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Two-dimensional optimal linear detector for slowly fluctuating radar signals in compound Gaussian clutter

A.H. Rafie and M.R. Taban*

Department of Electrical and Computer Engineering, Yazd University, Yazd, P.O. Box 89195-741, Iran.

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Abstract. In this paper, we propose a two-dimensional (2-D) Optimal Linear Detector (OLD) for radar target detection in compound Gaussian clutter, and obtain an explicit relation of its coefficients for slowly fluctuating targets. We assume that the samples of signal and clutter are correlated in both range and azimuth directions, and the target detection in each radar cell is implemented by a 2-D sample collection of the received signal. In most conventional detectors, in each pulsation interval, samples of the echo of each radar cell are passed through a matched filter along the range, and a pre-detection is performed; then, the binary results are integrated for successive echoes of that cell (along the azimuth). In fact, by applying the binary integration, we ignore the considerable correlation among 2-D data in the azimuth direction. In the proposed 2-D OLD detector, the correlation of signal and clutter in both range and azimuth directions is considered, aiming to improve the detection performance. Our simulations confirm that this detector outperforms the conventional one-dimensional OLD, as well as AND, OR, and "κ out of n" binary integrators.

1. Introduction

The coherent detection of targets by pulsed radar against a background of non-homogeneous and non-stationary unwanted clutter, due to echoes from the sea, land, or weather, is a problem of fundamental interest in the radar community [1]. Generally, the problem of radar target detection splits into two fundamental subproblems: formulating a model for the underlying interference (noise and clutter) and obtaining an implementable detection structure based on minimizing the loss relative to the optimum detection structure.

For many years, radar systems had relatively low resolution capabilities, and, hence, according to the central limit theorem, workers in the field concluded that the appropriate statistical model for clutter was the Gaussian. With this model in hand, the well-known optimal detector is matched filter.

Nowadays, in the case of high resolution radars and/or at low grazing angles, experimental evidence shows that the statistics of radar clutter significantly deviate from Gaussian behavior [2-7]. In these cases, a satisfactory fit of the clutter amplitude probability density function (pdf) can be achieved through bi-parametric families of distributions, namely, through pdfs containing a shape parameter in addition to the scale one [2,8-10]. Among these, we mention the most commonly adopted, namely, the Weibull and the K distributions [11,12,13]. These last pdfs are, in general, compatible with the so-called compound Gaussian model, which has received wide attention in the recent past due to its theoretical and physical justification [14,15]. However, some other compound Gaussian distributions have recently been used for

*. *Corresponding author. Tel.: +98 913 1519702;*
Fax: +98 351 8200144
E-mail addresses: rafie.amir@gmail.com (A.H. Rafie); mrtaban@yazd.ac.ir (M.R. Taban)

modeling the high resolution radars based on inverse Gaussian and inverse Gamma distributions for the texture component [16,17].

The baseband equivalent of a compound Gaussian clutter process can be deemed the product of two mutually independent processes: a fast varying complex zero-mean, possibly correlated Gaussian process, (the so-called speckle component), and a "more-slowly varying" nonnegative random process (the so-called texture component) [2,6,7,18,19].

One of the most important detectors, used in radar signal detection, is the linear detector. This detector usually has an acceptable performance, in addition to simplicity. North has showed that in the case of additive white Gaussian noise and a perfectly known signal, a linear filter, named the matched filter, maximizes its output Signal to Noise Ratio (SNR) [20]. Dwork [21] and Zadeh and Ragazzini [22] have proved that in the presence of colored Gaussian noise and for a perfectly known signal, the optimum detector is also the matched filter.

The structure of an optimal detector (if it exists) is usually more complicated than that of a linear detector. The linear detector has a known simple structure with unknown coefficients, and, hence, the goal is to determine the optimum coefficients so that the objective function of the detection criterion (the output SNR or probability of detection) is maximized. Such a detector is called an Optimal Linear Detector (OLD).

Nayebi and Aref in [23] have obtained the OLD for Gaussian signal detection in Gaussian noise, according to the Neyman-Pearson (N-P) criterion. In their work, assuming that the data is real, they show that in the case of a zero mean signal or perfectly known signal, the N-P criterion causes the SNR to maximize at the output of the linear detector. Similar to the approach achieved in Gaussian noise, in some other work, the OLD has been obtained for signal detection in non-Gaussian interference. Picinbono and Duvaut in [24] have studied OLD in Spherically Invariant Random Processes (SIRP) according to the deflection criterion, which is not necessarily the optimum criterion in radar applications. Taban et al. in [25,26] have proposed an OLD for slowly fluctuating target detection in pseudo-Gaussian (or SIRP) interference, according to the N-P criterion. In addition to some problems, such as considering data as real [23] and deflection criterion [24], in all previous works, just one sample from each echo has been used for detection. Mathematically, the sampling frequency, f_s, is selected as $1/\tau$, where τ is radar pulse width.

Usually, it is suitable to choose a higher sampling rate, so that several samples are taken from each echo. Then, a digital matched filter can be used over these samples to improve the signal to noise ratio. It is common to apply a matched filter-based detector on the samples of the echo of each pulse (along the range). Then, binary results are integrated for successive pulsation time (along the azimuth) by a binary integrator, such as "AND", "OR", or "κ-out-of-n" rule, to improve the detection performance [27]. However, this detector ignores the correlation between the echoes received from a target (and also from the clutter) in the successive pulses. A more powerful detector may be achieved if all the two-dimensional (2-D) collections of samples (in the range and the azimuth directions) are used in a 2-D OLD.

The aim of the work presented here is to provide a greater insight into the 2-D detection mechanism. With this in mind, we propose a two-dimensional optimal linear detector (2-D OLD) for slowly fluctuating targets in compound Gaussian clutter, when the covariance matrix of clutter is known. We suppose that the sampling frequency of the received signal in the range direction is an integer multiple of bandwidth. In other words, the number of received samples of a range cell is more than one in each pulsation. Samples of the received signal of a cell in range are collected in a 2-D matrix during the successive pulsation times (in azimuth). Indeed, we have a 2-D moving window that first moves one or more samples in the azimuth direction, then, slides along all of the samples in the range direction, consecutively. The detection algorithm is executed in each step of the window shift. In conventional detectors, each raw of this matrix (the samples of the same pulsation time) is detected separately and the binary results may be integrated for successive pulsation times in order to improve the detection performance. Undoubtedly, by applying binary integration, we ignore the considerable correlation among 2-D data in the azimuth direction. In the proposed 2-D OLD detector, this correlation is considered, aiming to improve the detection performance. In our approach to this problem, we have assumed that the radar uses I-Q sampling, so, in addition to the amplitude, the phase of the samples is also maintained and can be used in the coherent detection scheme.

