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Greedy spanner algorithms in practice

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Abstract. Spanners generated by the greedy algorithm - or greedy spanners - not only have good theoretical properties, like a linear number of edges, low degree and low weight, but also, as previous experimental results show, they are superior to spanners generated by other algorithms in practice. Because of the good properties of greedy spanners, they found several applications like in protein visualization.

The major issue in computing greedy spanners is the high time and space complexity of algorithms that compute it. To construct the greedy spanner on a set of n points, the original greedy algorithm takes $\mathcal{O}(n^3 \log n)$ time. In 2005, an improvement was proposed by Farshi and Gudmundsson [Lecture Notes in Computer Science, Vol. 3669, pages 556-567] that works much faster in practice, but later it was shown that it has same theoretical time complexity. In 2008, Bose et al. [Lecture Notes in Computer Science, Vol. 5124, pages 390-401] discovered a near-quadratic time algorithm for constructing greedy spanners. In this paper, we compare time complexity of these three algorithms for computing the greedy spanner in practice.

1. Introduction

A network on a set V of n points (in Euclidean space) can be modeled as an undirected graph G with vertex set V of size n and an edge set E of size m where every edge $e = (u, v)$ has a weight $wt(e)$. A network is called a geometric or Euclidean network if the weight of the edge $e = (u, v)$ is the Euclidean distance $|uv|$ between its endpoints u and v.

There are several ways to measure the quality of a network; one is the dilation or stretch factor of the network. For each pair of points $u, v \in V$, the ratio of the length of the shortest path between u and v in G, denoted by $\mathbf{d}_G(u, v)$, to the Euclidean distance between u and v is called the dilation between u and v in G. The length along a path is sum of lengths of all edges on the path. The maximum dilation between all pairs of vertices of G is called the dilation of G. Informally, in a network with low dilation, the amount of detour one might need to take to travel from one node to another node of the graph, when it is only allowed to travel along edges of the graph, is close to the actual Euclidean distance between the nodes.

Let $t \geq 1$ be a real number. A network G is called a t-spanner on vertices of G, if the dilation of G is at most t. In other words, a graph $G(V, E)$ is a t-spanner of V, if for each pair of vertices $u, v \in V$, there exists a path in G between u and v of weight at most $t \cdot |uv|$. We call such a path a t-*path* between u and v.

Obviously, with this measure of quality, we need to have a lot of edges in the graph to have good networks which means they are expensive to build. So the main objective of studying spanners is to construct sparse t-spanners with t sufficiently close to 1. To measure sparseness, one can consider different criteria like the weight of the spanner, i.e. sum of weights of all of its edges, the number of edges and the maximum

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degree. Spanners have applications in several fields: from robotics, network topology design, distributed systems and design of parallel machines to metric space searching [1], protein visualization [2] and broadcasting in communication networks [3].

There are several algorithms that given a set of points and a $t > 1$, compute a t-spanner of the point set in $\mathbb{R}^d$ (see [4]). Different algorithms generate spanners with different properties, most of them contain a linear number of edges, and some of them have low weight and degree. Next to theoretical results, algorithms are compared in experiments based on different properties, like number of edges, maximum degree and weight of generated graph and also the running time of them (see [1,5,6]).

The greedy algorithm is not only the simplest algorithm to generate a spanner, but the generated spanners also have a linear number of edges, constant degree and optimal weight in theory. Experiments also show that the greedy algorithm generates spanners which have superior properties compared to other algorithms [5]. The only problem with the algorithm is its time and space complexity, the original greedy algorithm requires cubic time and quadratic space complexity which makes it impossible to compute the greedy spanner. Greedy spanner is generated by the greedy algorithm, on sets of points that contain more than a few thousand points using the original greedy algorithm. In [5] Farshi and Gudmundsson proposed an improvement (we will refer to the improved variant as FG-Greedy) that worked well in practice, however, it was later shown [7] that this improvement does not improve the theoretical worst-case bound. In the same paper, Bose et al. introduced an algorithm that generates the greedy spanner in $\mathcal{O}(n^2 \log n)$ time using $\mathcal{O}(n^2)$ space. To the best of our knowledge, this algorithm had not been implemented before (Independently of our work, Bouts et al. [8] have recently implemented this algorithm).

Recently, Alewijnse et al. [9] proposed an algorithm that computes the greedy spanner in $\mathcal{O}(n^2 \log^2 n)$ time using linear space (note that all previous greedy algorithms use quadratic space). They also implemented the algorithm and showed the practical behaviour of the algorithm on point sets with different distributions and up to one million points which was a big progress since the highest number of points that the greedy spanner generated before this contains only 13,000 points [5]. They also showed that the expected time complexity of their algorithm is linear for uniformly distributed point sets. There is also another recent and independent work by Bouts et al. [8] that provides a theoretical framework for analyzing greedy spanner algorithms. In particular it provides improved analytical bounds for two of the algorithms discussed in this paper. For the FG-Greedy algorithm it provides a bound in terms of the spread of the point set; this resolves the question why the FG-Greedy performs well in practise and also explains its performance in the experiments on uniformly distributed data. For NQT-Greedy, the paper proves a tighter bound in terms of t. It also reports on some experiments on the three algorithms discussed in this paper.

