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A novel encoding scheme based evolutionary approach for the bi-objective grid patrol routing problem with multiple vehicles

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Abstract. In this paper, we investigate the Bi-objective Grid Patrol Routing Problem (BGPRP) in which multiple patrol vehicles have to cooperatively patrol several nodes for a given specific time during a certain planning horizon in a grid map. The BGPRP is an NP-hard problem and an extended application of the Periodic Vehicle Routing Problem (PVRP). However, unlike PVRP, in addition to minimizing the total routing distance, the considered BGPRP also aims to maximize the routing coverage of lanes. The BGPRP involves both the combination and permutation of nodes simultaneously. In this paper, an efficient encoding scheme is developed to tackle both the combination and the permutation of nodes simultaneously. Then, we apply an immune based evolutionary approach for solving the BGPRP with the objective of minimizing total routing distance. Finally, an integer programming approach is utilized to maximize the routing coverage of lanes. Numerical results of multiple patrol vehicles in a grid map show the performance of the approach.

1. Introduction

In Taiwan, residents are usually out of town or go abroad for several days during the Chinese festival. They may ask policemen to patrol their houses for specific times during a certain planning horizon; for example, once or twice among three days. Such a service is called a "festival patrol service" [1]. Practically, the policemen will receive applications of the patrol in advance, and then schedule the patrol routes for each day. Usually, policemen have two main objectives concerning their routing schedule. The first is to minimize the total routing distance of the patrol, and the second is to maximize the routing coverage of lanes when the first objective is achieved. In our previous paper [1], we studied a similar patrol routing problem. However, only one objective, i.e. minimizing the total routing distance of the patrol, was involved. In this paper, we attempt to investigate the bi-objective patrol routing problem. For simplicity, we assume that the considered lanes are in grid. Thus, the newly introduced bi-objective routing problem in this paper is named the "Bi-objective Grid Patrol Routing Problem" (BGPRP).

As known, the considered BGPRP is similar to the conventional Periodic Vehicle Routing Problem

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(PVRP) [2]. Nowadays, PVRP has attracted much attention and has many applications in practice, including collection of waste [3-5], collection of milk [6], maintenance operation of elevators [7], inventory of supermarket chains [8], product distribution [9], salesman routing for the lottery [10] etc. However, unlike PVRP, in addition to minimizing the total routing distance, the BGPRP also aims to maximize the routing coverage of lanes. Until now, as known, several approaches have been developed to solve the conventional PVRP, including classic heuristics, metaheuristics and mathematical programming based approaches, as follows:

(a) *Classic heuristics.* Researchers have proposed two-stage or three-stage methods to solve PVRP, e.g. [11-14]. Their methods considered the combination and permutation of nodes separately. In other words, their stage-1 of the approach was usually used to solve the combination problem of nodes, and then stage-2 of the approach was adopted to solve the permutation problem of nodes.

(b) *Metaheuristics.* Researchers have applied metaheuristics, tabu search, the genetic algorithm, etc. for solving PVRP, including [15-17]. However, their metaheuristics usually converge to local optimums, especially when the problem size is large. Additionally, several metaheuristics are very sensitive to the setting of parameters [18].

(c) *Mathematical programming based approaches.* Researchers have proposed mathematical programming approaches to solve PVRP, including [19-20]. However, when the problem size is larger, these approaches are time-consuming. Furthermore, Mourgaya and Vanderbeck [20] also reported the poor performance of these approaches.

More details of PVRP and their solving approaches are referred to in the excellent survey paper by Francis et al. [21].

In this paper, we will introduce a new Bi-objective Grid Patrol Routing Problem (BGPRP) and present an efficient encoding scheme which represents the combination and permutation of nodes in clusters simultaneously. Based upon the encoding scheme, we will apply an immune based evolutionary algorithm for solving the considered BGPRP. This paper is organized as follows. In Section 2, we define the notations and assumptions of BGPRP, and then present its brief mathematical formulation. A small example is given to illustrate the BGPRP. In Section 3, we present an efficient encoding scheme to tackle both the combination and the permutation of nodes simultaneously. Using the encoding scheme, an immune based evolutionary algorithm is developed to solve BGPRP for the first objective. Next, we further utilize an integer linear programming to maximize the routing coverage of lanes for the second objective. Numerical results will be reported and discussed in Section 4, and, finally, conclusions will be summarized in Section 5.