The remainder of this paper is organized as follows: in Section 2, firstly, sampling of the received signal in two directions of range and azimuth is studied. Then, the statistical modelling of target and clutter signals is presented. In Section 3, we propose the structure of a 2-D OLD and obtain its coefficients for non-fluctuating and slowly fluctuating targets. In Section 4, using computer simulation, the performance of the proposed detector is evaluated and compared with some conventional detectors, such as AND, OR and one-dimensional (1-D) OLD. Finally, conclusions are drawn in Section 5.

2. Data sampling and modelling of signal and clutter

Consider an active pulse radar with pulse width τ_0, partitioning range domain into the range cells with length ΔR corresponding to τ_0. The received signal is sampled at rate $f_s = P/\tau_0$, which means that in every pulsation interval, P samples are taken from each echo of the range cell. On the other hand, we use K successive echoes of each range Cell Under Test (CUT) for improving the detection performance. The details are seen in Figure 1.

In other words, we are dealing with a 2-D structure for the data samples of a CUT. Figure 2 shows the 2-D structure for a typical CUT containing the relevant indices. In this structure, index k, $1 \leq k \leq K$ denotes the number of echo from the CUT, and index p, $1 \leq p \leq P$ refers to the number of sample along the range direction. So we have a $K \times P$ matrix of CUT samples as $\mathbf{Y} = [y_{kp}]_{K\times P} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_P]$ to be processed by the detector.

Since the received samples have a 2-D structure, the correlation coefficients of these samples produce a three-dimensional matrix. However, the algebraic operation over the three-dimensional matrix is not as straightforward as that of 1- and 2-dimensional arrays. So, it is better to rearrange the received samples in a vector form as:

$$\mathbf{y}^T = [\mathbf{y}_1^T, \mathbf{y}_2^T, ..., \mathbf{y}_P^T], \tag{1}$$

where ^{T} represents transpose operator. Here, vector $\mathbf{y}$ has $N = K \times P$ elements containing all the samples of the received signal. In the following, we use this form of data for deriving the 2-D OLD.

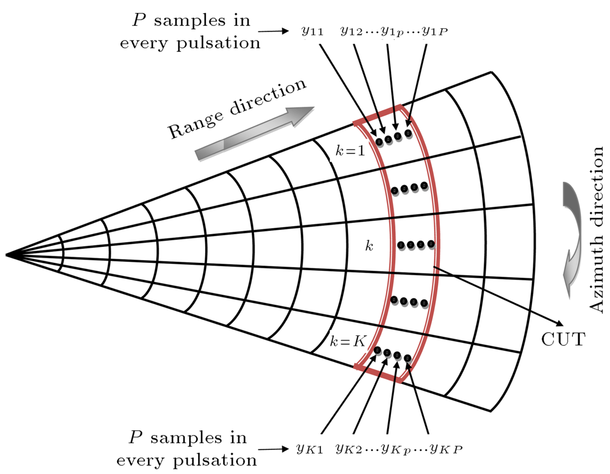

Figure 1. An spatial image of a range Cell Under Test (CUT) with K successive echoes, each echo consists of P samples in the range direction.

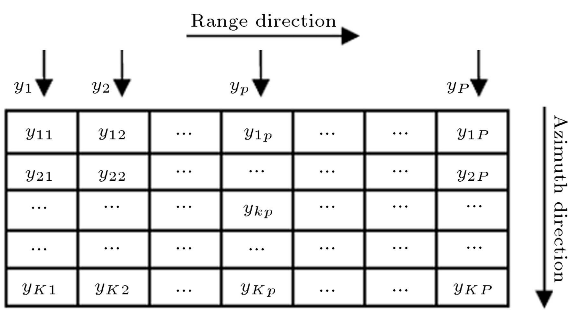

Figure 2. A $K \times P$ matrix of CUT samples that is used for detection.

2.1. Clutter model

The distribution used for the statistical modelling of non-Gaussian clutter is compound Gaussian, whose pdf is given by [28,29]:

$$f_{\mathbf{n}}(\mathbf{n}) = \int_0^\infty \frac{1}{\pi^N det(\mathbf{M}_0)\tau^N} \exp\left(-\frac{\mathbf{n}^H \mathbf{M}_0^{-1}\mathbf{n}}{\tau}\right) f_\tau(\tau)\, d\tau. \tag{2}$$

In the above equation, $\mathbf{n} = [\mathbf{n}_1^T, \mathbf{n}_2^T, ..., \mathbf{n}_P^T]^T$ is a zero mean $N \times 1$ vector representing clutter samples, due to K radar successive pulses and P samples from each pulse. The nonnegative random variable, τ (referred to as texture), with unit mean value represents the local power of the clutter in the CUT. Texture τ has pdf $f_\tau(\tau)$, which determines the non-Gaussian behavior of the clutter. Also, $\mathbf{M}_0 = E(\mathbf{n}.\mathbf{n}^H)$ is the $N \times N$ covariance matrix of $\mathbf{n}$, where $E(.)$ is the statistical expectation operator and ^{H} denotes the conjugate transpose (Hermitian). Exact evaluation of the covariance matrix, $\mathbf{M}_0$, and its relation with autocorrelation matrices of each vector, $\mathbf{n}_p$, $(\mathbf{M}_{pp}, 1 \leq p \leq P)$ and their cross correlation matrices, $\mathbf{M}_{ij}$, $(1 \leq i, j \leq P)$ are informative. These correlation matrices, $\mathbf{M}_{ij}$, have dimension $K \times K$, as follows:

$$\mathbf{M}_{ij} = E(\mathbf{n}_i.\mathbf{n}_j^H), \qquad 1 \leq i, j \leq P. \tag{3}$$

Regarding the above equation, it is clear that covariance matrix $\mathbf{M}_0$ is formed from $\mathbf{M}_{ij}$ matrices as follows:

$$\mathbf{M}_0 = \begin{pmatrix} \mathbf{M}_{11} & . & \mathbf{M}_{1P} \\ . & \mathbf{M}_{pp} & . \\ \mathbf{M}_{1P} & . & \mathbf{M}_{PP} \end{pmatrix}. \tag{4}$$

