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A novel approach in multi response optimization for correlated categorical data

R. Kamranrad and M. Bashiri*

Department of Industrial Engineering, Shahed University, Tehran, Tehran-Qom Freeway, Iran.

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Abstract. The main purpose of this paper is the optimization of multiple categorical correlated responses. So, a heuristic approach and a log-linear model have been used to simultaneously estimate the responses of surface parameters. Parameter estimation has been performed with the aim of maximizing the amount of concordance. Concordance means that the joint probability of occurrence of dependent responses in each treatment is more than other probabilities in the same treatment. The second step of this research is the optimization of multi correlated responses for categorical data using some practical meta-heuristic algorithms, including simulated annealing, Tabu search and the genetic algorithm. Using each meta-heuristic algorithm, the best controllable factors are selected to maximize the joint probability of success. Three simulated numerical examples with different sizes have been used to describe the proposed algorithms. Results show the superiority of joint success probability values in the Tabu search algorithm, compared to the other approaches.

1. Introduction

In the design of experiments, various problems are divided into specific classes considering the number and nature of the response variables (data). Problems with one response variable are called single-response problems, and problems with more response variables are called multi-response problems. These variables can also be divided into two general quantitative and categorical types. The aim of this classification is to assign and apply proper techniques for analyzing each problem, as using inappropriate analyzing methods for each type leads to invalid results. Making the best use of these methods depends on initial conditions, which should be checked for adequacy. For example, in the analysis of variance, the residuals should be normally distributed with a fixed variance, but categorical data do not follow the mentioned conditions. This subject was analyzed by Jaeger (2008), who applied analysis of variance for analyzing categorical data. The results showed that the mentioned approach has an incorrect interpretation for the categorical data, so, the proper tools should be applied [1]. With respect to the aforesaid explanations and details, various problems, with regard to their techniques, can be categorized into four classes of response; single and multi-continuous, and categorical. The mentioned categories and their related techniques for different problems are shown in Table 1.

Multi response problems (with continuous data) can be analyzed in two positions with independent responses. For the purpose of examining multi response problems with independent observations, it is possible to analyze single responses separately. Indeed, the previously implemented experiments not only do not include single response or independent responses, but many of them consist of more than one categorical response observation, so that the occurrence probability of each observation influences the likelihood of other observations occurring. Therefore, under this

*. *Corresponding author. Tel.: +98 21 51212092;*
Fax: +98 21 51212021
E-mail address: bashiri.m@gmail.com (M. Bashiri)

Table 1. Proper techniques for different categories of single and multiple responses problems.

Problem type	Single response		Multiple responses	
	Categorical	**Continuous**	**Categorical**	**Continuous**
	Chi-square Pearson	ANOVA	Chi-square Pearson	MANOVA
Analysis technique	Logistic regression (logit model)	Linear regression	Log-linear model	Desirability function Taguchi's loss function
	SN-ratio Taguchi			Classic methods

condition, we should apply techniques associated with categorical multi response problems (which can be defined as binary, nominal and ordinal). It is worth mentioning that the defined problem in this research, which is related to problems with some dependent categorical responses, belongs to a kind of binary response. In these problems, some methods, such as Chi- Square Pearson, log-linear model etc., can be implemented for analysis and optimization.

Log-linear models, as Generalized Linear Models (GLMs), are usually used in modeling cell counts of contingency tables. The models specify the expected count depending on the levels of categorical variable for that cell. The purpose of log-linear modeling is the analysis of association patterns. Log-linear models are of use primarily when there are at least two response variables. With a single categorical response, it is simpler and more natural to use logit models. When one variable is treated as a response and the others as explanatory variables, logit models for that response variable are equivalent to certain log-linear models [2]. In accordance with the above description, a log-linear model is used to analyze the contingency tables, which simply demonstrate the number of implemented observations for dependent responses. In the current research, the log-linear model has been used for analysis of an experiment including control variables with more than one response variable. In other words, the log-linear model can be treated as a log link function between control and response variables. So, the main problem is estimating the regression coefficients. As a result, in this research, a novel method has been proposed to estimate the parameters of a dependent categorical multi-response regression model. This paper is organized as follows: In the next section, previous related studies have been reviewed and discussed. In the third part, an innovative approach has been proposed for simultaneous log-linear parameter estimation for maximizing the number of model concordances in the dependent binary multi-response regression model. Then, some meta-heuristics have been proposed in optimization of the log-linear model. In section 5, some numerical analyses illustrate the proposed approach, and, finally, concluding remarks have been presented in Section 6.

2. Review of the literature

According to Table 1, there are some methods used to analyze a multiple response problem with continuous data. One of these methods is Taguchi's loss function. Chang (2008) used the data mining technique for dynamic multi response problems in designing the Taguchi experiment. He used an artificial neural network to construct the response function model and simulated annealing for determining the best collection of factors [3]. Datta et al. (2009) proposed the application of an anthrophy measuring technique, on the basis of the Taguchi approach, for resolving the correlated optimization problems. In their research, they used the principal component analysis technique for eliminating the available coherence between responses and converting them into independent responses [4]. Chang et al. (2009) used the weighted Taguchi loss function in optimization of process parameters for assessing the product income [5]. Another way to analyze the multi correlated continuous response problem can be the multivariate analysis of variance (MANOVA). Pal and Gauri (2010) examined the efficiency of different continuous methods for optimizing the independent multi-response problem using multinomial regression [6]. Chien and Bernard (2004) proposed a two stages process for selecting variables by combining MANOVA and the design of experiments [7].

