



Improved mathematical model for mass transfer in fractured reservoirs during the gas injection process

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Abstract. In fractured reservoirs with relatively low matrix permeability, i.e. small matrix block size with respect to capillary threshold height, diffusion becomes an important recovery mechanism. In this work, we have attempted to model the mass transfer between the matrix and fracture by considering a fractured porous media as a single matrix block with an adjacent fracture. An appropriate model should be applicable in the case of the matrix being saturated with both saturated and undersaturated oils. The proposed model presents such versatility. The model is a modification for the formulation of Jamili et al. [Jamili, A., Whillite, G.P. and Green, D.W., *Modeling Gas-Phase Mass Transfer Between Fracture and Matrix in Naturally Fractured Reservoir*, SPE132622 (2011)], which suffered from several drawbacks such as the use of the classical Fick's law, and of Hua and Whitson [Hua Hu, Whitson, C.H. and Yuanchang, Q.i., *A Study of Recovery Mechanism in a Nitrogen Diffusion Experiment*, SPE21893 (1991)] to calculate diffusion mass transfer coefficients between the fracture and matrix, and the use of Darcy's law to model convection mass transfer between the fracture and matrix. In this work, these drawbacks were surveyed and amended. Subsequently, the improved model has been validated through a gas injection experiment. Following model validation, the effects on the recovery rate of matrix permeability, initial gas saturation and injection rate are investigated. The numerical analysis showed that the effect of gas injection rate on the recovery rate is considerable.

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1. Introduction

It is common to develop the gas invaded zone within fractured reservoirs by injecting gas and maintaining reservoir pressure. Recovery in such reservoirs will be the result of a complex interplay of several mechanisms, such as viscous flow, extraction by molecular diffusion, gravity drainage, oil swelling, and capillary forces, where the main mechanism is generally gravity

drainage. However, in many cases, such as reservoirs with low permeability matrix, small matrix block size and high capillary pressure, gravity drainage may be very low or ineffective. Low permeability also results in inefficient viscous displacement. Diffusion is, therefore, the main recovery mechanism in these cases. Diffusion in fractured reservoirs, unlike in conventional reservoirs, can significantly affect the efficiency of gas injection in oil reservoirs and recycling in gas/condensate reservoirs. During gas injection in fractured reservoirs, the injected gas is inclined to flow through the fractures and may, therefore, push or vaporize the oil in the fractures before any significant gas penetration into the surrounding matrix. Physical

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diffusion, similar to gravity, results in a change in the path of the injected gas species from the fractures to the matrix, giving rise to a late breakthrough.

The gas injected into the fractures diffuses into the resident fluid inside the matrix and vaporizes the light oil fractions. The vaporized oil is then transported into the fracture by means of convection and diffusion. The process of diffusion/stripping has been described by Saidi [1]. Theoretically, the recovery of light and intermediate fractions of oil is total (providing sufficient quantities of gas are injected).

The parameters that influence diffusion include the nature of the injected gas, the composition of the oil in place, the presence of water, the fracture intensity, the rate of gas injection, and the geometry of the matrix blocks. Among those, the composition of the oil in place and the nature of the injected gas are the most important.

Various authors have discussed the effect of diffusion on oil recovery in fractured reservoirs [2–13]. Hua and Whitson [5] assumed laminar, incompressible, and steady state flow of injected gas in the fractures, while ignoring the convection mass transfer between the fracture and the matrix. They also assumed that inside the fracture, gas stream velocity and physical properties are constant and unaffected by diffusion. Finally, they proposed an equation for calculating mass transfer coefficients between the gas flowing in the fracture and the resident fluid inside the matrix block.

Jamili et al. [13] simulated experiments number M5 of Morel et al. [14] and number M25 of Le Romancer et al. [15]. They treated the fracture as a boundary condition for the matrix and discretized the compositional material balance equations that govern the flow of fluid in porous matrix by the finite difference method. The diffusion terms are modeled by Fick's law. The mass transfer coefficients between fracture and matrix were calculated by Hua and Whitson [5]. Convection between matrix and fracture was defined in the model based on Darcy's law. There, the results of the simulations matched the experimental data well.

In this work, we model the mass transfer between the matrix and fracture by considering fractured porous media as a single matrix block with an adjacent fracture. The proposed model is a modification of that of Jamili et al. [13].

Accurate prediction of diffusion coefficients, including the off-diagonal elements, is a key issue. We use the multicomponent-diffusion-coefficient model of Ghorayeb and Firoozabadi [16] to calculate the full diffusion matrix as a function of temperature, pressure, and composition. In this work, we use the model by Leahy Dios and Firoozabadi [17], in place of Hayduk and Minhas [18], in the model of Ghorayeb

and Firoozabadi [16] to predict the infinite dilution coefficients.

Viscosity and interfacial tension are calculated from Lohrenz et al. [19] and Parachor methods [20], respectively.

First, the governing equations, with regard to matrix and fracture fluid flow, are presented separately, and afterwards, the matrix/fracture interaction is applied by using appropriate equations to model the mass transfer through the matrix/fracture interface (or boundary).

We assumed that there is no oil initially in the fracture and the fracture is fully saturated with gas. When a fracture saturated in gas is in the vicinity of a matrix saturated with undersaturated oil, the gas-gas and oil-oil diffusions will not start because of phase discontinuity. The gas oil mass transfer is, therefore, crucial to model diffusion properly. The commonly used approach, which is based on the Film theory, assumes thermodynamic equilibrium at the gas-oil interface and the continuity of component fluxes across the interface. An appropriate model should be applicable in the case of the matrix being saturated with both saturated and undersaturated oils. The model suggested by Hoteit [21] presents such a versatility and is used in this paper.