The most common pdfs used for modelling the non-Gaussian clutter, namely, the Weibull and K distributions, are compatible with the model (2). Precisely, the Weibull amplitude pdf (apdf), as:

$$f_W(u) = a.b.u^{b-1}\exp(-a.u^b), \qquad u, a, b > 0, \tag{5}$$

is amenable to a compound-Gaussian representation in the range of its shape parameter $0 < b \leq 2$ [18]. Similarly, the K-distribution pdf, as:

$$f_K(u) = \frac{(a^{v+1}.u^v)}{(2^{v-1}.\Gamma(v))} K_{v-1}(a.u), \quad u \geq 0, \quad a, v > 0, \tag{6}$$

Range direction $\rightarrow$

$\mathbf{s}_1 \downarrow$		$\mathbf{s}_p \downarrow$		$\mathbf{s}_P \downarrow$
$A\alpha(1)e^{j(\varphi+0\Omega)}$	...	$A\alpha(p)e^{j(\varphi+0\Omega)}$	...	$A\alpha(P)e^{j(\varphi+0\Omega)}$
$A\alpha(1)e^{j(\varphi+\Omega)}$	...	...	...	$A\alpha(P)e^{j(\varphi+1\Omega)}$
.	...	$A\alpha(p)e^{j(\varphi+(k-1)\Omega)}$	...	.
.	...	...	...	.
$A\alpha(1)e^{j(\varphi+(K-1)\Omega)}$	...	$A\alpha(p)e^{j(\varphi+(K-1)\Omega)}$	...	$A\alpha(P)e^{j(\varphi+(K-1)\Omega)}$

Azimuth direction $\downarrow$

Figure 3. Samples of a target signal located in a CUT in range and in successive pulsation time directions.

can be regarded for any positive value of its shape parameter, v, as the apdf of a compound-Gaussian process, where $\Gamma(.)$ and $K_v(.)$ are the Gamma function and second-kind modified Bessel function of order v, respectively [18]. For both distributions, a is a scale parameter related to the common variance, σ^2, of the clutter quadrature components.

2.2. Signal model

Swerling described four statistical models for representing the fluctuation of radar targets [30]. These models are divided into the slowly and rapid fluctuating cases. We can extend the conventional models of fluctuating radar targets [1,30,31] to the case of a 2-D signal sampling in range and azimuth directions [32]. According to the slowly fluctuating target models (including Swerling I and III), samples of the (probable) target signal over each CUT can be shown in Figure 3.

Thus, for slowly fluctuating targets, the complex sample of the target signal, corresponding to the (k,p) sample of 2-D CUT, can be written as:

$$\mathbf{s}_{kp} = A\alpha(p)\exp(j(\varphi + (k-1)\Omega)),$$
$$1 \le k < K \ , \ 1 \le p < P, \tag{7}$$

where A and φ denote the random amplitude and phase of the target echo, respectively, and Ω is the phase due to the Doppler shift of target (f_d) normalized to the Pulse Repetition Frequency (PRF), as follows:

$$\Omega = 2\pi \frac{f_d}{\mathrm{PRF}}. \tag{8}$$

In Eq. (7), $\alpha(p)$ (only depends on the radar pulse shape and time of sampling) includes the amplitude of the emitted pulse samples. Although the time of arrival of the signal is unknown, the two-dimensional processing interval (data window) is consecutively moving in the range direction, sample by sample, and adaption between the filter and signal occurs in the signal location certainly. As previously described, by rearranging the signal matrix, we obtain a vector form of target signal $\mathbf{s}$ as follows:

$$\mathbf{s}^T = A\exp(j\varphi).[\alpha(1)\boldsymbol{\delta}^T, ..., \alpha(p)\boldsymbol{\delta}^T, ..., \alpha(P)\boldsymbol{\delta}^T], \tag{9}$$

where vector $\boldsymbol{\delta}$ depends only on the Doppler shift phase of target as:

$$\boldsymbol{\delta} = [1, e^{j\Omega}, e^{j2\Omega}, ..., e^{j(K-1)\Omega}]^T. \tag{10}$$

In this work, f_d (and equivalently Ω and $\boldsymbol{\delta}$) is assumed to be known. This is a rational assumption, because, using a filter bank in conventional radar systems is common and the value of f_d corresponding to each subband can approximately be assumed constant.

In a realistic radar scenario, the complex signal amplitude, $Ae^{j\phi}$, is unknown and fluctuates from scan to scan. We assume that phase φ is an uniformly random variable in the interval $[0, 2\pi)$ and the amplitude of the target samples has the pdf $f_A(a)$ which depends on the selected swerling model. A widespread choice for the target amplitude model is the so called, Swerling-I, wherein, modulus A is modeled as a Rayleigh random variable as:

$$f_A(a) = \frac{2a}{P_A} e^{-\frac{a^2}{P_A}} \qquad a \ge 0. \tag{11}$$

Here, P_A is mean square value of A. In, Swerling-III model, A is a chi random variable with degrees of freedom 4 pdf, as:

$$f_A(a) = \frac{8a^3}{{P_A}^2} e^{-\frac{2a^2}{P_A}} \qquad a \ge 0. \tag{12}$$

3. Two-dimensional optimal linear detector

In this section, we propose a novel detector called 2-D OLD for a slowly fluctuating target embedded in compound Gaussian interference (clutter plus noise). The problem of radar signal detection in a clutter domain environment can be posed using the following binary hypothesis test model:

$$\begin{cases} \mathcal{H}_0: & \mathbf{y} = \mathbf{n}, \\ \mathcal{H}_1: & \mathbf{y} = \mathbf{s} + \mathbf{n}, \end{cases} \tag{13}$$

where $\mathbf{y}$, $\mathbf{s}$, and $\mathbf{n}$ denote the $N \times 1$ complex vectors of the samples from the baseband equivalent of the received signal, target signal, and interference, respectively. So, we assume that under $\mathcal{H}_0$ hypothesis, the received signal contains only the interference, while, under $\mathcal{H}_1$, the target signal is also added to the interference in the received signal.

