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Supply chain network design for deteriorating items with discount on transportation cost

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Abstract. Distribution of deteriorating items is different from that of other items. This issue leads distributors to transport in lower volumes. On the other hand, one of the mechanisms that attract buyers to purchase items is discount; although a larger amount of order has a lower price for one item, it has a higher risk of deterioration. Despite the importance of the issue, previous researches on deteriorating items did not consider discount conditions in designing supply chain network. Hence, in this paper, balancing the cost of ordering and the cost of deterioration, with consideration of discount, through a new model is studied. The problem is solved for numerical examples with an improved meta-heuristic method composed of Simulated Annealing (SA) and Genetic Algorithm (GA) and the results are reported. Furthermore, a heuristic method for small scale problems is represented and compared with the introduced algorithm to analyze performance of the method. Finally, results show a significant difference between the costs of the models (with discount and without it).

1. Introduction

In recent years, many researchers are attracted to issues related to supply chain management [1]. One of the most important problems in this field is the proper design of supply chain. Supply Chain Network Design (SCND) problems are extended in different ways. These extensions include considering different types of facilities and multi echelon networks and multiple types of products, stochastic and dynamic parameters of demands and costs [2].

There are many studies which investigate designing supply chain network. Jayaraman [3] investigated the simultaneous relationship between management of inventory, facility location, and determination of transportation policy. The author suggested an integrated model for designing a distribution network which showed correlation of these three elements. Also, a mathematical model of mixed integer programming was proposed whose objective function was minimizing the total distribution costs related to the three decision variables (facilities location, inventory, and different options of transportation). Jayaraman and Pirkul [4] studied the problem of multi producers and multi-commodity facility location with the goal of minimizing total fixed costs of establishment of facilities and variable costs of dispatching and processing the commodities. Also, a heuristic Lagrangian-based method was proposed in this research. Ross and Jayaraman [5] worked on a type of problem which deals with designing the distribution network with multiple products, one producing center, multiple Distribution Centers (DCs), cross docking, and multiple markets. To solve this problem, simulated annealing algorithm is used. Garrido and Miranda [6] suggested an approach to integrate inventory control decisions in general models of facility location. In this research, demand is assumed stochastic, and a nonlinear mixed integer model and a heuristic algorithm are proposed to solve the problem.

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Amiri [7] investigated designing a distribution network in supply chain. For this purpose, a facility location model with single product and multiple manufacturers was extended. The facilities had multi-level capacity and the model determined the optimum level of capacity. A heuristic approach was proposed to solve the problem.

Deteriorating items such as foods, drugs, films, or drinks are damageable, perishable, or vaporizable and have typically an expiration date [8]. Sometimes deteriorating items cause time-constraints for distribution network in supply chain.

Many researchers have worked on this field. Ghare and Schrader [9] were the pioneers of this field. In their research, the effect of decaying products on the function of inventory systems was explained. Covert and Philip [10] designed an inventory control model for deteriorating items with variable deteriorating rates and supposed that shortage was not allowed. In their study, deteriorating rate was considered based on Weibull density function. Wee [11] proposed a model for determining the Economic Production Quantity (EOQ) for deteriorating items with fixed production rates and loss of shortage. Wee and Yang [12] suggested a multi-level mathematical model for deteriorating items. The model, economic order quantity in wholesale and retail, was concerned with the objective of simultaneous minimization of inventory costs at each level. Sarker et al. [13] designed a supply chain network for deteriorating items considering inflation rate and delay in payment and shortage with the goal of determining optimum ordering policy for customers. Tarantilis and Kiranoudis [14] proposed a meta-heuristic approach for routing distribution of deteriorating foods. In this research, a solution for the problem of fresh milk distribution in Greece was proposed. Law and Wee [15] suggested an inventory-production model for deteriorating and ameliorating items from the viewpoints of customer and producer. Deteriorating and ameliorating rates were calculated based on Weibull distribution. They developed a heuristic method to achieve the optimal solution. Chen and Chen [16] proposed an inventory control policy with periodic review. The objective function considered simultaneous replenishment planning and discount for deteriorating items. Periodic review was compared with fixed order quantity in the research. The problem was formulated as a dynamic planning problem and was solved with numerical searching techniques. The results showed that periodic review was better than fixed order quantity. Begum et al. [17] proposed an EOQ model for deteriorating items considering unit production cost, nonlinear demand, and shortage.