Regarding Table 1, logistic regression and the Taguchi SN-ratio are considered analysis tools and the optimization of problems with single categorical responses, which include many applications in the context of health and treatment, marketing, product design etc. Huang et al. (2009) used this model to specify the relation between individuals' maturity time and control variables, such as nutrition, gender, environment, etc. [8]. Wu and Yeh (2006) analyzed the comparative methods, such as Taguchi's accumulation analysis, the Nair scoring scheme and Jeng's weighted probability scoring scheme for optimizing the categorical data [9]. Zhu et al. (2008) used logistic regression to examine the influence of consumer attitude and mentality on designing the product. In this case, they examined customer opinion and requirements on a specific product, and conveyed them to the designer in order to make a reasonable relationship and connection between the product design and customer needs [10]. Also, Bashiri et al. (2011) optimized the response variable in ordinal logistic regression, making use of heuristic and meta-heuristic methods. They achieved the best probability event for a response with the highest ranking by determining the best level of control factors [11]. On the other hand, Yeh et al. (2009) proposed the method of parameter estimation of a binary logistic regression model for single response problems [12]. Other studies have been done for binary responses. For example, Bashiri et al. (2011) proposed the new association measurements approach for obtaining the maximum fitness of the model [13].

There are some studies on multi categorical responses problems. Goodman (1981) proposed association models under the title of local cross-ratios for $I * J$ tables. This model is known as the Row-Column Model [14]. After Goodman, Dale (1986) [15], and Molenberghs and Lesaffre (1977) represented a marginal model for categorical data under the title of marginal accumulative logit and global-cross ratios; use of these models makes it possible to calculate the parameters of joint probability for correlated responses [16]. Then, using Goodman's local and Dale's global models, Lapp et al. (1998) proposed an associated model for the ordinal response [17]. Glonek et al. (1994) presented a multivariate logistic model for two and three variables of the binary response. Also, they evolved and developed the aforesaid model for some discrete variables of nominal and ordinal response [18]. Agresti (2002) applied a log-linear model in contingency tables with two, three and four variables of categorical response, which shows the method for calculating the joint probabilities of dependent responses [2]. Biswas (2004) proposed a theory approach for random categorical variables with correlated patterns [19]. In this research, the procedure of assessing the correlation among dependent responses and parameters of joint probabilities was proposed. In the current study, after demonstrating a heuristic method for simultaneous parameter estimation of a correlated log-linear model, optimization of a multi response model will be performed with the aim of maximizing the joint probability of success occurrence. Table 2 summarizes previous studies regarding all types of problem, as well as continuous and categorical data.

According to Table 2, only two pieces of research have used the log-linear model in which, however, they do not develop an optimization algorithm. Also, other research into multi categorical data does not include optimization and parameter estimation. This research focuses on some new research related aspects comparing them to other studies, which have been depicted in Table 2.

Table 2. Performed studies on different techniques of all kinds of data.

Problem type	**Authors**	**Year**	**Model**				**Characteristics**			
			LR	**Tag**	**Logistic**	**Log-lin**	**CR**	**PE**	**Opt**	**MHA**
Continuous data	Chang [3]	2008	-	●	-	-	●	-	●	●
	Chang et al. [5]	2009	-	●	-	-	●	●	●	-
	Datta et al. [4]	2009	-	●	-	-	-	-	●	-
	Pal & Gauri [6]	2010	●	-	-	-	-	-	●	-
Single categorical response	Wu & Yeh [9]	2006	-	●	-	-	-	-	●	-
	Zhu et al. [10]	2008	-	-	●	-	-	-	●	-
	Yeh et al. [12]	2009	-	-	●	-	-	●	-	-
	Bashiri et al. [11]	2011	-	-	●	-	-	-	●	●
Multiple categorical responses	Goodman [14]	1981	-	-	-	-	●	-	-	-
	Dale [15]	1986	-	-	-	-	●	-	-	-
	Glonek et al. [18]	1994	-	-	-	-	●	-	-	-
	Molenberghs & Lesaffre [16]	1997	-	-	-	-	●	-	-	-
	Lapp et al. [17]	1998	-	-	-	-	●	-	-	-
	Agresti [2]	2002	-	-	-	●	●	-	-	-
	Biswas [19]	2004	-	-	-	●	●	-	-	-
	Present research	-	-	-	-	●	●	●	●	●

Note: LR: Linear Regression; Tag: Taguchi; Log-Lin: Log-Linear; CR: Correlated Responses; PE: Parameter Estimation; Opt: Optimization; MHA: Meta-Heuristic Algorithm.

3. Proposed approach for correlated binary responses

In this section, a novel method is proposed for simultaneous estimation of regression coefficients in multi binary correlated responses maximizing the number of concordances. The concordance for the occurrence of dependent responses means that the joint estimated probability for observed response values in each treatment is more than the others under the same conditions. For instance, consider the design of experiment with two correlated binary responses. Suppose that, in the first treatment, the observed responses values are 1 and 0, respectively. Concordance will occur if the joint estimated occurrence probability of (1,0) is more than the others (0,1),(1,1) and (0,0). So, in the proposed approach, coefficients estimation tries to maximize the number of concordances for all observations. As mentioned previously, the proposed approach is based on the log-linear model for analyzing and optimizing multi correlated categorical responses. Thus, a brief discussion of the log-linear model is presented in the next section.