The general structure of a coherent linear detector is as follows:

$$|\mathbf{W}^H.\mathbf{y}| \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} T. \tag{14}$$

Here, the complex weighting vector, $\mathbf{W}$, contains the detector coefficients and T is a threshold corresponding to the desired false alarm probability (P_{fa}). In OLD, vector $\mathbf{W}$ should be determined, so that the optimum detection performance is achieved. Regarding N-P criterion, we should find an optimum $\mathbf{W}$ for the test Eq. (14), so that it maximizes the detection probability (P_d) for every desired P_{fa}. Suppose that the output of the linear filter is a random variable, x, with absolute value, q, or, mathematically:

$$q = |x| = |\mathbf{W}^H.\mathbf{y}|. \tag{15}$$

We can use the following equations to calculate the P_d and P_{fa} of the detector:

$$\begin{cases} P_{fa} = \Pr(q > T|\mathcal{H}_0) \\ P_d = \Pr(q > T|\mathcal{H}_1) \end{cases} \tag{16}$$

For an interference with pdf $f_{\mathbf{n}}(\mathbf{n})$, these probabilities can be written as follows:

$$\Pr(q > T|\mathcal{H}_i) = \int_{q>T|\mathcal{H}_i} \cdots \int f_{\mathbf{n}}(\mathbf{n})\, d\mathbf{n}, \quad i = 0, 1, \tag{17}$$

and for a zero mean compound Gaussian interference, substituting Eq. (2) into Eq. (17) yields:

$$\Pr(q > T|\mathcal{H}_i) = \int_0^\infty \left(\int_{q>T|\mathcal{H}_i} \cdots \int \frac{1}{\pi^N \det(\tau \mathbf{M}_0)} \exp\left\{ -\mathbf{n}^H (\tau\mathbf{M}_0)^{-1}\mathbf{n} \right\} d\mathbf{n} \right) f_\tau(\tau)\, d\tau, i = 0, 1. \tag{18}$$

To calculate P_{fa} under $\mathcal{H}_0$, we have:

$$P_{fa} = \int_0^\infty \Pr(q > T|\mathcal{H}_0, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0)) f_\tau(\tau)\, d\tau. \tag{19}$$

The internal term $\Pr(q > T|\mathcal{H}_0, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0))$ is equal to P_{fa}, in the case of Gaussian interference with covariance matrix $\tau\mathbf{M}_0$, as below (see Appendix A):

$$\Pr(q > T|\mathcal{H}_0, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0)) = \exp(-\frac{T^2}{\tau P_n}), \tag{20}$$

where P_n is the interference power at the output of the linear filter, which can be easily calculated as:

$$P_n \triangleq \sigma_x^2 = \mathbf{W}^H\mathbf{M}_0\mathbf{W}, \tag{21}$$

where σ_x^2 is the variance of x. Substituting Eq. (20) into Eq. (19), we obtain:

$$P_{fa} = \int_0^\infty \exp(-\frac{T^2}{\tau P_n}) f_\tau(\tau)\, d\tau \triangleq H(\frac{T^2}{P_n}). \tag{22}$$

It can be easily shown that the $H(\zeta)$ is a strictly increasing function in ζ and, hence, is invertible. So, it is possible to obtain threshold T as follows:

$$\frac{T^2}{P_n} = H^{-1}(P_{fa}). \tag{23}$$

To calculate P_d from Eq. (18) under $\mathcal{H}_1$, we have:

$$P_d = \int_0^\infty \Pr(q > T|\mathcal{H}_1, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0)) f_\tau(\tau)\, d\tau. \tag{24}$$

The internal term $\Pr(q > T|\mathcal{H}_1, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0))$ is equal to P_d in the case of Gaussian interference with covariance matrix $\tau\mathbf{M}_0$ as below (see Appendix B):

$$\Pr\left(q > T|\mathcal{H}_1, \mathbf{n} \sim \mathcal{N}(\mathbf{0}, \tau\mathbf{M}_0)\right) = \int_{\Gamma_S} \mathcal{Q}\left(\sqrt{\frac{2}{\tau}}\gamma_W ; \sqrt{\frac{2}{\tau P_n}}T \right) f_{\mathbf{s}}(\mathbf{s})\, d\mathbf{s}, \tag{25}$$

where γ_W^2 is the temporal signal to interference power ratio at the output of the linear filter as follows:

$$\gamma_W^2 \triangleq \frac{P_s|\mathbf{s}}{P_n} = \frac{\mathbf{W}^H\mathbf{s}\mathbf{s}^H\mathbf{W}}{\mathbf{W}^H\mathbf{M}_0\mathbf{W}}, \tag{26}$$

and Γ_S and $f_{\mathbf{s}}(\mathbf{s})$ are the sample space and pdf of all random parameters of $\mathbf{s}$, respectively. In Eq. (25), $\mathcal{Q}(\alpha;\beta)$ is Marcum's Q-function, which is a strictly increasing function of the positive variable, α, for every constant and arbitrary value of β.

$$\mathcal{Q}(\alpha;\beta) = \int_\beta^\infty x \exp\left(-\frac{x^2+\beta^2}{2}\right) \mathcal{I}_0(\alpha x)\, dx, \tag{27}$$

where $\mathcal{I}_0(.)$ is the first-kind modified Bessel function of order zero. Substituting Eq. (25) into Eq. (24) and replacing $\frac{T^2}{P_n}$ with that of Eq. (23), we have:

$$P_d = \int_0^\infty \int_{\Gamma_S} \mathcal{Q}\left(\sqrt{\frac{2}{\tau}}\gamma_W ; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})} \right) \times f_{\mathbf{s}}(\mathbf{s})\, d\mathbf{s} f_\tau(\tau)\, d\tau. \tag{28}$$

So, according to the N-P criterion and the strictly increasing characteristic of the $\mathcal{Q}$- function in γ_W, we should maximize P_d for a desired P_{fa} in order to achieve the 2-D OLD.

3.1. Perfectly known signal

In this case, Eq. (28) is simplified as below:

$$P_d = \int_0^\infty \mathcal{Q}\left(\sqrt{\frac{2}{\tau}}\gamma_W ; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})} \right) f_\tau(\tau)\, d\tau, \tag{29}$$

and γ_W^2 becomes equal to the average signal to interference ratio (SIR). Since $\mathcal{Q}\left(\sqrt{\frac{2}{\tau}}\gamma_W; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})}\right)$ is a strictly increasing function in γ_W, if we can find a vector, $\mathbf{W}$, which maximizes γ_W for each arbitrary value of τ, this $\mathbf{W}$ maximizes P_d too. Therefore, the optimum value of vector $\mathbf{W}$ is the one which maximizes the following equation:

$$\mathbf{W}_{opt} = \arg\left(\max_{\mathbf{W}}\left(\frac{\mathbf{W}^H\mathbf{s}\mathbf{s}^H\mathbf{W}}{\mathbf{W}^H\mathbf{M}_0\mathbf{W}}\right)\right). \tag{30}$$

Using Cauchy-Schwartz inequality, we obtain:

$$\mathbf{W}_{opt} = \mathbf{M}_0^{-1} \cdot \mathrm{s}\square. \tag{31}$$

Regarding the above equation, we conclude that for a 2-D known signal, the 2-D OLD is equivalent to the 2-D matched filter.