In this paper we compare the Bose et al. greedy algorithm with two other algorithms that generate greedy spanner, based on their running time in practice. We did not consider the linear space algorithm introduced by Alewijnse et al. [9].

We should mention that there exist an algorithm with near-linear worst-case time complexity, namely approximate greedy algorithm [4,10], that generates spanners with theoretical properties similar to the greedy spanner but, as previous experiments show (see [5]) in practice it is far worse than the greedy spanner especially when t is close to 1 and points are uniformly distributed.

The paper is organized as follows. First we briefly describe the implemented algorithms together with the theoretical bounds and implementation details. In Section 3 we discuss the experimental results. Finally, we discuss possible improvements and future research.

Throughout the paper, t will be assumed to be a small constant. In the experiments we used values of t between 1.02 and 2, because in most applications t is close to 1. Also for larger values of t there are other options like the Delaunay triangulation which is known to have dilation 1.998 [11]. Also, we know that any 2-spanner is a t-spanner for each $t > 2$.

2. Greedy spanner algorithms

Here, we give a short description of each of the algorithms implemented together with their theoretical bounds. For a detailed description of each algorithm, considered in this section, please refer to the book by Narasimhan and Smid [4] and papers by Farshi and Gudmundsson [5], Bose et al. [7] and Bouts et al. [8].

2.1. The original greedy algorithm

The greedy algorithm was introduced independently by Althöfer et al. [12] and Bern in 1989, and since then, other researchers worked on it (see [10,13-17]). The graph constructed using the greedy algorithm will be called a greedy graph or a greedy spanner. Note that, as we will see, if no pair of points in the input set has same distance, then the greedy spanner is unique. Otherwise, we can have several greedy spanners. But, if we fix an order on pairs with equal distances, then again we can assume that the greedy spanner is unique.

Intuitively, the original greedy algorithm constructs the graph in a natural way, like producing the road network between cities generated by humans. It starts to check cities that are close to each other and adds a road between them if there is no short route between them on existing roads, and continues to check pairs that are farther apart. More formally, the original algorithm starts with a graph with no edges. Then, it considers all pairs of points in increasing order based on the distance between a pair of points. In other words, it sorts all pairs of points with respect to their increasing distances, and then processes them in that order. To process an edge (u,v), it performs a shortest path query in G between u and v, and if there is no t-path between u and v in G, i.e. the length of the shortest path between u and v is more than $t.|uv|$, then (u,v) is added to G, otherwise it is discarded. Since the original greedy algorithm checks $\binom{n}{2}$ pairs of points, and for each pair, it performs one shortest path query (using Dijkstra's algorithm implemented by Fibonacci heap), which takes $\mathcal{O}(\frac{n}{t-1}+n\log n)$ time (since there are $\mathcal{O}(\frac{n}{t-1})$ edges in G), the time complexity of the algorithm is $\mathcal{O}(\frac{n^3}{t-1}+n^3\log n)$ and it uses $\mathcal{O}(n^2)$ space.

The greedy approach can also be used to prune a given graph $G=(V,E)$, that is, instead of considering all pairs of points (see Algorithm 1), the algorithm only considers those which are endpoints of the edges in E.

2.2. The FG-greedy algorithm

Because of the cubic time complexity of the original greedy algorithm, it is practically impossible to construct greedy spanner on point sets with more than a few thousand points. For example, based on the experiments in [5,18], constructing the greedy 2-spanner on a set of 4,000 uniformly distributed points in the Euclidean plane, using the original greedy algorithm, takes more than 12,000 seconds and constructing 1.1-spanner on the same point set takes almost 42,000 seconds.

To make it possible to do experiments on larger point sets, Farshi and Gudmundsson [5,18] proposed a simple technique to improve the running time of the original greedy algorithm. The new algorithm, which we will call FG-greedy algorithm, works similar to the original greedy algorithm, except it uses a matrix to store the length of the shortest path between pairs of points. The matrix is updated when the algorithm performs a single-source shortest path query. Each time the algorithm wants to check if there is a t-path between a pair of points, it first checks the matrix to see if the path length between the points is less than t times the Euclidean distance between the points. If the path length is small enough, it skips the edge without performing the single-source shortest path query. Otherwise, it performs the shortest path query and then it updates the distance matrix and then decides whether the edge should be added to the graph or not (see Algorithm 2). Note that the distance matrix is not up-to-date, so if the distance

```
Input: V and t > 1.
Output: t-spanner G = (V, E).
E' : = all pairs of points in V sorted increasingly based on their distances;   /* ties are broken arbitrarily */
E := ∅;
G := (V, E);
foreach (u, v) ∈ E' do                                                          /* in sorted order */
  if Shortest Path Length (G, u, v) > t.|uv| then
    E := E ∪ {(u, v)};
  end
end
return G = (V, E);
```