On the other hand, to attract more customers, vendors utilize some mechanisms such as discount on price for a large amount of order. Some researchers considered discount in designing supply chain. Tsao and Lu [18] investigated designing a supply chain with considering cost discount. Also some researchers, such as Tang and Yang [19] and Tang et al. [20], studied supply chain network design for deteriorating items; but, there seems to be a gap in designing supply chain network for deteriorating items with consideration of discount.

The paper is structured as follows. In Section 2, SCND for deteriorating items is modeled. Section 3 introduces an improved meta-heuristic algorithm composed of simulated annealing algorithm and genetic algorithm. In Section 4, a numerical example is surveyed and the two methods (SA and GA) are compared. Finally, conclusions are drawn in Section 5.

2. Development of the model

The model, developed in this paper, considers a three-stage supply chain network consisting of a supplier and many customers and many DCs. Any customer's demand could be assigned to any DC, if a DC has enough inventory. Schematic of the supply chain is illustrated in Figure 1.

The main objective, in this paper, is cost reduction for all parts of the supply chain. Main variables are decision making about establishing a DC, assigning demand, and selecting a price level for that DC.

2.1. Assumptions and notations

In the model, following assumptions are considered:

1. DCs have sufficient inventory for a specific item;
2. The rate of discount announced by any DC is different;

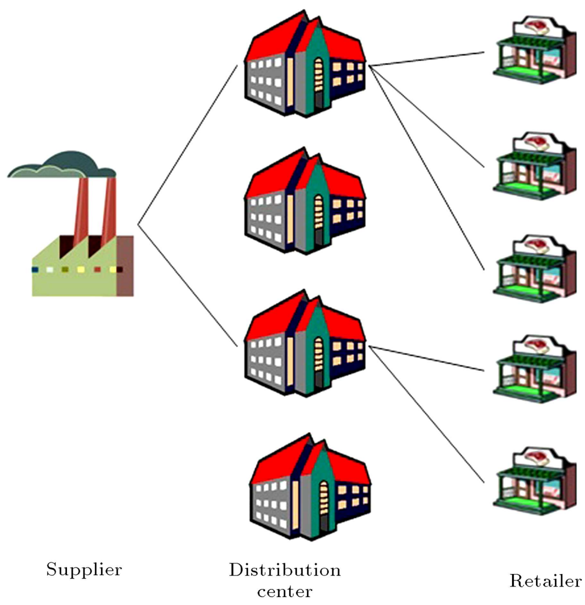

Figure 1. The schematic of the supply chain network.

3. Shortage is not allowed;
4. Deterioration occurs when the items arrive at the DCs;
5. Deterioration rate is constant and called (is small).

Notations are defined as follow:

$I_j(t)$	Inventory level of DC j
a_i	Demand rate of customer i
A_i	Fixed cost of each order
Q_j	Amount of each order
T_i	Planning period for customer i
TC_j	annual inventory cost
h	Fixed holding cost per unit of item
p	Fixed deterioration cost per unit of item
c_{jn}	The shipment cost that DC$_j$ chooses from price level n
f_j	The fixed cost of locating a DC at candidate site j
$x_j = 1$	If potential site j is chosen and 0, otherwise
$y_{ij} = 1$	If demand at customer i is assigned to DC$_j$ and 0, otherwise
$z_{jk} = 1$	If DC$_j$ chooses price level k and 0, otherwise
$d_{j,n}$	nth quantity break point for DC j.

