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Research Note

Development of empirical equations for prediction of modulus of elasticity for monodisperse metallic foams

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Abstract. Metallic foams are a class of lightweight materials that show high potential for different industrial applications such as automotive and aerospace engineering. However, many factors have prevented metallic foams from being fully utilized in industrial applications. One main factor is that the influences of the porous structure on the mechanical properties of metallic foams are not well known yet. In this paper, a finite element model was used to analyze monodisperse closed cell aluminum foam in order to determine the relationship between its elastic modulus, porosity and pore diameter, and a nonlinear relationship was found between them. In this regard, an empirical equation was developed and implemented for the prediction of modulus of elasticity versus porosity and pore diameter. Comparing experimental and analytical approaches by other researchers with those obtained in this research shows that agreement exists within 1 to 3 percent. Therefore, this model could be further used for other porous structures. It was also found that a similar empirical equation with new coefficients can also be used to predict the modulus of elasticity of cast iron and steel foams.

1. Introduction

Metallic foams are solid, low density, cellular materials containing a volume fraction of gas-filled pores [1]. Because of their unique properties, such as low density, high energy absorption capacity, high specific strength to density ratio and high elastic modulus to weight ratio, they have recently gained considerable attention in fields where low weight, high stiffness and high energy absorption are strict design parameters [2,3,4]. Especially, metallic foams are considered to be an interesting material class in automotive and aerospace industries as they are excellent materials for weight reduction and safety purposes [5-11]. Metallic foams are usually characterized by their cell topology: open cell and closed cell [12]. Open cell foams contain connected pores which usually occupy more than 75% of the whole volume, while closed cell foams consist of isolated gas-filled pores [13]. In recent years, many different processes have been developed for manufacturing metallic foams. Some of these methods include Solid Laser Sintering (SLS) and melt foaming. These methods are undergoing continuous improvements to produce higher quality materials and to achieve better predictability of their properties [4,7,14,15].

Because of the already available infrastructure for using steel and aluminum foams in industries, they are far more convenient than other materials for use in previously mentioned industrial applications [3]. Using metallic foams in industrial applications demands the

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full characterization of their mechanical properties such as modulus of elasticity, plateau stress, crush load and relative density. Full mechanical characterization of metallic foams is quite a difficult task as the foams properties are a function of its base material, cell topology, relative density, cell size and cell shape [4]. In this regard, metallic and polymeric foams have been subject to numerous experimental, analytical and theoretical investigations.

Nammi et al. used a finite element model to determine the crush resistance of closed cell aluminum foams [16]. Wang et al. determined the plateau stress and crushing behavior of aluminum foam filled hat sections using theoretical analysis [17]. De Giorgi et al. compared finite element analysis with experimental results to determine a suitable numerical model for FEA. They proved that Kelvin cells are not accurate enough and suggested elliptical cells defined by random dimensions, which were in good agreement with experimental results [18]. Experimental studies indicate that at low porosities, mechanical properties of metallic foams can be described using the following simple scaling equations:

$$\frac{E^*}{E} = \alpha(\frac{\rho^*}{\rho})^n, \tag{1}$$

where E^* and E are the modulus of elasticity of the foam and its base material, and ρ^* and ρ are their densities, respectively [19]. It should be noted that mechanical properties of metallic foams could generally be described using Eq. (1). Simone and Gibson used finite element analysis to determine the equivalent elastic modulus of closed cell aluminum foams as a function of relative density. They extracted a second-order relationship for the elastic modulus, as follows:

$$\frac{E^*}{E} = \varphi^2(\frac{\rho^*}{\rho})^2 + (1-\varphi)\frac{\rho^*}{\rho}, \tag{2}$$

where E and ρ are elastic modulus and density of aluminum, E^* and ρ^* are those of the foam, and φ is the volume fraction of solid contained in the cell edges [20]. Although this model has often been used as a basis for aluminum foam analysis, it lacks consideration of the effect of cell size on the equivalent elastic modulus. However, Hai-Jun et al. studied the behavior of Al-Si closed-cell aluminum foam experimentally and reached relationships between the modulus of elasticity and relative density of the foam that could approximately be explained using the Gibson-Ashby model [21].