3.2. Slowly fluctuating target

The optimum value of $\mathbf{W}$ is the one which maximizes P_d in Eq. (28), where γ_W is given by Eq. (26). For slowly fluctuating targets, $\mathbf{s}\mathbf{s}^H$ can be obtained from Eq. (9) as $\mathbf{s}\mathbf{s}^H = A^2\mathbf{\Lambda}$, where:

$$\mathbf{\Lambda} = \begin{bmatrix} \alpha^2(1)\boldsymbol{\delta}\boldsymbol{\delta}^H & \dots & \alpha(1)\alpha(P)\boldsymbol{\delta}\boldsymbol{\delta}^H \\ \vdots & \ddots & \vdots \\ \alpha(P)\alpha(1)\boldsymbol{\delta}\boldsymbol{\delta}^H & \dots & \alpha^2(P)\boldsymbol{\delta}\boldsymbol{\delta}^H \end{bmatrix}. \tag{32}$$

Since matrix $\mathbf{\Lambda}$ is known and positive definite, we can decompose it as $\mathbf{\Lambda} = \mathbf{L}\mathbf{L}^H$. Hence, substituting $\mathbf{s}\ \mathbf{s}^H = A^2\mathbf{L}\mathbf{L}^H$ into Eq. (26), we achieve ${\gamma_\mathbf{W}}^2 = A^2 \cdot \lambda_W^2$, where λ_W^2 is defined as:

$$\lambda_W^2 = \frac{\mathbf{W}^H\mathbf{L}\mathbf{L}^H\mathbf{W}}{\mathbf{W}^H\mathbf{M}_0\mathbf{W}}. \tag{33}$$

In this case, we can rewrite Eq. (28) as:

$$P_d = \int_0^\infty \left[\int_0^\infty \mathcal{Q}\left(\sqrt{\frac{2}{\tau}}A\lambda_W; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})}\right) f_A(A)\,dA\right] f_\tau(\tau)\,d\tau \tag{34}$$

Here, $f_A(A)$ is the pdf of the amplitude of the target samples and depends on the selected Swerling model.

$\mathcal{Q}\left(\sqrt{\frac{2}{\tau}}A\lambda_W; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})}\right)$ is a strictly increasing function in λ_W for each arbitrary value of τ and A for a constant P_{fa}. According to Eq. (34), since P_d is the average of the non-negative function $\mathcal{Q}\left(\sqrt{\frac{2}{\tau}}A\lambda_W; \sqrt{\frac{2}{\tau}H^{-1}(P_{fa})}\right)$ on the domain of two non-negative variables, A and τ, with non-negative weight function, $f_A(A)f_\tau(\tau)$, we can easily show that P_d is also a strictly increasing function in λ_W. Hence, if we find a $\mathbf{W}$ which maximizes λ_W, this $\mathbf{W}$ also maximizes P_d. Therefore, the optimum value of vector $\mathbf{W}$ is the one which maximizes the following equation:

$$\mathbf{W}_{opt} = \arg\left(\max_{\mathbf{W}}\left(\frac{\mathbf{W}^H\mathbf{L}\mathbf{L}^H\mathbf{W}}{\mathbf{W}^H\mathbf{M}_0\mathbf{W}}\right)\right). \tag{35}$$

Using Cauchy-Schwartz inequality, we obtain:

$$\mathbf{W}_{opt} = \mathbf{M}_0^{-1} \cdot \mathbf{L}\square \tag{36}$$

Considering Eq. (35) shows that in the case of a slowly fluctuating target and compound Gaussian interference, the 2-D OLD is a detector which maximizes the output SIR.

4. Simulation results

In this section, we evaluate the performance of 2-D OLD using the Monte Carlo simulation. We first compare the detection performance of the proposed detector with that of four conventional AND, OR, κ-out-of-n and 1-D OLD detectors. Then, we illustrate the sensitivity of 2-D OLD, with respect to the parameters of clutter and target signal.

In our discussion, we use the following notations for calling the related parameters or variables in the simulations: SP = clutter shape parameter, Swer = number of swerling model, Dist = clutter distribution (W for Weibull and K for K distributed clutter), F = target Doppler shift normalized to PRF ($\frac{f_d}{\mathrm{PRF}}$), SCBW_A and SCBW_R = power spectral bandwidths of clutter samples in azimuth and range directions, respectively, normalized to PRF, SIR= signal to interference power ratio ($= \frac{E(|\mathbf{s}|^2)}{E(|\mathbf{n}|^2)}$), K = number of successive reflected echoes from a cell (or number of pulses in the coherent processing interval), and P = number of extracted samples from each radar range cell in every pulsation.

We firstly notice that the clutter covariance matrix, $\mathbf{M}_0$, is a Toeplitz matrix and its spectrum shape is Gaussian [10]. Hence, we assume that the correlation matrix, $\mathbf{M}_0$, is completely determined in terms of the normalized azimuth and range power spectral bandwidths of clutter, SCBW_A and SCBW_R. Commonly, the clutter samples are more correlated in an azimuth direction in comparison with the range direction. Also, for simulation purposes, the baseband equivalent of the transmitted waveform is a rectangular (coherent) pulse train.

For obtaining the detection performance curves, each simulation is statistically repeated with 10^5 iterations. In each iteration, we first generate a $K \times P$ matrix of interference (clutter) samples with compound Gaussian distribution and Gaussian correlation in both range and azimuth directions using the methods proposed in [15,18], corresponding to the CUT data under

H_0 hypothesis. Afterwards, another $K \times P$ matrix is produced by adding the interference matrix to a $K \times P$ matrix of signal samples (generated based on Eq. (9)) for providing the CUT data under H_1 hypothesis. Both CUT data matrices are processed by all the detection algorithms.

After implementing all iterations, we determine an ordered thresholds set. For each detection method, we compute two P_{fa} and P_d sets corresponding to the thresholds set using the detection processing output under H_0 and H_1, respectively. The Receiver Operating Characteristic (ROC) curves can easily be obtained by these P_{fa} and P_d sets. The detection processing output of the proposed detector is obtained using Eq. (14). In the other detectors, first, a Matched Filter (MF) corresponding to $\alpha(t)$ is acted on each raw of the CUT data (in range). Then, in the 1-D OLD, all K results obtained by MF are integrated (in azimuth) and compared with the threshold. In the AND, OR, and "κ out of n" detectors, each of the K results obtained by MF is compared with a common threshold, separately, and K initial decisions are fused by AND, OR, or "κ out of n" rules for a final decision.

At first, the performances of five previously mentioned detectors are evaluated and compared with each other in Figures 4-7. To this end, the ROC curves are used for comparing the performances of the detectors. As seen, the 2-D OLD outperforms others for various values of parameters of clutter and signal. Among others, AND and OR detectors have the best and worst performance, respectively.

