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# Supply chain channel coordination under sales rebate return policy contract using simulation optimization

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## KEYWORDS

Supply chain coordination;  
Return policy with sales rebate and penalty contract;  
Simulation;  
Price- and effort-dependent demand.  
SRP: Sales Rebate and Penalty;  
RSRP: Return policy with Sales Rebate and Penalty;  
LRSRP: Limited return policy with Sales Rebate and Penalty.

**Abstract.** This paper proposes a Stackelberg game-based approach for channel coordination for a supply chain consisting of one supplier and two competing retailers facing stochastic demand that is sensitive to both sales effort and retail price. In the proposed approach, the supplier, as the leader, defines the contract format and parameters, and the retailers determine the order quantity, retail price and sales effort. The literature primarily focuses on the design of the contract parameters to ensure channel coordination, whereas much less attention is given to the analysis of conditions supporting contract implementation. This study focuses on implementation of the Return policy with a Sales Rebate and Penalty (RSRP) contract as a coordination mechanism. The negotiation and trading procedure among supply chain members is modeled using a simulation optimization-based decision support tool. The possibility that retailers impose their own preferences that disturb channel coordination after signing the RSRP contract is analyzed, and a new Limited Return policy with a Sales Rebate and Penalty (LRSRP) contract, which helps the supplier guarantee channel coordination and control retailer decisions, is proposed.

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## 1. Introduction

The objective, in this paper, is to examine the use of sales rebates or penalties as contract incentives and their impact on channel relationships and decision-making among supply chain member firms. Supply chain channel coordination and decision-making are generally approached from two different relationship perspectives; decentralized or centralized, although hybrid shared partnerships exist. Decentralized supply chain relationships involve several self-interested

members who use distributed local decision-making optimization strategies in order to make decisions considering their own preferences, their own constraints and their own objectives, and who only care about their monetary payoffs (generally, profits). This focus on monetary payoffs results in the well-known phenomenon of double marginalization, where each member's profit is optimized but the overall integrated supply chain channel profit margin is suboptimal [1].

Centralized supply chains involve a single decision-maker who possesses information regarding the overall supply chain, and the supply chain members operate under a global decision-making optimization strategy. In general, better overall supply chain economic performance results, although the global optimization strategy causes economic penalties for

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some local members. This is due to increased levels of information sharing among all supply chain members, and centralized planning and control, which ensures effective channel coordination. This supports not only aligning the preferences, constraints and objectives of the individual supply chain members, but also supports overall system-wide performance. In centralized decision-making scenarios, supply chain contracts are coordination mechanisms that help to incentivize the independent members to engage and participate in centralized supply chain channel coordination.

In this research, the focus is centralized channel coordination exploring the use of channel sales rebate or penalties as sales incentives, specifically in a two-tier supply chain, where the supply chain configuration consists of a single supplier and two competing retailers.

Coordination contract models have been studied and developed in recent years [1-8]. It is now becoming popular to model supply chains as agent-based simulation systems, or to use discrete event simulation to learn more about their behavior or investigate the implications of alternative configurations [9]. Yu et al. [10] studied the impact of supply disruption on the supply chain system using simulation, where the researchers used two different distribution functions of random variables to characterize the disruption and its impact.

Many previous studies have investigated contracts for channel coordination within a supply chain, whereas much less emphasis has been placed on implementation of these contracts and the necessary supporting conditions for their maintenance. The focus in this paper is the implementation of a Return policy with a Sales and Rebate Penalty (RSRP) contract in a supply chain consisting of one supplier and two competing retailers facing stochastic demand, which is sensitive to both sales effort and retail price. The negotiation procedure among supply chain members is model optimized via simulation. The possibility of retailers imposing their own preferences after instituting the RSRP contract, which may potentially upset channel coordination, is analyzed, and a new Limited Return policy with a SRP (LRSRP) contract, which helps the supplier to ensure channel coordination, is proposed. In fact, in a RSRP contract, it is possible for retailers to readjust their order quantities, retail prices and sales efforts within the parameters of the contract.

In this research, the presence of two competing retailers with definite opinions on order quantity, effort level and retail price, and the stochastic nature of demand, make it difficult to model and solve the channel coordination problem analytically, due to its inherent complexity. Simulation optimization is used to model the stochasticity of demand and addresses this supply chain channel coordination problem. A major

advantage in using simulation is in its ability to model the uncertainty, interaction and non-linearity of relationships between the different members. This allows relaxation of some simplifying assumptions made by previous researchers to increase the tractability of the problem, such as assuming independent deterministic demand, a monopoly market and no advertising, to name a few.

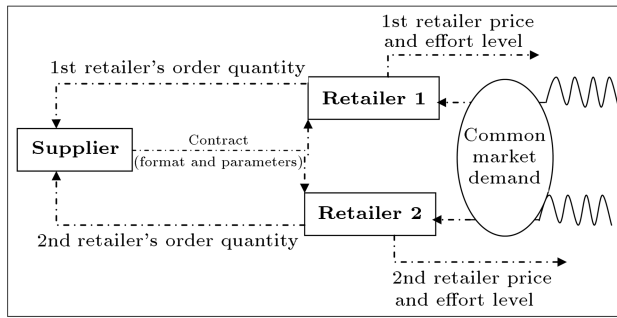
The main contributions of this research are the following. First, the channel coordination problem in a supply chain consisting of one supplier and two competing retailers facing stochastic demand that is sensitive to both sales effort and retail price, has not been studied in the literature to date. Second, by analyzing the implementation of the RSRP contract, a new flexible LRSRP contract that can achieve appropriate channel coordination in the supply chain is proposed. Finally, the proposed contract design methodology, consisting of simulation optimization, can aid in investigating the coordination problem.