Nammi et al. constructed two finite element models for aluminum foam: Cruciform-pyramidal and cubic-spherical. They determined the equivalent elastic modulus of each model as follows [16]:

$$\frac{E^*}{E}(\text{cruciform} - \text{pyramidal}) = 0.69(\frac{\rho^*}{\rho})^{1.03}, \tag{3}$$

$$\frac{E^*}{E}(\text{cubic} - \text{spherical}) = 0.63(\frac{\rho^*}{\rho})^{1.04}. \tag{4}$$

Konstantinidis et al. constructed an analytical three-dimensional model for closed cell aluminum foams considering the cell geometry (Circular, elliptic, rectangular and square). They gained the equivalent elastic modulus for each model having $\rho^*/\rho = 0.6$. They also compared their results with quasi-static experimental data gained from elliptical cell shaped foams [22]. Gong et al. used an extended Mori-Tanaka model to predict the elastic behavior of closed cell aluminum foams having different cellular structures. The model was then compared with experimental data gained from AlCuMn alloy and found out to be a good approximation of aluminum foam models [23].

According to Cao et al., elastic modulus and compressive strength of foams depend not only on the porosity, but also on the cell size of the selected foam [24]. Foams containing different sized/shaped pores (polydisperse materials) are usually difficult to be analyzed using theoretical or numerical methods because of having different pore sizes/shapes. However, there is another kind of porous material - called monodisperse material - which contains pores having a single size and shape. For these foams, the elastic modulus can be determined as a function of porosity and pore size. A technique used for making monodisperse foams is by blowing gas at a constant pressure into the foaming solution through a circular nozzle. If the gas flow rate is low enough, bubbles are generated in a series of quasi-static states. This procedure is illustrated in Figure 1. The bubble size of the foam

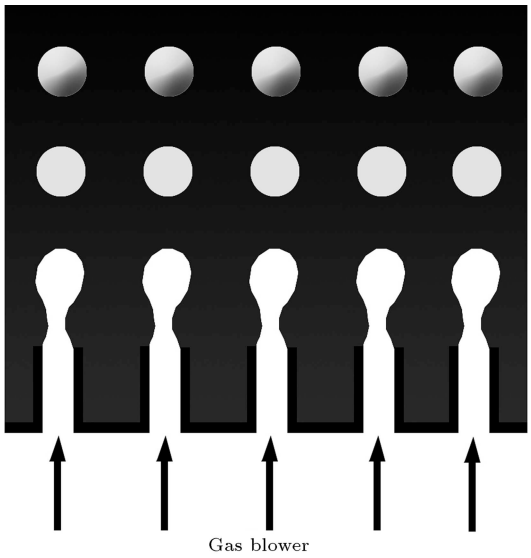

Figure 1. Production of monodisperse aluminum foam using gas-blowing technique.

made by this technique has the order of magnitude of millimeters [25].

In this paper, three-dimensional rectangular structure models are introduced to represent monodisperse aluminum foams having different porosities and pore sizes. By using finite element analysis, a static compression load is applied to each model and displacement (and, therefore, the equivalent elastic modulus) of each model is determined. A suitable formula representing the equivalent elastic modulus as a function of porosity and pore size of the models is suggested by using a curve-fitting method. The formula is then compared to results obtained by experimental [11,19] and analytical [23] findings. Finally, in a similar manner for steel and grey cast iron foams, new empirical equations are derived.

2. Modeling

Finite element analysis of the model is performed using commercial FEM software. The basic structure for the analysis includes a 20*20*10 mm block of aluminum having equal sized pores with uniform distribution. Zhang et al. [19] studied blocks of commercially available closed-cell aluminum foams and found out that the pore diameters vary between 0.3 and 2.4 millimeters. Therefore, in order to make an acceptable assumption of pore size, spherical pores with four different diameters of 1.5, 1.75, 2 and 2.25 millimeters were studied in this research. As shown in Figures 2 and 3, for achieving a more realistic model and higher porosities, spherical pores were distributed in the structure using a space-filling algorithm. Analysis is performed based on different pore size and porosity in the model.

A 3-dimensional 20-node rectangular element is used for the analysis of the structure. The elements are refined until convergence is achieved. As the imposed boundary condition, it is assumed that the bottom surface of the structure is fixed. A static compressive force of 1000 Newtons is applied to the top surface and the displacement of the top surface is obtained (Figure 4).

In order to find the quality and repeatability of any numerical experiment, grid independency of results must be investigated. Achieving grid convergence confirms that a finer mesh will not improve the solution. In this study, grid independence was investigated by refining meshes until the difference between solutions from the two finest meshes was less than 3 percent. Figure 5 shows the results of grid independence analysis for the model with 1.5 mm pores.

To determine the equivalent elastic modulus of each model, a similar sized solid model (20*20*10 mm) is analyzed using the same 20-node rectangular element. With the same boundary conditions applied to the bottom surface and a 1000 Newtons compressive force applied to the top surface, by changing the elastic modulus of the model and making the displacement of the top surface equal to the displacement of each porous model, the equivalent elastic modulus is achieved. This process is applied to all models and the results are obtained.