Some notations are related to inventory management; to illustrate those parameters, Figure 2 depicts inventory level at DC$_j$:

2.2. Model

After introducing variables and parameters, the problem can be defined as locating DCs and determining the assignment of demands of retailers to them so as to satisfy the demands of the retailers. Any retailer's demand could be assigned to any DC, and if a DC has enough inventory, it can support all the retailers. Each of the DCs announces a price level from the levels defined for discounting.

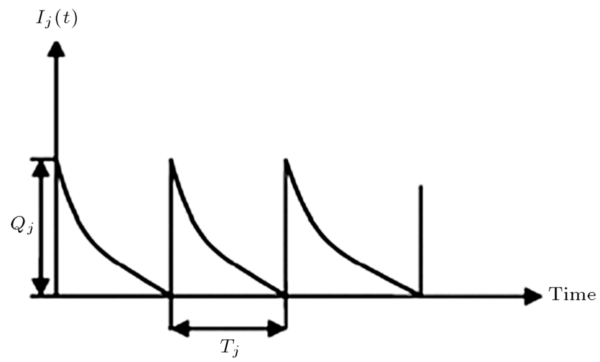

Figure 2. Inventory level at DC$_j$.

Suppose that D_j is defined as the total demand rate served by DC$_j$, or $D_j = \sum_i a_i$.

Transportation cost with consideration of discount is:

$$c_{jn} = \begin{cases} c_{j1}, D_j < d_{j1} \\ c_{j2}, d_{j1} \leq D_j - d_{j1} < d_{j2} \\ \vdots \\ c_{jn}, d_{j,n-1} \leq D_j - d_{j,n-1} < d_{j,n} \end{cases} \tag{1}$$

Note that discount is incremental. For the incremental discount policy, the discount applies only for quantities exceeding the break points. Inventory change in t is equal to the summation of item deterioration and consumption of the item. This means:

$$\frac{dI_j(t)}{dt} + \theta I_j(t) = -D_j,$$
$$0 \leq t \leq T_j. \tag{2}$$

Similar to D_j, suppose that c'_j is the shipment cost that DC$_j$ finally chooses to place from k price level. Therefore:

$$TC_j = \frac{hD_j}{\theta^2 T_j}\left(e^{\theta T_j} - \theta T_j - 1\right) + \frac{A_j}{T_j}$$
$$+\frac{p(Q_j - D_j T_j)}{T_j} + c'_j \frac{Q_j}{T_j}. \tag{3}$$

In Eq. (3), the first term is holding cost, the second term is the cost of placing order, the third term is the deterioration cost, and the fourth term is the shipment cost.

Holding cost and deteriorating cost, used in Eq. (3), are obtained as follows:

Inventory level at t can be obtained as follows:

$$I(t) = e^{-\int \theta dt}\left(\int -De^{\int \theta dt} + dt + c\right)$$
$$= c^{-\theta t}\left(\int -De^{\theta t}dt + c\right) = e^{-\theta t}\left(-De^{\theta t} + c\right)$$
$$= -\frac{D}{\theta} + e^{-\theta t}c. \tag{3.1}$$

Using boundary condition ($I(T) = 0$):

$$I(t) = \frac{D}{\theta}\left(\frac{e^{\theta T} - e^{\theta t}}{e^{\theta t}}\right),$$
$$0 \leq t \leq T. \tag{3.2}$$

And, finally, holding cost based on $I(t)$ is:

$$h\int_0^T I(t)dt = h\int_0^T \frac{D}{\theta}\left(\frac{e^{\theta T}-e^{\theta t}}{e^{\theta t}}\right)dt$$
$$= h\frac{D}{\theta}\int_0^T \left(e^{\theta(T-t)}-1\right)dt$$
$$= h\frac{D}{\theta}e^{\theta T}\int_0^T e^{\theta t}dt - h\frac{D}{\theta}T$$
$$= h\frac{D}{\theta^2}e^{\theta T}\left(1-e^{-\theta T}\right) - h\frac{D}{\theta}T$$
$$= h\frac{D}{\theta^2}\left(e^{\theta T}-1\right) - h\frac{D}{\theta}T$$
$$= h\frac{D}{\theta^2}\left(e^{\theta T}-\theta T-1\right). \quad (3.3)$$

The amount of deteriorating is the difference between amount of order and amount of demand in period T. Thus, deteriorating cost can be obtained from multiplying p by this amount.