3.1. Log-linear model

In this model, the joint probabilities of correlated responses are calculated according to Eq. (1):

$$\pi_{ij} = \frac{\mu_{ij}}{\sum\sum\mu_{ab}}, \tag{1}$$

where, μ_{ij} is the joint expected value of correlated responses. The concept of the above-mentioned equation means that the joint probability parameter of two response variables that exist in i & j classification would be calculated by dividing the expected value of the same class from one treatment to the expected values of all classifications of the same treatment from each design of experiment. Therefore, joint probabilities in each treatment of the design of the experiment will have $\pi_{ij} \geq 0$ and $\sum_i \sum_j \pi_{ij} = 1$ limitations [2].

3.2. Generalized log-linear model

The general model of an ordinary log-linear model, described for a correlated model, is written in Eq. (2):

$$\text{Log}\mu_{ij} = X\beta, \tag{2}$$

in such a way that X demonstrates the control variable matrices, and β are the model parameters that should be estimated. By observing Eq. (2), the following result can be obtained:

$$\mu_{ij} = \exp(X\beta).$$

So, in order to calculate the joint probability values, we can apply Eq. (3) [17]:

$$\pi_{ij} = \frac{\exp(X\beta)}{\sum_a\sum_b \exp(X\beta)}. \tag{3}$$

In the following section, the proposed approach for simultaneous parameter estimation is presented.

3.3. Steps of simultaneous parameter estimation

After getting acquainted with the log-linear model, in this section, the steps of parameter estimation for problems with two correlated binary response variables are proposed. Under this circumstance, for each experimental treatment, four expected joint probabilities will be calculated, and the maximum is used for response value prediction. In other words, each treatment requires estimating four different parameters for calculating the maximum joint probability. Parameters can be estimated using an iterative method. The steps in parameter estimation using the heuristic method are described as follows:

1. In the first stage, an initial random value is considered for the model. Also, the ordinary least square method can be used to acquire the initial parameters for all four joint probabilities.
2. Parameter estimation is improved in each treatment, based on maximum likelihood, using the Yeh et al. (2009) [12] method. However the stopping condition of the proposed approach differs from the Yeh et al. approach. Thus, the heuristic algorithm terminates when the number of concordances do not change in the specified number of iterations. In other words, the heuristic approach tries to maximize the likelihood estimation and, then, continues to maximize the number of concordances. Figure 1 illustrates the pseudo code of the heuristic algorithm for simultaneous estimation of log-linear model parameters which has been proposed by Kamran Rad and Bashiri (2012) [20].

4. Multi response optimization for correlated binary responses

After estimating the parameters of a multi correlated binary response problem and constructing the prediction models, in this section, we will describe optimization of the problem with the aim of maximizing the joint probability value for success responses using simulated annealing, Tabu search and the genetic algorithm. Since each manner of searching the explained methods is different, it is essential to examine all three metaheuristic algorithms.

4.1. Proposed Simulated Annealing for optimization of Correlated Binary Responses (SA-CBR)

On the basis of simulating the thermal operation, this algorithm is one of the most famous meta-heuristic

1. Estimate the initial parameter using least square
2. Obtain the linear regression function matrix $\eta(ij) = X\beta_{ij}$, such that X is the control factors matrix and β is the estimated parameter
3. Calculate the joint expected values (E_{ij}) and Diag Matrix (W_{ij}) for correlated responses
4. Calculate the Q index in each iteration with Eq. (4)

$$Q(ij) = \eta(ij) + \left(\hat{W}(ij)\right)^{-1}(y(i) - E(ij))$$
for $i = 1, 2, \quad j = 1, 2$ (4)

5. Estimate the new parameters in each iteration with Eq. (5).

$$\beta^{(k+1)} = \left(X^T W(ij)X\right)^{-1} X^T W(ij)Q(ij)$$
for $i = 1, 2, \quad j = 1, 2$ (5)

In Eq.(5), X^T is the transposing of X matrix.

6. Calculate the joint probability for each treatment

Calculate the number of concordance.

Figure 1. Pseudo code of heuristic algorithm for simultaneous estimation of log-linear model based on Yeh et al. [12].

algorithms ever established. Thermal operation simulation is a local search method which can be applied to optimize discrete and continuous problems. Like other methods of meta-heuristics, this method can evade the trap of local optimum. Simulated annealing contains some parameters; T is the algorithm temperature in the intended iteration, and $Prob$ is the condition of acceptance for movement in the thermal operation. The proposed meta-heuristic algorithm utilizes this relation. The algorithms of simulated annealing in estimating the pseudo-linear regression parameters have been used before by Zheng and Zhang (2005), producing acceptable results [21]. The proposed pseudo-code code of the SA algorithm is shown in Figure 2.

This algorithm terminates in the frozen state. Then, the best value of controllable variable which leads to generation of the highest value of success joint probability is selected.

4.2. Proposed Tabu Search for optimization of Correlated Binary Responses (TS-CBR)

The algorithm of the Tabu search is one of the most efficient methods of problem optimization, specifically problems with discrete data. So, an algorithm has been proposed for correlated binary responses based on the Tabu search, which is called (TS-CBR). The pseudo code of the proposed approach has been shown in Figure 3.

4.3. Proposed Genetic Algorithm for optimization of Correlated Binary Responses (GA-CBR)

Genetic algorithms are very different to other optimization procedures. In these algorithms, the design space should be converted into genetic space. Therefore, genetic algorithms work with a series of coded variables. Due to the randomization nature of genetic algorithms, the produced responses can be good, bad or even impossible. Therefore, determining proper parameters plays an important role in acquiring an acceptable response in a limited or scant period of time. It includes selecting the primary population, calculating the fitness function, selecting parents, choosing crossover children and replacing the children with parents. The initial pseudo-code of the genetic algorithm for correlated binary responses is illustrated in Figure 4.

