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## A new super-efficiency model in the presence of both zero data and undesirable outputs

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**Abstract.** In 2013, Guo and Wu (“A complete ranking of DMUs with undesirable outputs using restrictions in DEA models”, Mathematical and Computer Modeling, Vol. 58, Nos. 5-6, pp. 1102-1109) proposed a model for ranking Decision Making Units (DMUs) in the presence of undesirable outputs. In this paper, we show that their model can be infeasible when some of the input data are zero. We also extend the super-efficiency model proposed by Lee and Zhu in 2012 (“Super-efficiency infeasibility and zero data in DEA”, European Journal of Operational Research, Vol. 216, No. 2, pp. 429-433) in the presence of undesirable output. Our proposed model is feasible when input and/or output data are nonnegative. A numerical example addresses the applicability of the proposed model.

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### 1. Introduction

The need for environmental and natural resource protection imposes many restrictions on companies to measure and reduce undesirable output, like smoke pollution or waste water. There is growing concern about undesirable outputs that have a direct effect on human beings and their environment, and, thus, their consideration is an important topic for research.

Data Envelopment Analysis (DEA) has become one of the most frequently applied tools for measuring the relative efficiency of peer Decision Making Units

(DMUs) that have multiple inputs and outputs. The DEA first introduced by Charnes et al. [1] is called the CCR model, which works under Constant Returns to Scale (CRS). The CCR model was later adjusted by Banker et al. [2] by adding convexity constraints to make it possible to work under Variable Returns to Scale (VRS), which is called the BCC model. The DEA has been used in many settings, for example, selecting an Enterprise Resource Planning (ERP) software [3], enhancing standard performance practices [4], technology selection [5], suppliers selection [6,7], and tube-bending processes [8]. Nevertheless, traditional DEA models fail to rank the DMUs with the same efficiency score, and, thus, researchers have been seeking ways to rank both efficient and inefficient DMUs. Alirezaee and Afsharian [9], for the first time, proposed a novel model for the complete ranking of DMUs through a new index called the “balance index”, which is based on the DEA model. Alirezaee and Afsharian [9] discussed that the  $p$ th DMU is efficient when its profit by shadow prices becomes zero. In this situation, the profit of other DMUs is equal to or less than zero, so, the current DMU dominates other DMUs in this profit

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competition. Wu et al. [10] discussed that the balance index is unstable. Therefore, they developed a modified model via introducing the “maximal balance index” in order to have a unique ranking.

The super-efficiency method, first proposed by Andersen and Petersen [11], is one of the ranking methods in DEA. The super-efficiency method omits the DMU under evaluation from the data set. Therefore, efficient DMUs obtain efficiency scores larger than or equal to 1, according to model orientation. Also, inefficient DMUs get the same efficiency scores as those calculated by the CCR model. One of the main and common problems which occur in the super-efficiency method is that it becomes infeasible under variable returns to scale. Lee et al. [12] developed a two stage process to rectify the VRS infeasibility problem. To make the model feasible when zero data are present in inputs, Lee and Zhu [13] extended the work of Lee et al. [12]. As discussed by Lee et al. [12], it should be noted that zero output data will not cause infeasibility in output oriented super-efficiency models and the output side of constraints will always be satisfied.

Guo and Wu [14] extended a DEA model to consider undesirable outputs, along with a complete ranking of DMUs. They used restrictions to recognize a unique ranking of DMUs via the new “maximal balance index” based on the optimal shadow price. (The shadow price is the change, per unit of the right hand side (RHS) of a constraint, in the objective value of the optimal solution obtained through relaxing the constraint. That is, it is the marginal utility of increasing the RHS of constraint, or the marginal cost of reducing the RHS of constraint.) However, their model is infeasible when some of the inputs are zero. There are various papers on undesirable outputs in DEA, including Sueyoshi and Goto [15], Jahanshahloo et al. [16], Liang et al. [17], Noorizadeh et al. [18], Azadi and Farzipoor Saen [19], and Farzipoor Saen [20]. Yang and Pollitt [21] incorporated both undesirable outputs and uncontrollable variables into DEA. Ebrahimnejad [22] created an equivalence relation between multiple objective linear programming and the output-oriented Banker, Charnes, and Cooper (BCC) model, in the presence of undesirable factors and fuzzy data. Barros [23] incorporated both desirable and undesirable outputs in the same model to evaluate the net efficiency.