Algorithm 1. The original greedy algorithm.

```
Input: V and t > 1.
Output: t-spanner G = (V, E).
foreach (u, v) ∈ V^2 do
  Weight (u, v) := ∞;
end
E' : = all pairs of points in V sorted increasingly based on their distances;   /* ties are broken arbitrarily */
E := ∅;
G := (V, E);
foreach (u, v) ∈ E' do                                                          /* in sorted order */
  if Weight (u, v) > t.|uv| then
    Compute the single-source shortest path with source u in G;
    foreach w ∈ V do
      update Weight (u, w) and Weight (w, u);
    end
    if Weight (u, v) > t.|uv| then
      E := E ∪ {(u, v)};
    end
  end
end
return G(V, E);
```

Algorithm 2. The FG-greedy algorithm.

between a pair of points is not small enough, it updates the matrix for the pair of points and then makes its decision.

As experiments show, [5,18], this improved algorithm uses around 35 and 100 seconds to compute 2-spanner and 1.1-spanner on a set of 4,000 uniformly distributed points which shows a big improvement. Using this improvement, the greedy spanner on point sets containing up to 13,000 points was constructed.

Note that, as shown by Bose et al. [7], this improvement does not improve the worst-case theoretical time complexity of computing greedy spanners.

2.3. The NQT-greedy algorithm

In 2008, Bose et al. [19] proposed a near-quadratic time algorithm to compute the greedy spanner which was based on the FG-greedy algorithm. Their algorithm makes the following changes. It partitions the $\binom{n}{2}$ pairs of distinct points in V into buckets, such that within each bucket, distances differ by at most a factor of two; then, the algorithm processes the buckets one after another.

Consider the current bucket contains all pairs whose distances are in the interval $[L, 2L)$. For each point u of V, it performs the bounded Dijkstra's algorithm with source u and distance $2tL$, and stores all operations performed by the Dijkstra's algorithm in a stack. Thus, for each vertex v, such that $\mathbf{d}_G(u,v) \leq 2tL$, the value of $\mathbf{d}_G(u,v)$ is known, which is stored as $weight(u,v)$ in the distance matrix. When the algorithm adds an edge (u,v) to the greedy spanner, it takes all points, p, for which the distance between p and one of the endpoints of the edge (u,v) is less than $(t-\frac{1}{2})L$ and updates corresponding distances. To update the distances, instead of running the bounded Dijkstra's algorithm with source p and distance $2tL$ from scratch, it uses the stack stored with p to *undo* the execution of the bounded Dijkstra's algorithm (in the graph prior to the insertion of the edge (u,v)) until the minimum key in the priority queue is at most $\min((t-\frac{1}{2})L, L)$. Then, it restarts Dijkstra's algorithm from this state, using the graph that contains the new edge (u,v), and terminate as soon as the minimum key in the priority queue is larger than $2tL$; during the execution, it stores the sequence of all operations in the stack associated with p.

A detailed description of the algorithm is given in Algorithms 3-5. The time complexity of this algorithm is $\mathcal{O}(n^2 \log n)$ and it uses $\mathcal{O}(n^2)$ space [7].

2.3.1. Implementation

The implementations of the first two algorithms are straight-forward. Implementing the third algorithm is a bit tricky, because of the undo/redo operations that have to be done when performing the Dijkstra's algorithm. We used some (extra) data structures to be able to do this operation without increasing the complexity of the algorithm.

We implemented the algorithms in C++ and compiled the code using -o argument. The data structures needed were implemented using the pseudo-codes of [20], some of which are with minor changes. To keep the graph, we used an adjacency matrix and to be able to find all neighbours of a vertex, we also kept all neighbours of each vertex in a linked list. Using this, we can check adjacency between vertices in constant time and we can traverse all neighbours of a vertex in time proportional to the degree of the vertex. The priority queue is implemented using a min-heap (see [20]), and to make it possible to find an specific vertex in the priority queue in constant time, we maintain an array of pointers which keep a pointer to vertex i in the priority queue in its ith entry of the array. The stack is implemented using a linked list. For the list of pairs of points, we use a structure that stores indices of the points with their Euclidean distance.

The experiments were done on two different machines. For point sets up to 4,000 points in the Euclidean plane, the code is compiled using G++ version 4.6 and runs on a Intel E2200 (2.2 GHz x2) machine with 3 GB of RAM and Ubuntu 12.04 operating system. For larger point sets, we needed more memory, so we ran the code on a Intel Xeon E5620 (2.4 GHz) with 24 GB RAM and Debian 6.0.3 operating system and G++ version 4.4.

The experiments are done on uniformly distributed point sets; for point sets up to 4,000 points the range of each coordinate is between 0 and 10,000, and for larger point sets, the range of each coordinate is between 0 and 30,000. For each case, we run the algorithms on 3 to 10 different point sets and we take average between them and use maximum and minimum values between them to see how much the difference between different point sets is.