3. Results and discussion

Using the procedure described in the previous section, the equivalent elastic modulus of monodisperse alu-

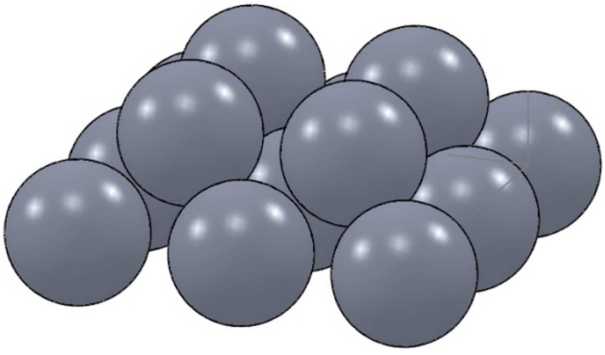

Figure 2. Pore exposure in the block.

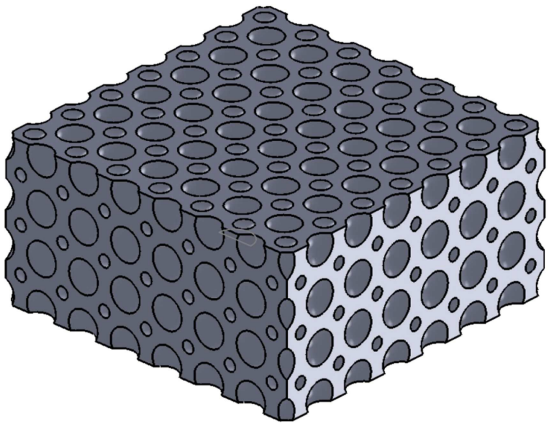

Figure 3. Typical model used for analysis.

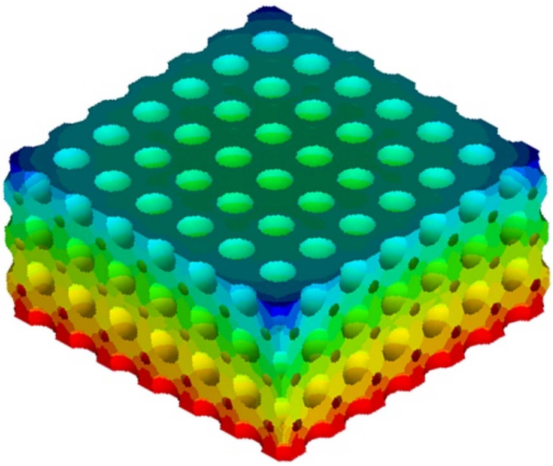

Figure 4. The deformed model.

minum foam for each pore diameter is determined and plotted, with respect to porosity, into a single diagram (Figure 6(a)-(d)).

By plotting all results into a three-parameter diagram (Figure 7) and using the curve-fitting method, a suitable formula showing the relationship between the elastic modulus, pore size and porosity is obtained as follows:

$$\frac{E^*}{E} = \frac{-1.442}{d} + 1.275\exp(-1.464r) + \frac{1.097}{d^2}, \qquad (5)$$

where r is the porosity and d is the pore diameter in mm.

The results of findings in this paper are compared with the normalized elastic modulus versus the porosity of porous aluminum with equal sized spherical pores represented by Gong et al. [23] (Figure 8). It is clear that the trend of the two curves is in good agreement with 5.1% difference. The experimental results made by Konstantinidis et al. [22] for spherical pores compared to data obtained from the present formula are presented in Table 1 for the 40% porosity. These comparisons indicate that the obtained formula has the potential for use as a reference in determining the mechanical properties of metallic foams. Although this model is only valid for monodisperse metallic foams with spherical pores, future studies could work on different porous structures to obtain a more general understanding of porous materials and their properties.