2. The Bi-objective Grid Patrol Routing Problem (BGPRP)

2.1. Notations

T	Planning horizon (days).
n	Number of nodes to be patrolled in the network during period T.
N_i	Set of nodes to be patrolled on the ith day; $1 \leq i \leq T$.
N	Total number of nodes to be patrolled in the network during period T; $N = \lvert N_1\rvert + \lvert N_2\rvert + \cdots + \lvert N_T\rvert$.
c_i	Number of patrol vehicles on the ith day; $1 \leq i \leq T$.
C	Number of patrol vehicles during period T; $C = (c_1, c_2, \cdots, c_T)$.
S_{ij}	Sequence of nodes to be patrolled by vehicle j on the ith day. $S_{ij} = (s_{ij0}, s_{ij1}, s_{ij2}, \cdots, s_{ijk}, s_{ij0})$. $1 \leq i \leq T$, $k = \lvert N_i\rvert$, $1 \leq j \leq c_i$ and $S_{ij} \subseteq \{1, 2, \cdots, n\}$. Note that $s_{ij0} \equiv$ the start/end node, i.e. the police station in the network.
v_t	Given the number of visits (patrol times) for node t during period T; $1 \leq v_t \leq T$ and $1 \leq t \leq n$.
v	$v = (v_1, v_2, \cdots, v_n)$, where $v_1 + v_2 + \cdots + v_n = N$.
$L(S_{ij})$	Minimal routing distance of vehicle j on the ith day based upon S_{ij}; $1 \leq i \leq T$ and $1 \leq j \leq c_i$.
$d(t_1, t_2)$	Minimal distance between node t_1 and node t_2; $t_1 \neq t_2$ and $t_1, t_2 \in \{1, 2, \cdots, n\}$.
X_{ijt}	Set of patrolled lanes (segments) from node t to node $t+1$ for vehicle j on the ith day based upon S_{ij}; $1 \leq i \leq T$ and $1 \leq j \leq c_i$.

2.2. Assumptions

(a) The considered lanes form a grid network;

(b) Each node is patrolled, at most, once per day;

(c) For day i, vehicle j follows the scheduled nodes of S_{ij} to patrol; $1 \leq i \leq T$ and $1 \leq j \leq c_i$;

(d) For each day, there is at least one node patrolled for each vehicle. That is, $S_{ij} \neq \phi$;

(e) The first objective of BGPRP is to minimize the summation of routing distance $L(S_{ij})$ and the second objective aims to maximize the routing coverage of lanes when the first objective is achieved; $1 \leq i \leq T$ and $1 \leq i \leq T$.

2.3. Mathematical programming model of BGPRP

The first objective of BGPRP is to minimize the total routing distance $L(S_{ij})$, and its mathematical programming model can be formulated as follows.

Problem 1. The 1st objective:

$$\text{Min} \quad \sum_{i=1}^{T} \sum_{j=1}^{c_i} L(S_{ij}), \tag{1}$$

$$\text{s.t.} \quad L(S_{ij}) = \min \sum_{k=0}^{|N_i|} d(s_{ijk}, s_{ij,k+1}),$$

$$1 \leq i \leq T, \qquad 1 \leq j \leq c_i \qquad (s_{ij|N_i|+1} \equiv s_{ij0}), \tag{2}$$

$$\sum_{i=1}^{T} |N_i| = \sum_{t=1}^{n} v_t, \qquad 1 \leq |N_i| \leq n, \tag{3}$$

$$S_{ij} \subseteq \{1, 2, \cdots, n\}, \qquad 1 \leq i \leq T, \qquad 1 \leq j \leq c_i. \tag{4}$$

The objective function (1) aims to minimize the total routing distance of the patrol. Eq. (2) is the minimal routing distance of nodes by patrol vehicle j on the ith day, based upon S_{ij}. Eq. (3) confines the total times of patrols for each day. Eq. (4) shows the decision variable:

$$S_{ij} = (s_{ij0}, s_{ij1}, s_{ij2}, \cdots, s_{ijk}, s_{ij0}) \subseteq \{1, 2, \cdots, n\},$$

i.e. the sequence of nodes to be patrolled by vehicle j on the ith day. Note that Eq. (4) consists of both the combination and permutation of nodes. More specifically, once $v = (v_1, v_2, \cdots, v_n)$ is given in Eq. (3), one has to:

(i) Determine the combinations of nodes (i.e., N_i), and then;

(ii) Based upon N_i, find the permutations of nodes (i.e., S_{ij}), such that the objective function (1) is minimized.

When Problem 1 is solved and the first objective is obtained, then the considered BGPRP has to solve the following Problem 2, such that the routing coverage of lanes is maximized.

Problem 2. The 2nd objective:

$$\text{Max} \quad \left| \bigcup_{ijt} X_{ijt} \right|, \tag{5}$$

s.t. corresponding lanes in shortest routes.

Remarks

(a) X_{ijt} is the set of patrolled grid lanes (segments) from node t to node $t+1$ for vehicle j on the ith day, based upon S_{ij}; $1 \leq i \leq T$, $1 \leq j \leq c_i$, and $|\bigcup_{ijt} X_{ijt}|$ is the union of all patrolled grid lanes, based upon the routes obtained in the first objective. Therefore, the objective (5) is to maximize the routing coverage of lanes.

(b) If there are r_t possible shortest routes between nodes t and node $t+1$, then, one of the following two cases has to be adopted for maximizing the routing coverage of lanes.