The proposed model is validated by a gas injection experiment (experiment no. M5 of Morel et al. [14]). Following model validation, the effects on recovery rate of matrix permeability, initial gas saturation and injection rate are investigated.

2. Mathematical model

In this work, a fractured porous media is considered as a single matrix block with an adjacent fracture. This approach is a fine-scale representation of a naturally fractured reservoir, since it allows one to study the fluid flow between the fracture and the matrix block.

2.1. The defects of the model of Jamili et al. [13] and modifications

There are several drawbacks within the model of Jamili et al. [13]:

- (a) Using Classical Fick's law: The classical Fick's law is the most utilized model in reservoir engineering literature and in commercial and academic reservoir simulators [2,22–25]. Current practice is to use this model in the context of effective diffusivity, where diffusion in a multicomponent mixture is assumed to behave as pseudo binary [3]. This model is elegant and simple from a computational point of view and may provide reasonable results for many applications. However, it may not honor the equi-molar condition which states that the total diffusion flux must be zero. In some cases, this

model may fail to provide an even qualitatively correct description of diffusion behavior [26]. The failing of the classical Fick's law is a result of neglecting the dragging effect that is described by the off-diagonal elements in the diffusion coefficient matrix [27]. Krishna and Standart [28] argued that the classical Fick's law could be only valid for cases with special conditions, such as ideal binary mixtures and ideal multicomponent mixtures having diffusion coefficients that can be regarded as equal. Nevertheless, these conditions, which specify the validity range for the classical Fick's law, are sufficient but may not be necessary. The bottom line is that it is difficult to anticipate when Fick's law does and does not work. Petroleum fluids, even at low pressures, are not ideal because of the diversity in size of molecules. A realistic model of diffusion should include both diagonal and off-diagonal entries in the matrix of diffusion coefficients. To overcome this issue, we use the generalized Fick's law.

- (b) Using Hua and Whitson [5] to calculate diffusion mass transfer coefficients between fracture and matrix: There are several assumptions in the work of Hua and Whitson [5]: (1) Gas stream velocity and physical properties are constant along the fracture and unaffected by occurring mass transfer between the matrix and fracture; (2) Incompressible and steady state flow along the fracture; (3) The diffusion terms are modeled by classic Fick's law; and (4) The convection mass transfer between the matrix and fracture is ignored.

The Hua and Whitson relationship to calculate mass transfer coefficients is inadequate, because the applied assumptions are not always satisfied. Therefore, we must do our best to model diffusion mass transfer by applying more reliable assumptions.

- (c) Using Darcy's law to model convection mass transfer between fracture and matrix: We know that matrix block geometry is an efficient parameter on mass transfer between matrix and fracture, therefore, Darcy's law is inefficient for modeling convection mass transfer. A transfer function that regards the effect of matrix block geometry on mass transfer by shape factor is definitely better than Darcy's law. The shape factor is related to the geometry and size of the unfractured matrix blocks and can be used as a tuning parameter against well pressure tests or experimental data.

2.2. Governing equations

2.2.1. Fluid flow through matrix

The governing equations for the three-phase (gas/oil/water) compositional flow within the porous matrix are obtained by the species-balance equations,

overall material balance, Darcy's law, generalized Fick's law, the thermodynamic equilibrium between the phases and the constraint equations.

Species-balance equations. Material balance equations govern the transport of each component in oil and gas by convection and diffusion mechanisms:

$$\begin{aligned} & \nabla \cdot \left(x_c \rho_o \frac{k k_{ro}}{\mu_o} (\nabla P_o - \gamma_o \nabla D) \right. \\ & \quad \left. + y_c \rho_g \frac{k k_{rg}}{\mu_g} (\nabla P_g - \gamma_g \nabla D) \right) \\ & \quad + \nabla \cdot \left(\varphi \rho_o s_o \sum_{k=1}^{n_c-1} D_{ck}^o \nabla \cdot x_k \right. \\ & \quad \left. + \varphi \rho_g s_g \sum_{k=1}^{n_c-1} D_{ck}^g \nabla \cdot y_k \right) + q_{D,fm,c} + q_{C,fm,c} \\ & = \frac{\partial}{\partial t} (\phi (\rho_o s_o x_c + \rho_g s_g y_c)), \\ & c = 1, 2, \dots, n_c - 1. \end{aligned} \quad (1)$$

The first derivative term represents the convection mechanism in the oil and gas phases, which is formulated by Darcy's law. The second term represents the diffusion mechanism. The diffusion fluxes are modeled by the generalized Fick's law. $q_{D,fm,c}$ is the diffusion rate of c between the matrix and fracture that occurs at the fracture/matrix boundary. $q_{C,fm,c}$ is the convection rate of c between the matrix and fracture that occurs at the fracture/matrix boundary. In this work, the convection term, $q_{C,fm,c}$, is defined by the multiphase transfer function as:

$$\begin{aligned} q_{C,mf,c} = \sigma_{sf} V_{bm} & \left[\left(\rho_o x_c \frac{k_x k_{ro}}{\mu_o} \right)_m [P_f - P_{om}] \right. \\ & \left. + \left(\rho_g y_c \frac{k_x k_{rg}}{\mu_g} \right) [P_f - P_{gm}] \right], \end{aligned} \quad (2)$$

where σ_{sf} is the matrix shape factor that is calculated as [29]:

$$\sigma = \frac{1}{V_{bm}} \sum_{j=1}^n \frac{A_j}{l_j}. \quad (3)$$

Overall material balance equation. Summing Eq. (1) for all the species results in an overall material balance equation of the form:

$$\nabla \cdot \left(\rho_o \frac{k k_{ro}}{\mu_o} (\nabla P_o - \gamma_o \nabla D) + \rho_g \frac{k k_{rg}}{\mu_g} (\nabla P_g - \gamma_g \nabla D) \right)$$

$$+ \sum_{c=1}^{n_c} q_{D,fm,c} + \sum_{c=1}^{n_c} q_{C,fm,c} = \frac{\partial}{\partial t} (\phi(\rho_o s_o + \rho_g s_g)). \quad (4)$$