The procedure described for determining the equivalent elastic modulus of porous aluminum is also

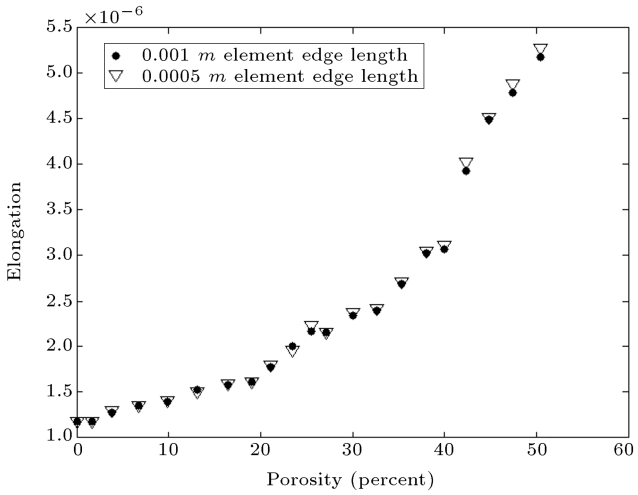

Figure 5. Comparison between results of structural analysis using two consecutive steps of mesh refinement.

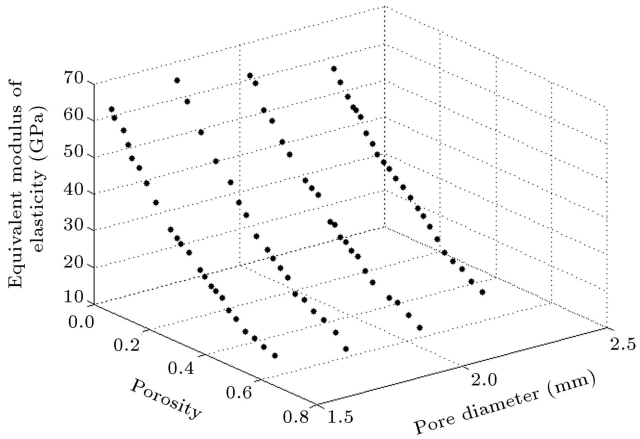

Figure 7. Three-parameter diagram featuring all results.

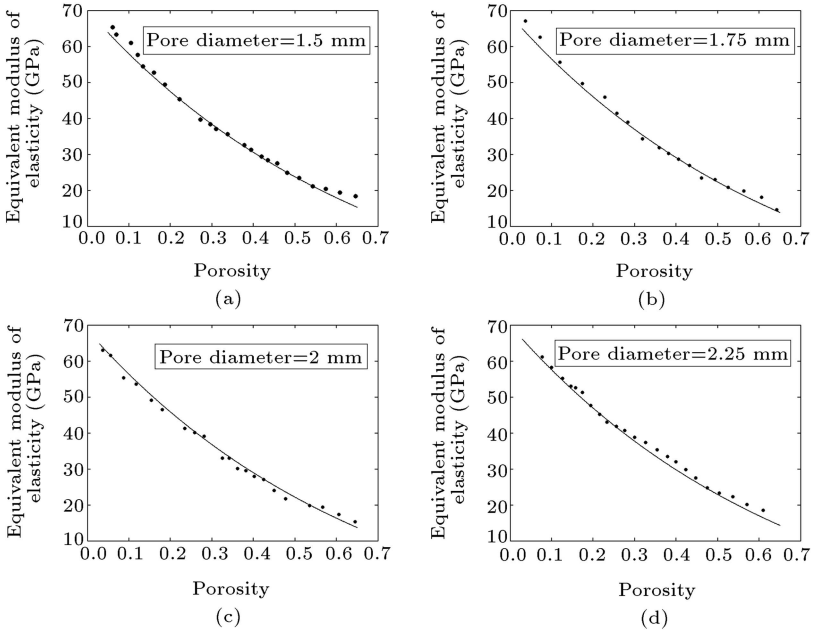

Figure 6. Equivalent elastic modulus of porous aluminum having pore diameters of 1.5, 1.75, 2 and 2.25 mm with respect to porosity.

Table 1. Comparison between the elastic modulus found by the present model and Konstantinidis et al. [22].

Present work (d=1.5 mm)	31 GPa
Present work (d=1.75 mm)	29.5 GPa
Present work (d=2 mm)	29.3 GPa
Present work (d=2.25 mm)	29.8 GPa
Result obtained by Konstantinidis et al. [22]	30.9 GPa

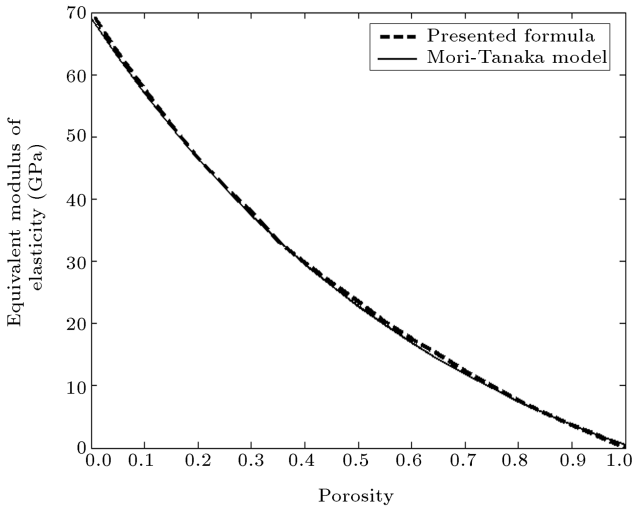

Figure 8. Comparison between Mori-Tanaka analytical results and presented numerical formula.