The remainder of this paper is organized as follows. In Section 2, the problem, model assumptions and notations are described. The proposed analytical models for cases of decentralized and centralized supply chains are presented in Section 3. Coordination models for RSRP and LRSRP contracts are proposed in Section 4. The proposed simulation optimization-based methodology for contract design is presented in Section 5. Section 6 illustrates the proposed methodology via a numerical example, and Section 7 concludes the work, including discussion of future directions of this research effort.

## 2. Problem description

This research investigates a channel coordination problem in a two-tier supply chain composed of a supplier and two competing retailers. The retailers face a random demand that is sensitive to not only sales effort, but also retail price. The supplier, with knowledge of the demand, defines the format of the contract and the requisite parameters to achieve the overall best performance of the supply chain that achieves a win-win situation for all participants in the contract. The information and logic flow of the sequence of events, shown pictorially in Figure 1, is as follows:

- I. The supplier offers a contract to two retailers for an upcoming selling season.
- II. The two competing retailers, upon reviewing and agreeing to the terms of the supplier's contract, determine their individual order quantities, retail prices and levels of effort.
- III. The supplier production and delivery of the product to the two retailers are scheduled to occur before the selling season.



**Figure 1.** Information and logic flow in the two-tier supply chain with a single supplier and two competing retailers.

- IV. The selling season commences and actual demand for the product at the retailers is then observed.
- V. Transfer payments are made between the supplier and the two retailers, according to the terms of the contract.

### 2.1. Modeling assumptions and notation

The proposed model has some important characteristics and assumptions that affect the modeling and analysis procedure. This section also summarizes important notations necessary for the proposed mathematical model. The assumptions are as follows:

- The decision model is a single-period model;
- The model is based on price- and effort-dependent stochastic (random) demand;
- The product being supplied to the retailer has a relatively short lifecycle;
- The supplier has enough capacity to satisfy the retailers;
- The retailers competing factors affect the demand;
- The two retailers are equally powerful to compete in one common market;
- Any end-of-period inventory is considered as salvage.

The notations that represent the variables and parameters used in the proposed model formulation are as follows:

#### Cost and price parameters

- $S_s$  : Salvage value per unit at the end of period at the supplier;
- $G_s$  : Goodwill cost at supplier per shortage unit;
- $C_s$  : Production cost per unit at the supplier;
- $S_r$  : Salvage value per unit at the end of period at the retailers;
- $G_r$  : Goodwill cost at retailers per shortage unit;

- $C_r$  : Marginal cost at the retailers per unit;
- $W$  : Wholesale price.

#### Decision variables for the retailers

- $P_{r1}$  : Retail price at Retailer 1;
- $P_{r2}$  : Retail price at Retailer 2;
- $e_{r1}$  : Effort level of Retailer 1 to promote sales;
- $e_{r2}$  : Effort level of Retailer 2 to promote sales;
- $Q_{r1}$  : Order quantity for Retailer 1;
- $Q_{r2}$  : Order quantity for Retailer 2.

#### Contract decision variables for the supplier

- $\beta$  : Return credit (buyback rate);
- $T$  : Sales target;
- $\tau$  : Rebate or penalty;
- $L$  : Limit of returned merchandise.

#### Performance measures

$\Pi_j^i$  : expected profit of supply chain member  $j$  in the case of contract/non-contract  $i$ , where  $j = \{r1: \text{Retailer 1}, r2: \text{Retailer 2}, s: \text{Supplier}, sc: \text{Supply Chain}\}$  and  $i = \{\text{RSRP: return policy with sales and rebate contract, LRSRP: limited return policy with sales and rebate contract, } C: \text{centralized supply chain, } DC: \text{decentralized supply chain}\}$ .

### 2.2. Modeling price- and effort-dependent demand

The price-dependent demand for two competing retailers, as defined in [11], is given by:

$$D_{ri}(P_{ri} + P_{rj}) = \alpha - \lambda P_{ri} + \gamma P_{rj},$$

$$i = 1, 2 \quad \text{and} \quad j = 3 - i, \quad (1)$$

where  $P_{ri}$  denotes the price at retailer  $i$ ,  $\alpha$  represents the original demand of the common market for retailer  $i$ ,  $\lambda$  represents retailer  $i$  store-level factor that affects consumer sensitivity to retail price, and  $\gamma$  denotes the competitive factor. Since demand is inherently an uncertain parameter, a random variable,  $\mu$ , with density function,  $f$ , and cumulative distribution function,  $F$ , is added to Eq. (1), or:

$$X_i = D_{ri}(P_{ri}, P_{rj}) + \mu,$$

$$i = 1, 2 \quad \text{and} \quad j = 3 - i. \quad (2)$$

Although Eq. (2) does not satisfy all competitive market conditions, it can and has been used as an acceptable price-dependent demand function [12]. However, it is important to note here that in real markets, pricing levels directly impact the competitive factor. For instance, a rise of \$1 in high selling prices of a

product decreases the primary demand for that product more than in the case of a rise of \$1 when the selling price is lower. It is also important to note here that different pricing levels directly impact the store-level factor. When the selling prices at the retailers surpass market thresholds, then the retailers will lose more of their customers than when the selling prices at the retailers do not exceed those market thresholds. Eq. (2) does not consider the price-dependent nature of the competing factor and the store-level factor.