Water material balance. It is assumed that hydrocarbon and water phases are insoluble, therefore, water transportation occurred only via the convection mechanism through the porous matrix:

$$\nabla \cdot \left(\rho_w \frac{kk_{rw}}{\mu_w} (\nabla P_w - \gamma_w \nabla D) \right) = \frac{\partial}{\partial t} (\rho_w s_w). \quad (5)$$

Chemical equilibrium. The hydrocarbon phases are considered to be in chemical equilibrium through the porous matrix. Chemical equilibrium is stated by equality between the fugacities of each species in both oil and gas phases:

$$f_{c,o} = f_{c,g} \quad c = 1, 2, \dots, n_c. \quad (6)$$

Capillary pressure. The relationships between the gas, oil and water pressures are governed by the capillary pressures:

$$P_{cog} = P_g - P_o, \quad (7)$$

$$P_{cow} = P_o - P_w. \quad (8)$$

Constraint equations. The sum of mole fractions within the hydrocarbon phases is equal to one. The sum of the saturations of gas, oil and water is also equal to one:

$$\sum_{c=1}^{n_c} x_c = 1, \quad (9)$$

$$\sum_{c=1}^{n_c} y_c = 1, \quad (10)$$

$$s_o + s_g + s_w = 1. \quad (11)$$

The multiphase compositional flow through porous media is governed by Eqs. (1) to (11). This system of equations consists of the $(2n_c + 6)$ equation and the $(2n_c + 6)$ unknown $(P_o, P_g, P_w, s_o, s_g, s_w, x_c, y_c, c = 1, 2, \dots, n_c)$.

2.2.2. Fluid flow through fracture

We assumed that there is no oil initially in the fracture and that the fracture is fully saturated with gas. The injected gas diffuses into the porous matrix, through gas and liquid phases. This causes oil to be vaporized and then transported by convection and diffusion to the gas flowing in the fracture.

The concentration gradient is usually used to calculate the diffusion mass transfer at the matrix/fracture boundary. The flowing gas inside the

fracture also affects the mass transfer between the fracture and matrix. Therefore, for mass transfer calculations, not only the concentration gradient but also the effect of flowing gas inside the fracture must be considered. Using laminar flow theory, the species mass balance inside the fracture is as follows:

$$\begin{aligned} \nabla \cdot (y_c \rho_g v) - \nabla \cdot \left(\rho_g \sum_{k=1}^{n_c-1} D_{ck}^g \nabla \cdot y_k \right) - q_{C,mf,c} \\ - q_{D,fm,c} = \frac{\partial}{\partial t} (y_c \rho_g), \quad c = 1, 2, \dots, n_c - 1. \end{aligned} \quad (12)$$

Summing Eq. (12) for all species results in an overall Material balance equation:

$$\nabla \cdot (\rho_g v) - \sum_{c=1}^{n_c} q_{C,mf,c} - \sum_{c=1}^{n_c} q_{D,mf,c} = \frac{\partial}{\partial t} (\rho_g). \quad (13)$$

2.2.3. Initial conditions

It is assumed that the system is initially under gravity and chemical equilibriums and that the fluid is distributed uniformly through the matrix; therefore, there are convection and diffusion flow through the porous matrix.

According to Darcy's law and gravity equilibrium assumption:

$$(\nabla P_p - \gamma_p \nabla D) = 0 \quad p = w, o, g. \quad (14)$$

For a horizontal plane $\nabla D = 0$, therefore, $\frac{\partial P_p}{\partial x}$ and $\frac{\partial P_p}{\partial y}$ are equal to zero. This states that the pressure is constant in a horizontal plane at time zero. For a vertical plane $\nabla D = 1$, therefore, $\frac{\partial P_p}{\partial z} = \gamma_p$ suggesting that the vertical pressure distribution is given by the column weight. Accordingly, if pressure at a reference height is given, then pressure at any point in the model can be determined.

According to the generalized Fick's law and chemical equilibrium assumption:

$$\sum_{k=1}^{n_c-1} D_{ck}^o \nabla \cdot x_k = 0, \quad c = 1, 2, \dots, n_c - 1, \quad (15)$$

$$\sum_{k=1}^{n_c-1} D_{ck}^g \nabla \cdot y_k = 0, \quad c = 1, 2, \dots, n_c - 1. \quad (16)$$

This results in no composition gradient within the matrix; hence $\nabla x_c, \nabla y_c = 0$ ($c = 1, 2, \dots, n_c$). If fluid composition and PT (pressure and temperature) conditions are known, one can determine the composition and saturation of hydrocarbon phases through flash calculations.

2.2.4. Boundary conditions

In this work, a fractured porous media is considered as a single matrix block with an adjacent fracture. Many researchers seal some sides of the matrix [5,9,13–15]. This is to say that there are fractures contacting some sides (hence, a mass transfer between the matrix and fracture), while other sides are in no contact with any fracture (no mass transfer between the matrix and fracture). So, there are two kinds of boundary for a porous matrix block, namely, flow boundaries and no-flow boundaries, also known as sealed boundaries.

Sealed boundaries. The diffusion and convection fluxes for any species and any phases (oil, gas and water) are zero.