Solving Eq. (3) is difficult because of the existence of exponential functions; hence, using Taylor extension and neglecting the third and larger powers of θ and for small values of θ, the following term can be used instead [21]:

$$e^{\theta} \cong \frac{2+\theta t}{2-\theta t}. \quad (4)$$

By substituting Eq. (4) in Eq. (3), and then taking derivative from Eq. (3) (convexity of the Eq. (3) is proved in Appendix A), optimal annual inventory cost will be:

$$TC_j^* = \sqrt{\frac{2D_jA_j}{h+\theta p+\theta c'_j}} + c'_jD_j. \quad (5)$$

Now with the assumption that D_j and c'_j are known, the model is formulated as follows:

$$\min \sum_j \left(f_jx_j + \left(\sum_{i\in I} a_i \left(\sum_k c_{jk}z_{jk}\right) y_{ij}\right) + \sqrt{\sum_{i\in I} 2a_iA_j(h+\theta p+\theta\sum_k c_{jk}z_{jk})y_{ij}}\right).$$

Subject to:

$$\sum_j y_{ij} = 1, \quad i\in I$$
$$y_{ij} \leq x_j, i\in I$$
$$z_{jk} \leq x_j$$
$$\varepsilon z_{j1} < D_j \leq d_{j,1}z_{j,1}$$
$$d_{j,1}z_{j,2} < D_j \leq d_{j,2}z_{j2}$$
$$\vdots$$
$$d_{j,n-1}z_{jn} < D_j < Mz_{jn}$$
$$x_j, y_{ij}, z_{jk} \in \{0,1\}, \quad (6)$$

where M is a big number.

The objective function minimizes the sum of deterioration cost, holding cost, fixed cost of establishing DCs, order cost, and transportation cost. The first constraint ensures that demand at customer i is assigned to one of the DCs. The second and the third constraints guarantee that if a DC is not established, y_{ij} and z_{jk} will be equal to zero. Other constraints show different levels of prices with respect to discount.

The first term of the objective function and the first and second constraints can be considered as a UFLP. UFLP is an Np-hard problem [22]. The problem has extra constraints and is nonlinear which leads to a more complex problem. Hence, meta-heuristic methods are developed to solve the problem.

3. Improved meta-heuristic algorithm

Meta-heuristic methods are generally divided into two classes: constructing and improving methods. Constructing methods first generate the initial solutions and then improve them, but the improving methods need initial solutions to improve them to gain the best possible solutions.

One of the most applied meta-heuristic methods is simulated annealing. This improving method is a generic probabilistic one for the global optimization problem of locating a good approximation to the global optimum in a large search space. Simulated annealing is a process wherein the temperature is reduced slowly, starting from a random search at high temperature, eventually becoming a pure greedy descent as it approaches zero temperature.

This notion of slow cooling is implemented in the SA algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space [23]. Accepting worse solutions is a fundamental property of meta-heuristics, because it allows for a more extensive search for the optimal solution.

Another method, frequently used in the literature, is genetic algorithm. This algorithm, as an improving method, is routinely applied to generate acceptable solutions to optimization and search problems [24]. In a genetic algorithm, a population of candidate solutions to an optimization problem is evolved toward better solutions. Each candidate solution has a set of properties which can be mutated and altered; the evolution usually starts from a population of initial solutions. In each generation, the fitness of every individual in the population is evaluated; the more fitted individuals are stochastically selected from the current population. Then, using mutation, a new population is created. It is, then, used in the next iteration of the algorithm.

To achieve better solutions, sometimes it is required to combine two or more meta-heuristics. In this paper, an improved meta-heuristic algorithm with two parts is developed. The SA outputs are applied as the initial solutions for GA. Using these initial solutions and other mechanisms, such as mutation and crossover, GA improves initial solutions and reaches better solutions for the problem. Details of the improved method are discussed below.