5. Numerical analysis

In this section, some simulated numerical examples with different sizes will be detailed for better illustration of the proposed approaches. In the first example, seven treatments with five controllable variables have

```
Initial parameter
        1. Estimate the best parameter for responses
        2. Select an initial solution (control variables)
Repeat
        3. Calculate the joint probabilities using Eq. (3)
              If the computed joint probability value is better than the previous value
                  Then go to next step
              Else with new probability of exp(−Δ/T) go to next step;
                  Such that Δ is the difference of probabilities
        4. If interior loop is less equal of max interior loop OR exterior loop is less
           equal of max exterior loop
              Stop
                     Else
                          Go to Step 3
Until stop criterion is true.
```

Figure 2. Pseudo code of the simulated annealing algorithm for optimization of correlated binary responses (SA-CBR).

```
Initial solution
      2.  Select the best estimated parameter from the results of heuristic
          algorithm
     3.  Select an initial solution (control variables)
Repeat
     4.  Calculate the joint probability values using Eq. (3)
     5. If the computed joint probability value is better than the previous value
                      Then go to the next step
     6.              Else move to the best control variables neighborhood that
        isn't Tabu and go to step 3
     7. If iter= max iter
          Stop
        Else go to step 5
Print values of best _x & best _p11.
```

Figure 3. Pseudo code of the Tabu search algorithm (TS-CBR).

```
Initial population
        1.  Specify the certain number of initial control variables
        2. Calculate the joint probabilities using the random selected control variables and
           specify the best joint probability
        3. Select the best controllable set with regard to its probability
Repeat
       Selection
        4. Select the two control variable sets among the initial population randomly
        5. Calculate the joint probability of success for two random selected sets
        6. Select the best control variable sets with regard to the max joint probability of success
        Cross over and mutation
        7. Create the random number between 1 to all number of treatments (for example h)
        8. Set the new control variable matrix with h first treatments from control variables set
           1 and rest treatments are putting from control variables set 2 for new matrix 1
        9. Set the new control variable matrix with h first treatments from control variables set
           2 and rest treatments are putting from control variables set 1 for new matrix 2
        10. Calculate the joint probability of success using the new matrix 1 and 2
         Replacing parents with child
                    If computed joint probability value is better than the previous value
                           Then replace new matrix with previous matrix
                     Else
                       Save the previous matrix as good control variables set
Until the run iteration is equal to certain number of iteration
```

Figure 4. Pseudo code of the genetic algorithm (GA-CBR).

been simulated. The second example consists of 10 treatments and 7 controllable variables, and the last example consists of 15 treatments with 10 controllable variables. Also, in these three examples, two binary correlated responses have been simulated. It is worth mentioning that the results of the third example have been described in detail and the others are reported only.

5.1. Simultaneous parameter estimation of correlated binary responses for simulated examples

Each controllable variable has been defined as five levels in the three following examples, and the limits and values of each of the responses and controllable variables are illustrated in Tables 3 to 5.

It should be said that the algorithms proposed in this research have been performed by MATLAB software, and the following results have been produced. In order to demonstrate and prove the accuracy and correctness of the values of joint probability, joint expected observations have been proposed for the third example. After implementing the algorithm of regression coefficient estimation, the expectation and joint probability of the third example have been shown in Tables 6 to 8.

Table 3. The simulated experimental result for the first example.

Controllable variables						Response variables	
Trt	**X1**	**X2**	**X3**	**X4**	**X5**	**Y1**	**Y2**
1	1	2	4	2	1	1	0
2	4	3	1	2	3	1	1
3	3	5	2	3	1	0	0
4	4	2	5	4	2	1	1
5	2	3	4	1	2	1	0
6	2	1	2	2	4	0	1
7	3	4	3	2	3	0	1

Here, some analyses of fractioned experimental design effects on parameter estimation have been studied. First, the numbers of predictor variables, by elimination of X10 and X9 from the main third example, have been reduced. In these cases, the

Table 4. The simulated experimental result for the second example.

Controllable variables								Response variables	
Trt	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**Y1**	**Y2**
1	2	5	1	3	5	1	4	0	1
2	3	5	1	2	4	5	2	1	0
3	3	1	2	4	1	3	3	0	1
4	4	2	1	2	5	1	2	1	1
5	1	3	4	1	2	5	3	1	0
6	2	5	3	2	4	3	1	1	0
7	3	1	3	5	1	2	5	1	1
8	4	2	3	5	2	1	3	1	1
9	3	4	2	1	4	2	5	0	1
10	2	1	4	3	2	1	5	1	0

log-linear parameters have been estimated and the following results have been obtained. Comparing the new estimated parameters with the values of Table 6, it is clear that, as the main example, the absolute values of intercept coefficients of fractioned experiments with 9 and 8 predictor variables are more than other coefficients. Also, comparison of the variances of these estimated parameters show that by whatever number the predictor variable is reduced, then, the variance of fractioned experiments would increase. Another case for the effect of the main example fraction on parameter estimation is including reduction of the number of levels of predictor variables. Also, in this case, the log-linear parameters are estimated and the determined results show that, unlike the first case, by reducing the numbers of predictor variable levels from five to three, the absolute values of all intercept coefficients are not higher than the other coefficients of the predictor variables. But, the variance value of this case is less than that of the main example.