Therefore, the convection flux at the boundary is:

$$\rho_p \frac{kk_{rp}}{\mu_p} (\nabla P_p - \gamma_p \nabla D) = 0, \quad p = w, o, g, \quad (17)$$

and the diffusion flux is:

$$\sum_{k=1}^{n_c-1} D_{ck}^o \nabla \cdot x_k = 0, \quad c = 1, 2, \dots, n_c - 1, \quad (18)$$

$$\sum_{k=1}^{n_c-1} D_{ck}^g \nabla \cdot y_k = 0, \quad c = 1, 2, \dots, n_c - 1. \quad (19)$$

Flowing boundaries. At these boundaries, mass exchange occurs between the matrix and fracture. It is assumed that there is no oil initially in the fracture and that the fracture is fully saturated with gas. When a fracture saturated in gas is in the vicinity of a matrix saturated with undersaturated oil, the gas-gas and oil-oil diffusions will not start because of phase discontinuity. The gas-oil mass transfer is, therefore, crucial to model diffusion properly. The commonly used approach, which is based on the Film theory, assumes thermodynamic equilibrium at the gas-oil interface and continuity of component fluxes across the interface.

Hoteit and Firoozabadi [10] suggested using the cross-phase equilibrium concept at the matrix-fracture interface in discrete fracture models. This concept assumes that the fracture gas is in thermodynamic equilibrium with the matrix oil within a thin region adjacent to the fracture. With this approach, a thin, two-phase region is introduced between the fracture blocks and the matrix blocks (Figure 1). Therefore, gas-in-gas and oil-in-oil diffusions between the matrix and the fracture blocks can occur via the two-phase region. The introduced region should be thin enough to reduce the gridding effect. It was found that at reservoir scale with the fracture aperture around 1 mm, for example, the two-phase gridblock thickness should

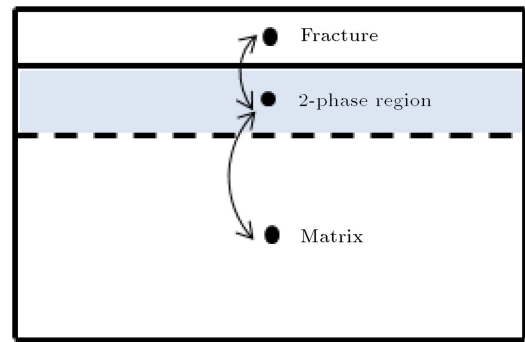


Figure 1. Cross flow equilibrium concept: Gas-gas and oil-oil diffusions occur across two-phase region [21].

be in the range of 10 cm [10]. This approach provides reasonable accuracy and is simple to apply in single-porosity and discrete-fracture models. However, it is not clear how to apply this approach in the context of dual-porosity models [21].

In this work, the approach proposed by Hoteit [21] is used to model mass transfer at the matrix/fracture interface (or boundary).

In an isothermal system, it is assumed that there is a thin transition region at the gas-oil contact where the two fluids totally mix and, consequently, are in chemical equilibrium. If no reaction occurs at the interface, the continuity of component molar fluxes across the interface holds. Consider the situation where a fracture gridblock saturated with gas is adjacent to a matrix gridblock saturated with oil. A sketch of the two blocks is shown in Figure 2. We note that the gas-oil interface coincides with the matrix-fracture interface, i.e., the gridblock boundary.

An appropriate model should be applicable in the case of the matrix being saturated with both saturated and undersaturated oils. The proposed model presents such versatility.

According to Hoteit [21], the following is considered for the governing equations of mass transfer at the flowing boundary:

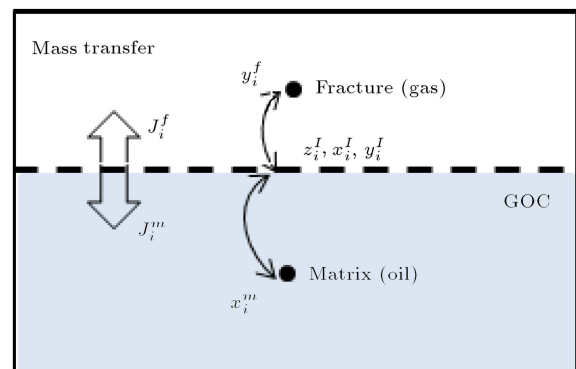


Figure 2. Mass transfer at the gas-oil interface: Gas-gas and oil-oil diffusion occur across a thin film at the interface [21].

1. The diffusion and convection molar fluxes defined from both sides of the boundary are continuous, therefore:

- (a) Continuity of diffusion fluxes across the boundary:

$$(J_c^g + J_c^o)_m = J_{cf}^g, \quad c = 1, 2, \dots, n_c - 1. \quad (20)$$

By applying generalized Fick's law to Eq. (20), we have:

$$\begin{aligned} & \left(\varphi \rho_g s_g \sum_{k=1}^{n_c-1} D_{ck}^g \frac{\partial y_k}{\partial x} \right)_m + \left(\varphi \rho_o s_o \sum_{k=1}^{n_c-1} D_{ck}^o \frac{\partial x_k}{\partial x} \right)_m \\ &= - \left(\rho_g \sum_{k=1}^{n_c-1} D_{ck}^g \frac{\partial y_k}{\partial x} \right)_{f, x=0} \\ & c = 1, 2, \dots, n_c - 1, \end{aligned} \quad (21)$$

where:

$$\left(\frac{\partial y_k}{\partial x} \right)_f = \frac{y_{k,mf} - y_{kf}}{\frac{F_a}{2}}, \quad (22)$$

$$\left(\frac{\partial x_k}{\partial x} \right)_m = \frac{x_{km} - x_{k,mf}}{\frac{\Delta x}{2}}, \quad (23)$$

$$\left(\frac{\partial y_k}{\partial x} \right)_f = \frac{y_{k,mf} - y_{kf}}{\frac{F_a}{2}}, \quad (24)$$