In this paper, the attempt is to improve the price-dependent stochastic demand function, such that competitive factors and store-level factors are updated through different price threshold levels. It is assumed that the store-level factor,  $\lambda_{ik}$ , and the competitive factor,  $\gamma_{il}$ , change at three different price intervals, i.e.  $(\eta_1, \eta_2, \eta_3)$ . A new competitive price- and effort-dependent stochastic demand function is presented as follows:

$$\begin{aligned}
 D_{ri}(P_{ri}, P_{rj}, e_{ri}, e_{rj}) &= \alpha - \lambda_{ik}(P_{ri} - \eta_k - 1) \\
 &+ \gamma_{il}(P_{rj} - P_{ri}) + \Psi e_{ri} - \psi e_{rj}, \\
 i &= 1, 2, \quad j = 3 - i \quad \text{and} \quad k = 1, 2, 3, \\
 \text{IF } P_{ri} &\geq \eta_3 \quad \text{then} \quad K = 3, \\
 \text{IF } P_{rj} &\geq \eta_3 \quad \text{then} \quad l = 3, \\
 \text{IF } \eta_2 &\leq P_{ri} < \eta_3 \quad \text{then} \quad K = 2, \\
 \text{IF } \eta_2 &\leq P_{rj} < \eta_3 \quad \text{then} \quad l = 2, \\
 \text{IF } P_{ri} &< \eta_2 \quad \text{then} \quad K = 1, \\
 \text{IF } P_{ri} &< \eta_2 \quad \text{then} \quad l = 1, \\
 \lambda_{i1} &< \lambda_{i2} < \lambda_{i3}, \quad \gamma_{i1} < \gamma_{i2} < \gamma_{i3}, \\
 \eta_1 &< \eta_2 < \eta_3, \\
 X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \varepsilon) &= D_{ri}(P_{ri}, P_{rj}, e_{ri}, e_{rj}), \quad (3)
 \end{aligned}$$

where the notation is similar to that used in the traditional price-dependent stochastic demand function, and  $\eta_k$  is the market selling price threshold. The market is sensitive to the three-level incremental rise in price at the retailers. Therefore, the decision-makers can appraise at the price threshold levels,  $\eta_k(\eta_1, \eta_2, \eta_3)$ . The variable  $\lambda_{ik}$  is the store-level factor at retailer  $i$ , which is influenced by the product selling price at the retailer. The variable,  $\gamma_{il}$ , is a competitive factor, which is affected by the selling price of a comparable product at the rival retailer. The variable,  $e_{ri}$ , represents a single effort level at retailer  $i$  that

shows the retailer activities that promote sales, and  $\Psi$  and  $\psi$  represent the effect of the retailer's sales effort level, and the rival retailer's sales effort level impact on primary demand at the retailer, respectively. The function,  $X_i$ , considers the randomness in market demand. Here,  $\mu$  is a random variable that represents the stochastic feature of the market demand, and  $f$  and  $F$  represent the probability density function and cumulative distribution function of  $\mu$ , respectively.

### 3. Supply chain models for channel coordination

#### 3.1. Decentralized supply chains

In the case of decentralized supply chains, the retailers determine their local order quantities, retail prices and levels of sales effort regarding common market demand characteristics and costs. The supplier has no influence over the decisions of the retailers, and the retailers make operational and logistical (i.e., inventory and scheduling) decisions based on their own local preferences, constraints and objectives (i.e., profits). So, now, formulation of the decentralized supply chain channel coordination problem begins with the following notations. Let:

$S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})$  represents the expected sales of retailer  $i$  or  $j$ .

$I_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})$  represents the expected surplus inventory of retailer  $i$  or  $j$ , and

$GW_{ri}$  represents the value equivalent to the loss of goodwill in business at retailer  $i$  due to an inventory shortage.

The specific functions above are expressed in Eqs. (4), (5) and (6), respectively:

$$\begin{aligned}
 S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) &= \begin{cases} Q_{ri} & \text{IF } Q_{ri} \leq X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) \\ X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) & \text{IF } Q_{ri} > X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) \end{cases} \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 L_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) &= \begin{cases} Q_{ri} - S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) & \text{IF } Q_{ri} > X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) \\ 0 & \text{Otherwise} \end{cases} \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 GW_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) &= \begin{cases} X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) - Q_{ri} & \text{IF } Q_{ri} < X_i(P_{ri}, P_{rj}, e_{ri}, e_{rj}, \mu) \\ 0 & \text{Otherwise} \end{cases} \quad (6)
 \end{aligned}$$

The retailer profit function is given in Eq. (7). Here,  $g(e_{ri})$  represents each retailer,  $i$ , cost of achieving effort level,  $e_{ri}$ . The retailer profit per unit is the revenue from selling a unit of a new product and a salvage unit to customers, minus the unit cost of lost goodwill, the cost of promoting sales and the cost of ordering units of the product from the supplier:

$$\begin{aligned}\Pi_{ri}^{DC} = & P_{ri}(S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})) \\ & - C_r(Q_{ri}) - G_r(GW_{ri}) - W(Q_{ri}) - g(e_{ri}) \\ & + S_r(L_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})) \\ i = 1, 2 \quad \text{and} \quad j = 3 - i.\end{aligned}\quad (7)$$

Thus, the optimal policy for each retailer is obtained by maximizing the expected profit function, which is shown in Eq. (8):

Maximize:

$$\begin{aligned}E[\Pi_{ri}^{DC}(P_{ri}, e_{ri}, Q_{ri})] \quad i = 1, 2 \quad \text{and} \quad j = 3i, \\ P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj},\end{aligned}$$

S.t:

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj} \geq 0. \quad (8)$$

### 3.2. Centralized supply chains

In centralized supply chains, the entire supply chain system can be considered a single entity, whose expected profit needs to be optimized. So, the objective is to define order quantities, sales efforts and retail prices in order to maximize the profit function of the overall supply chain. Hence, the decision variables are set, such that satisfy Eq. (10):

$$\begin{aligned}\Pi_{sc}^c = & P_{ri}S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) \\ & + S_rL_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) - G_rGW_{ri} \\ & - C_rQ_{ri} - g(e_{ri}) \\ & + P_{rj}S_{rj}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) \\ & - G_rGW_{rj} - C_rQ_{rj} - g(e_{rj}) \\ & + S_rL_{rj}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) \\ & - C_s(Q_{ri} + Q_{rj}) - G_sGW_s, \\ i = 1, 2 \quad \text{and} \quad j = 3i.\end{aligned}\quad (9)$$

Maximize:

$$E[\Pi_{sc}^c] \quad P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}$$

S.t:

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj} \geq 0. \quad (10)$$

It has been shown that in the case of centralized supply chains, the expected profit at each retailer is not maximized, as decisions are made in the best interest of all members of the supply chain; in effect, sharing the profit returns among all supply chain members. However, in the case of decentralized supply chains, retailers make marketing and logistics decisions only considering their own interests [13]. This is often the reason for retailers not entering into contracts where part of the agreement is to accept centralized decisions that consider all members in the entire supply chain.