Table 6. Values of parameters estimation in the heuristic method of the third example.

Control variables	$\beta 11$	$\beta 00$	$\beta 10$	$\beta 01$
Intercept	-7.5286	-1.9144	-11.8804	-1.8190
X1	0.4697	0.3392	1.0510	0.3810
X2	0.1896	-0.4787	0.3835	-0.5265
X3	0.4817	-0.4378	1.4680	-0.4434
X4	-0.6256	0.5782	-1.3409	0.5883
X5	-0.5699	0.9447	-1.3490	0.9941
X6	0.0751	0.2701	-0.0201	0.2796
X7	-0.1064	-0.0804	-0.5117	-0.1095
X8	0.2726	0.4620	0.2278	0.4986
X9	1.1082	-0.5995	2.4195	-0.6476
X10	1.0279	-0.7403	1.8455	-0.7506

Using estimated parameters (Table 6 parameters), we will be able to calculate the joint expected observation values relevant to this example. There are 15 treatments in the third example, each treatment of which has four joint expected observations. These values are shown in Table 7.

Considering the values of Table 7, it is possible to calculate the joint probability values for each treatment of the third example. The joint probability values

Table 5. The simulated experimental result for the third example.

Controllable variables											Response variables	
Trt	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**	**X7**	**X8**	**X9**	**X10**	**Y1**	**Y2**
1	3	1	5	3	3	4	2	1	2	1	0	1
2	2	5	2	2	2	1	2	3	2	4	1	0
3	1	3	5	4	4	2	3	4	3	2	0	1
4	4	2	1	2	5	2	3	1	5	3	1	1
5	3	4	2	5	2	1	2	2	4	3	1	0
6	2	1	4	2	1	5	2	1	4	2	1	0
7	5	4	3	1	2	3	4	5	2	1	1	1
8	1	1	3	4	2	4	3	1	3	5	1	0
9	2	4	5	3	2	4	1	3	1	1	0	1
10	3	4	5	2	3	5	5	1	2	3	1	0
11	1	4	3	2	4	3	1	3	5	2	1	0
12	3	1	4	1	4	2	1	4	2	3	1	1
13	1	5	3	1	2	4	3	4	2	1	0	0
14	4	3	5	3	4	2	5	1	4	3	1	0
15	5	3	1	4	3	1	2	3	1	5	0	1

Table 7. Values of expected observations in the third example.

Trt	$\mu 11$	$\mu 00$	$\mu 10$	$\mu 01$
1	0.0301	1.5633	0.0382	1.9734
2	0.9413	0.0162	4.7414	0.0157
3	0.0772	0.7862	0.0600	0.8837
4	0.8514	0.5886	4.1309	0.6669
5	0.4765	0.0827	2.6292	0.0802
6	1.8781	0.0276	136.9051	0.0273
7	0.7156	0.8875	2.6926	1.0772
8	0.7060	0.0346	2.7041	0.0347
9	0.0372	0.5120	0.0381	0.5943
10	0.6085	0.0488	3.9055	0.0479
11	1.1589	0.1075	18.7428	0.1069
12	0.6227	1.1257	4.6003	1.6252
13	0.1206	0.1266	0.0768	0.1242
14	1.7839	0.0680	69.3473	0.0684
15	0.2433	1.2827	0.1188	1.7407

Table 8. Joint probabilities and concordance position for two correlated binary responses in the third example.

Trt	$\pi 11$	$\pi 00$	$\pi 10$	$\pi 01$	Concordance position
1	0.0084	0.4377	0.0106	0.5474	C[a]
2	0.1647	0.0028	0.8295	0.0030	C
3	0.0427	0.4351	0.0332	0.4890	C
4	**0.1365**	**0.0944**	**0.6622**	**0.1069**	**D**[b]
5	0.1458	0.0253	0.8044	0.0245	C
6	0.0135	0.0002	0.9861	0.0002	C
7	**0.1332**	**0.1652**	**0.5011**	**0.2005**	**D**
8	0.2029	0.0099	0.7772	0.0100	C
9	0.0315	0.4333	0.0332	0.5030	C
10	0.1320	0.0106	0.8470	0.0104	C
11	0.0576	0.0053	0.9314	0.0057	C
12	**0.0781**	**0.1412**	**0.5769**	**0.2038**	**D**
13	0.2691	0.2824	0.1714	0.2721	C
14	0.0250	0.0010	0.9730	0.0011	C
15	0.0719	0.3789	0.0351	0.5142	C

[a]C: Concordance; [b]D: Discordance.

and the concordance position for each treatment of the third example are shown in Table 8.

As shown in Table 8, there are twelve concordances from 15 observations. The concordance can be determined by comparing the computed joint probabilities with the joint probability of the observed response values of Table 5. As an example, for the fourth treatment, the greatest value of joint probability is related to response (1,0), whereas the observed response for such treatment is (1,1), using Table 5, and it is a discordance. For validation of the estimation heuristic algorithm, two methods, such as measure of association, and comparison with the case of independence between responses, can be used. Measures of association can be considered a means of examining model fitness by the use of a number of existing concordances. These measures consist of "Somers' D, Kendall's Tau-a & Goodman- Kukal Gamma", which is about the defined 0 and 1. Whenever its value approaches 1, the model enjoys the higher fitness [21]. It is worth pointing out that, under the condition of independency of two or more variables, the values of joint probability will be equal to the product of each of the individual probabilities. Using the above-mentioned procedures, the efficiency of the proposed heuristic algorithm will be specified. The number of concordances and values of measures of association, in two states of independency and dependency, between variables, for each of the three examples, is given in Table 9.