The first part of the improved algorithm is SA. SA is applied to generate initial solutions for the GA. After determining the initial and the final temperatures for the algorithm, the cooling mechanism is selected as a geometric procedure. The stopping condition is defined as reaching a proper number of iterations (MaxIt). Eq. (7) shows a geometric cooling function. Alpha is the geometric multiplier and T_f and T_0 are the initial and final temperatures.

$$\text{Alpha} = (T_f/T_0)\hat{}(1/\text{Maxlt}). \tag{7}$$

Also, to diversify movement in the solution space, usually a worse solution is accepted by a probability. In this paper, the probability function is applied based on Eq. (8). If a random number between zero and one is smaller than this probability, the new solution is accepted as the current solution.

$$P = \exp\left((\text{bestcost} - \text{cost})/T\right). \tag{8}$$

The main loop of the algorithm is introduced in Box I. This algorithm is executed for the number of the initial solutions required to run GA. Initial solutions by SA are better than the randomized ones.

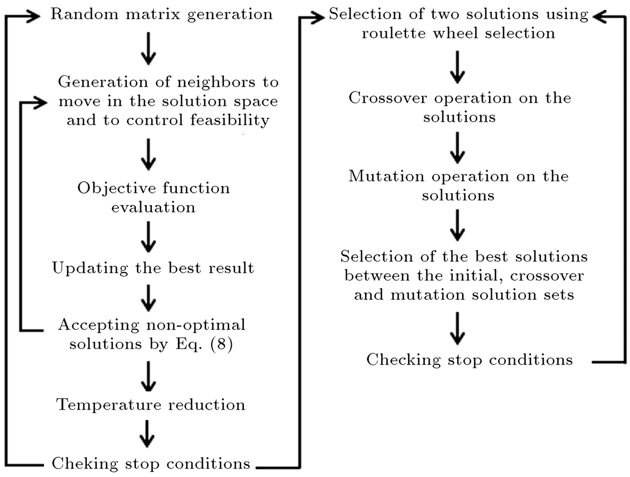

Figure 3. The meta-heuristic algorithm.

Genetic algorithm applied in the second step of the algorithm is shown in Box II.

Thus, utilizing the initial solutions from SA and improving them by GA helps to gain better global solutions with a better value for objective function. Figure 3 briefly shows the improved algorithm.

In the algorithm shown in Box II, as mentioned above, genetic algorithm is used to improve the results of SA algorithm. In the next section, results of some numerical examples are surveyed.

4. Numerical example

As mentioned before, the problem, studied in this paper, is determining supply chain features, such as location of DCs and assigning demands to each established DC. The conditions considered for the supply chain are deterioration of the items and discount on transportation costs.

The supply chain consists of 3 echelons, one supplier and some DCs and retailers. Some problem instances, consisting of small, medium, and large size

```
For each iteration
        Construct an empty matrix and fill it with random variables
        // Using it, locating the DCs and selecting the levels of the price can be performed
        // Dimensions of the matrix are related to the number of potential DCs and the number of
        // retailers, and also the number of levels announced for discount.
        Use the neighborhood mechanisms to move in the    solution space and control the feasibility of
         the new solution,
        // Shift and swap the mechanism used in this section
        Calculate the objective function for this solution
        Substitute this solution with the best solution rec- ognized so far, under the following conditions:
        // if the objective function of new solution is better than the best objective function, found so far
        // if the above condition has not occurred, with the probability equal to Eq. (8), substitution is allowed
        Temperature= Temperature × Alpha
End
```

Box I

```
For each iteration
    Use roulette wheel method to select two initial solutions
    Perform crossover on the two solutions selected
    // Hold the results in a set (set 1)
    // Check feasibility
    Perform mutation on some of the initial solutions
    // Hold the results in a separate set (set 2)
    // Check feasibility
    From initial solutions, set 1 and set 2, select the best solutions with the better values for
    objective function
End
```

Box II

Table 1. Data for the small scale problem instances.

		c_1	d_1	c_2			D
	DC_1	10	3000	6		Retailer 1	800
						Retailer 2	1000
DCs	DC_2	12	2000	6	Retailers	Retailer 3	1200
						Retailer 4	900
	DC_3	12	4000	5		Retailer 5	1100

Table 2. Results from solving the small size problem instances.