- (b) Continuity of convection fluxes across the boundary:

$$\begin{aligned} & \left[\left(\rho_o \frac{k_x k_{ro}}{\mu_o} \frac{\partial P_o}{\partial x} \right) + \left(\rho_g \frac{k_x k_{rg}}{\mu_g} \right) \frac{\partial P_g}{\partial x} \right]_m \\ &= \sigma_{sf} V_{bm} \left[\left(\rho_o \frac{k_x k_{ro}}{\mu_o} \right)_m [P_f - P_{om}] \right. \\ & \quad \left. + \left(\rho_g \frac{k_x k_{rg}}{\mu_g} \right)_m [P_f - P_{gm}] \right], \end{aligned} \quad (25)$$

where:

$$\frac{\partial P_o}{\partial x} = \frac{P_{om} - P_{boundary}}{\frac{\Delta x}{2}}, \quad (26)$$

$$\frac{\partial P_g}{\partial x} = \frac{P_{gm} - P_{boundary}}{\frac{\Delta x}{2}}. \quad (27)$$

2. The hydrocarbon phases are under chemical equilibrium, therefore:

$$\begin{aligned} & f_{c,o}(P, T, x_{c,mf}, c = 1, 2, \dots, n_c - 1) \\ &= f_{c,g}(P, T, y_{c,mf}, c = 1, 2, \dots, n_c - 1) \\ & c = 1, 2, \dots, n_c. \end{aligned} \quad (28)$$

This system of equations consists of $2n_c$ equations and $2n_c$ unknowns ($P_f, V, y_{c,mf}, x_{c,mf}, c = 1, 2, \dots, n_c - 1$).

3. Numerical scheme

The differential equations governing compositional multiphase flow in porous media are presented in the previous section. Some of these equations are nonlinear. The numerical technique replaces all derivatives by the finite difference approximations resulting in a set of nonlinear algebraic equations. Then, the resultant equations are linearized and solved by the iterative Newton-Raphson method.

The Young and Stephenson [30] numerical method is used as the numerical scheme in this work, which is an IMPESC (implicit pressure/explicit saturation composition) type numerical model. Young and Stephenson [30] defined W and F as $\rho_w s_w$ and $\rho_o s_o + \rho_g s_g$, respectively. According to Young and Stephenson [30], $W, F, P_{mo}, P_f, V, z_{mc}, y_{mc}, y_{fc}, c = 1, 2, \dots, n_c - 1$ are considered primary variables, and $s_{om}, s_{gm}, s_{wm}, x_{cm}, c = 1, 2, \dots, n_c - 1$ as secondary variables. Transmissibilities and diffusion terms in the flow equations are evaluated explicitly. The values of the primary and secondary variables in each time step are used as the initial guess for next time step.

In this work, matrix and fracture gridding are performed first. The gridding is carried out in such a way that no gridblock is found partially in the matrix and fracture.

As mentioned earlier, the matrix/fracture interaction is accounted for through matrix/fracture interface calculations. Since fluid flow through the matrix and fracture are not independent, their governing equations must be solved simultaneously. Therefore, the Jacobian matrix structure is slightly different to that of Young and Stephenson [30].

The gridblock types can be recognized, with respect to the number of unknowns:

- a) $2n_c + 6$ unknowns ($P_o, P_g, P_w, s_o, s_g, s_w, x_c, y_c, c = 1, 2, \dots, n_c$) for each matrix gridblock containing two hydrocarbon phases;
- b) $n_c + 4$ unknowns ($P_p, P_w, s_p, s_w, x_c, c = 1, 2, \dots, n_c, p = \text{oil or gas}$) for each matrix gridblock containing a single hydrocarbon phase;
- c) $n_c + 1$ unknowns ($P_f, y_c, c = 1, 2, \dots, n_c$) for each fracture gridblock containing the gas phase.

To ascertain convergence and stability, the equations and unknowns must be arranged within the Jacobian matrix, in proper order. The fracture flow equations were added to the Young and Stephenson [30] structure in different ways. It was observed that for the same initial guess, the precision and convergence of the solutions are nearly the same, suggesting that the fracture flow equations can be added to the Young and Stephenson [30] structure arbitrarily. In this method, the Newton/Raphson scheme is used to solve

for primary variables in each gridblock. The secondary variables are then evaluated using primary variables.

4. Model validation

The proposed model is validated by a gas injection experiment (experiment no. M5 of Morel et al. [14]). The experiment is designed in one dimension. It is performed to model the mass exchange between the flowing gas in the fracture and the fluid saturating the horizontal matrix block. Species recovery and gas saturation along the matrix block are measured. Local saturation through the porous media is measured by the gamma-ray attenuation method [14]. Jamili et al. [13] conducted this experiment, assuming that since nitrogen injection is a three component system, the absorption coefficient, densities and molar compositions of hydrocarbon phases change continually. This result in local saturation is impossible to calculate exactly without having local fluid properties. To override the problem, they calculated saturations for two extremes:

- Matrix fluid is a binary mixture of methane and pentane;
- Matrix fluid is a binary mixture of nitrogen and pentane.

Obviously, the real process is an intermediate state between (a) and (b).

Table 1 shows the input data used for simulation. The relative permeability and capillary pressure data are presented in Table 2.

Binary interaction coefficients are set to zero. The Peng-Robinson equation of state is used for the phase behavior description. The pressure at the matrix/fracture boundary is assumed to be constant during simulation and is set equal to the injection pressure of 1479 psia. The capillary pressure (Table 2) is

Table 2. Relative permeabilities and capillary pressure [14].