With regard to Table 9, it should be said that three examples with measures of association of 0.5714, 0.50, and 0.6 enjoy an acceptable fitness in the proposed heuristic method. Likewise, a smaller number of concordances in the state of independence among the variables is indicative of the discordance of the suitable efficiency of this method for determining the joint probability of simultaneous occurrence of responses of one treatment from the example. Therefore, making use of the above procedures, it is possible to consider the heuristic method of simultaneous estimation of parameters for correlated binary responses as an effective method. In the following section, estimated parameters

Table 9. Number of concordances of three examples with heuristic estimation and independent single response methods.

		Method		Measure of association	
Example	**Criteria**	**IWV**	**HM**	**IWV**	**HM**
First	Con	2	5	0	0.5714
	Discon	5	1		
	Tied	0	1		
Second	Con	3	7	0	0.5
	Discon	7	2		
	Tied	0	1		
Third	Con	4	12	0	0.6
	Discon	11	3		
	Tied	0	0		

Note: HM: Heuristic Method;

IWV: Independence Within Variables;

Con: Concordance; Discon: Discordance.

are used to find optimal controllable factors by the proposed meta-heuristics.

5.2. Optimization of multi-responses problem with correlated binary responses for simulated examples

In this section, the proposed meta-heuristics of SA-CBR, TS-CBR and GA-CBR are used to optimize the three simulated examples. The initial parameters of SA-CBR are $T = 10,000$ (high temperature) and $T0 = 0.01$ and $R = 0.995$ (cooling ratio). It is worth mentioning that each of the three proposed meta-heuristic algorithms for the third example has been implemented 10 times, and the produced results are included in Tables 10 to 12.

In the genetic algorithm, the initial population has been selected randomly from the ten populations. Indeed, the selected population is the initial control factor matrix. Using these ten populations, the maximum value of the joint probability of success has been calculated.

The above-mentioned steps have been performed for design of the first and second example, and the final results have been compared in Tables 13 and 14, respectively. The mentioned tables contain the best treatment between the existing treatments also. The comparison shows that the proposed meta-heuristic algorithms can perform better than existing experiments.

With respect to the presented results in Table 13, it can be seen that maximum values of the joint probabilities in two methods of SA-CBR and Ts-CBR have equal values. At present, regardless of consideration of the time of processing by MATLAB software, both the two aforesaid methods are pertinent and proper for optimizing the binary multi-response problem, as well as correlated and dependent responses. In order to pass agreeable and impartial judgment between the performances of the two procedures, the following examples will be examined by considering larger dimensions. Table 14 shows the best level of controllable variables, in addition to the highest value of success joint probability for the third example,

Table 10. Results of optimizing the third example using the SA-CBR algorithm.

Run	Best level of controllable variables										Max P(1,1)	Computational time (sec)
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10		
1	5	4	5	2	4	3	5	5	5	5	0.7437	8.435875
2	2	4	3	5	1	2	2	5	5	3	0.7355	9.222657
3	5	2	2	1	2	3	3	4	3	5	0.7252	7.680367
4	2	2	1	1	3	2	1	2	1	5	0.7472	9.356642
5	4	1	4	2	4	2	2	5	4	2	0.6111	7.968220
6	5	5	1	2	1	1	3	2	4	2	0.7401	9.478968
7	**3**	**3**	**5**	**4**	**3**	**4**	**3**	**3**	**3**	**3**	**0.7497**	**8.695766**
8	1	1	4	5	4	3	2	1	1	4	0.7131	12.285350
9	3	2	1	1	5	2	3	5	4	2	0.7423	11.266097
10	4	2	1	1	4	5	3	4	4	3	0.7202	8.932846

Table 11. Results of optimizing the third example using TS-CBR.

Run	Best level of controllable variables										Max P(1,1)	Computational time (sec)
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10		
1	1	5	1	2	2	1	5	4	1	5	0.7704	35.184831
2	1	5	1	2	1	2	5	1	1	4	0.7821	39.462379
3	1	5	1	5	2	3	5	2	3	5	0.7144	35.828080
4	1	5	1	4	2	1	5	3	2	54	0.7371	33.767990
5	1	5	1	3	1	1	4	1	1	4	0.7099	34.127304
6	1	5	1	3	1	5	5	3	1	5	0.7755	34.086139
7	2	5	1	1	1	2	5	1	1	3	0.6917	33.859022
8	1	5	1	1	4	2	5	2	2	5	0.7162	34.587007
9	**1**	**5**	**1**	**3**	**1**	**2**	**5**	**3**	**1**	**5**	**0.7891**	**34.011828**
10	1	5	1	1	1	2	5	2	1	3	0.7321	34.329037

Table 12. Results of optimizing the third example using GA-CBR.

Run	Best level of controllable variables										Max P(1,1)	Computational time (sec)
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10		
1	4	3	1	3	1	4	2	2	2	1	0.6416	11.323056
2	4	3	4	4	3	3	1	3	2	4	0.6531	10.808063
3	4	3	1	5	4	2	2	1	5	3	0.5235	13.444903
4	4	4	1	2	2	3	1	4	1	1	0.6843	8.654253
5	4	1	3	3	1	5	5	5	1	1	0.6832	11.530855
6	1	2	1	4	1	5	5	2	2	5	0.5793	9.469021
7	2	2	1	3	4	3	3	1	5	2	0.6078	11.880108
8	1	2	1	5	1	5	3	3	2	1	0.5866	11.116903
9	**2**	**3**	**3**	**1**	**4**	**5**	**1**	**5**	**3**	**3**	**0.7268**	**10.693.50**
10	2	2	4	4	4	1	5	4	2	5	0.7194	8.853593

Table 13. The best level of controllable variables and success joint probability for the first example.