	SA	GA	IMA
Objective function	75859	75859	75859

problems, are tested with the improved algorithm. To show performance of the improved algorithm, it is applied on each of the problem instances. The contribution of the two portions of the proposed algorithm in solution quality is shown. In these problems, the deterioration rate is 0.01, deterioration cost is 5, holding cost is 10 (per unit), fixed cost of each order is 500, and establishing cost for each DC is 10000.

First, a small problem is studied. This problem consists of one supplier, three DCs and five retailers; each DC has two separate levels of price. Data for this problem is presented in Table 1.

Under an identical time and number of iterations, Table 2 shows the results from the SA, the GA, and the Improved Meta-heuristic Algorithm (IMA).

According to the results of the algorithms, only the first DC is economic to establish and others are not chosen by the algorithms.

It is very difficult to solve the problem, manually, because the number of cases prevents calculation without software. But, there is an easy way to approach the answer which is proposed below.

Lemma 1. In addition to the results from the meta-heuristics, for small sizes and under the assumption that holding cost and deterioration cost are negligible, some results can be guessed. For each DC, there are two prices and one break point of discount.

There are three major components of the problem that impact the solutions. The first component is the price of the first level of discount (c_1). The second component is the difference between price of the second and the first levels of discount ($c_1 - c_2$). The third component is the break point (d). These components, together, determine that which DCs can be established. Smaller values for c_1, d and larger values for $c_1 - c_2$ are desirable. Hence, the index below can show the desirability for the choice of DCs.

$$V_j = \frac{c_{j1} - c_{j2}}{c_{j1} \times d_j}, \qquad c_{j1} > c_{j2}. \tag{9}$$

Based on the above points, the steps below help to solve the problem.

Find the smallest price in the first level of discount between all DCs and name break point of discount in this DC (DC_l): d_l.

If the sum of demands is larger than d_l, order d_l from DC_l. For the other DCs, two cases can happen:

1. Break point of discount of other DCs is larger than d_l. Order from DC_l;
2. Break point of discount of other DCs is equal or smaller than d_l. With V_j, select the next DC to order.

As an example, it is shown that just DC_1 should be established from three DCs explained in the small instance problem. For this problem, Lemma 1 is applied below.

The smallest price in the first level of discount between all DCs is 10 in DC_1, thus: $d_l = 3000$. Therefore, order 3000 from DC_1.

In DC_2, break point is 2000, and in DC_3 it is 4000. Thus, we use the index which is explained. V_j's are calculated as: $V_1 = 0.000133$, $V_2 = 0.00025$, $V_3 = 0.000146$. Hence, choose DC_1 for the remained orders. This lemma cannot be used for the large problems and problems with many discount break points.

The next problem instances, considered in this paper, show performance of the proposed algorithm in a larger size. In this scale of problem, 30 DCs and 100 retailers comprise the supply chain. Also three levels of price are considered.

Parameters of the problem are determined as below:

- Values for retailer demands are uniformly drawn from [200, 400];
- Values for the first level of transportation cost are uniformly drawn from [5, 8];
- Values for the second level of transportation cost are uniformly drawn from [9, 12];
- Values for the third level of transportation cost are uniformly drawn from [13, 15];
- Values for the first break point of discount are uniformly drawn from [800, 1300];
- Values for the second break point of discount are uniformly drawn from [1300, 2200];
- Values for other parameters are the same as those for the small size problem.

Results of solving the problem with 3 meta-heuristic algorithms are presented in Table 3. SA solves the problem with $T_0 = 1000$ and $T_f = 0.01$, and the number of iterations in each temperature is 100. GA is applied to the problem with crossover rate of 0.8 and mutation rate of 0.3. The improved algorithm first solves the problem with 5 matrices which is made by SA. Under an identical time, Table 3 gives the results from each method.