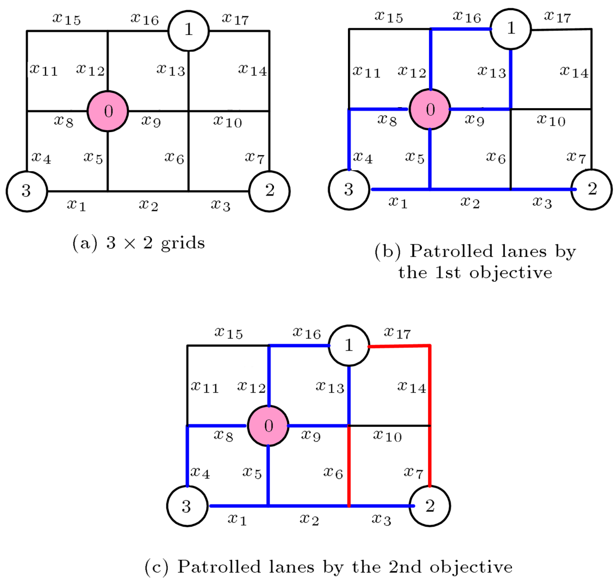
 (i) When the number of patrols from node t to node $t+1$ in the first objective is larger than, or equal to r_t, all possible patrolled lanes of the set, from node t to node $t+1$, have to be patrolled.

 (ii) When the number of patrols from node t to node $t+1$ in the first objective is less than r_t, we may use 0-1 integer programming to maximize the routing coverage of lanes in the second objective.

Next, we illustrate a small example of BGPRP and show the solving process for Problems 1 and 2, respectively.

2.4. Example 1

Problem 1. The 1st objective: Consider Figure 1 as an example. Assume that $n = 3$, $v = (2, 2, 3)$, $T = 3$ and $C = (1, 1, 1)$. Consider the feasible solution, $S_{11} = (0, 1, 3, 0)$, $S_{21} = (0, 1, 2, 3, 0)$ and $S_{31} = (0, 2, 3, 0)$. Note that, for this solution, node 1 is patrolled twice (on days 1 and 2, respectively), node 2 is patrolled twice (on days 2 and 3, respectively), and



(a) 3 × 2 grids

(b) Patrolled lanes by the 1st objective

(c) Patrolled lanes by the 2nd objective

Figure 1. The example of network with 2 × 3 grids.

node 3 is patrolled three times (once per day). Thus, $L(S_{11}) = 2+4+2 = 8$, $L(S_{21}) = 2+3+3+2 = 10$ and $L(S_{31}) = 3+3+2 = 8$ if the length of each grid lane (segment) is one. Thus, the total routing distance for this feasible solution is $8+10+8 = 26$.

Consider another feasible solution, $S_{11} = (0,1,2,3,0)$, $S_{21} = (0,3,0)$, and $S_{31} = (0,1,2,3,0)$. Thus, $L(S_{11}) = 2+3+3+2 = 10$, $L(S_{21}) = 2+2 = 4$ and $L(S_{31}) = 2+3+3+2 = 10$. The total routing distance for this feasible solution is $10+4+10 = 24$. Clearly, this feasible solution is superior to the former.

Problem 2. The 2nd objective: Suppose that $S_{11} = (0,1,2,3,0)$, $S_{21} = (0,3,0)$, and $S_{31} = (0,1,2,3,0)$ are obtained from the first objective; we may further utilize the following arguments and approach to maximize the routing coverage of lanes.

(a) Since we have to patrol twice from node 0 to node 1 by this solution, i.e., in S_{11} and S_{31}, and there are two possible shortest routes between these two nodes, grid lanes (segments) x_9, x_{12}, x_{13}, x_{16} have to be patrolled (see Figure 1(a)), or:

$$x_9 = x_{12} = x_{13} = x_{16} = 1. \tag{6}$$

Note that $x_k \in \{0,1\}$ indicates whether the kth lane is selected or not.

(b) Similarly, we have to patrol twice from node 2 to node 3, and there is only one shortest route between these two nodes. Thus, we have:

$$x_1 = x_2 = x_3 = 1. \tag{7}$$

(c) Similarly, we have to patrol four times from node 0 to node 3 (or node 3 to node 0), and there are two possible shortest routes between these two nodes. Thus, we have:

$$x_1 = x_4 = x_5 = x_8 = 1. \tag{8}$$

Based upon the above arguments of Eqs. (6)-(8), we have:

$$x_1 = x_2 = x_3 = x_4 = x_5 = x_8 = x_9 = x_{12} = x_{13} = x_{16} = 1, \tag{9}$$

which are depicted in blue in Figure 1(b).