Algorithm	Best level of controllable variables					Max P(1,1)	Computational time (sec)
SA-CBR	**4**	**3**	**4**	**5**	**2**	**0.7241**	**3.029388**
TS-CBR	**2**	**1**	**5**	**5**	**1**	**0.7241**	**7.825848**
GA-CBR	5	2	2	4	5	0.5726	2.412095
Between existing treatments	2	1	2	2	4	0.512	0.519133

Table 14. Best level of controllable variables and value of success joint probability for the second example.

Algorithm	Best level of controllable Variables							Max P(1,1)	Computational time (sec)
SA-CBR	5	4	2	5	3	3	4	0.8219	4.060960
TS-CBR	**1**	**4**	**5**	**1**	**2**	**3**	**1**	**0.8261**	**15.35039**
GA-CBR	5	5	3	4	4	4	5	0.5543	6.718625
Between existing treatments	2	1	4	3	2	1	5	0.1916	1.516953

making use of three meta-heuristic algorithms and one heuristic algorithm.

According to the current results available in Table 14, again, it could be observed that the values of success of joint probabilities arisen from the three meta-heuristic algorithms enjoy a considerable difference from the same value in the proposed heuristic method. However, it is obvious that, as the dimensions of the problems increase, the value of success of the joint probability arisen from the meta-heuristic of the Tabu Search will be little more than the algorithm of simulated annealing. This little difference does not prove or indicate that the Tabu search is superior to simulated annealing. Due to this matter, judgment will be made dependently, subject to the third example with a larger dimension, compared to the second example. Therefore, the glossaries of the produced results are given in Table 15 for the third example.

5.3. Validation of the proposed meta-heuristic algorithms

According to the estimated coefficients, the optimal controllable factors can be determined by exact algorithms (using Lingo 8.0) and the proposed meta-heuristic algorithms. So, in this section, some more simulated experiments were analyzed for validation of the proposed approach. The max P(1,1) values of 30 numerical examples have been calculated using SA-CBR, TS-CBR, GA-CBR and an exact algorithm, whose results have been reported in Table 16. Note that it has been supposed that both responses are of the

Table 15. Best level of controllable variables and value of success joint probability for the third example.

Algorithm	Best level of controllable variables										Max P(1,1)	Computational time (sec)
SA-CBR	3	3	5	4	3	4	3	3	3	3	0.7497	8.695766
TS-CBR	**1**	**5**	**1**	**3**	**1**	**2**	**5**	**3**	**1**	**5**	**0.7891**	**34.011828**
GA-CBR	2	3	3	1	4	5	1	5	3	3	0.7268	10.693050
Between existing treatments	4	3	5	3	4	2	5	1	4	3	0.2691	1.204306

Table 16. Exact and meta-heuristic results comparison for 30 numerical examples.

Numerical example	Max P(1,1)			Exact P(1,1)	Gap	Max time of meta-heuristics(s)	Exact time (min)
	SA-CBR	TS-CBR	GA-CBR				
6 treats	0.8540	**0.9862**	0.6401	1.0000	0.0138	4.2866	09.45
	0.8828	0.9266	0.7291				
	0.8675	0.9558	0.7276				
7 treats	0.7195	0.7241	0.5130	0.7388	0.0592	7.8258	11.49
	0.6971	0.7195	0.4989				
	0.7045	**0.7241**	0.4923				
8 treats	0.8921	**0.9971**	0.8779	1.0000	0.0029	10.9697	15.12
	0.9100	0.9859	0.8898				
	0.9083	0.9363	0.7368				
10 treats	0.8104	**0.8256**	0.5279	0.9033	0.0777	15.9890	23.33
	0.7885	0.7911	0.4855				
	0.7926	0.8144	0.4956				
12 treats	0.8712	**0.9726**	0.8827	0.9945	0.0219	19.2516	31.19
	0.8830	0.8919	0.8806				
	0.8093	0.8948	0.8119				
14 treats	0.8180	**0.8961**	0.7684	—	—	29.8081	$T > 45$ mins
	0.8688	0.8931	0.7855				
	0.8279	0.8643	0.7091				
15 treats	0.7437	**0.7755**	0.6842	0.7941	0.0186	33.7689	″
	0.7355	0.7371	0.6916				
	0.7472	0.7321	0.6531				
18 treats	0.8154	0.8975	0.8830	—	—	45.6636	″
	0.8843	0.8946	0.8910				
	0.8722	**0.9014**	0.8003				
20 treats	0.8681	0.8985	0.8059	—	—	59.3527	″
	0.9033	**0.9179**	0.8585				
	0.9132	0.8267	0.8463				
24 treats	0.8402	0.9257	0.8992	—	—	78.3536	″
	0.8962	**0.9298**	0.9013				
	0.9047	0.9325	0.9318				

LTB (the larger, the better) type. Also, the analysis of variance (ANOVA) technique has been used in order to show the existing superiority of the three meta-heuristic algorithms towards each other. So, we use Minitab software to examine the meaningful differences between them.