The initial solution has intensive effect on the results of simulated annealing algorithm, and if there is not a good initial solution for SA, final results may not be suitable. In this paper, because of the general definition of the problem, and random selection of initial solution, final SA results are not desirable and must be combined with GA for improvement. The combination is performed properly and makes remarkable improvement on the output of SA. In other words, contribution of the GA in improvement of the SA results is about 69% and the combination of SA with GA also improves the results of GA by 23%.

Table 3. Results from 3 meta-heuristic algorithms to solve large scale problems.

	SA	GA	SA-GA
Objective function	618740	192740.4	147295
Time (sec)	32	21	46

Table 4. Effect of the number of outputs of SA, used to run GA, on the time and quality of solutions.

SA outputs used for GA	Objective function	Time (Sec)
5	147295	46
10	146261	55
20	141711	68
30	130909	145
50	130909	250

Table 4 shows sensitivity analysis of output numbers of solutions obtained by SA used as GA initial solutions. The table shows the effect of using outputs of SA for GA. As the number of SA outputs increases, solutions will be better, but execution time increases. This issue is shown, graphically, in Figures 4 and 5.

Results indicate that the best solution tends to assign demands to the minimum possible number of DCs. On the other hand, costs, such as deterioration, holding, and establishing costs, balance the number of established DCs.

Improved algorithm is used to evaluate some other problem instances. Results are shown in Table 5.

According to the results of Table 5, performance of the improved algorithm is depicted in Figure 6.

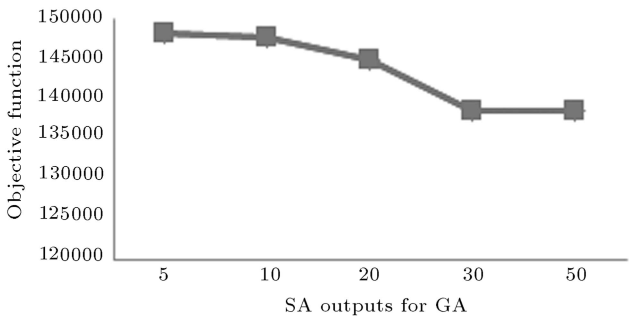

Figure 4. Effect of different numbers of SA outputs, used for GA, on the objective function.

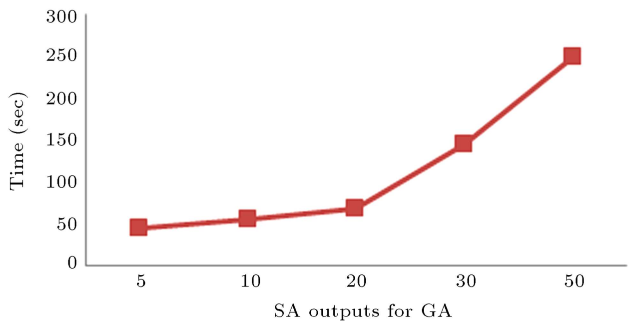

Figure 5. Effect of different numbers of SA outputs, used for GA, on the running time.

Table 5. Results from meta-heuristic algorithms to solve some problem instances.

Problem no.	Problem size		Objective function based on SA	Objective function based on GA	Objective function based on SA-GA	% Improvement of IMA over SA	% Improvement of IMA over GA
	DC	Retailer					
1	3	5	75859	75859	75859	-	-
2	5	10	184435.2	101339.145	91574.92	50	9
3	10	30	265867.4	119221.999	104433.4	61	12
4	15	50	320155.5	123897.255	103004.7	68	17
5	30	100	618740	192740.4	147295	76	24