In this paper, we extend the work of Lee and Zhu [13], so that it can deal with undesirable outputs. Also, it is shown that the developed model is always feasible when there are zero data. The main contribution of this study is to develop a novel super-efficiency DEA model for ranking DMUs in the presence of undesirable outputs.

The rest of this paper is organized as follows. In

Section 2, Alirezaee and Afsharian [9] and Guo and Wu's [14] models are discussed. Section 3 presents Lee and Zuo's [13] model. In Section 4, the proposed super-efficiency model is formulated. The numerical examples are presented in Section 5, and concluding remarks are given in Section 6.

## 2. The models proposed by Alirezaee and Afsharian [9] and Guo and Wu [14]

Charnes et al. [1] proposed Model (1).

$$\begin{aligned} \max \text{EFF}^o &= \sum_{r=1}^s u_r y_r^o, \\ \text{Subject to } &\sum_{r=1}^s u_r y_r^j - \sum_{i=1}^m v_i x_i^j \leq 0, \\ &j = 1, \dots, n \\ &\sum_{i=1}^m v_i x_i^o = 1, \\ &u_r, v_i \geq 0, \end{aligned} \quad (1)$$

where  $m$  is the number of inputs, and  $s$  is the number of desirable outputs. The  $x_i^j$  is the amount of  $i$ th input of the DMU $_j$  ( $j = 1, \dots, n$ ),  $y_r^j$  is the amount of  $r$ th desirable output of the DMU $_j$ . Let  $(x^o, y^o)$  denote the input and output vector of the DMU under evaluation. The  $i$ th input of the DMU $_o$  is denoted as  $x_i^o$ , and the  $r$ th output of the DMU $_o$  is denoted as  $y_r^o$ . The  $u_r$  is the weight related to the  $r$ th desirable output, and  $v_i$  is the weight related to the  $i$ th input.

Alirezaee and Afsharian [9] considered the variables in Model (1) as shadow prices. Also, they considered  $\sum_{r=1}^s u_r y_r^j$  and  $\sum_{i=1}^m v_i x_i^j$  as total revenue and total cost for  $j$ th DMU, respectively. They introduced the following constraint to address the profit for  $j$ th DMU:

$$\begin{aligned} \sum_{r=1}^s u_r y_r^j - \sum_{i=1}^m v_i x_i^j &\leq 0, \\ j &= 1, \dots, n. \end{aligned}$$

Using these expressions, for a DMU, a balance index is produced which is the sum of quantities of the profit restriction of other DMUs. Hence, the DMU $_o$  is efficient when its profit by shadow price equals zero. Thus, in this condition, the profit of every other DMU is equal to or less than zero.

With a simple numerical example, Wu et al. [10] showed that the proposed model by Alirezaee and Afsharian [9] cannot completely rank all the DMUs,

and claimed that the model is not stable. Here, instability means that there are still ties among efficient DMUs. Guo and Wu [14] developed Model (2) by introducing the new maximal balance index, which determines a unique ranking of DMUs in the presence of undesirable output.

$$\begin{aligned} \max \quad & \left( \sum_{i=1}^m v_i w_i + \sum_{t=1}^k \eta_t h_t - \sum_{r=1}^s u_r q_r \right), \\ \text{Subject to} \quad & \sum_{r=1}^s u_r y_r^j - \sum_{i=1}^m v_i x_i^j - \sum_{t=1}^k \eta_t b_t^j \leq 0, \quad \forall j, \\ & \sum_{i=1}^m v_i x_i^o + \sum_{t=1}^k \eta_t b_t^o = 1, \\ & \sum_{r=1}^s u_r y_r^o = \text{EFF}^o, \\ & u_r, v_i, \eta_t \geq 0, \quad \forall r \forall i \forall t, \end{aligned} \quad (2)$$

where  $w_i$  ( $i = 1, \dots, m$ ),  $q_r$  ( $r = 1, \dots, s$ ), and  $h_t$  ( $t = 1, \dots, k$ ) are the sum of  $i$ th input,  $r$ th desirable output, and the  $t$ th undesirable output, respectively.  $\eta_t$  ( $t = 1, \dots, k$ ) represents the weight of undesirable outputs. The superscripts ( $j$ ) and ( $o$ ) indicate the  $j$ th DMU ( $j = 1, \dots, n$ ) and the DMU under evaluation, respectively. Also,  $x_i^j$  ( $i = 1, \dots, m$ ),  $y_r^j$  ( $r = 1, \dots, s$ ), and  $b_t^j$  ( $t = 1, \dots, k$ ) indicate the amount of  $i$ th input,  $r$ th desirable output, and  $t$ th undesirable output for DMU $_j$ , respectively.