S_g	K_{ro}	K_{rg}	P_{cog} (psi)
0	1	0	2.22865
0.1	0.9	0.0002	2.3548
0.2	0.586	0.004	2.48095
0.3	0.316	0.02	2.6071
0.4	0.153	0.045	2.71223
0.5	0.063	0.1	2.83838
0.55	0.037	0.15	2.93306
0.6	0.02	0.21	3.0276
0.65	0.0096	0.3	3.15375
0.7	0.0039	0.5	3.2789
0.8	0	0.9	3.99475

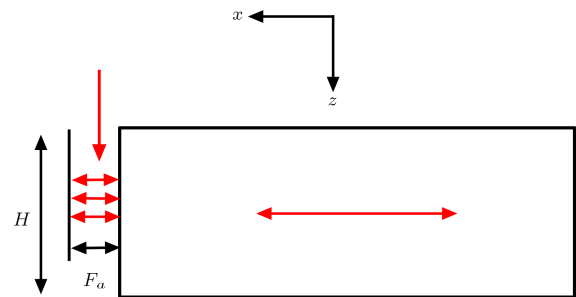


Figure 3. Experiment no. M5 of Morel et al. [14].

reported at reference interfacial tension (2.9 dynes/cm) and must be modified with respect to local interfacial tension:

$$P_c = P_c^{\text{ref}} \left(\frac{\sigma}{\sigma^{\text{ref}}} \right). \quad (29)$$

The system is a one-dimensional two phase fluid system saturated with three components. Figure 3 shows the schematic of the system. All sides of the core are

Table 1. Models input for simulation, experiment no. M5 of Morel et al. [14].

Rock material	Paris basin chalk
Core length (m)	0.357
Core cross section (m ²)	0.032×0.032
Core porosity	0.40
Core permeability (md)	2
Water saturation (%)	0
Oil residual saturation (%)	0.2
Pressure (psi)	1479
Temperature (°C)	38.5
Initial gas saturation (%)	0.25
Mole fraction C_1	0.524
Mole Fraction C_5	0.476
N_2 flow rate in the fracture (cm ³ /hr)	4 until 14.4 days, then 16

closed except the left, where a stream of nitrogen is passed at a constant rate. Red arrows depict the possible directions of flow. The fluid flows through the porous matrix in the x direction only. Injected gas flows inside the fracture in the z direction, and mass exchange between the matrix and fracture occurs in the x direction. Thus, flow inside the fracture is two-dimensional.

According to Figure 3, the boundary conditions for fluid flow through the fracture are as follows:

$$y_c = y_{c,inj} \text{ at } z = 0$$

$$y_c = y_{c,mf} \text{ at } x = 0$$

$$\frac{\partial y_c}{\partial x} = 0 \text{ at } x = F_a \text{ (} c = 1, 2, \dots, n_c \text{),} \quad (30)$$

where $y_{c,mf}$ is the mole fraction of component c inside the gas phase at the matrix/fracture boundary; $y_{c,inj}$ is the mole fraction of component c inside the gas phase at the injection point; and F_a is fracture width at the x direction.

It was assumed that the linear velocity in Eqs. (12)-(13) is constant (equal to injection linear velocity) and unaffected by mass transfer between the matrix and fracture.

In this work, the system is first simulated by the proposed model, the results of which are compared to the experimental data reported by Jamili [13]. Following model validation, the effects of matrix permeability, initial gas saturation and injection rate on recovery rate are investigated.

The core was simulated with 20 grids in the x direction. The fracture was simulated with 5 grids in the z -direction. All runs were performed on a 2.5 GHZ, Core 2 Due PC. Runtime was less than 10 minutes for all runs.

Figures 4 and 5 show the methane and pentane recovery, respectively. As can be seen, the simulation results well match the experimental data, indicating the very good performance of the proposed model in

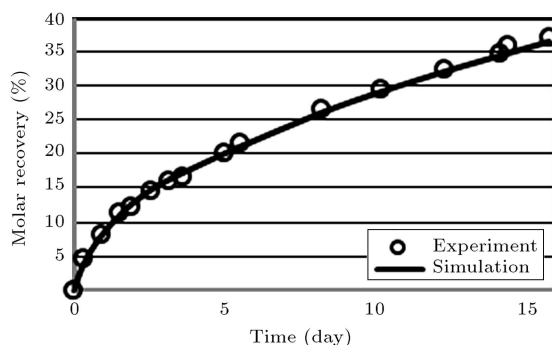


Figure 4. Variation of the methane molar recovery through simulation and experimental results; the experiment no. M5 of Morel et al. [14].

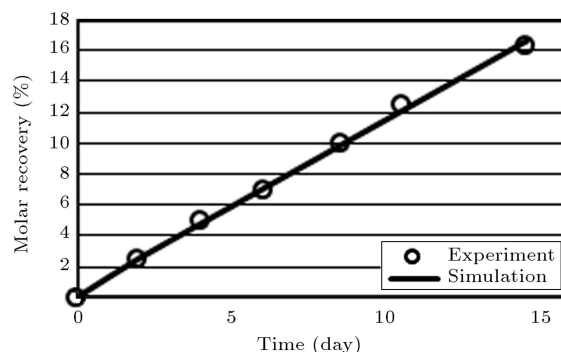


Figure 5. Variation of the pentane molar recovery through simulation and experimental results; the experiment no. M5 of Morel et al. [14].