Note that all algorithms mentioned here produce exactly the same networks, so we do not need to compare the networks generated by different algorithms.

3. Running time comparison

The results of the experiments are presented in Tables 1, 2 and 3 for different point sets and different t's. The time in all tables and diagrams are in seconds.

The first thing one can see in the tables is that the FG-greedy is faster than NQT-greedy algorithm in practice for all t's which are big enough. We expected this result, since the amount of overhead of the NQT-greedy algorithm is higher than the FG-greedy algorithm.

But, for low values of t, as shown in Figure 1, the NQT-greedy algorithm runs faster than the FG-greedy algorithm. The improvement is 50 percent for $t = 1.02$

Input: V and $t > 1$.
Output: t-spanner $G = (V, E)$.

foreach $u \in V$ **do**
    $weight\ (u, u) := 0$;
end
foreach $(u, v) \in V^2$ with $u \neq v$ **do**
    $weight\ (u, v) := \infty$;
end
E' := list of all pairs of distinct points in V, sorted in non-decreasing order of their distances;
$i := 1$;
while $E' \backslash (\bigcup_{k-1}^{i-1} E_k) \neq \varnothing$ **do**
    L_i : = distance of the shortest pair in $E' \backslash (\bigcup_{k=1}^{i-1} E_k)$;
    E_i : = sorted list of all pairs in $E' \backslash (\bigcup_{k=1}^{i-1} E_k)$ whose distances are in $[L_i, 2L_i)$;
    $i := i + 1$;
end
$l := i - 1$;
$E := \varnothing$;
$G := (V, E)$;
foreach $u \in V$ **do**
    PQ_u := priority queue storing all $v \in V$ with key $weight\ (u, v)$;
    τ_u := empty stack;
end
for $i := 1, \cdots, l$ **do**
    foreach $u \in V$ **do**
        if $i > 1$ **then**
            DIJKSTRA-Undo $\left(\tau_u, PQ_u, \left(t - \frac{1}{2}\right) L_{i-1}\right)$;
        end
        DIJKSTRA-Bounded $(G, u, 2tL_i, PQ_u, \tau_u)$;
    end
    foreach $(u, v) \in E_i$ (in sorted order) **do**
        if $weight\ (u, v) > t.|uv|$ and $weight\ (v, u) > t.|uv|$ **then**
            $E := E \cup \{(u, v)\}$;
            foreach $p \in V$ **do**
                if $|pu| < \left(t - \frac{1}{2}\right) L_i$ or $|pv| < \left(t - \frac{1}{2}\right) L_i$ **then**
                    if $weight\ (p, u) + |uv| < weight(p, v)$ **then**
                        DIJKSTRA-Undo $\left(\tau_p, PQ_p, \min\left(\left(t - \frac{1}{2}\right) L_i, L_i\right)\right)$;
                        in PQ_p, decrease the key of v to $weight\ (p, u) + |uv|$;
                        $weight\ (p, v) := weight(p, u) + |uv|$;
                        DIJKSTRA-Bounded $(G, p, 2tL_i, PQ_p, \tau_p)$;
                    end
                    if $weight\ (p, v) + |uv| < weight(p, u)$ **then**
                        DIJKSTRA-Undo $\left(\tau_p, PQ_p, \min\left(\left(t - \frac{1}{2}\right) L_i, L_i\right)\right)$;
                        in PQ_p, decrease the key of u to $weight\ (p, v) + |uv|$;
                        $weight\ (p, u) := weight(p, v) + |uv|$;
                        DIJKSTRA-Bounded $(G, p, 2tL_i, PQ_p, \tau_p)$
                    end
                end
            end
        end
    end
end
return $G(V, E)$;

Algorithm 3. The Near-Quadratic Time (NQT) greedy algorithm.

Input: graph G, vertex s, real number $L > 0$, priority queue PQ, stack τ.
Output: using PQ, continue Dijkstra's algorithm with source s until all shortest-path distances in G which are at most L have been computed; the algorithm stores all operations in τ (the pseudocode does not explicitly mention this).

while the minimum key in PQ is at most L **do**
    delete the element u with minimum key from PQ;
    foreach node v adjacent to u in G **do**
        if $weight\ (s, u) + |uv| < weight(s, v)$ then
            in PQ, decrease the key of v to $weight\ (s, u) + |uv|$;
            $weight\ (s, v) := weight(s, u) + |uv|$
        end
    end
end

Algorithm 4. A bounded version of the Dijkstra's algorithm.

```
Input: stack τ, priority queue PQ, real number L > 0.
while the minimum key in PQ is larger than L do
    pop the top element c from τ;
    undo the changes based on the information in c;
end
```

Algorithm 5. The algorithm to undo previous operations of the Dijkstra's algorithm.

Table 1. The running time (in seconds) of different greedy algorithms for different t's and point sets up to 4,000 points.