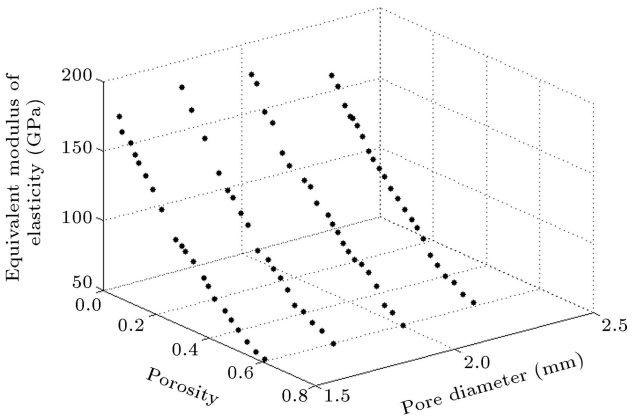

Figure 9. Results for steel foam.

applied to steel and gray cast iron in order to find a relationship for E as a function of r and d. It is found that the same form of relationship is applicable for steel and gray cast iron foams with different coefficients. Figures 9 and 10 present three-dimensional plots for the modulus of elasticity of steel and gray cast iron versus porosity and pore size. The modulus of elasticity formulation in terms of porosity and pore size for steel and grey cast iron is found to be as follows:

$$\frac{E^*}{E} = \frac{-0.751}{d} + 1.195\exp(-1.558r) + \frac{0.651}{d^2}, \qquad (6)$$

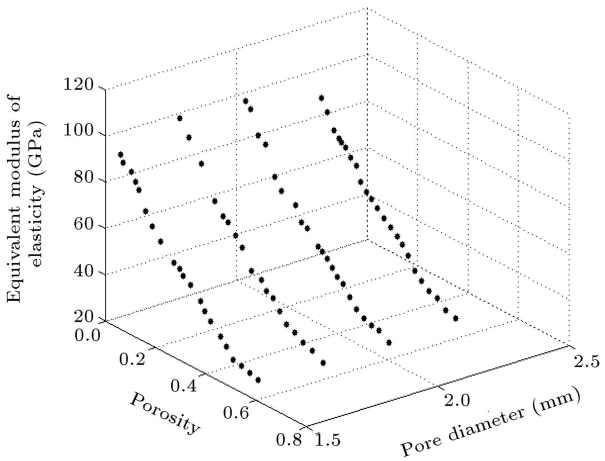

Figure 10. Results for gray cast iron foam.

for steel, and:

$$\frac{E^*}{E} = \frac{-0.526}{d} + 1.139\exp(-1.718r) + \frac{0.422}{d^2}, \qquad (7)$$

for gray cast iron.

Comparing the results of the empirical equations and numerical findings indicates a maximum difference of 5 and 10 percent for steel and grey cast iron foams, respectively. The predicted formulation for aluminum, steel and grey cast iron indicates that a similar trend with proper coefficients could be associated with more monodisperse metallic foams.

4. Conclusion

In this paper, empirical equations are developed to evaluate the modulus of elasticity of aluminum, cast iron and steel foams as a function of porosity and pore diameter. Using the developed formula, the predicted modulus of elasticity of aluminum foam and other materials lies within 1 to 3 percent of the analytical and/or experimental findings. These results indicate that general formulas could be obtained that describe the mechanical properties of different classes of metallic foams.

The developed formulae can be generalized to other monodisperse metallic foams as well. Results of this study indicate that general models could be implemented to describe the mechanical properties of metallic foams based on their porous structures, without the immediate need for laboratory testing. Much more work still needs to be done on this subject in order to achieve more general formulas that can be used directly to describe the mechanical behavior of metallic foams.

Nomenclature

E^*	Modulus of elasticity of metallic foam (Pa)
E	Modulus of elasticity of foams base metal (Pa)
ρ^*	Foam density (kg/m^3)
ρ	Density of foams base metal (kg/m^3)
φ	Volume fraction of solid contained in foams cell edges
d	Diameter of monodisperse pores (mm)
r	Porosity of the monodisperse foam

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