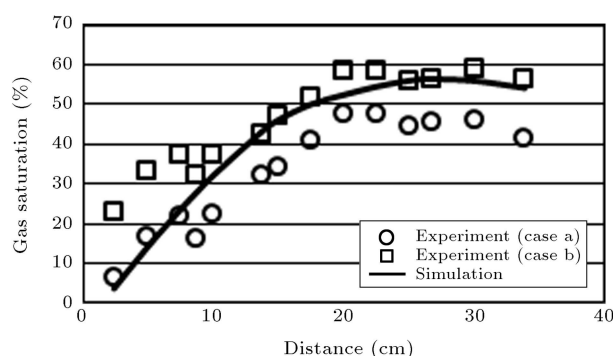


Figure 6. Variation of the gas saturation distribution along the core at $t = 8$ days through simulation and experimental results; experiment no. M5 of Morel et al. [14].

predicting the production behavior of methane and pentane.

Figure 6 shows gas saturation distribution along the core after 8 days. As expected, the saturation distribution is placed between two extremes. This also indicates that the proposed model predicts saturation distribution successfully.

Values of the diffusion coefficients in liquid and gas phases at a corresponding composition for the matrix gridblock adjacent to the fracture at 99.77 atm and 311.65 k are presented in Tables 3 and 4.

5. Numerical analysis and discussion

Following model validation, the effects of matrix permeability, initial gas saturation and injection rate on recovery rate are investigated.

5.1. The effect of matrix permeability on recovery rate

0.1, 0.5 and 2 md are selected to investigate the permeability effect on recovery rate.

This permeability range of magnitude selected for diffusion remains a main production mechanism.

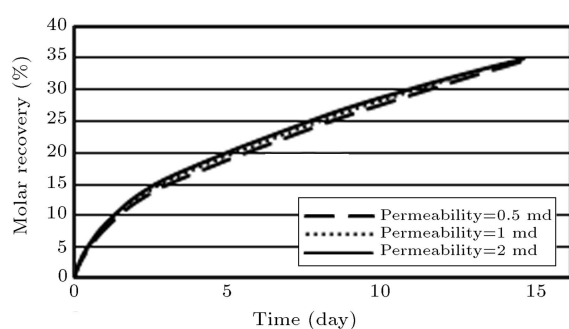
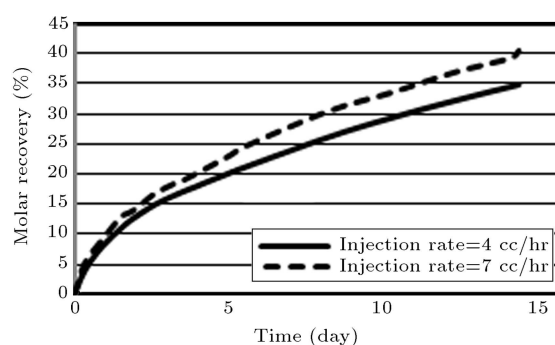
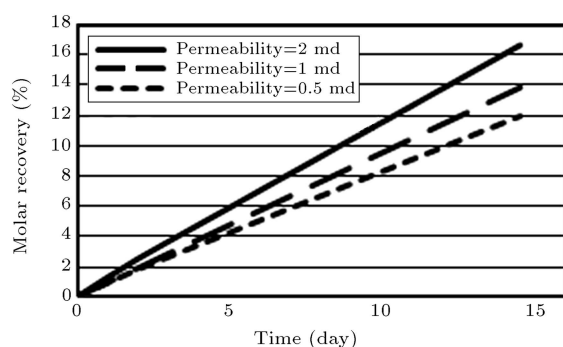
Figure 7 shows the permeability effect on the

Table 3. Molecular diffusion coefficients (cm²/sec) in gas phase, 99.77 atm and 311.65 k.

Components	C_1	C_5	N_2
C_1	2.154×10^{-3}	6.35×10^{-4}	-1.12×10^{-5}
C_5	-1.22×10^{-5}	1.10×10^{-3}	-1.35×10^{-5}
N_2	-1.39×10^{-3}	-8.60×10^{-4}	7.58×10^{-4}
Composition (mol%)	62.79	4.58	32.63

Table 4. Molecular diffusion coefficients (cm²/sec) in liquid phase, 99.77 atm and 311.65 k.

Components	C_1	C_5	N_2
C_1	1.23×10^{-3}	5.94×10^{-4}	-3.43×10^{-5}
C_5	1.92×10^{-5}	1.06×10^{-3}	-1.53×10^{-7}
N_2	5.12×10^{-5}	4.51×10^{-4}	1.06×10^{-3}
Composition (mol%)	28.26	64.32	7.42

**Figure 7.** Variation of the methane molar recovery rate through simulation results (permeability effect), experiment no. M5 of Morel et al. [14].**Figure 9.** Variation of the methane molar recovery rate through simulation results (gas injection rate effect); experiment no. M5 of Morel et al. [14].**Figure 8.** Variation of the pentane molar recovery rate through simulation results (permeability effect), experiment no. M5 of Morel et al. [14].

methane recovery rate. The methane recovery rate decreases negligibly with a decrease in matrix permeability. This negligible decrement is due to the convection role in methane production. Since methane is produced by diffusion mainly [13] and there is no significant relationship between permeability and diffusion, the change in methane recovery due to change in matrix permeability is not considerable.

Figure 8 shows the permeability effect on the pentane recovery rate. Since the main mechanism for

pentane production is convection [13] and the effect of permeability on convection is direct (Darcy's law) with an increase in permeability, the pentane recovery rate increases considerably.

5.2. The effect of injection rate on recovery rate

To investigate the effect of gas injection rate on recovery, simulations were performed with two injection rates of 4 and 7 cc/hr.

As seen in Figure 9, the recovery rate of methane increases as the injection rate increases. This is due to the effect of gas convection inside the fracture on the mass exchange between the matrix and fracture. The mass transfer coefficients between the matrix and fracture increase by increasing the injection rate.