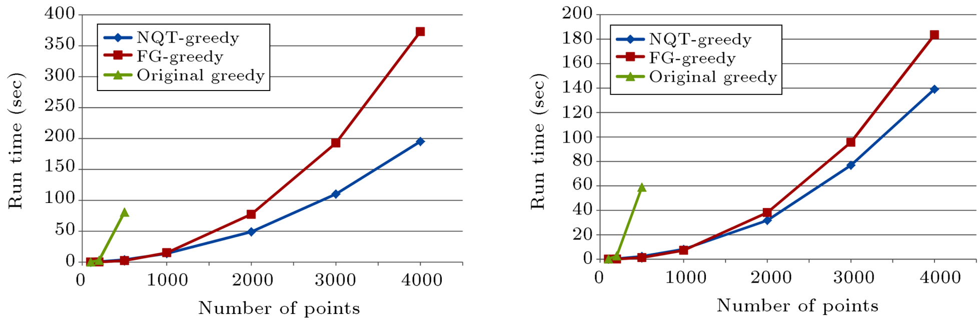
t	# Points	NQT-greedy			FG-greedy			Original greedy		
		Average	**Min**	**Max**	**Average**	**Min**	**Max**	**Average**	**Min**	**Max**
1.05	100	0.10	0.10	0.11	0.03	0.02	0.03	0.28	0.27	0.29
1.1	100	0.07	0.07	0.08	0.02	0.02	0.02	0.26	0.25	0.27
1.2	100	0.05	0.05	0.06	0.01	0.01	0.02	0.24	0.24	0.25
1.4	100	0.05	0.04	0.05	0.01	0.00	0.01	0.23	0.22	0.24
1.6	100	0.04	0.04	0.04	0.01	0.01	0.01	0.21	0.21	0.22
1.8	100	0.04	0.04	0.05	0.00	0.00	0.01	0.21	0.20	0.21
2	100	0.04	0.03	0.04	0.01	0.00	0.02	0.22	0.20	0.24
1.05	500	2.30	2.10	2.38	1.31	1.24	1.35	58.85	56.28	60.05
1.1	500	1.56	1.35	1.66	0.77	0.75	0.79	44.56	43.79	45.58
1.2	500	1.22	1.09	1.29	0.54	0.52	0.58	40.14	38.90	41.53
1.4	500	1.05	1.01	1.08	0.38	0.36	0.39	36.42	36.10	36.86
1.6	500	1.00	0.96	1.04	0.32	0.31	0.34	34.47	33.84	35.31
1.8	500	0.96	0.90	0.98	0.30	0.28	0.31	34.87	33.19	38.15
2	500	0.95	0.93	0.97	0.28	0.27	0.29	34.05	32.83	38.02
1.05	1000	8.13	7.60	8.49	7.29	7.02	7.43	712.34	698.98	722.84
1.1	1000	6.50	5.96	6.98	4.47	4.18	4.83	529.41	512.33	546.82
1.2	1000	4.95	4.39	5.32	2.81	2.79	2.87	407.11	398.36	426.22
1.4	1000	4.25	4.08	4.48	1.92	1.87	2.06	342.29	335.75	362.38
1.6	1000	4.05	3.92	4.10	1.57	1.54	1.64	311.99	306.22	323.72
1.8	1000	3.83	3.71	3.96	1.40	1.36	1.45	298.28	295.92	304.04
2	1000	3.86	3.74	3.92	1.33	1.29	1.39	290.52	287.81	296.97
1.05	2000	31.78	30.48	34.13	38.17	37.60	39.18	-	-	-
1.1	2000	24.98	23.20	28.02	23.10	22.19	24.04	5766	5667	5907
1.2	2000	20.16	18.28	22.09	14.18	13.85	14.37	4503	4431	4612
1.4	2000	18.19	17.44	18.91	9.28	8.87	9.71	3605	3541	3639
1.6	2000	16.76	15.77	17.32	7.71	7.22	8.04	3285	3096	3450
1.8	2000	16.25	15.52	16.60	6.59	6.52	6.64	2939	2913	2977
2	2000	15.58	15.16	15.89	6.14	5.98	6.30	2904	2849	2999
1.05	4000	139.11	127.08	147.13	183.60	179.70	188.93	-	-	-
1.1	4000	107.15	99.64	112.47	111.72	109.23	113.11	-	-	-
1.2	4000	92.76	89.40	96.43	70.27	68.84	72.31	-	-	-
1.4	4000	81.78	80.03	83.01	47.09	46.26	47.69	-	-	-
1.6	4000	71.85	69.63	75.01	38.46	37.17	39.59	-	-	-
1.8	4000	70.34	67.66	71.34	34.17	33.36	34.66	-	-	-
2	4000	68.54	68.22	69.30	31.55	30.75	32.37	-	-	-

Table 2. The running time (in seconds) of different greedy algorithms for different t's and point sets between 4,000 and 10,000 points.

