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Analytic investigation of the effects of condensation shock on turbulent boundary layer parameters of nucleating flow in a supersonic convergent-divergent nozzle

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Abstract. Under the influence of intense expansion and supersonic acceleration of the steam flow in a divergent channel, the instability of the flow intensifies. In the lack of external surfaces, this non-equilibrium state causes nucleation and consequent growth of formed nuclei. Due to the release of latent heat from condensation to the supersonic flow at the location of nucleation, an increase in pressure is developed in this small region, which is known as condensation shock. In this research, the effects of this shock on boundary layer parameters are investigated. First, the water vapor flow that has the capability of nucleation is modeled analytically, as adiabatic, inviscid and one dimensional, and then, using the mathematical equations of laminar and turbulent boundary layer parameters and the inviscid-viscous Interaction method. The results of this analytical modeling show that although the influence of the boundary layer on the expansion flow is limited, it still causes approximately 3% increase in the diameter of the water droplets. However, the effects of two-phase flow on the boundary layer parameters at the location of the condensation shock are considerable. The major novelty of this research is determining the quantification and qualification of these effects.

1. Introduction

Due to the relatively small thickness of the boundary layer, modeling of real flows in supersonic expanding divergent-convergent channels can be undertaken by several methods. Based on the boundary layer theory first presented by Prandtl, due to the existence of viscosity in real flows, the flow can be divided into a boundary layer region near the surface and an inviscid flow, which allocates the majority of the path (especially for expanding flows). By this approach, the set of flow equations is transformed into two sections, namely, boundary layer equations (laminar and turbulent) and Euler (two-phase flow), as found in [1-5].

Expanding the one-dimensional model to two or three-dimensional versions is very complicated (compared to conventional numerical methods). Despite the vast research done in this area, the analysis and exact modeling of the two-phase nucleating phenomenon (without considering parameters such as complex flow geometry), for the reasons below, are still required for development of the mathematical (exact) one-dimensional nucleating flow model:

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a) For verification and validation of conventional numerical 2D and 3D methods (Time Marching techniques), under conditions where test cases are not available or do not exist (which, due to serious lack of test cases in this area, is usual). Using the theoretical results of the exact solution of the 1D model, it is possible to compare 2D and 3D models (by implementing considerations) and substitute 1D results with the explained method for laboratory data.

b) Work is still needed on improving the nucleation and growth equations (which are two important parts of modeling two-phase gas-liquid flow). After applying the improvements, they can be implemented in a 1D mathematical model and the improved results can be compared to available test cases, because, in these kinds of improvement, only the two-phase phenomenon should be analyzed and studied to validate the proposed models.

c) Other sources of uncertainty in modeling two-phase gas-liquid flow are the thermo-physical properties at different locations of the flow (such as the surface tension of tiny droplets and the physical and thermodynamic properties of unstable supercooled vapor at the nucleation region) and also a suitable state equation for the non-equilibrium supercooled vapor. Therefore, there is still a need for improving the values of thermo-physical properties, and, using the 1D model, we can validate the new values for thermo-physical properties using available test cases.

In this research, the adiabatic flow of supersonic inviscid two-phase vapor is modeled in one dimension using Euler equations. The pressure ratio is selected in such a way that the flow will be free of aerodynamic shocks and the fluid remains supersonic in the diverging section of the channel. It is to be noted that at the shock region, which happens after the throat of the channel, only the shock of condensation occurs. This is a considerable pressure increase in a small length of channel. But, despite this increase of pressure, the fluid remains supersonic two-phase [6-8]. It is to be noted that the flow is single phase in the convergent section.

For calculating the boundary layer parameters, such as δ, δ^*, θ and C_f, first, the inviscid two-phase flow is solved and then the results of this solution are used as initial input data for the integral equations of the boundary layer. Afterwards, using the inviscid-viscous interaction method (used in some references, for example [3-5]), the effects of the boundary layers on two-phase flows and, especially, the effects of the condensation shock on boundary layer parameters, are calculated, which is the novelty of this research.

In the following sections, first, the solution for inviscid two-phase liquid-gas flow equations and the inviscid-viscous interaction method are described. Then, the analytical method for calculating the laminar and turbulent boundary layer parameters is explained. Finally, the results of the modeling are presented.

2. Inviscid two-phase analytic model

In this model, the flow is first considered as a single-phase vapor flow. After passing the Wilson point, numerous scattered tiny droplets are created from nucleation, and the flow can be analyzed by either Lagrange-Euler or Euler-Euler general methods.