Increasing the injection rate also increases the recovery rate of pentane (Figure 10), although not as effectively as is the case for methane.

5.3. The effect of initial gas saturation on recovery rate

The initial gas saturation of the matrix block depends on the initial pressure of the matrix. To investigate the effect of this parameter, the initial pressure is

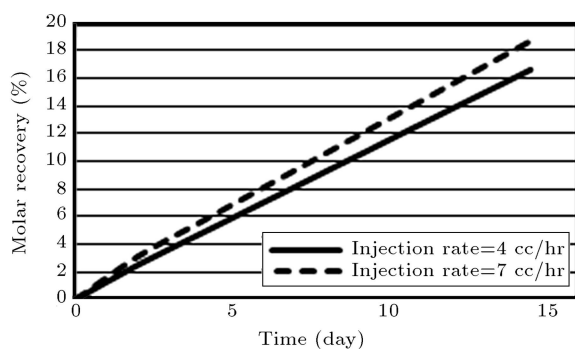


Figure 10. Variation of the pentane molar recovery rate through simulation results (gas injection rate effect); experiment no. M5 of Morel et al. [14].

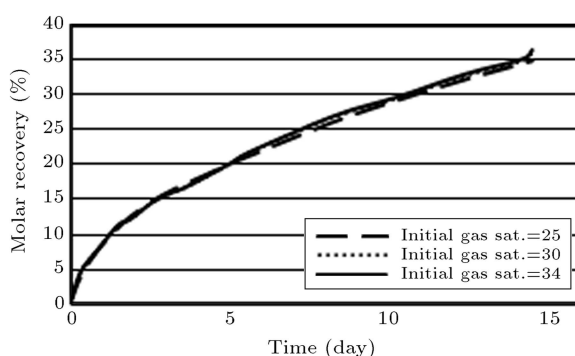


Figure 11. Variation of the methane molar recovery rate through simulation results (initial gas saturation effect); experiment no. M5 of Morel et al. [14].

changed and the initial gas saturation is calculated by performing flash calculation. The simulation is done for initial gas saturations of 0.25, 0.30 and 0.34.

Figure 11 shows the effect of change in initial gas saturation on methane recovery rate. Increasing initial gas saturation increases the methane recovery rate negligibly. Two kinds of diffusion process occur at the matrix/fracture boundary, namely, gas-gas diffusion and oil-gas diffusion. With an increase in gas saturation, the gas-gas diffusion increases and oil-gas diffusion decreases. As the role of the former is more significant, the recovery rate of methane increases.

Figure 12 shows the effect of change in the initial gas saturation on pentane recovery rate. It can be seen that an increase in initial gas saturation has a small effect on decreasing the pentane recovery rate. As mentioned before, the main mechanism of pentane recovery is oil convection, and this decreases with a decrease in oil saturation (relative permeability effect). Thus, there is a decrease in pentane with an increase in the initial gas saturation. This is in accordance with the results obtained by Morel et al. [14], which can be seen as another factor in validation of the model.

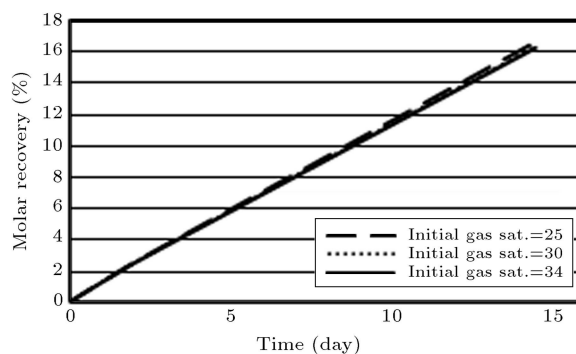


Figure 12. Variation of the pentane molar recovery rate through simulation results (initial gas saturation effect); experiment no. M5 of Morel et al. [14].

6. Conclusions

In order to obtain a better model for mass exchange between the fracture and matrix during gas injection, several modifications were proposed to the Jamili et al formulation [13].

The validity of the new formulation was verified using experimental data. Then, effects of matrix permeability, gas injection rate and initial gas saturation on recovery rate were investigated. The following results were obtained:

- The effect of permeability on the recovery rate of pentane is significant, since the main production mechanism of pentane is by convection.
- The effect of permeability on the recovery rate of methane is negligible, as the main production mechanism is diffusion for methane.
- The effect of initial gas saturation on the recovery rate of methane and pentane is negligible.
- The effect of gas injection rate on the recovery rate of methane and pentane is considerable. An increase in the injection rate leads to greater recovery rates for both methane and pentane. The effect is more noticeable in case of methane, due to diffusion being its main production mechanism.

Acknowledgment

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Nomenclature

D	Depth, m
D_{ij}	Diffusion coefficient, $\text{m}^2.\text{s}^{-1}$
f	Fugacity, Pa
F_a	Fracture width, m

k_r	Relative permeability
n_c	Number of components
P	Pressure, Pa
P_c	Capillary pressure, Pa
P_{cog}	Oil/gas capillary pressure, Pa
P_{cow}	Oil/water capillary pressure, Pa
s	Saturation
v	Linear velocity, m.s^{-1}
V	Bulk volume, m^3
VF	Vapor fraction
x	Liquid mole fraction
y	Gas mole fraction

Greek letters

γ	Specific weight, Pa.m^{-1}
μ	Viscosity, Pa.s
ρ	Molar density, mol.m^{-3}
σ	Interfacial tension, N.m^{-1}
ϕ	Porosity

Subscripts

b	Bulk
c	Component
C	Convection
D	Diffusion
f	Fracture
g	Gas
m	Matrix block
mf	Matrix/fracture boundary
o	Oil
w	Water

Superscripts

g	Gas
o	Oil
ref	Reference

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