(d) We have to patrol twice from node 1 to node 2 and there are three possible shortest routes between these two nodes. Thus, based upon Figure 1(b):

(i) Route 1: If we select route 1, then $x_7 = x_{14} = x_{17} = 1$ (3 new lanes will be patrolled);

(ii) Route 2: If we select route 2, then $x_7 = x_{10} = 1$ (2 new lanes will be patrolled);

(iii) Route 3: If we select route 3, then $x_6 = 1$. (1 new lane will be patrolled);

(iv) Patrol twice from node 1 to node 2:

$$Z_{121} + Z_{122} + Z_{123} = 2, \tag{10}$$

where $Z_{12k} \in \{0,1\}$ indicates whether the kth route between node 1 and node 2 is selected or not. Thus, by Route 3, we have:

$$Z_{123} \geq x_6, \quad x_6 \quad \text{and} \quad Z_{123} \in \{0,1\}. \tag{11}$$

That is, if Route 3 is not selected, then lane x_6 will not be patrolled. Similarly, by Relation (1) and Route 2, we have:

$$Z_{121} + Z_{122} \geq x_7, \qquad Z_{122} \geq x_{10},$$

$$Z_{121} \geq x_{14}, \qquad Z_{121} \geq x_{17}, \tag{12}$$

$$Z_{121}, Z_{122}, Z_{123}, x_7, x_{10}, x_{14}, x_{17} \in \{0,1\}. \tag{13}$$

Finally, based upon Relations (10)-(12), we have to solve Problem 2, as follows, for maximization of the routing coverage of lanes.

$$\text{Max} \quad \left| \underset{ijt}{\cup} X_{ijt} \right| = x_6 + x_7 + x_{10} + x_{14} + x_{17}, \tag{14}$$

$$\text{s.t.} \quad \text{Relations (10) -- (12)}. \tag{15}$$

Solving Relations (14) and (15), we have:

$$Z^*_{121} = Z^*_{123} = 1, \qquad \text{and}$$

$$x^*_6 = x^*_7 = x^*_{14} = x^*_{17} = 1, \tag{16}$$

which are depicted in red in Figure 1(c). Therefore, by Eqs. (9) and (16), the maximal routing coverage (%) of lanes is computed as (10+4)/17=0.8235 or 82.35%.

3. Immune based evolutionary approach

3.1. Steps of the immune algorithm

In this section, we will present an immune based evolutionary approach to obtain the first objective of BGPRP. To shorten this explanation, we refer to Weissman and Cooper [22] and Huang [23] for the details of an immune system and its mechanism.

The main steps of an immune based algorithm are as follows [24,25]:

Step 1. Generate an initial population of strings (antibodies) randomly;

Step 2. Evaluate each individual in the current population and calculate the corresponding fitness value for each individual by Relation (1);

Step 3. Select the best W individuals with the best fitness values;

Step 4. Clone the best W individuals (antibodies) selected in Step 3. Note that the clone size for each selected individual is an increasing function of the affinity with the antigen;

Step 5. The set of the clones in Step 4 will suffer the genetic operation process, i.e. crossover and mutation [26];

Step 6. Calculate the new fitness values of these new individuals (antibodies) from Step 5. Select those individuals who are superior to the individuals in the memory set, and then replace the inferior individuals in the memory set with them. While the memory set is updated, the individuals will be eliminated if their structures are too similar;

Step 7. Check the stopping criterion of maximum generations. If not, stop, then, go to Step 2. Otherwise, go to the next step;

Step 8. Stop. The optimal or near optimal solution(s) can be obtained from the memory set.

3.2. The new encoding scheme

Our coding of string is based upon the permutation of $\{1, 2, \cdots, n * \max|v_t|\}$, where $\max|v_t|$ is the maximum number of patrols of nodes in the network during period T; $1 \leq v_t \leq T$. Next, we use the following efficient encoding scheme to convert any permutation of $\{1, 2, \cdots, n * \max|v_t|\}$ into a feasible route of nodes over the period, T.

The main steps of the encoding procedure are as follows [1]:

Step 1. Generate an initial permutation of string P randomly from $\{1, 2, \cdots, n * \max|v_t|\}$;

Step 2. Divide P into n parts equally, and then rank the numbers into R for each part;

Step 3. Shade the first v_t rank numbers (R) of node t for each part, $t = 1, 2, \cdots, n$. Let $w = 0$, $S_{ij} = \phi$, $i = 1, 2, \cdots, T$ and $1 \leq j \leq c_i$;

Step 4. Compute all R_new by the following two rules:

(a) Rule 1: R_new = 1, if its corresponding c_R is 1 or R is not shaded;

(b) Rule 2: R_new = $k + 1$ if:

(i) R is shaded;

(ii) Its corresponding $c_R > 1$; and

(iii) Its corresponding $\text{mod}(P, c_R) = k$.

Step 5. $w = w + 1$. Find t, such that $P(\text{node } t) = w$, and append node t into S_{R,R_new} when its corresponding R is shaded, where S_{R_new} is the sequence of nodes to be visited on the Rth day on patrol vehicle R_new. Ignore this step when its corresponding R is not shaded. Repeat Step 5 until $w = n * \max|v_t|$. Finally, append the start point and the end point in every S_{ij}.

Table 1. The encoding example for $v = (2, 2, 3)$, $C = (1, 1, 2)$, and $T = 3$.