It is shown that when some of the inputs are zero, the obtained results from this model will be infeasible.

For example, consider the numerical example in Table 1. The obtained results from Model (2) are shown in Table 2. Since there is a zero number in input 2, Table 2 depicts that Model (2) is infeasible for DMU $_B$  and DMU $_C$ .

This paper focuses on finding a feasible solution for all the DMUs, some of which may have zero data in inputs and/or outputs.

### 3. The model proposed by Lee and Zhu [13]

The problem of super-efficiency in DEA is that it becomes infeasible when some of the input data are zero. The previous proposed models work only when data are positive. To overcome this shortcoming, Lee and Zhu [13] extended the models proposed by Lee et al. [12] and Chen and Liang [14], which can be feasible when some of the inputs have zero data. Lee and Zhu [13] proposed Model (3) as follows:

$$\min \quad \tau + M^* \left( \sum_{r=1}^s B_r + \sum_{i=1}^m t_i \right),$$

$$\text{Subject to} \quad \sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j x_i^j - t_i x_i^{\max} \leq (1 + \tau) x_i^o$$

$$(I = 1, 2, \dots, m),$$

$$\sum_{\substack{j=1 \\ j \neq 0}}^n \lambda_j y_r^j \geq (1 + \tau) y_r^o \quad (r = 1, 2, \dots, s),$$

**Table 1.** The data set taken from Lee and Zhu [13].

DMU	Input 1	Input 2	Desirable output	Undesirable output 1	Undesirable output 2
A	2	1	1	1	2
B	1	2	1	2	3
C	1	4	2	1	2
D	2	3	1	3	0
E	3	0	1	1	3

**Table 2.** The results of Model (2).

DMU	Maximal balance index	Results of Model (2)
A	$6u_1 - 9v_1 - 10v_2 - 8\eta_1 - 10\eta_2$	-0.6
B	$6u_1 - 9v_1 - 10v_2 - 8\eta_1 - 10\eta_2$	Infeasible
C	$6u_1 - 9v_1 - 10v_2 - 8\eta_1 - 10\eta_2$	Infeasible
D	$6u_1 - 9v_1 - 10v_2 - 8\eta_1 - 10\eta_2$	-13.75
E	$6u_1 - 9v_1 - 10v_2 - 8\eta_1 - 10\eta_2$	-1.07

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j = 1 \quad (j = 1, 2, \dots, n),$$

$$\lambda_j \geq 0, \quad j \neq o \quad \beta_r, t_i \geq 0, \quad \tau \text{ is unrestricted.} \quad (3)$$

The  $\lambda_j$ , often referred to as the “intensity” variable, is used to make an analytical linkage among all DMUs. The first and second constraints of Model (3) ensure that if there are zero data in inputs and/or outputs, the proposed model is feasible. In Model (3), the third constraint is referred to as the “normalization” constraint. Note that the  $M$  in Model (3) is a user-defined large positive number which Cook et al. [24] set equal to 105. Chen [25] shows that the super-efficiency can be regarded as input saving/output surplus achieved by an efficient DMU. Based upon Lee et al. [12], Cook et al. [24], and Lee and Zhu [13],

the input saving index is defined as  $\frac{\sum_{i \in I} \left( \frac{1+t_i^*}{1} \right)}{|I|}$ , when  $\{I|t_i^*\}$  is not empty, and the output surplus index is defined as  $\frac{\sum_{r \in R} \left( \frac{1}{1-\beta_r^*} \right)}{|R|}$ , when  $\{O|\beta_r^*\}$  is not empty.