### 4. Supply chain coordination

Supply chain contracts are coordination mechanisms defining the mutual sharing of the risks and rewards of coordination in order to satisfy supply chain members who take a part in channel coordination. A contract is viewed to effectively coordinate the supply chain, if the supply chain members' decisions lead not only to optimal local performance, but also to optimal system-wide performance. One approach to achieving channel coordination, which is a common method cited in the open literature, is to, first, set the local decision variables to the values for a centralized case, then, to investigate a contract scheme, where the risks and benefits are mutually shared across the supply chain, while simultaneously satisfying the individual members who enter into the agreement [14-16].

It has been shown that when considering price- and effort-dependent stochastic demand, classical contracts such as revenue sharing or buyback do not lead to channel coordination [1,14]. Here, two competing retailers add more complexity to the problem and coordination conditions. He et al. [1] suggest that a combined contract consisting of a Return policy with Sales or Rebate Penalty (RSRP) can coordinate a supply chain with a supplier and a retailer. In the proposed coordination model by He et al. [1], it is assumed that the supplier offers the proposed contract to the retailer, and the retailer in return agrees to set the order quantity, the effort to promote sales and the retail price equal to that in the case of a centralized decision. In this paper, it is shown that this is not, in fact, a realistic assumption. We propose an improved Return policy with Sales and Rebate Penalty (RSRP) contracting scheme by adding a limit,  $L$ , for returned products, which determines the number of unsold products that the supplier accepts from the retailer, and payment of  $\beta$  dollars for each of them. The proposed simulation optimization decision support tool, which assures the successful implementation of the LRSRP contract, is described in Section 5.

#### 4.1. Return policy with Sales and Rebate Penalty (RSRP) contract

Under an RSRP contract, the supplier establishes a product target,  $T$ , for the retailer. If the retail sales are beyond the target, the supplier will offer a rebate, i.e. a reward of  $\tau$  for each unit of product sold above  $T$ . Otherwise, the retailer has to pay the supplier a penalty, i.e. a payment of  $\tau$  for each unit of product unsold below  $T$ . In addition, the retailer returns the unsold units of product at the end of the selling season to the supplier and receives a payment of  $\beta$  for each product.

In this trading procedure, the supplier defines the contract parameters, consisting of  $\beta$ ,  $\tau$ ,  $T$ , and the retailers, with knowledge of the contract parameters, decide their order quantities, sales efforts and retail prices. Setting the values of the retailers' decision variables equal to those for a centralized case, the supplier pursues a contract setting on the condition that the supply chain members' profit is at least equal to that which would be realized in a decentralized case. It is a real assumption that the supplier searches for a contract that better realizes the supplier's profit. The retailers adjust their own decisions on order quantities, retail prices and sales efforts, seeking the settings that maximize their profit (see Eq. (12)):

$$\begin{aligned} \Pi_{ri}^{\text{RSRP}} = & P_{ri}(S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})) \\ & - C_r(Q_{rj}) - G_r(GW_{ri}) - W(Q_{ri}) \\ & - g(e_{ri}) + \beta(L_{ri}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj})) \\ & - \tau(S_{ri}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) - T) \end{aligned}$$

$$i = 1, 2 \quad \text{and} \quad j = 3i. \quad (11)$$

Maximize:

$$E[\Pi_{ri}^{\text{RSRP}}(P_{ri}, e_{ri}, Q_{ri})],$$

$$i = 1, 2 \quad \text{and} \quad j = 3 - i,$$

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}.$$

S.t:

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj} \geq 0. \quad (12)$$

#### 4.2. Limited Return policy with SRP (LRSRP) contract

As previously mentioned, the RSRP contract allows retailers to not follow the centralized decisions on order quantities, retail prices and sales efforts, since the supplier guarantees the unsold units of products remaining at the retailer after the sale season. Furthermore, the supplier rewards the retailers; the more units the

retailers sell, the more they receive awards. So, it may be most profitable for the retailers to increase their sales efforts or decrease the retail prices in order to sell as many units of their products as they can. In this research, we, in fact, suggest a limitation for the return guarantee made by the supplier. This limitation acts as a mechanism to restrict retailer influence to reduce supplier profit margins. This limit,  $L$ , determines the upper limit for the number of unsold units of product that are guaranteed by, and returned to, the supplier, thus limiting transfer payments,  $\beta$ , to the retailer for each returned unit. This limitation makes the retailers take into consideration the supplier and, ultimately, the system-wide supply chain channel profit in their decisions on order quantities, effort levels and prices, which is shown in Eq. (14):

$$\begin{aligned} \Pi_{ri}^{\text{LRSRP}} = & P_{ri}(S_{ri}(Q_{ri}, P_{ri}, P_{rj}, e_{ri}, e_{rj})) \\ & - C_r(Q_{rj}) - G_r(GW_{ri}) - W(Q_{ri}) - g(e_{ri}) \\ & + \beta(\max[L_{ri}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj}), L]) \\ & - \tau(S_{ri}(Q_{rj}, P_{ri}, P_{rj}, e_{ri}, e_{rj}) - T), \end{aligned}$$