Node (i)	**1**			**2**			**3**		
P	3	9	2	5	8	4	7	6	1
R (day)	**2**	**3**	1	**2**	**3**	1	**3**	**2**	**1**
R_new (vehicle)	**1**	**2**	1	**1**	**1**	1	**2**	**1**	**1**

3.3. Example 2 (BGPRP with multiple patrol vehicles)

Consider the example in Figure 1(a) again. In this example, we assume $n = 3$, $v = (2, 2, 3)$, $T = 3$, and $C = (1, 1, 2)$. For this example, the length of each string of the immune based algorithm is given by $n * \max|v_t| = 3 * \{\max(2, 2, 3)\} = 3 * 3 = 9$.

Step by step details of the encoding procedure are shown below:

Step 1: Assume that the $P = (3, 9, 2, 5, 8, 4, 7, 6, 1)$ is generated randomly from $\{1, 2, \cdots, 9\}$;

Step 2: Divide P into $3(= n)$ parts, and then rank the numbers for each part as shown in Table 1. Since there are three numbers in each part, the rank ranges from 1 to 3 in row R (day);

Step 3: Because $v = (2, 2, 3)$, we shade the first v_t rank numbers of node t, as shown in Table 1; $t = 1, 2, 3$. $w = 0$. $S_{11} = S_{21} = S_{31} = S_{32} = \phi$;

Step 4: Since $C = (1, 1, 2)$, all R_new are computed, as those shown in Table 1;

Step 5: $w = w + 1 = 1$. $P(\text{node } 3) = 1$, and its rank is shaded, so append 3 to S_{11}; $S_{11} = (1)$;

Step 5: $w = w + 1 = 2$. $P(\text{node } 1) = 2$, but, its rank is not shaded, so ignore;

Step 5: $w = w + 1 = 3$. $P(\text{node } 1) = 3$, and its rank is shaded, so append 1 to S_{21}; $S_{21} = (1)$;

Step 5: $w = w + 1 = 4$. $P(\text{node } 2) = 4$, and its rank is not shaded, so ignore;

Step 5: $w = w + 1 = 5$. $P(\text{node } 2) = 5$, and its rank is shaded, so append 2 to S_{21}; $S_{21} = (1, 2)$;

Step 5: $w = w + 1 = 6$. $P(\text{node } 5) = 6$, and its rank is shaded, so append 3 to S_{21}; $S_{21} = (1, 2, 3)$;

Step 5: $w = w + 1 = 7$. $P(\text{node } 3) = 7$, and its rank is shaded, so append 3 to S_{32}; $S_{32} = (3)$;

Step 5: $w = w + 1 = 8$. $P(\text{node } 2) = 8$, and its rank is shaded, so append 2 to S_{31}; $S_{31} = (2)$;

Step 5: $w = w + 1 = 9$. $P(\text{node } 1) = 9$, and its rank is shaded, so append 1 to S_{32}; $S_{32} = (3, 1)$ and $w = n * \max|v_t| = 9$, stop. Finally, adding the start point and the end point, we have $S_{11} = (0,1,0)$, $S_{21} = (0,1,2,3,0)$, $S_{31} = (0,2,0)$ and $S_{32} = (0,3,1,0)$ by the permutation $P = (3,9,2,5,8,4,7,6,1)$.

Note that if any set of S_{ij} is empty (i.e., it violates the assumption), we may assign a penalty to this solution.

4. Numerical results and discussions

In this section, we test our immune based evolutionary approach for the new BGPRP with multiple patrol vehicles. Consider the 8×8 square grid network in Figure 2. Suppose that $T = 3$ (days) and $v = (3,2,3,1,2,3,2,3,3,1,2,1,2,3,2)$, i.e. nodes 4, 10, and 12 have to be patrolled once, nodes 2, 5, 7, 11, 13, and 15 have to be patrolled twice, and the other nodes have to be patrolled three times during three days. In addition, the rectilinear distance between any two adjacent nodes is set to one. In the numerical experiment, we vary the number of patrol vehicles from 1 to 2 for each day, and apply our immune based evolutionary approach for all combinations of patrols vehicle. That is, we test our approach for $C = (1,1,1)$, (1,1,2), (1,2,2), and (2,2,2), respectively. For each case, we experiment 500 times using the immune based evolutionary approach and collect all top solutions for the first objective, i.e. the minimization of total routing distance. And then, based upon these top solutions, we further apply the integer programming approach stated in Sections 2.3 and 2.4 for maximizing the routing coverage of lanes for the second objective. All numerical results are computed by Intel-Pentium

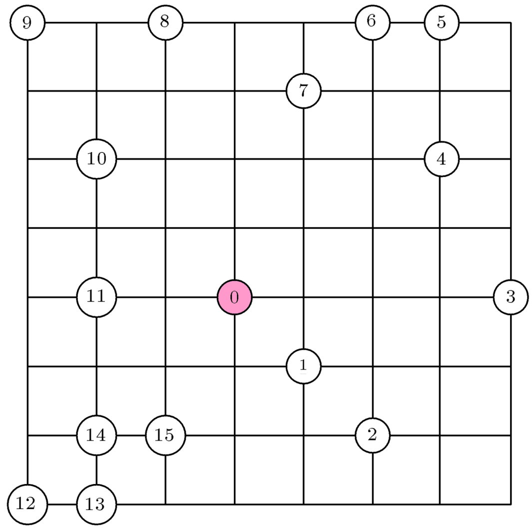


Figure 2. The square network with 8×8 grids (0=the start/end point).