# Points	t	NQT-greedy			FG-greedy		
		Average	Min	Max	Average	Min	Max
4000	1.2	79.16	71.01	82.58	52.21	50.69	53.14
5000	1.2	125.99	117.36	132.01	97.25	95.49	98.51
6000	1.2	189.66	172.03	210.59	141.62	138.16	144.86
8000	1.2	341.87	300.64	369.78	272.96	262.92	288.20
10000	1.2	552.69	503.64	588.29	454.21	445.26	461.40
4000	1.6	64.94	62.61	66.01	30.86	30.26	31.16
5000	1.6	100.85	96.85	106.63	51.03	50.61	52.25
6000	1.6	154.50	147.87	156.55	75.44	74.99	76.41
8000	1.6	271.16	257.74	284.87	145.73	142.05	151.29
10000	1.6	445.26	423.18	459.23	245.15	241.75	249.70
4000	2	58.27	57.22	59.71	25.43	24.87	25.90
5000	2	94.62	92.88	95.91	41.49	41.10	41.91
6000	2	140.05	137.80	141.40	61.55	60.84	62.41
8000	2	252.35	245.62	259.85	119.15	116.86	121.22
10000	2	408.48	400.75	412.49	199.58	195.75	201.54

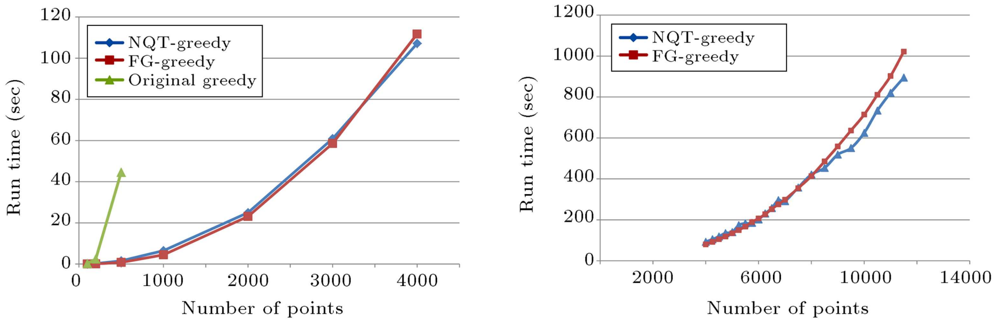
Table 3. The running time (in seconds) of different greedy algorithms for $t = 1.1$ and point sets between 4,000 to 12,000 points.

# Points	NQT-greedy			FG-greedy		
	Average	Min	Max	Average	Min	Max
4000	92.69	81.87	99.2	80.23	79.47	81.59
4250	105.15	96.39	110.82	92.47	91.05	93.23
4500	118.79	110.23	123.87	105.09	104.34	106.1
4750	135.53	132.75	138.93	118.97	118.46	119.6
5000	140.13	140.13	140.13	134.53	134.53	134.53
5250	173.96	171.19	176.72	150.21	148.36	152.06
5500	183	164.25	198.06	166.96	166.63	167.24
5750	185.38	172.36	195.22	189.27	185.58	194.11
6000	201.9	186.54	230.86	207.02	205.94	207.97
6250	231.24	212.48	241.87	227.25	225.71	229.01
6500	257.32	249.68	272.27	252.62	248.89	254.79
6750	296.72	280.39	307.89	275.88	275.69	276.05
7000	291.66	262.57	309.64	298.93	294.64	304.43
7500	357.79	313.06	381.77	354.96	353.2	355.94
8000	421.41	398.03	440.03	413.96	411	417.98
8500	453.82	421.89	489.69	485.4	479.95	489.27
9000	518.84	495.74	538.07	559.1	553.38	563.87
9500	550.65	538.63	562.8	636.53	630.95	643.44
10000	624.94	593.38	649.05	714.04	709.55	719.32
10500	733.63	695.52	768.48	810.66	805.91	814.42
11000	820.26	748.16	886.68	902.61	899.47	906.71
11500	895.16	837.77	977.79	1022.11	996.05	1052.36



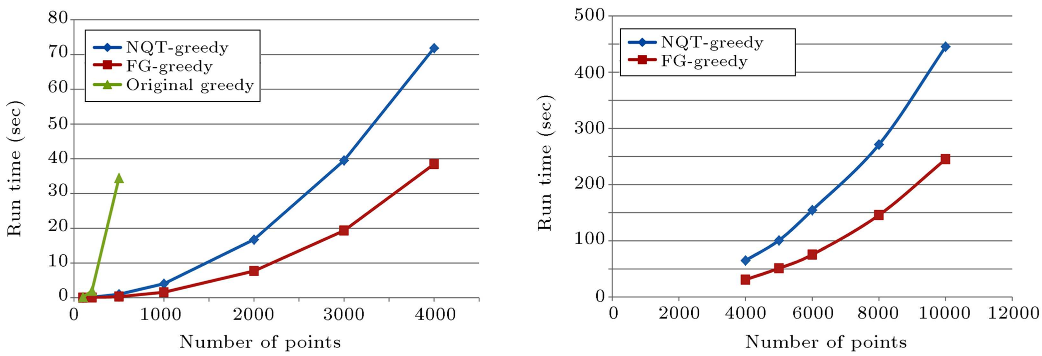
(a) $t = 1.02$ (a) $t = 1.05$

Figure 1. Comparing running time of the greedy algorithms for $t = 1.02$ and $t = 1.05$ for point sets up to 4,000 points.