$$i = 1, 2 \quad \text{and} \quad j = 3i. \quad (13)$$

Maximize:

$$E[\Pi_{ri}^{\text{LRSRP}}(P_{ri}, e_{ri}, Q_{ri})],$$

$$i = 1, 2 \quad \text{and} \quad j = 3 - i$$

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}$$

S.t:

$$P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj} \geq 0. \quad (14)$$

## 5. Proposed methodology

The complexity of the model, interactive effects of retailer decisions and random characteristics of demand make it difficult for the proposed problem to be solved analytically. Simulation modeling has been used quite successfully to study processes that are too complex or ill-defined to permit the application of analytical model formulation and/or evaluation. In this research, simulation optimization is embedded in a proposed decision support methodology. This assists the supplier in a centralized supply chain to tune the parameters of a contracting scheme in such a manner that satisfies the retailers participating in, and bound by, the contract. Also, the proposed decision support tool allows one to simulate the real Stackelberg differential game conditions that exist between the supplier and the retailers. In general, Stackelberg differential game

theoretic models have been used quite effectively to study supply chain competition and hierarchical or sequential decision-making in supply chains and marketing channel [17–22]. In a supplier Stackelberg game model, the supplier is the leader and the retailer is the follower in determining pricing strategies. In a retailer Stackelberg game model, the retailer is the leader and the supplier is the follower. This research considers the supplier the leader and designer of the contract, and the retailers are the followers, deciding on terms such as order quantities, sales efforts and retail prices.

The proposed contract designing methodology helps the supplier construct and then offer a contract to retailers that guarantees effective channel coordination, in which, supply chain members mutually share the risks and rewards of the coordination in order to maximize the local profit returns of the supply chain members, as well as system-wide profit. The steps of the proposed methodology are explained as follows:

- I. Given the market demand, the decentralized supply chain is simulated and the retailers' decisions on order quantities, retail prices and levels of sales effort are determined.
- II. Considering the decision variables determined in step I, the supplier-level, retailer-level and supply chain-level profits for the decentralized supply chain are calculated.
- III. Next, the centralized supply chain is simulated and the centralized decisions on order quantity, retail prices and sales efforts are determined.
- IV. Considering the decision variables determined in step III, the supplier-level, retailer-level and supply chain-level profits for the centralized supply chain are calculated.
- V. After setting the decision variables to the values assuming centralized control, decisions on the contract parameters,  $T$ ,  $\beta$  and  $\tau$ , are made, considering the preferences, constraints and objectives of all supply chain members.
- VI. After setting the contract parameters to the values determined in step V, the retailers' decisions on order quantities, retail prices and sales efforts are determined.
- VII. Considering the retailers new decisions at step VI, and contract parameters found in step V, the profits of all members in the chain are computed.
- VIII. The limit,  $L$ , for the unsold unit return policy is determined, where  $L$  is the maximum value or interval that does not let the retailers perturb the mutual channel coordination. In fact, steps VI and VII need to be iteratively repeated for different values of  $L$  in order to examine the goodness of the  $L$  that limits the influence of

the retailers' decisions on the supplier's expected profits.

### 5.1. Simulation model

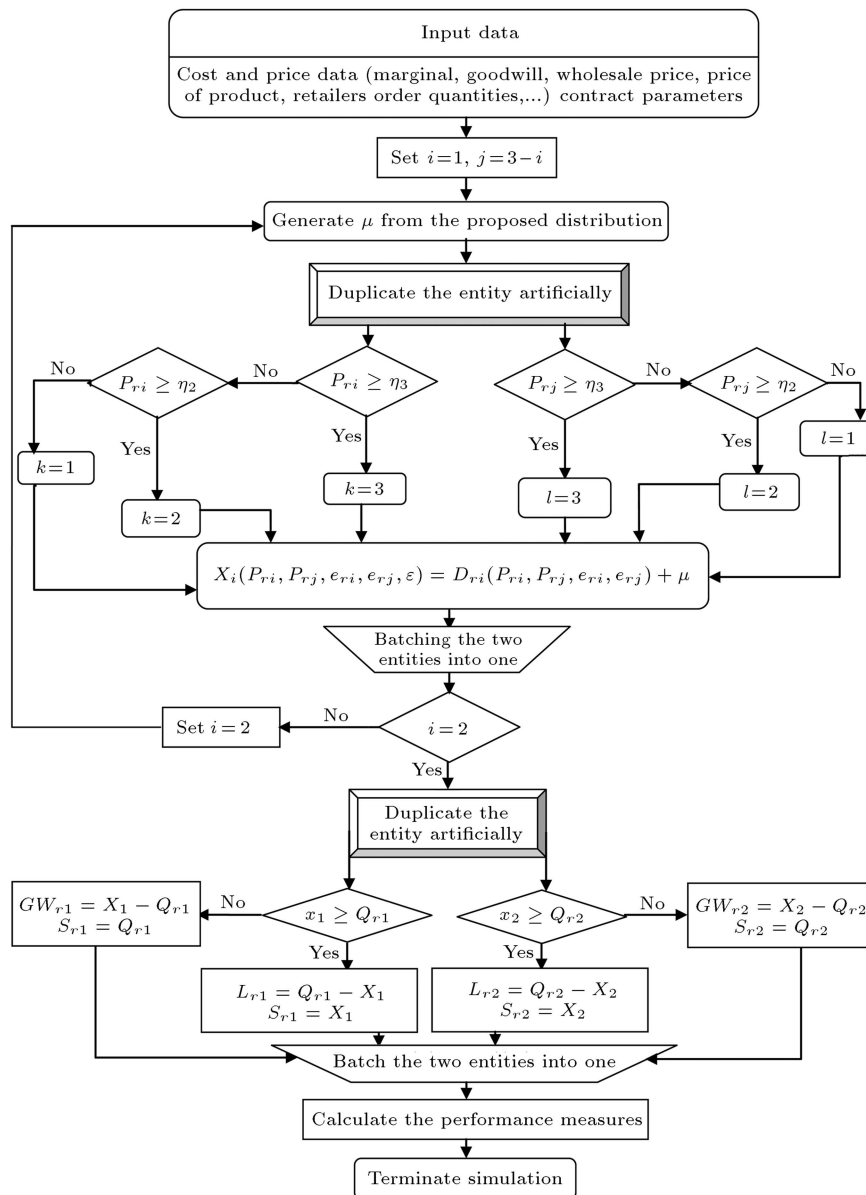
The mathematical model formulations presented in Sections 2 and 3 are too complex to solve analytically to identify an optimal or a near optimal set of contract parameters to coordinate the supply chain. The added complexity is due to the stochastic nature of the market demand ( $\mu$ ) and its interaction between the decision variables ( $P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}$ ). Simulation can effectively model the real-world systems [23]. In this research, a simulation model is used to model the Stackelberg game conditions in a supply chain consisting of one supplier and two retailers, and then to compute the objective functions of the complex system. Arena® simulation software by Rockwell Automation is used to build the simulation model of the supply chain with a stochastic price-and effort-dependent competitive market demand shown earlier in Figure 1 [24]. Figure 2 shows the general conceptual logic of the proposed simulation model.