Input savings index:

$$\hat{l} = \begin{cases} 0, & \text{if } I = \phi \\ \frac{\sum_{i \in I} \left( \frac{1+t_i^*}{1} \right)}{|I|}, & \text{if } I \neq \phi \end{cases}$$

Output surplus index:

$$o = \begin{cases} 0, & \text{if } R = \phi \\ \frac{\sum_{r \in R} \left( \frac{1}{1-\beta_r^*} \right)}{|R|}, & \text{if } R \neq \phi \end{cases}$$

where  $R = \{o|\beta_r^* > 0\}$  and  $I = \{i|t_i^* > 0\}$  are based upon Model (3), and  $|R|$  and  $|I|$  are the cardinality of the set of  $R$  and  $I$ , respectively. Thus, the super-efficiency score can be expressed as follows:

$$\tilde{\theta} = 1 + \tau^* + o + \hat{l}.$$

#### 4. Proposed model

An undesirable output has an undesirable result in the production process. A common way for measuring the efficiency of DMUs with undesirable outputs is to treat them as inputs, because, basically, inputs and undesirable outputs incur costs for a DMU. Hence, DMUs usually want to decrease input and undesirable output as much as possible.

In this section, we treat the undesirable output as input and extend a new super-efficiency model in the existence of undesirable output when some of the inputs and/or output data are zero.

Let us define  $x_{ij}$  ( $i = 1, 2, \dots, m$ ) as the amount of input,  $i$ , used by DMU <sub>$j$</sub> ,  $y_{rj}$  ( $r = 1, 2, \dots, s$ ) as the

amount of desirable output,  $r$ , produced by DMU <sub>$j$</sub> , and  $b_{tj}$  ( $t = 1, 2, \dots, k$ ) as the amount of undesirable output,  $t$ , produced by DMU <sub>$j$</sub> . The sets of inputs and outputs defined in this way are assumed to characterize the Production Possibility Set (PPS) as follows:

$$T = \left\{ (x, y, b) : \sum_{j=1}^n \lambda_j x_{ij} \leq x_i, \quad i = 1, \dots, m; \right. \\ \left. \sum_{j=1}^n \lambda_j y_{rj} \geq y_r, \quad r = 1, \dots, s; \sum_{j=1}^n \lambda_j b_{tj} \leq b_t, \right. \\ \left. t = 1, \dots, k; \lambda_j \geq 0, \quad j = 1, \dots, n \right\}.$$

We denote  $x_i^{\max} = \max_{o=1}^n \{x_i^o\}$  and  $b_t^{\max} = \max_{o=1}^n \{b_t^o\}$ . Thus, we can write the relative efficiency of DMU <sub>$k$</sub>  using Model (4).

$$\min \tau + M^* \left( \sum_{r=1}^s B_r + \sum_{i=1}^m t_i + \sum_{t=1}^k \eta_t \right),$$

$$\text{Subject to } \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_i^j - t_i x_i^{\max} \leq (1 + \tau) x_i^o,$$

$$(i = 1, 2, \dots, m),$$

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j b_t^j - \eta_t b_t^{\max} \leq (1 + \tau) b_t^o, \quad (t = 1, 2, \dots, k),$$

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j y_r^j \geq (1 - B_r) y_r^o, \quad (r = 1, 2, \dots, s),$$

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j = 1, \quad (j = 1, 2, \dots, n),$$

$$\lambda_j \geq 0, \quad j \neq o, \beta_r, t_i, \eta_t \geq 0, \quad \tau \text{ is unrestricted.} \quad (4)$$

In Model (4), each DMU which obtains a higher efficiency score is selected as the best DMU. The first, second, and third constraints of Model (4) ensure that if there are zero data in inputs, undesirable outputs and/or desirable outputs, the proposed model is feasible. Model (4) is based upon Variable Return to Scale (VRS) and has an additional convexity constraint defined by limiting the summation of multiplier weights equal to 1. Model (4) deals with undesirable outputs. The rationale behind the negative sign,  $\eta_t b_t^{\max}$ , in the left hand side of the second constraint is as follows.

The first is unit invariance. Assume that the  $t$ th undesirable output is scaled by factor  $b$ . The second constraint of Model (4) can be re-written as follows:

$$\sum_{\substack{j=1 \\ j \neq o}}^n b\lambda_j b_t^j - b\eta_t b_t^{\max} \leq b(1 + \tau)b_t^o \quad t = 1, 2, \dots, k, \quad (5)$$

which is equal to the original constraint. This means that the optimal feasible solution of Model (4) is unit invariant.

The second reason is that it will not be zero when  $b_t^o$  is zero. If we substitute  $\eta_t b_t^{\max}$  with  $\eta_t b_t^o$ , then  $\eta_t b_t^o$  will be zero when  $b_t^o$  is zero.