In the Euler-Euler approach, the nucleation and droplet growth equations are added as two extra Eulerian equations to the Eulerian conservation of mass, energy and momentum. Finally, by solving a set of equations in the Eulerian space, the characteristics of the flow are obtained.

Young [9] has described the advantages of the Lagrange–Euler model, the most important of which are:

- There is no requirement for a tiny grid at the location of nucleation for calculation of the main flow properties.
- This approach offers a step by step and more accurate solution of the dynamics governing the droplets due to the Lagrangian nature of this phenomenon.

In this research, the Lagrange-Euler model is used, which employs the improved nucleation equations and integration of the droplet growth equation in Lagrangian coordinates by following the movement path of droplets. On the other hand, the gas dynamic equations governing the two-phase flow, including conversation of mass, energy and momentum, are imposed in an Eulerian framework. Finally, two series of Lagrangian equations, including nucleation and growth, and three series of Eulerian equations, including conservation of mass, energy and momentum, and also the auxiliary state equation are simultaneously combined. The characteristics of the flow, including the parameter values of the liquid phase, are obtained, as further explained in a previous study in [6].

For obtaining the Eulerian continuum equations governing the flow, a one-dimensional control volume is used. After conducting arithmetic operations, the conservation of mass momentum and energy are obtained as follows:

$$\frac{d\rho_G}{\rho_G} + \frac{dA}{A} + \frac{dU_G}{U_G} + \frac{dM_L}{M - M_L} = 0, \tag{1}$$

$$\frac{dP}{P} = -\frac{f\rho_G U_G^2}{2PD_e}dx - \frac{(M)U_G}{AP}\frac{dU_G}{U_G}, \tag{2}$$

$$\frac{dT_G}{T_G}+\frac{P}{\rho_G C_P T_G}\left(1-\frac{Y}{X}\right)\frac{dP}{P}+\left(1-\frac{Y}{X}\right)\frac{dP}{P}$$
$$+\frac{U_G^2}{C_P T_G}\frac{dU_G}{U_G}-\frac{L}{C_P T_G}\frac{dM_L}{M}=0, \quad (3)$$

where M_L is the mass of the liquid phase, L is the latent heat of vapor, U_G is the vapor's velocity and X and Y will be defined in the following. It is to be noted that for the inviscid solution, the friction factor parameter (f) is not considered.

One of the most successful models proposed for calculating the nucleation equation is Hall's comparative method [10,11]:

$$J_{\text{Hale}}=J_0\exp\left(-\frac{16\pi}{3}\Omega^3\left(\frac{T_C}{T}-1\right)^3/(\ln S)^2\right), \quad (4)$$

where J_0 and Ω are 10^{26} cm^{-3}s^{-1} and 1.47, respectively, and S is the vapor's super-saturation ratio $[P/P_s(T_G)]$. This model gives better results where other classic methods do not work very well.

For calculating the droplet's growth rate, the model proposed by Bakhtar and Zidi [12] is used. In this model, the equations governing the growth of the droplets in different flow regimes, including free molecular and continuous regimes, are obtained as follows:

$$L\frac{dm_r}{dt}=4\pi r^2\frac{\lambda}{r\left[1/(1+2\Lambda\text{Kn})+3.78\text{Kn}/\text{Pr}\right]}(T_L-T_G). \quad (5)$$

In this equation, the heat transfer coefficient (α_r) is calculated by:

$$\alpha_r=\frac{\lambda}{r\left[1/(1+2\Lambda\text{Kn})+3.78\text{Kn}/\text{Pr}\right]}, \qquad \Lambda=0.75, \quad (6)$$

where Kn is the Knudsen number and is the ratio of average free molecular flow to the droplet diameter, λ is the heat conductivity coefficient, and m_r is the mass of the droplet.

To complete the set of mentioned equations, the auxiliary state equation of real vapor is needed. In this research, the viral state equation with three viral coefficients is used, which in the differential form, is:

$$\frac{dP}{P}-X\frac{d\rho_G}{\rho_G}-Y\frac{dT_G}{T_G}=0, \quad (7)$$

where coefficients X and Y are defined as:

$$X=\frac{\rho_G}{P}\left(\frac{\partial P}{\partial\rho_G}\right)_{T_G}$$
$$=\frac{1+2B_1\rho_G^2+3B_2\rho_G^2+4B_3\rho_G^3}{1+B_1\rho_G^1+B_2\rho_G^2+B_3\rho_G^3}, \quad (8)$$

$$Y=\frac{T_G}{P}\left(\frac{\partial P}{\partial\rho_G}\right)_{\rho_G}$$
$$=1+\frac{\rho_G T_G}{1+B_1\rho_G^1+B_2\rho_G^2+B_3\rho_G^3}$$
$$\times\left[\frac{dB_1}{dT_G}+\rho_G\frac{dB_2}{dT_G}+\rho_G^2\frac{dB_3}{dT_G}\right]. \quad (9)$$