### 5.2. Proposed simulation optimization decision support methodology

The optimization of simulation models often deals with a situation in which the interest is to find which of a large number of sets of model specifications lead to optimal output performance [25]. One of the popular methods to optimize simulated systems is using metaheuristics. In this mechanism, the simulation model is treated as a black-box, i.e. only the inputs and outputs of the simulation model are observed. At each iteration, the metaheuristic optimizer chooses a set of values for input variables and uses the output value generated by the simulation model to make a decision regarding selection of the next trial solution, with the goal of finding optimal values for decision variables.

In this study, OptQuest® optimization software by OptTek Systems, Inc. is used to find the optimal or near-optimal set of supply chain contract parameters. OptQuest, which is integrated with many commercial software packages, including Arena, combines the metaheuristics of a Tabu search, scatter search and neural network into a single, composite search algorithm, to provide maximum efficiency in identifying new scenarios.

In this problem, OptQuest searches for the best set of six decision variables ( $P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}$ ) in cases of centralized and decentralized supply chain controls in order to maximize overall supply chain and retailer profits, subsequently (Figure 3). Then, the simulation model is run using these decision variables, and the outputs are recorded in each supply chain case. After setting the decision variables to their values, in the case of a centralized supply chain, and then using



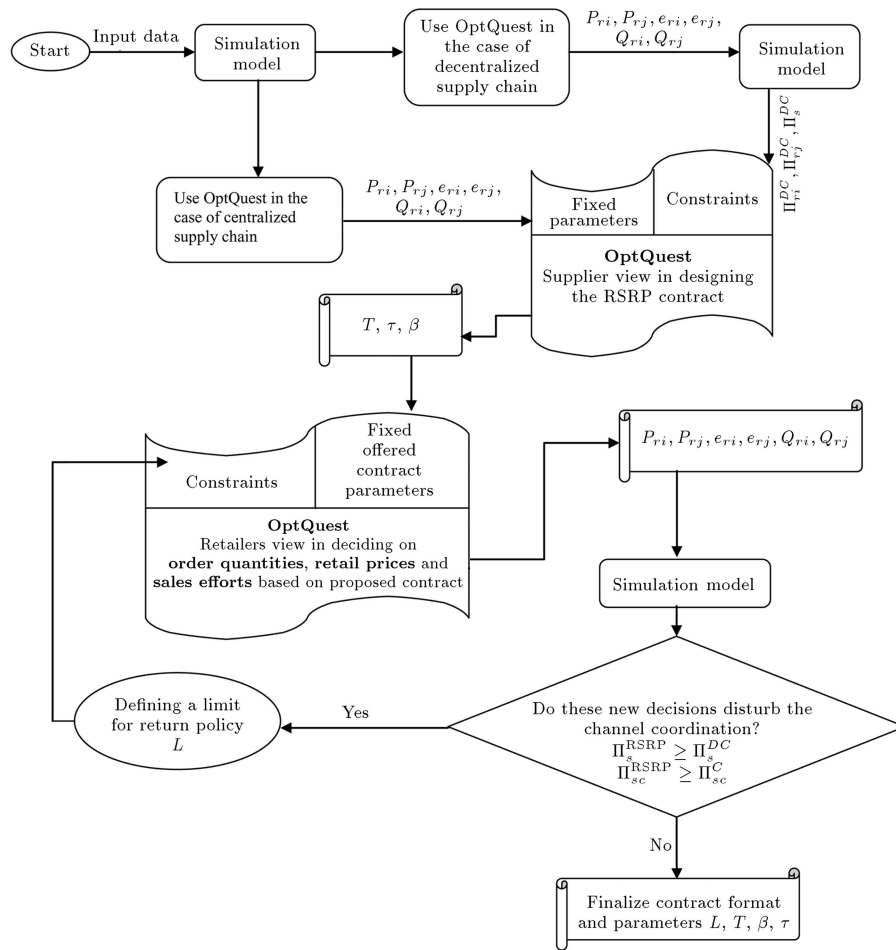
**Figure 2.** The conceptual illustration of the proposed simulation model.

the outputs from the case of a decentralized supply chain as constraints for the optimization model, OptQuest searches for the best set of contract parameter values (i.e.,  $T$ ,  $\beta$  and  $\tau$ ) that coordinate the supply chain. Then, considering these contract parameter values, the retailers' new decisions on order quantities, retail prices and sales efforts that potentially increase their profits under the assigned contract are optimized. Simulating the retailers' decisions on order quantities, retail prices and sales efforts based on the proposed RSRP contract, the supplier investigates whether the retailers' new decisions perturb the channel coordination. If yes, a limitation on return policies that restricts the retailers' influence on jeopardizing the coordination by local decisions is sought. In the numerical example presented in Section 6, the flexi-

bility and effectiveness of this proposed methodology is shown.