The ROC of these detectors are shown in Figures 4 and 5 for a Weibull clutter with SCBW$_R$ = 0.2 and 0.5, respectively, when $P = 10$, $K = 5$, SIR = 5 dB, Swer = 1, $F = 0.1$, SCBW$_A$ = 0.15, and $\kappa = 4$. Through these figure, we see that increasing SCBW$_R$ or decreasing the correlation of clutter samples in a range direction improves the performances of all detectors without any change in their ranks. Nevertheless, this is not a regular rule, because the structure of all detectors depends on the covariance matrix of interference (related to SCBW$_A$ and SCBW$_R$).

Similar comparison between the detectors performance is implemented in Figures 6 and 7 for a K-distributed clutter with SCBW$_A$ = 0.2 and 0.12, respectively, when $P = 15$, $K = 6$, SIR = 0 dB,

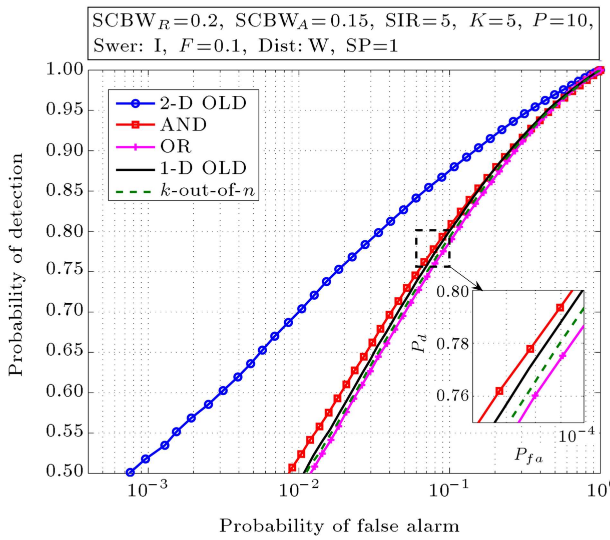

Figure 4. Performance comparison of the 2-D OLD, 1-D OLD, AND and OR detectors in Weibull clutter. The parameters are set to: $P = 10$, $K = 5$, SIR = 0 dB, F = 0.1, SCBW$_A$ = 0.15, SCBW$_R$ = 0.2 and $\kappa = 4$.

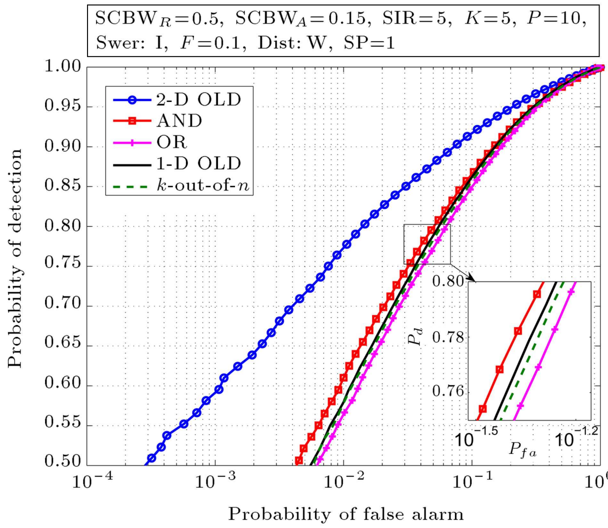

Figure 5. Performance comparison of the 2-D OLD, 1-D OLD, AND and OR detectors in Weibull clutter. The parameters are set to: $P = 10$, $K = 5$, SIR = 0 dB, F = 0.1, SCBW$_A$ = 0.15, SCBW$_R$ = 0.5 and $\kappa = 4$.

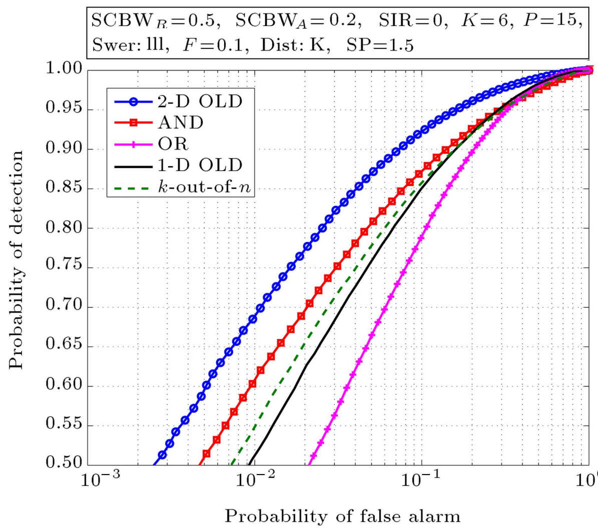

Figure 6. ROC comparison of the 2-D OLD, 1-D OLD, AND and OR detectors in K-distributed clutter. The parameters are set to: $P = 15$, $K = 6$, SIR = -5 dB, SP = 1.5, Swer = 3, F = 0.1, SCBW$_R$ = 0.5, SCBW$_A$ = 0.2 and $\kappa = 5$.

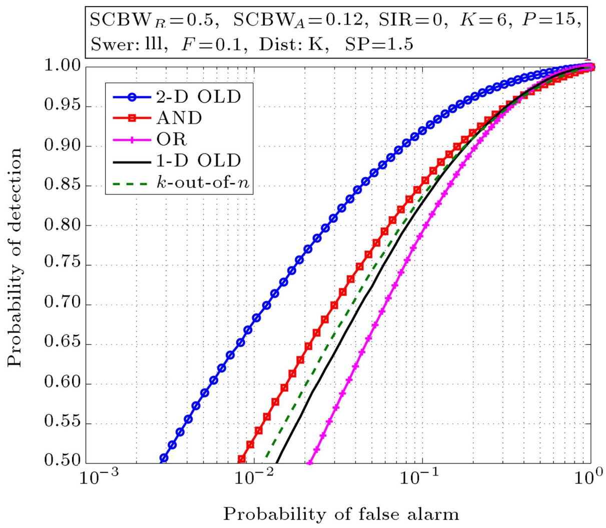

Figure 7. ROC comparison of the 2-D OLD, 1-D OLD, AND and OR detectors in K-distributed clutter. The parameters are set to: P = 15, K = 6, SIR = -5 dB, SP = 1.5, Swer = 3, F = 0.1, SCBW$_R$ = 0.5, SCBW$_A$ = 0.12 and κ = 5.