(a) $t = 1.1$ (b) $t = 1.1$

Figure 2. Comparing running time of the greedy algorithms for $t = 1.1$.



(a) $t = 1.6$ (b) $t = 1.6$

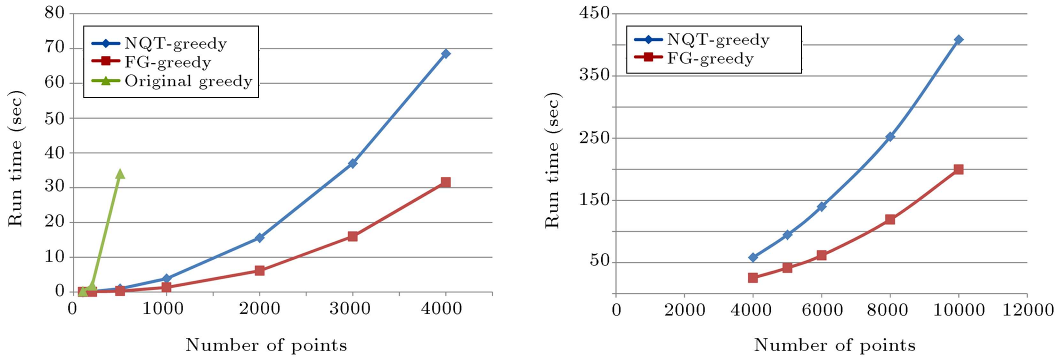
Figure 3. Comparing running time of the greedy algorithms for $t = 1.6$.

on a point set with 4,000 uniformly distributed points, and for $t = 1.05$, this improvement is 40 seconds on the same set.

As shown in Figure 2 and Table 3, the FG-greedy and the NQT-greedy algorithm take almost the same time, but the NQT-greedy runs faster for large enough point sets, say sets of more than 8,000 points.

For larger values of t, say $t \geq 1.2$, the FG-greedy algorithm is faster, and the difference increases when t grows. However, as one can see in Figures 3 and 4, the running time of the NQT-greedy algorithm grows slower than the running time of the FG-greedy algorithm, which means that one can expect that the NQT-greedy algorithm works faster if the point set is large enough, say with a few million points which is consistent with the analytical bounds of the algorithms on uniformly distributed points. We were not able to do experiments on larger point sets because they would have needed a lot of memory. For applications with large point sets, the space complexity is one of the major bottlenecks, so we suggest to use an algorithm with linear-space complexity introduced recently (see [8,9]). The time complexity of this linear-space algorithm is slightly more than the NQT-greedy algorithm, but because of its low space complexity, it works much better on large point sets.

From the application perspective, one might want to know, for his special case, which algorithm works faster. To see this, based on experiments, there is a diagram in Figure 5, which shows which algorithm



(a) $t = 2$ (b) $t = 2$

Figure 4. Comparing running time of the greedy algorithms for $t = 2$.

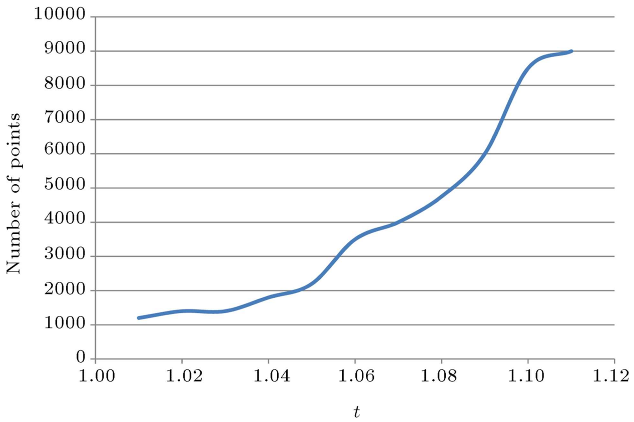


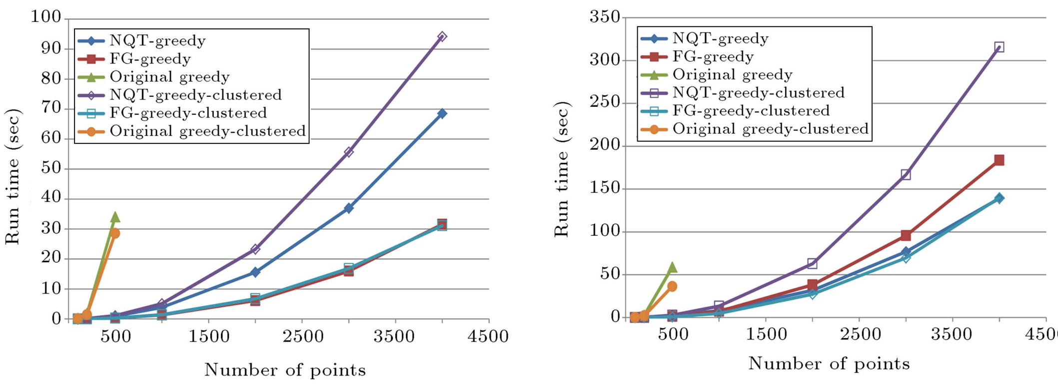
Figure 5. In the cases above the graph, the NQT-greedy algorithm runs faster than the FG-greedy algorithm.