## 6. Numerical example

In this section, a numerical example is presented to illustrate how the developed contract designing algorithm, using simulation optimization, can help the supplier to offer a contract to the competing retailers. The simulation model input and control parameters and respective values are listed in Table 1. OptQuest is used to find the decision variable values ( $P_{ri}, P_{rj}, e_{ri}, e_{rj}, Q_{ri}, Q_{rj}$ ) for both decentralized and centralized supply chain profit functions, Eqs. (8) and (10), respectively. The number of independent simulation replications is set to 10,000 to assure the



**Figure 3.** Simulation optimization based decision support methodology.

**Table 1.** Parameters set used in the numerical example.

$S_s : 0$	$G_s : 10$	$C_s : 12$
$W : 20$	$S_r : 0$	$\alpha : \text{UNIF}(200, 300)$
$\lambda_{i1} : 10$	$\lambda_{i2} : 15$	$\lambda_{i3} : 30$
$\eta_1 : 23$	$\eta_2 : 26$	$\eta_3 : 30$
$\yen : 20$	$\psi : 5$	$g(e_{ri}) = \Omega e_{ri}^2 / 2$
$\mu : \text{NORM}(0, 30)$	$C_r : 2$	$G_r : 14$
$\gamma_{i1} : 5$	$\gamma_{i2} : 10$	$\gamma_{i3} : 15$
$\Omega : 100$		

reliability and the desired precision of the results. Some of the decision variables, such as  $Q_{ri}$  and  $Q_{rj}$ , are set as a discrete variable to represent the market condition. The resulting set of decision variable values is shown in Table 2.

Setting the decision variables to their values in the case of a centralized supply chain and using the simulation results from the case of a decentralized supply chain as constraints for the optimization model, Eq. (12), OptQuest is used to search for the best set of contract parameters ( $T, \tau, \beta$ ). Table 3 presents the set of RSRP contract parameters.

Signing the RSRP contract, retailers investigate the possibilities of improving their profits by readjusting the decisions on order quantity, sales effort and retail price. It is shown in Table 4 that RSRP contract conditions may allow retailers to improve their own expected profits, while reducing the supplier's expected profits, by refining the details of the contract.

As shown in Table 4, the new decisions made by the retailers considering contracts' format and parameters, endanger the supplier's profit. It may even cause the supplier's profit to decrease in comparison with the decentralized supply chain case. In order to limit the retailers' potential to reduce the supplier's planned profit, when adjusting the contract, we suggest that the supplier put a limit,  $L$ , for the return policy, which means that the supplier only guarantees a limited amount of unsold products that are returned with the payment of  $\beta$  for each unit of product. To identify the most appropriate  $L$ , retailers' reactions to the different values of  $L$  are evaluated in order to identify a reliable interval for  $L$  that can assure channel coordination. The results from different  $L$  values are summarized in Table 5.

As shown in Table 5, the return policy limit,

**Table 2.** Centralized and decentralized supply chain control.

Case	Retailer decision						Expected profit with 95% confidence interval ( $\alpha = 0.05$ )			
							$\Pi_{r1}^i$	$\Pi_{r2}^i$	$\Pi_s^i$	$\Pi_{sc}^i$
	$P_{r1}$	$P_{r2}$	$e_{r1}$	$e_{r2}$	$Q_{r1}$	$Q_{r2}$	Mean $\pm$	Mean $\pm$	Mean $\pm$	Mean $\pm$
							half-width	half-width	half-width	half-width
Decentralized										
supply chain ( $i : DC$ )	32.96	32.37	2.33	1.22	164	154	768.95 $\pm$ 12.90	773.96 $\pm$ 12.68	2544.00 $\pm$ 4.72	4086.91 $\pm$ 25.57
Centralized										
supply chain ( $i : SC$ )	29.58	30.49	2.64	2.30	239	209	660.97 $\pm$ 13.71	617.03 $\pm$ 15.36	3584.00 $\pm$ 3.68	4862.00 $\pm$ 28.99

**Table 3.** RSRP contract parameters defined by supplier.

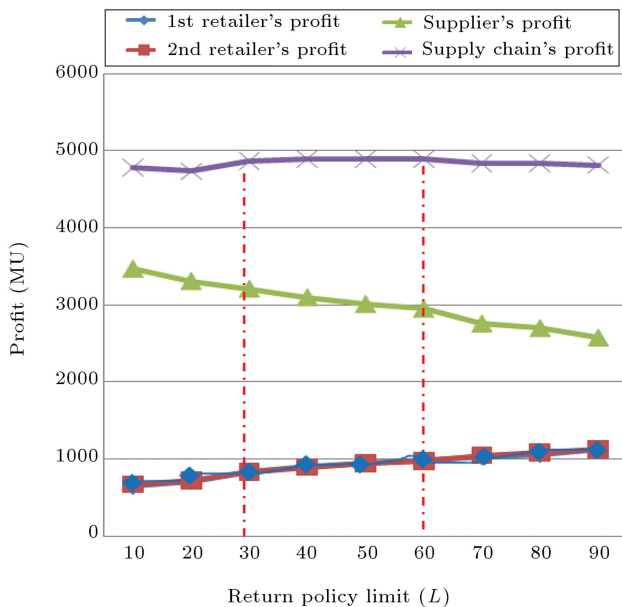
Contract	Contract parameters			Expected profit with 95% confidence interval ( $\alpha = 0.05$ )			
				$\Pi_{r1}^{RSRP}$	$\Pi_{r2}^{RSRP}$	$\Pi_s^{RSRP}$	$\Pi_{sc}^{RSRP}$
				Mean $\pm$	Mean $\pm$	Mean $\pm$	Mean $\pm$
				half-width	half-width	half-width	half-width
RSRP contract	207	8.01	17	1075.83 $\pm$ 10.03	811.34 $\pm$ 11.2	2856.23 $\pm$ 10.30	4743.39 $\pm$ 30.84