By calculating the amount of nucleation and also the growth of droplets, the mass of the liquid phase in each calculation cell can be obtained. Then, by simultaneous solving of the differential equations of conservation and vapor auxiliary state using the fourth order Runge-Kutta, all characteristics of the adiabatic inviscid two-phase flow are obtained.

3. Inviscid-viscous interaction method

One of the effects of the boundary layer on inviscid flow is the limited displacement of the flow streamlines, particularly in expansion flows. Based on this, a model is proposed where the wall of the channel changes with the displacement thickness of the boundary layer in such a way that in the adiabatic flow, the mass and momentum conservation equations hold during the growth of the boundary layer. The basis of the displacement model is calculation of the boundary layer parameters and, particularly, its displacement thickness.

In this method, the calculations of the boundary layer and the main flow field (inviscid) must be integrated periodically, in which the type of integration is defined by the boundary layer effects and its condition. In situations where the boundary layer does not separate from the surface, the direct method can be used [1,4]. In this method, the distribution of velocity over the surface, which is obtained by solving the main inviscid flow field, along with other characteristics of the flow, are used as inputs to the solution of the boundary layer equations, and, by solving the boundary layer equations, the displacement thicknesses are obtained. By adding the calculated displacement thicknesses to the initial channel wall (reducing the channel width), the inviscid solution is performed. Again, by solving the inviscid two-phase flow field and imposing its results on the laminar and turbulent boundary layers, the displacement thicknesses are calculated and compared with their previous value. If the difference is not significant, results of the inviscid solution and also the results of the boundary layer equations are accepted. Otherwise, again, by adding to the displacement thickness of the initial channel wall, the inviscid solution is performed. This computational process is repeated until the obtained displacement thicknesses are close to its value from the previous step, as shown in Figure 1.

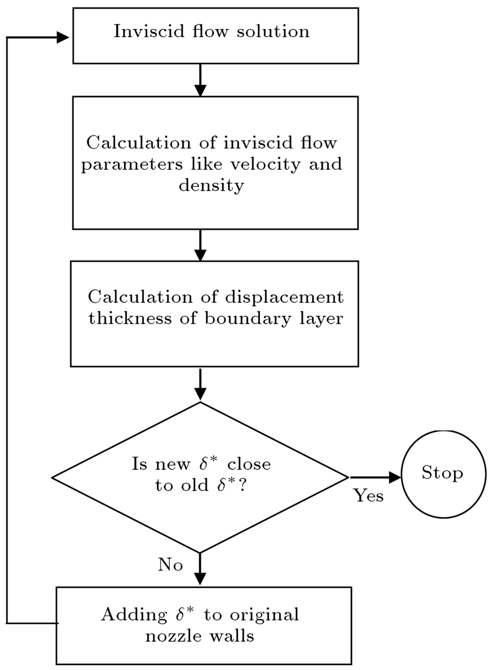

Figure 1. Algorithm of the inviscid-viscous interaction method.

Although the innovative viscous-inviscid method is from the 1970's, due to the expansive nature of the flow in expanding channels and steam turbines, (in other words, small boundary layer thickness) this method is still used in studies [3]. As explained before, the goal of this research is to strengthen and improve the 1D exact solution for steam, two-phase flow. In this paper, using the boundary layer equations suggested by Green, and the viscous-inviscid interaction method, the 1D mathematical modeling of the two-phase flow has been improved. The problem at this stage is not the large amount of calculations and time-consuming code, and the implication of the boundary layer with the possible method was consulted only.

Assuming subsonic single phase and laminar flow in the converging section, the thickness of the boundary layer or the boundary layer displacement thickness is small (relative to the diverging section). On the other hand, the flow at the throat is choked and the Mach is unity. With these two assumptions, it is assured that the mass flow rate is stabilized.

Following this approach, the viscous effects of the boundary layer on the main flow can be considered without direct solving of the Navier-Stokes equations of the whole two-phase field. As mentioned before, one of the main advantages of this method is that there are no numerical errors normally produced by the complete solution of the Navier-Stokes equation using any specific turbulent model.