As can be seen from Table 16, the gap between the results of the proposed algorithms and the exact solution is very low, but the exact method needs more computational time. This comparison proves the efficiency of the proposed solution algorithms. Also, an analysis of variance and a pair wise comparison have been made for better analysis. The results have been reported in Table 17.

Table 17. ANOVA table for three proposed meta-heuristic results.

Source	DF	SS	MS	F	P
Factor	2	0.2342	0.1171	9.55	0.001
Error	87	1.0671	0.0123		
Total	89	1.3013			

Individual 95% CIs for mean based on Pooled St Dev

Level	N	Mean	St Dev
SA-CBR	30	0.8277	0.0713
TS-CBR	30	0.8563	0.0940
GA-CBR	30	0.7366	0.1512

Tukey 95% simultaneous confidence intervals

SA-CBR subtracted from

	Lower	Center	Upper
TS-CBR	-0.0395	0.02862	0.0968
GA-CBR	-0.1592	-0.0910	-0.0229

TS-CBR subtracted from

GA-CBR	-0.1878	-0.1197	-0.0515

With regard to Table 17, it is clear that there is a meaningful difference between the three meta-heuristic algorithms. Likewise, results related to the confidence interval using the "Tukey" index show the superiority of the Tabu search over the other meta-heuristic algorithms. These statements have been shown in Figures 5 and 6.

By observing Figure 5, it is discovered that the Tabu search algorithm is relatively superior to the simulated annealing algorithm, while, in comparison with genetic algorithms, these two algorithms contains a much higher average of joint probability. Moreover, the

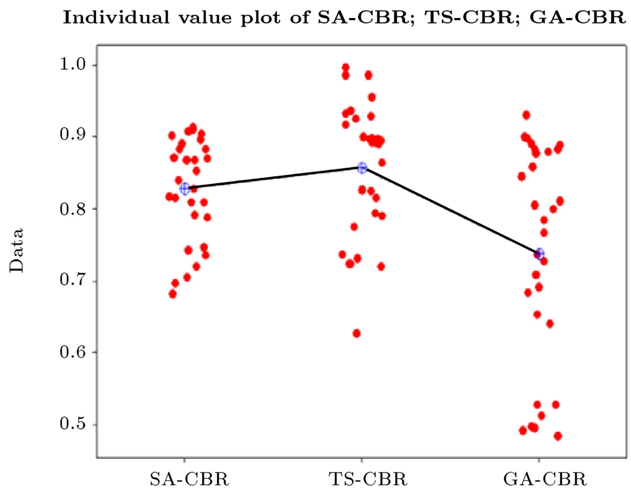

Figure 5. A diagram for examining the values of success joint probabilities on average for three proposed meta-heuristic algorithms.

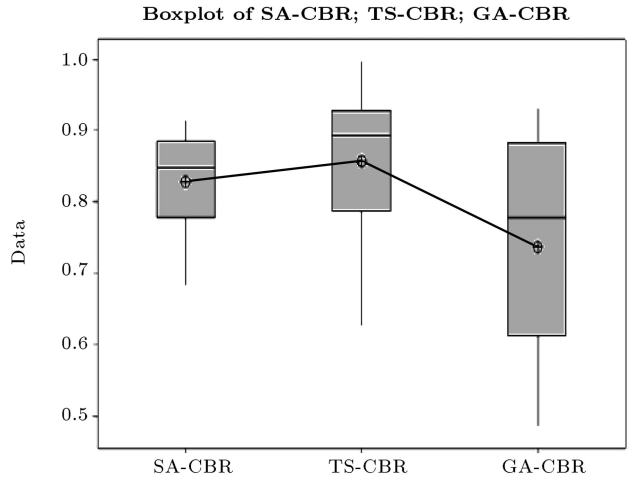

Figure 6. Box diagram of values of success joint probabilities for 3 proposed meta-heuristic algorithms.

produced box diagram displayed in Figure 6 confirms the above mentioned subject matter.

6. Concluding remark

In this research, some novel methods of solving and optimizing the multiple correlated binary responses problem have been proposed. In the beginning, a heuristic algorithm is presented for simultaneous estimation of log-linear model parameters, based on Yeh et al. [12]. After coefficient estimation, three meta-heuristic algorithms of SA-CBR, TS-CBR and GA-CBR have been proposed to find optimized controllable variables. In these three algorithms, the fitness function was defined in the form of maximizing the joint success probabilities. The proposed approaches were examined by some simulated examples, and the comparison of results from the proposed approaches and the exact solution shows the efficiency of the proposed solution approaches. Moreover, an analysis of variance shows that TS-CBR has better performance than the others for optimization of binary correlated multiple response problems. Studies on simultaneous parameter estimation and optimization of multi ordinal and nominal responses can be mentioned as future research. Analysis of problems with different types of categorical response variable can also be a future research topic. Also, researchers can maximize the association measurements by selecting the best controllable factor sets.

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Biographies

Reza Kamranrad holds BS and MS degrees in Industrial Engineering from Iran University of Science and Technology, and Shahed University, Iran, respectively, and is currently a PhD degree student of Industrial Engineering at Shahed University, Iran. He is also member of the Iranian Industrial Engineering Association. His research interests include design of experiments and statistical process control.

Mahdi Bashiri holds a BS degree in Industrial Engineering from Iran University of Science and Technology, and MS and PhD degrees in the same field from Tarbiat Modares University, Tehran, Iran. He is currently Associate Professor at Shahed University, Iran. His research interests include facility location, design of experiments and multiple response optimizations.