Similarly, based upon Lee et al. [12] and Cook et al. [24], the undesirable output saving index is defined as  $\frac{\sum_{t \in T} \left( \frac{1 + \eta_t^*}{1} \right)}{|T|}$ , when  $\{t | \eta_t^*\}$  is not empty.

$$\hat{t} = \begin{cases} 0, & \text{if } T = \phi \\ \frac{\sum_{t \in T} \left( \frac{1 + \eta_t^*}{1} \right)}{|T|}, & \text{if } T \neq \phi \end{cases}$$

where  $T = \{o | \eta_r^* > 0\}$  is based upon Model (4), and  $|T|$  is the cardinality of the set of  $T$ .

Then, we can write the super-efficiency score as follows:

$$\tilde{\theta} = 1 + \tau^* + o + \hat{l} + \hat{t}.$$

The efficiency measure,  $\tilde{\theta}$ , is separated into four parts: the radial efficiency, the input saving index, the desirable output surplus index, and the undesirable output saving index, which are denoted by  $1 + \tau^*$ ,  $\hat{l}$ ,  $o$ , and  $\hat{t}$ , respectively. Cook et al. [24] denote  $B^*$  as the optimal solution and argue that if, and only if,  $B^* > 0$ , the conventional model of super-efficiency is infeasible. Based upon Cook et al. [24], if  $\tau^* > 0$  or  $(1 + \tau^* > 1)$  for Model (3) or (4), then the DMU under evaluation should increase its inputs and decrease its outputs to reach the efficient frontier formed by the remaining DMUs. If  $\tau^* < 0$  or  $(1 + \tau^* < 1)$ , then the DMU under evaluation should decrease its inputs and outputs to reach the efficient frontier.

## 5. Numerical examples

### 5.1. Example 1

Consider again the data set in Table 1. These data are partially taken from Lee and Zhu [13]. Table 3 reports the results from our model, Lee and Zhu [13], and Guo and Wu [14].

The results are different. For instance, consider the DMUD depicted in Table 3. Note that the rank of DMUD in our model is 2, while its rank in Lee and Zhu's [13] model is 5. As seen, the model proposed by Guo and Wu [14] cannot produce feasible results. As a result, the rankings are significantly changed when the undesirable outputs are taken into account. Therefore, Table 3 shows that our proposed model not only considers undesirable outputs, but also deals with zero data effectively.

### 5.2. Example 2

To further display the superiority of our model against previous work, we review 27 Japanese electric power company data sets taken from Sueyoshi and Goto [15] (see Table 4; the data related to 27 Japanese power plant (DMUs)). Based on Sueyoshi and Goto [15], the data sets consist of two inputs (i.e. the total amounts of assets and the total amount of labor costs), two desirable outputs (i.e. the total amount of sales and the number of customers) and an undesirable output (i.e. the total amount of CO<sub>2</sub> emission). The sample period is from 2006 to 2008. These companies produce more than 25% CO<sub>2</sub> emission of all Japan. Table 5 presents the super-efficiency scores. This table indicates that the super-efficiency scores obtained from Models (3) and (4) are different.

Table 5 displays the methodological differences between two approaches. Note that the model proposed by Lee and Zhu [13] does not consider undesirable outputs and, therefore, the ranking results are different from our proposed model (Model (4)), which considers undesirable outputs. For example, consider the different ranking results of the Chugoko electric power company in 2006. By running Model (3), the rank of the Chugoko electric power company is 5, while, by running Model (4), its rank is 7, the difference

Table 3. Ranking changes.

DMU	Super-efficiency		Super-efficiency		Super-efficiency score	
	score obtained from our model	Rank	score obtained from Lee and Zhu [13] model	Rank	obtained from Guo and Wu [14] model	Rank
A	1.22	4	1	4	-0.6	-
B	1.4	5	1.4	3	Infeasible	-
C	3.5	1	3	1	Infeasible	-
D	2.54	2	0.6	5	-13.75	-
E	2.25	3	1.9166	2	-1.07	-