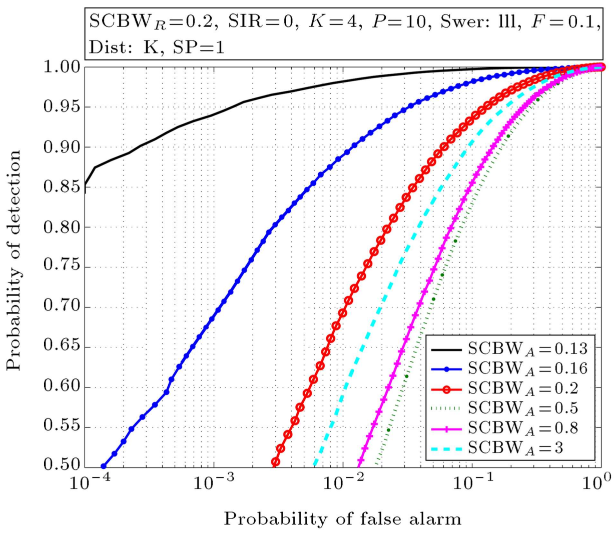

Figure 8. Performance evaluation of the 2-D OLD in K-distributed clutter for different values of SCBW$_A$. The parameters are set to: K = 4, P = 10, SIR = 0 dB, SP = 1, Swer = 3, F = 0.1 and SCBW$_R$ = 0.2.

SP = 1.5, Swer = 3, F = 0.1, SCBW$_R$ = 0.5, and κ = 5. Here, we see that when SCBW$_A$ decreases or correlation between samples in the azimuth direction increases, the performance of all detectors slightly degrades. Similarly, this observation also is not a regular rule. Figure 8 illustrates the sensitivity of 2-D OLD, with respect to the power spectral bandwidth of clutter samples in the azimuth direction (or SCBW$_A$). The simulation was run with: K = 4, P = 10, SIR = 0 dB, Dist = K, SP = 1, Swer = 3, F = 0.1, SCBW$_R$ = 0.2 and different values of SCBW$_A$. It is seen that for SCBW$_A$ = 0.13 (the biggest correlation), the 2-D

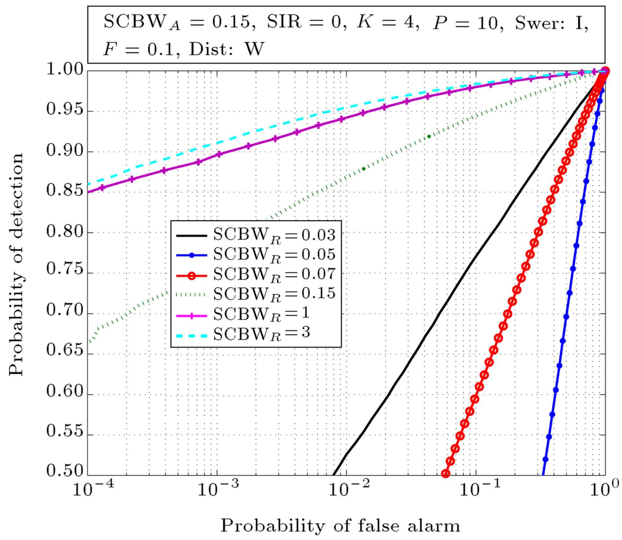

Figure 9. Performance evaluation of the 2-D OLD in Weibull clutter for different values of SCBW$_R$. The parameters are set to: K = 4, P = 10, SIR = 0 dB, Dist = W, SP = 1, Swer = 1, F = 0.1 and SCBW$_A$ = 0.13.

OLD has the best performance. Although increasing SCBW$_A$ from 0.13 to 0.5 degrades the performance, further increase of SCBW$_A$ from 0.5 improves the performance, unlike the previous procedure.

In Figure 9, the effect of the power spectral bandwidth of clutter samples in the range direction (or SCBW$_R$) is investigated on the 2-D OLD performance for: K = 4, P = 10, SIR = 0 dB, Dist = W, SP = 1, Swer = 1, F = 0.1, SCBW$_A$ = 0.15 and different values of SCBW$_R$. It is observed that, at first, increasing SCBW$_A$ from 0.03 to 0.05 decreases the performance, but, by increasing SCBW$_A$ from 0.05 to 1, the performance considerably improves.

We have also used the curves of probability of detection (P_d) versus SIR for a desired constant, P_{fa}, to evaluate the detector performance. Figure 10 illustrates the sensitivity of 2-D OLD, with respect to the number of samples in each pulsation (P), compared with the 1-D OLD, wherein: K = 5, P_{fa} = 10^{-3}, Dist = W, SP = 1, Swer = 1, F = 0.1, SCBW$_A$ = 0.12 and SCBW$_R$ = 0.3. The curves illustrate that increasing P, significantly increases the performance of the detector. Precisely, in comparison between 1-D OLD and 2-D OLD performances, the 1-D OLD requires an additional SIR of about 19 dB to obtain P_d = 0.9 for a constant P_{fa} = 10^{-3} in the case of P =15.

All the above curves show that the 2-D OLD largely outperforms the conventional 1-D OLD for all parameters, which has been shown in our simulations.

5. Conclusion

In this paper, we realized a two-dimensional optimal linear detector. Briefly speaking, the most important

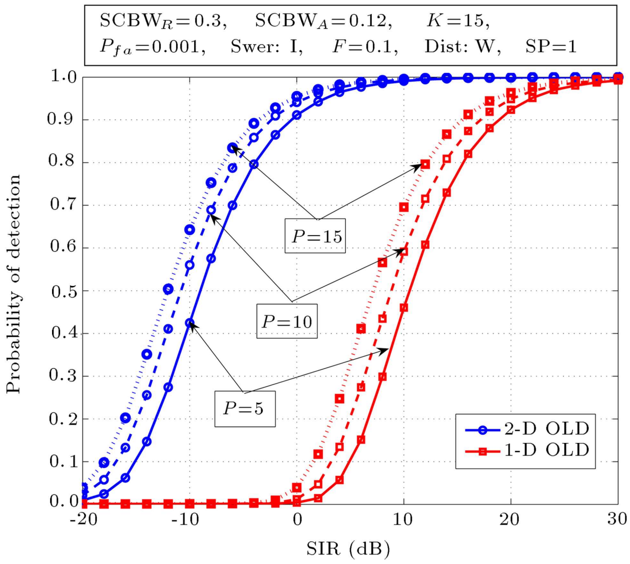

Figure 10. Performance comparison of the 2-D OLD and 1-D OLD in Weibull clutter for different values of P. The parameters are set to: $K = 5$, $P_{fa} = 10^{-3}$, SP = 1, Swer = 1, $F = 0.1$, SCBW$_A$ = 0.12 and SCBW$_R$ = 0.3.

difference between conventional and 2-D OLDs is that the latter uses sample correlation in both range and azimuth directions completely. But, the conventional OLD, because of its structure, ignores the sample correlation in the range direction. With high sampling along the range direction, correlation of the clutter samples in the range direction is significant and by considering this correlation, the detection performance will improve significantly. Simulation results demonstrate that the superiority of 2-D OLD in comparison with 1-D OLD is maintained for various values of signal and clutter parameters. This superiority is due to using sample correlation in both azimuth and range directions.