**Table 4.** Retailer decisions based on the offered RSRP contract.

Contract	Retailer decision						Expected profit with 95% confidence interval ( $\alpha = 0.05$ )				
							$\Pi_{r1}^{\text{RSRP}}$	$\Pi_{r2}^{\text{RSRP}}$	$\Pi_s^{\text{RSRP}}$	$\Pi_{sc}^{\text{RSRP}}$	
RSRP contract	$\beta = 17$	$P_{r1}$	$P_{r2}$	$e_{r1}$	$e_{r2}$	$Q_{r1}$	$Q_{r2}$	Mean $\pm$	Mean $\pm$	Mean $\pm$	Mean $\pm$
	$\tau = 8.01$							half-width	half-width	half-width	half-width
	$T = 207$	29.39	29.03	2.72	2.18	264	258	$1118.33 \pm 13.24$	$1124.66 \pm 12.62$	$2573.59 \pm 12.42$	$4816.58 \pm 38.06$

**Table 5.** Retailers decision based on LRSRP contract.

Coordination with contract	Retailer decision considering offered LRSRP contract						Expected profit with 95% confidence interval ( $\alpha = 0.05$ )			
LRSRP contract parameters							$\Pi_{r1}^{\text{LRSRP}}$	$\Pi_{r2}^{\text{LRSRP}}$	$\Pi_s^{\text{LRSRP}}$	$\Pi_{sc}^{\text{LRSRP}}$
$T = 207$ $\tau = 8.01$ $\beta = 17$	$P_{r1}$	$P_{r2}$	$e_{r1}$	$e_{r2}$	$Q_{r1}$	$Q_{r2}$	Mean $\pm$ half-width	Mean $\pm$ half-width	Mean $\pm$ half-width	Mean $\pm$ half-width
$L$										
10	29.34	29.14	3.22	2.66	236	239	646.14 $\pm$ 15.01	669.47 $\pm$ 17.72	3465.56 $\pm$ 6.20	4781.17 $\pm$ 26.53
20	29.34	29.14	3.21	2.67	236	229	710.74 $\pm$ 14.03	715.17 $\pm$ 14.06	3307.17 $\pm$ 4.29	4733.09 $\pm$ 24.88
30	29.41	29.13	3.02	2.43	243	236	828.60 $\pm$ 15.63	830.22 $\pm$ 15.32	3207.01 $\pm$ 4.68	4865.82 $\pm$ 28.97
40	29.41	29.13	2.89	2.27	245	238	902.01 $\pm$ 15.35	893.41 $\pm$ 15.23	3097.10 $\pm$ 5.22	4892.52 $\pm$ 30.61
50	29.41	29.13	2.89	2.27	245	238	945.31 $\pm$ 14.18	936.83 $\pm$ 14.05	3010.38 $\pm$ 6.14	4892.52 $\pm$ 30.61
60	29.41	29.13	2.89	2.26	245	238	977.60 $\pm$ 13.11	968.79 $\pm$ 13.04	2946.30 $\pm$ 7.26	4892.69 $\pm$ 30.64
70	29.38	29.04	2.72	2.26	260	260	1028.54 $\pm$ 16.11	1046.42 $\pm$ 16.17	2757.24 $\pm$ 8.33	4832.20 $\pm$ 37.56
80	29.38	29.04	2.72	2.26	260	260	1058.32 $\pm$ 15.10	1077.62 $\pm$ 15.14	2696.27 $\pm$ 9.44	4832.20 $\pm$ 0.02
90	29.39	29.03	2.72	2.18	264	258	1118.33 $\pm$ 13.24	1124.66 $\pm$ 12.62	2573.59 $\pm$ 12.42	4816.58 $\pm$ 38.06



**Figure 4.** Effects of retailers' decisions based on offered LRSRP contract on channel coordination.

$L$ , plays an important role in the proposed contract. Keeping  $L$  between 30 and 60 assures that the retailers are inclined to participate in the contract with the supplier profit assurance in the channel coordination. As shown in Figure 4, defining the return policy limit,  $L$ , lower than 30 or higher than 60, perturbs the balance of profit returns at the retailers or supplier, respectively.

## 7. Conclusions

In this paper, using simulation optimization, a Limited Return policy with SRP (LRSRP) contract tuning methodology has been developed. This methodology assures channel coordination in a supply chain, consisting of one supplier and two competing retailers facing stochastic demand, which is sensitive to both sales effort and retail price. Due to the stochastic nature of market demand and interaction between the decision variables, supply chain members behavior is simulated to effectively analyze the supply chain situation in the presence of contracts. The proposed methodology can support the supplier's decisions on the contract format and parameters by simulating the retailers' reaction to the offered contract.

The proposed algorithm considers real-world conditions in the contracting procedure and lets the supplier design a reliable LRSRP contract that assures channel coordination. It has been shown that the RSRP contract provides retailers with a situation in which they can increase their own profits by adjusting supplier profitability. Adding a limit to the return policy in the RSRP contract, defined through an iterative procedure, during the proposed

contract designing algorithm, addresses the weaknesses of the RSRP contract and leads to a more reliable contract.

There are many interesting avenues for future research. In this research, a two-tier supply chain configuration is assumed. This contracting approach can be extended to consider a three-tier supply chain, consisting of a manufacturer, distributor and retailer facing a price- and effort-dependent stochastic demand.

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