**Table 4.** The data set of Japanese electric power companies.

Variables	Electric power company	Input 1	Input 2	Desirable output 1	Desirable output 2	Undesirable output 2
		Total assets (100 billion JPY)	Labor cost (100 billion JPY)	Total sales (100 GWh)	Number of customers (100000)	CO <sub>2</sub> emission (100000 ton)
2006	Hokkaido	14.3	0.8	315.1	39.0	156.6
	Tohoku	37.1	1.3	809.5	76.7	408.3
	Tokyo	129.2	4.6	2876.2	280.7	1073.0
	Chubu	52.9	1.4	1326.9	103.9	519.2
	Hokuriku	14.8	0.5	282.0	20.8	113.8
	Kansai	61.9	2.1	1472.6	132.8	520.0
	Chugoku	24.8	1.2	612.6	52.1	394.7
	Shikoku	13.8	0.5	281.6	28.5	108.3
2007	Kyushu	37.9	1.4	844.0	83.5	306.0
	Hokkaido	14.6	0.6	324.4	39.2	150.8
	Tohoku	36.8	1.4	840.7	76.7	357.0
	Tokyo	130.6	3.4	2974.0	283.2	976.0
	Chubu	52.4	1.5	1374.8	104.4	637.8
	Hokuriku	14.8	0.4	293.0	20.8	128.8
	Kansai	61.4	2.1	1504.2	133.4	498.1
	Chugoku	25.3	1.1	635.8	51.9	425.4
2008	Shikoku	13.6	0.6	292.7	28.3	103.6
	Kyushu	37.8	1.4	880.8	83.8	316.0
	Hokkaido	15.6	0.5	318.4	39.4	167.8
	Tohoku	36.8	1.5	811.0	76.8	397.9
	Tokyo	129.9	4.8	2889.6	284.9	1265.0
	Chubu	51.1	1.9	1297.3	104.6	646.7
	Hokuriku	14.2	0.5	281.5	20.8	185.2
	Kansai	62.4	2.4	1458.7	134.0	549.9
	Chugoku	26.1	1.1	612.2	51.9	430.7
	Shikoku	13.5	0.7	287.0	28.3	114.6
	Kyushu	38.3	1.4	858.8	84.0	341.0

being significant. Table 5 reveals that by neglecting the existence of undesirable outputs, the ranking results will be wrong.

## 6. Concluding remarks

It is well-known that the traditional radial VRS super-efficiency model becomes infeasible when some of the inputs are zero. In the real world, the DMUs may produce two sorts of output, including desirable output and undesirable output. The numerical example shows that the proposed model by Guo and Wu [14] can be infeasible in some cases. To overcome this shortcoming, we extended the work of Lee and Zhu [13]. It was discussed that the proposed model can rank all DMUs producing undesirable outputs. Also, it is shown that the results are feasible when zero data exist in input or

output data. The main contributions of this paper are as follows:

- For the first time, a new super-efficiency model for ranking DMUs in the presence of undesirable outputs is developed;
- The proposed model is feasible when input or output data are zero;
- The proposed model deals with both undesirable outputs and zero data, simultaneously.

Further research can be done based on the results of this paper, one of which is as follows. In this paper, the data set are assumed to be deterministic. However, in the real world, some of the data might be stochastic. Developing a model considering undesirable

Table 5. Ranking results.

	Electric power company	Our model	Rank	Lee and Zhu [13] model	Rank
2006	Hokkaido	1.2867	2	1.2867	2
	Tohoku	0.9474	9	0.9475	9
	Tokyo	1.1500	4	0.1500	4
	Chubu	1.3606	1	1.3604	1
	Hokuriku	1.0017	8	1.0006	7
	Kansai	1.0776	5	1.0145	6
	Chugoku	1.0342	7	1.0342	5
	Shikoku	1.2397	3	1.2060	3
2007	Kyushu	1.0414	6	0.9911	8
	Hokkaido	1.2580	1	1.2580	1
	Tohoku	0.9345	9	0.9322	9
	Tokyo	1.0220	6	1.0750	3
	Chubu	1.1418	2	1.1418	2
	Hokuriku	1.0500	5	1.0500	7
	Kansai	1.0561	4	0.9820	6
	Chugoku	1.0200	7	1.0200	6
2008	Shikoku	1.0518	3	1.0735	4
	Kyushu	0.9839	8	0.9716	8
	Hokkaido	1.5297	1	1.5297	1
	Tohoku	0.9454	9	0.9448	8
	Tokyo	1.1070	5	1.0753	4
	Chubu	1.1215	3	1.1215	2
	Hokuriku	1.0719	6	1.0719	5
	Kansai	1.1164	4	0.9470	7
	Chugoku	0.9878	7	0.9819	6
	Shikoku	1.3725	2	1.0793	3
	Kyushu	0.9836	8	0.9197	9

and stochastic outputs, and zero data, is an interesting topic to pursue for researchers.

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