IV 1.86 GHz PC and programs are coded in MATLAB 2006. Numerical results are summarized and reported in Table 2. The grid lanes that must be patrolled in the first objective and the grid lanes selected by the integer programming approach are depicted in green and red, respectively, in Figures 3-6 for various numbers of vehicles. The best results of routing distance and routing coverage for various numbers of vehicles are depicted in Figure 7.

From Table 2 and Figures 3-7, we observe that:

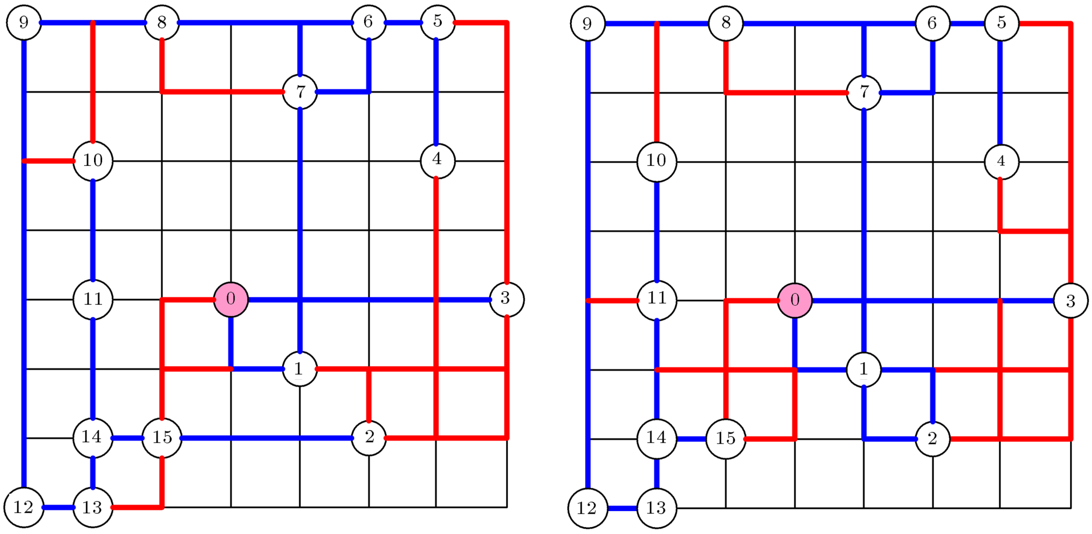
(a) With the increase in the number of patrol vehicles, the total routing distance (1st objective) increases. For example, the best objective values of total

Table 2. The top-2 solutions of 8×8 BGPRP.

No. of vehicles (C)	1st objective (distance)	2nd objective (coverage)	Top-2 solutions		
			Day 1	**Day 2**	**Day 3**
(1,1,1)	96	0.5982	(A) 1,2,3,5,6,7,8,9,10,11,14,15	(A) 1,3,4,5,6,8,9,11,14,13,15	(A) 1,7,6,8,9,12,13,14,2,3
	96	0.5982	(A) 15,14,13,11,10,9,8,6,5,3,2,1	(A) 14,11,9,8,6,7,1,2,3	(A) 15,14,13,12,9,8,7,6,5,4,3,1
(1,1,2)	98	0.6071	(A)15,14,13,12,9,8,6,7,3,2,1	(A)15,13,14,11,10,9,8,7,6,5,3,2,1	(A) 11,14,9,8,6,5,4,3 (B) 1
	98	0.5982	(A) 1,2,3,5,6,7,8,9,11,14,15	(A) 1,2,3,4,5,6,8,9,10,11,13,14	(A) 1 (B) 3,6,7,8,9,12,1
(1,2,2)	104	0.6607	(A) 1,2,3,4,5,6,7,8,9,12,13,14,11	(A) 7,6,8,9 (B) 1,3,2,14,13,15	(A) 3,5,6,8,9,10 (B) 11,14,15,1
	104	0.6518	(A) 15,14,13,12,9,8,7,6,5,4,3,1	(A) 8,9,7,6,5,3 (B) 11,14,15,2,1	(A) 10,9,8,6,3,2,1 (B) 11,13,14
(2,2,2)	108	0.6696	(A) 1 (B) 14,9,8,7,6,5,3	(A) 11,9,8,6,5,4,3 (B) 1,2,14,13,15	(A) 11,12,13,14,15 (B) 10,9,8,7,6,3,2,1
	108	0.6607	(A) 15,14,13,12,9,8,6,5,3 (B) 1	(A) 7 (B) 15,14,11,10,9,8,6,5,4,3,2,1	(A) 9,8,6,7,3,2,1 (B) 11,13,14