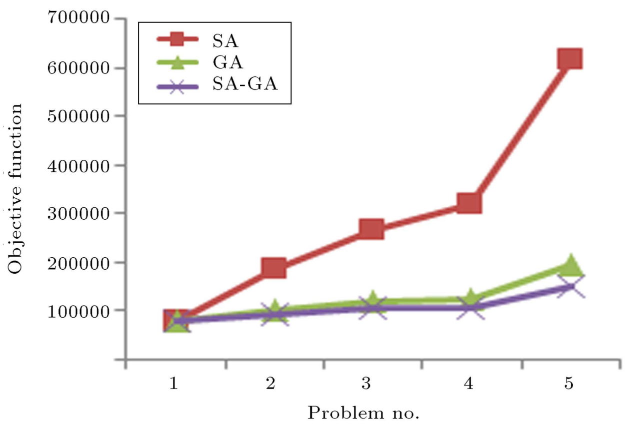

Figure 6. Comparison between results of algorithms in different problem instances.

5. Conclusion and future research

Supply chain management is developed in many industries as a concept and is performed, properly, at those industries. One of the most important parts of actualization of this concept is designing a network for the supply chain. To consider more aspects of real world in designing the supply chain, sometimes discount is announced on price. Previous studies did not consider discount in designing supply chain networks for transporting deteriorating items, and therefore, there were no solutions for such problems. Also, tuning the parameters of algorithms and designing an improved algorithm, based on the conditions of such problems, to solve the model are another contributions of the paper.

In this paper, first, a supply chain for deteriorating items was modeled, and then, it was solved by three meta-heuristics: SA, GA, and a combination of them. GA performed better than SA for problems defined based on the model; additionally, the proposed algorithm performed better than the others. SA, because of its nature and use of random initial solution, performed inappropriately; hence, application of an accessory method seems reasonable in addition to SA. Also, for the small size problems, a manual approach was proposed in the paper which helped to reach the solution. Results of the improved algorithm showed reasonable expectations, such as assigning more demands to DCs, announcing lower prices, and lower break points of discount, from the problem. As an area for future research, study of the proposed model in real world cases, considering shortage with efficient algorithms, can be recommended.

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Appendix A

Here, convexity of Eq. (3) is proved. The first derivative of the function (rewritten) is as follows:

$$TC_j' = -\frac{hD_j}{\theta^2}\left(\frac{e^{\theta T_j}-\theta T_j-1}{T_j^2}\right) + \frac{hD_j}{\theta^2}\left(\frac{\theta e^{\theta T_j}-\theta}{T_j}\right) - \frac{A_j}{T_j^2} - \frac{pQ_j}{T_j^2} - \frac{c_j' Q_j}{T_j^2}.$$

And the second derivate of the function (rewritten) is as follows:

$$TC_j'' = -\frac{hD_j}{\theta^2}\left(\frac{T_j(\theta e^{\theta T_j}-\theta)-2(e^{\theta T_j}-\theta T_j-1)}{T_j^3}\right) + \frac{hD_j}{\theta^2}\left(\frac{\theta^2 e^{\theta T_j}T_j-\theta e^{\theta T_j}+\theta}{T_j^2}\right) - \frac{2A_j}{T_j^3} - \frac{2pQ_j}{T_j^3} - \frac{2c_j' Q_j}{T_j^3}.$$

It can be rewritten as:

$$TC_j'' = \frac{hD_j}{\theta^2}\left(\frac{e^{\theta T_j}(\theta^2 T_j^2 - 2\theta T_j + 2) - 2}{T_j^3}\right) - \frac{2A_j}{T_j^3} - \frac{2pQ_j}{T_j^3} - \frac{2c_j' Q_j}{T_j^3}.$$

It can be expressed that the convexity of function can be proved through the following relation:

$$TC_j^* = \frac{hD_j}{\theta^2}\left(e^{\theta T_j}((1+\theta T_j)^2+1)-2\right) - \left(2A_j + 2pQ_j + 2c_j' Q_j\right).$$

T_j and θ are positive, then:

$$\left(e^{\theta T_j}((1+\theta T_j)^2+1)-2\right) \geq 0.$$

Thus for the small values of θ, TC_j is convex. Hence, derivation and use of minimum function as objective function are rational.

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