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Appendix A. P_{fa} of linear detector for Gaussian interference

The pdf of a zero mean complex Gaussian random vector with covariance matrix $\mathbf{M}_0$ is as:

$$f_{\mathbf{n}}(\mathbf{n}) = \frac{1}{\pi^N det(\mathbf{M}_0)} \exp\Big(- \mathbf{n}^H \mathbf{M}_0^{-1} \mathbf{n} \Big). \tag{A.1}$$

According to Eq. (15), the random variable, $x = \mathbf{W}^H.\mathbf{y}$, will be Gaussian, too, under $\mathcal{H}_0$. Assuming signal vector $\mathbf{s}$ as a known vector, we can calculate mean m_x and variance σ_x^2 of x easily, as follows:

$$m_x = \begin{cases} 0 & \text{for} \quad \mathcal{H}_0, \\ \mathbf{W}^H \mathbf{s} & \text{for} \quad \mathcal{H}_1, \end{cases} \tag{A.2}$$

$$\sigma_x^2 = \mathbf{W}^H \mathbf{M}_0 \mathbf{W}, \qquad \text{for} \qquad \mathcal{H}_0 \text{ and } \mathcal{H}_1. \tag{A.3}$$

So, random variable x has the following pdf:

$$f_x(x) = \frac{1}{\pi \sigma_x^2} \exp\left(- \frac{(x - m_x)^H (x - m_x)}{\sigma_x^2} \right). \tag{A.4}$$

As a result, random variable $q = |x|$ has a pdf equal to [33]:

$$f_q(q) = \frac{2q}{\sigma_x^2} \exp\Big(- \frac{1}{\sigma_x^2}(q^2 + |m_x|^2) \Big) \mathcal{I}_0\Big(\frac{2|m_x|}{\sigma_x^2} q \Big), \tag{A.5}$$

in which $\mathcal{I}_0(.)$ is first-kind modified Bessel function of order zero. Under $\mathcal{H}_0$, q has simply a Rayleigh distribution as follows:

$$f_q(q|\mathcal{H}_0) = \frac{2q}{P_n} \exp\left(- \frac{q^2}{P_n} \right), \tag{A.6}$$

where P_n is the interference power at the detector output, which is calculated as:

$$P_n = \sigma_x^2 = \mathbf{W}^H \mathbf{M}_0 \mathbf{W}. \tag{A.7}$$

Substituting Eq. (A.6) into Eq. (16), P_{fa} can be easily calculated as:

$$P_{fa} = \int_T^\infty \frac{2q}{P_n} \exp\left(- \frac{q^2}{P_n} \right) dq = \exp\left(- \frac{T^2}{P_n} \right). \tag{A.8}$$

Appendix B. P_d of linear detector for Gaussian interference

Under $\mathcal{H}_1$, and for perfectly known signal $\mathbf{s}$ in zero mean Gaussian interference with covariance matrix, $\mathbf{M}_0$, random variable x is still Gaussian, but with a non-zero mean. So, random variable q has the Rician distribution with power $P_n + P_s$ as follows:

$$f_q(q|\mathcal{H}_1) = \frac{2q}{P_n} \exp\left(-\frac{(q^2 + P_s)}{P_n}\right) \mathcal{I}_0\left(\frac{2\sqrt{P_s}}{P_n} q\right), \tag{B.1}$$

where:

$$P_s = |m_{x|\mathcal{H}_1}|^2 = |\mathbf{W}^H\mathbf{s}|^2 = \mathbf{W}^H\mathbf{s}\mathbf{s}^H\mathbf{W}. \tag{B.2}$$

Substituting Eq. (B.1) into Eq. (16), P_d of the detector is calculated as:

$$P_d = \int_T^\infty \frac{2q}{P_n} \exp\Big(-\frac{(q^2 + P_s)}{P_n}\Big)\mathcal{I}_0\left(\frac{2\sqrt{P_s}}{P_n} q\right) dq$$

$$= \mathcal{Q}\Big(\sqrt{\frac{2}{P_n}} m_x ; \sqrt{\frac{2}{P_n}} T\Big), \tag{B.3}$$

and substituting P_n and m_x from Eqs. (21) and (A.2) in Eq. (B.3), we have:

$$P_d = \mathcal{Q}\left(\sqrt{2}\gamma_W ; \sqrt{\frac{2}{P_n}T}\right), \tag{B.4}$$

where γ_W^2 is the signal to interference power ratio at the detector output as follows:

$$\gamma_W^2 = \frac{P_s}{P_n} = \frac{\mathbf{W}^H\mathbf{s}\mathbf{s}^H\mathbf{W}}{\mathbf{W}^H\mathbf{M}_0\mathbf{W}}. \tag{B.5}$$

We can easily extend Eq. (B.4) for random signal $\mathbf{s}$ as below:

$$P_d = \int_{\Gamma_S} \mathcal{Q}\left(\sqrt{2}\gamma_W ; \sqrt{\frac{2}{P_n}T}\right) f_{\mathbf{s}}(\mathbf{s})\, d\mathbf{s}, \tag{B.6}$$

where $f_{\mathbf{s}}(\mathbf{s})$ and Γ_S are the pdf and sample space of the random parameters of $\mathbf{s}$.

Biographies

Amir Hossein Rafie was born in Khorramabad, Iran, in 1982. He received his BS degree (Electrical Engineering) in 2005, and his MS degree (communications engineering) in 2008, from Yazd University, Yazd, Iran. His main research interests are statistical signal processing and radar signal detection.

Mohammad Reza Taban was born in Isfahan, Iran, in 1968. He received his BS degree in 1991 from Isfahan University of Technology (IUT), Isfahan, Iran, his MS degree in 1993 from Tarbiat Modarres University, Tehran, Iran, and his PhD degree in 1998 from IUT, all in Electrical Engineering. He joined the Yazd University faculty, Yazd, Iran, in 1999, and became Associate Professor in 2008. His main research interests are statistical signal processing, detection, and estimation. Dr. Taban has published more than 62 technical papers in international and national journals and conferences.