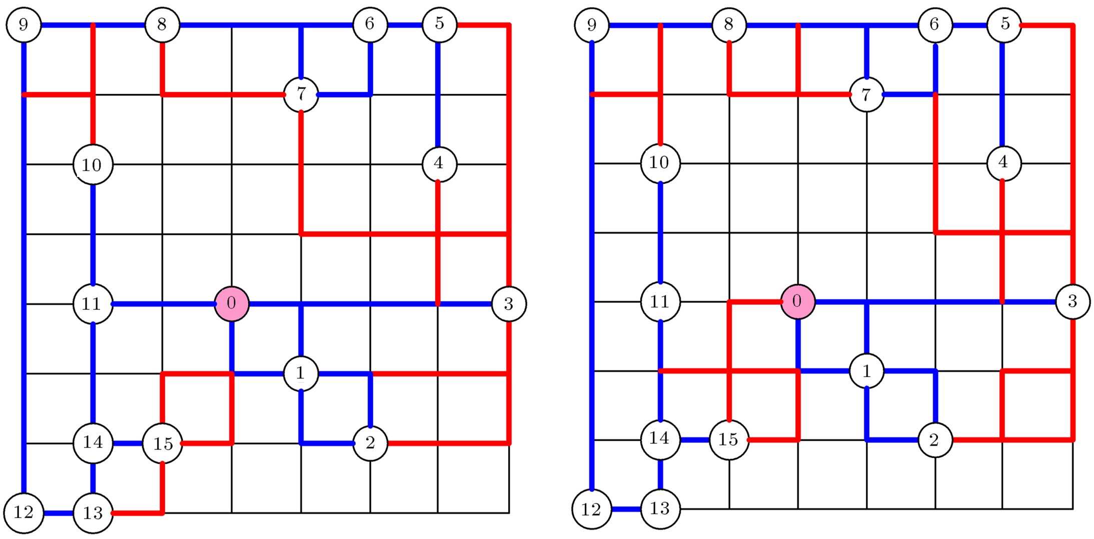
(1) Based upon 500 experiements of immune based algorithm.

(2) IA parameters: population=300, maximum generations=500, affinity=0.25, crossover=0.96, and mutation=0.01.



(a) Distance=96, coverage=(38+29)/112=0.5982 (b) Distance=96, coverage=(39+28)/112=0.5982

Figure 3. Top-2 solutions of patrolled lanes for $C = (1, 1, 1)$.



(a) Distance=98, coverage=(38+30)/112=0.6071 (b) Distance=98, coverage=(36+31)/112=0.5982

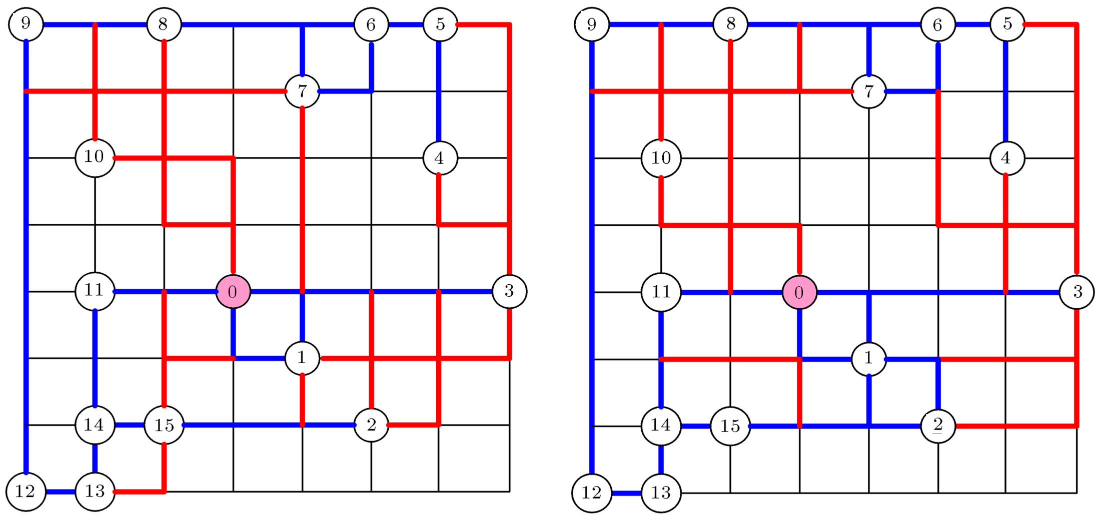
Figure 4. Top-2 solutions of patrolled lanes for $C = (1, 1, 2)$.

routing distance for the cases of patrol vehicles (1,1,1), (1,1,2), (1,2,2) and (2,2,2) are 96, 98, 104 and 108, respectively. This implies that more vehicles will increase the total routing distance.