works faster based on the number of points and the value of t. In all cases that lie below the graph, the FG-greedy algorithm runs faster and above the graph the NQT-greedy algorithm runs faster.

To see how sensitive are the greedy algorithms to the distribution of input point sets, we tested the algorithms on clustered point sets too. To make a clustered point set with n points, we selected $\sqrt{n}$ uniformly distributed squares and in each square we put $\sqrt{n}$ points. As seen in Figure 6, the FG-greedy algorithm works almost the same for uniform and clustered point sets for $t = 2$. For lower values of t, say $t = 1.05$, the FG-greedy algorithm is faster on clustered point sets. However, the NQT-greedy algorithm is slower on clustered point sets and as t decreases, the difference between running time of the algorithm on uniformly distributed point sets and clustered point sets increases. As seen in Table 4, the ratio of the time used by the NQT-greedy algorithm to the time used by the FG-greedy algorithm decreases when we decrease t or increase n. However, its decrease rate is small, but one can expect that for sufficiently large point sets and for low values of t, the NQT-greedy algorithm runs faster than the FG-greedy algorithm.

4. Space usage comparison

As seen in the algorithms, the FG-greedy algorithm needs more space compared to the original greedy algorithm, since it uses a matrix to store distance between all pairs of points in the generated graph.



(a) $t = 2$ (b) $t = 1.05$

Figure 6. Comparing running time of the greedy algorithms for $t = 2$ and $t = 1.05$ between uniformly distributed and clustered point sets.

Table 4. The running time (in seconds) of different greedy algorithms for clustered point sets (1st and 2nd columns) and the ratio between them (3rd column).

t	# Points	NQT-greedy	FG-greedy	NQT-greedy/FG-greedy
1.05	1000	13.35	4.83	2.76
1.05	2000	62.82	27.30	2.30
1.05	3000	166.79	69.66	2.39
1.05	4000	315.82	139.89	2.26
1.1	1000	9.88	3.44	2.87
1.1	2000	44.44	19.86	2.24
1.1	3000	105.89	45.49	2.33
1.1	4000	202.25	87.90	2.30
1.4	1000	5.37	1.81	2.96
1.4	2000	25.40	9.86	2.58
1.4	3000	59.70	23.09	2.59
1.4	4000	107.88	43.85	2.46
1.8	1000	4.96	1.46	3.40
1.8	2000	22.72	7.20	3.16
1.8	3000	49.97	17.96	2.78
1.8	4000	93.61	32.38	2.89
2	1000	5.09	1.46	3.50
2	2000	23.26	6.75	3.44
2	3000	55.68	16.85	3.30
2	4000	94.24	31.12	3.03

Table 5. The space usage of different greedy algorithms for different t's and point sets up to 4,000 points.

# Points	Algorithm	$t = 1.05$	$t = 1.1$	$t = 1.2$	$t = 1.5$	$t = 2$
$n = 2000$	Original greedy	61.6 MB	61.5 MB	61.4 MB	61.4 MB	61.3 MB
	FG greedy	92.2 MB	92.0 MB	92.0 MB	91.9 MB	91.9 MB
	NQT greedy	547.8 MB	511.7 MB	479.6 MB	451.7 MB	440.9 MB
$n = 4000$	FG greedy	367.3 MB	367.1 MB	366.9 MB	366.8 MB	366.7 MB
	NQT greedy	2.2 GB	2 GB	1.9 GB	1.8 GB	1.7 GB

The NQT-greedy algorithm uses extra space to keep all operations performed by Dijkstra's algorithm with source at any point to be able to make undo and fixing the shortest path computation after adding an edge to the graph. It is therefore expected that this algorithm uses a large amount of memory. As shown in Table 5, the experiment shows this behaviour, but the memory used by the NQT-greedy algorithm is much more than what is expected. Based on the experiments, memory usage of the FG-greedy algorithm is 1.5 times the memory used by the original greedy algorithm. This amount blows up to 5-6 times more space when using the NQT-algorithm. So the main obstacle of using the NQT-greedy algorithm is its memory usage, and any improvement on this will be the main interesting future work.

5. Future works

As for future works, one can study improving the space complexity of the NQT-greedy algorithm, or improving the worst-case time complexity of one of the recently introduced linear-space algorithms, see [8,9,21]. Another interesting question is to design an algorithm for constructing the greedy spanner in I/O-model which is an interesting model for processing big data.

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