart. These equations are extracted from Asadzadeh et al. [27] and Jearkpaporn et al. [26], respectively. Note that these robust scales based on the Holt-Winter residuals (calculated from Eq. (2)) are determined and referred to as RH-MAD and RH-IQR in this paper, respectively. These scales are calculated based on the ordinary and the robust Holt-Winter residuals under both clean and contaminated data and are shown in Table 6. In this table, the MAD and IQR scales are determined under different values of p and d for EWMA, chi-square, and the proposed combined EWMA/χ control chart. Moreover, these scales are obtained under different explained situations through 5000 simulation runs. Note that in the case of "Robust clean data", the clean data are used in the robust Holt-Winter method and the output residuals are applied to calculate these robust scales.

According to Table 6, MAD and IQR of robust clean data are slightly more than these values for the clean data. However, with increase in the p and d values (in the contaminated data), these scale values decrease, while using the proposed robust Holt-Winter model leads the MAD and the IQR scales close to the values of these scales under the clean data. Similar results are obtained for the other individual and combined control charts.

5. Conclusion and future research

In this paper, we proposed a new combined control chart based on ordinary and robust Holt-Winter models

Table 6. RH-MAD and RH-IQR values in the presence of (p) percent of outliers and the difference between the values of (d).

	p	d	Clean data		Robust clean data		Contaminated data		Robust data	
			MAD	IQR	MAD	IQR	MAD	IQR	MAD	IQR
EWMA Statistic	0.00	0.0	0.0194	0.0248	0.0219	0.0300				
	0.05	0.3					0.0097	0.0130	0.0145	0.0190
		0.4					0.0151	0.0193	0.0313	0.0398
		0.5					0.0041	0.0153	0.0540	0.0675
	0.1	0.3					0.0109	0.0195	0.0334	0.0472
		0.4					0.0090	0.0117	0.0259	0.0354
		0.5					0.0073	0.0094	0.0148	0.0156
Chi-square Statistic	0.00	0.0	1.1178	1.4125	2.1530	3.0879				
	0.05	0.3					0.8981	1.0496	1.7242	2.0062
		0.4					1.5501	1.6609	1.7880	2.0663
		0.5					0.7878	1.6825	1.9579	2.1981
	0.1	0.3					1.1264	1.5023	1.2343	2.2680
		0.4					1.1323	2.0513	1.2965	2.2323
		0.5					0.5833	0.07467	1.9450	2.4224
Combined EWMA/χ^2 statistics	0.00	0.0	1.2962	3.9003	2.7480	4.5382				
	0.05	0.3					0.0059	2.3102	1.6786	3.3976
		0.4					0.0218	2.7250	1.0797	4.2728
		0.5					0.2305	2.7675	1.6950	4.5918
	0.1	0.3					0.8235	2.9286	0.9914	4.3462
		0.4					1.0217	2.5386	1.2209	3.9970
		0.5					0.0725	3.0123	1.5999	4.7956

to monitor the autocorrelated simple linear profile in the presence of outliers. The results of the combined control chart performance based on the ordinary Holt-Winter model showed that this model was not effective under the contaminated data. Hence, we proposed the control chart based on the robust Holt-Winter model to decrease the effects of outliers. To show the better performance of the combined control chart based on robust Holt-Winter model, we first compared ARL_0 and ARL_1 under the clean and contaminated data. The results showed that the robust model could increase the ARL_0 of the contaminated data close to clean situation. Also, ARL_1 of the combined control chart with contaminated data under different shifts in the intercept and slope parameters approached ARL_1 with clean data when the robust model was applied. Additionally, we proposed the commonly used robust scales based on the robust Holt-Winter model called RH-MAD and RH-IQR to show the better performance of the robust combined control chart in the monitoring of the autocorrelated simple linear profile. The results also showed the better performance of the RH-MAD and RH-IQR scales than that of the values for these scales based on ordinary Holt-Winter model. This paper covered the AR(1) autocorrelated simple linear profile in the presence of outliers. Extending the proposed method for more complicated time series models such as MA, ARMA, and ARIMA could be a fruitful area for future research. In addition, one can propose a method to account for the presence of contaminated data when there is autocorrelation between profiles.

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