(b) With the increase in the number of patrol vehicles, the total routing coverage of lanes (2nd objective) increases. For example, the best 2nd objective values for the cases of patrol vehicles (1,1,1), (1,1,2), (1,2,2) and (2,2,2) are 0.5982, 0.6071, 0.6607, and 0.6696, respectively. This implies that more vehicles will increase the total routing coverage of lanes.

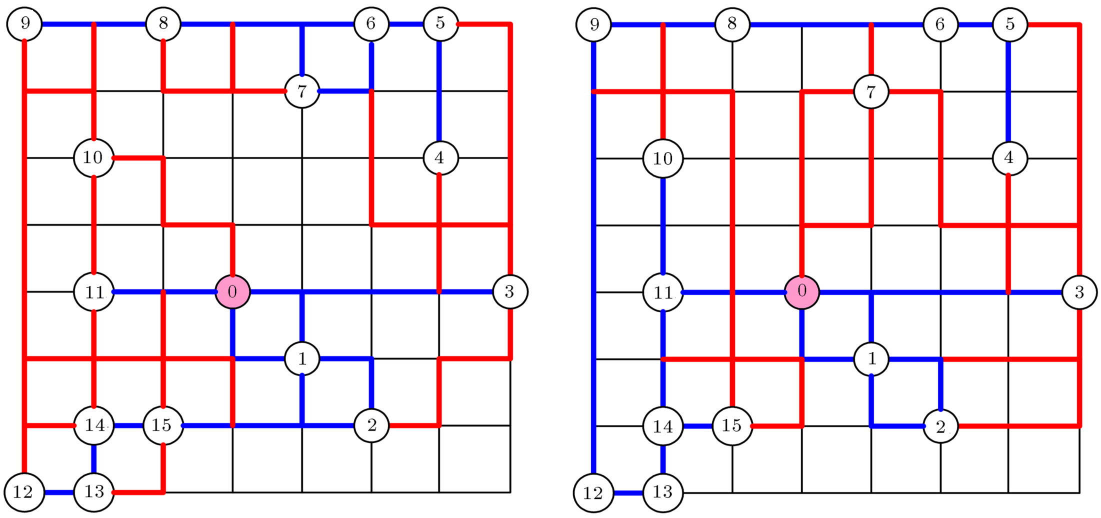
(c) The effect of an increase in vehicles does not seem to be equally significant for the total routing coverage of lanes, especially when there are few vehicles or more vehicles. For example, the total routing coverage of lanes only increases from 0.5982 to 0.6071 for cases of patrol vehicles (1,1,1) and (1,1,2), and only increases from 0.6607 to 0.6696 for cases of patrol vehicles (1,2,2) and (2,2,2). However, it increases from 0.6071 to 0.6607 for cases of patrol vehicles (1,1,2) and (1,2,2).

(d) The applied immune based approach can obtain multiple best solutions for the BGPRP. For example, as shown in Figure 3, the top-2 solutions, with the 1st objective value of 96 and the 2nd objective of coverage rate 0.5982, are different for the case of $C = (1, 1, 1)$. This further implies that the applied immune based approach can obtain multiple optimal solutions for the BGPRP.



(a) Distance=104, coverage=(35+39)/112=0.6607 (b) Distance=104, coverage=(38+35)/112=0.6518

Figure 5. Top-2 solutions of patrolled lanes for $C = (1, 2, 2)$.



(a) Distance=108, coverage=(29+46)/112=0.6696 (b) Distance=108, coverage=(35+39)/112=0.6607

Figure 6. Top-2 solutions of patrolled lanes for $C = (2, 2, 2)$.

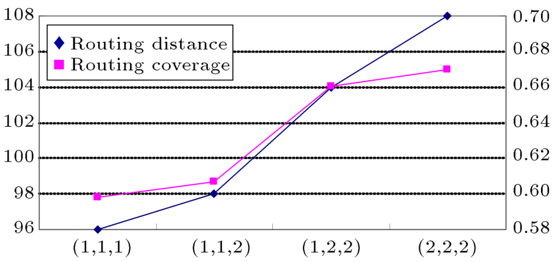


Figure 7. The routing distance and routing coverage for various C.

5. Conclusions

In this paper, we have:

(a) Investigated a new BGPRP in which several nodes have to be patrolled for a given number of times during a certain planning horizon.

(b) Presented an efficient encoding scheme to tackle both the combination and the permutation of nodes simultaneously, and then developed an efficient immune based evolutionary approach for solving the BGPRP.

(c) Shown the numerical results of instances with various numbers of patrol vehicle. Numerical results indicate that with the increase in the number of patrol vehicles, both the total routing distance (1st objective) and the total routing coverage of lanes (2nd objective) increase.

In future research, one may adopt other artificial intelligence approaches in solving Problem 1. Additionally, if possible, one may attempt to combine Problems 1 and 2 and solve the considered bi-objective problem directly.

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