Calculating δ^* is feasible in the laminar boundary layer using an integration method and in the turbulent boundary layer based on Green's integration methods. This will be explained in the following sections.

4. Solving boundary layer in laminar region

Integrating the basic equations of laminar boundary layer using the continuity equation, and performing a series of mathematical operations, the following integral is obtained [13]:

$$\theta^2 = \frac{0.45 v}{\overline{U}_e^6} \int_0^x U_e^5 dx, \tag{10}$$

where θ is the momentum thickness, v is kinematic viscosity, U_e is the edge of boundary layer velocity and the bar over U_e denotes the average value over an increment (Δx).

For calculation of δ^* and C_f, displacement thickness and skin friction coefficient, respectively, the following equations are used:

$$\delta^* = \theta . H(m), \tag{11}$$

$$C_f = \frac{2v}{\theta . U_e} l(m). \tag{12}$$

Assuming that $m = \frac{-\theta^2}{v}\frac{dU_e}{dx}$, Cebeci and Cousteix [13] proposed the following equations for $l(m)$ and $H(m)$:

$$l(m) = (m + 0.09)^{0.62}, \tag{13}$$

$$H(m) = \begin{cases} 2.61 - 3.75m + 5.24m^2 & m \geq 0 \\ 2.088 + \frac{0.0731}{(m+0.14)} & m < 0 \end{cases} \tag{14}$$

5. Compressible boundary layer in turbulent region

Comprehensive work has been done on analyzing the turbulent boundary layer flow. The first integration analysis was proposed by Prandtl. Subsequently, many researchers, such as Head and Patel [14] and Green et al. [15], made extensive efforts to develop a mathematical and exact method for estimating the growth of the turbulent boundary layer to which, considering the expanding flow, these analytical methods are applicable.

Some researchers [1,4] used Green's approach for calculating the parameters of the turbulent boundary layer using the inviscid-viscous interaction method. By introducing three main equations, namely, the momentum integral equation, the entrainment equation and the lag-entrainment equation, green and coworkers proposed a method for predicting the conditions of the turbulent boundary layer theory. Later, based on the turbulent energy, Green and coworkers proposed an ordinary differential equation for shear stresses, which, eventually, converts into a differential equation for the lag-entrainment. By simultaneous integration of these three equations, the values of the three independent parameters; transformed shape factor ($\bar{H}$), momentum thickness (θ) and entrainment coefficient (C_E), are calculated.

The momentum integral equation:

$$\frac{d}{dx}(r\theta) = r\frac{C_f}{2} - (H + 2 - Ma^2)\frac{r\theta}{U_e}\frac{dU_e}{dx}. \tag{15}$$

Entrainment equation:

$$r\theta\frac{d\bar{H}}{dx} = \frac{d\bar{H}}{dH_1}\left[rC_E - H_1\left\{r\frac{C_f}{2} - (H+1)\frac{r\theta}{U_e}\frac{dU_e}{dx}\right\}\right]. \tag{16}$$

Lag-entrainment equation:

$$\theta\frac{dC_E}{dx} = F\left\{\begin{array}{l} \frac{2.8}{H+H_1}\left[(C_\tau)^2_{EQ_o} - \lambda(C_\tau)^{\frac{1}{2}}\right] \\ \quad + \left(\frac{\theta}{U_e}\frac{dU_e}{dx}\right)_{EQ} \\ -\frac{\delta}{U_e}\frac{dU_e}{dx}\left(1 + 0.075\ \mathrm{Ma}^2\frac{1+0.2\mathrm{Ma}^2}{1+0.1\mathrm{Ma}^2}\right) \end{array}\right\}. \tag{17}$$

In the above equations, $\bar{H}$ is the transformed shape factor, r is the curvature radius, C_τ is the shear stress factor and λ is the overall scaling factor. Subscripts o and EQ refer to the values at zero gradient pressure and equilibrium flow, respectively. For calculating λ, the following equation can be used:

$$\lambda = 1 + \frac{7}{3}\mathrm{Ma}^2\left(1 + \frac{H_1}{\bar{H}}\right)\frac{\delta}{U_e}\frac{dU_e}{dx}. \tag{18}$$

Solving the above equations requires a number of auxiliary equations for calculating C_τ, C_f, H, H_1 and F. The auxiliary equations proposed by Green are as follows:

$$H_1 = 3.15 + \frac{1.72}{\bar{H} - 1} - 0.01\left(\bar{H} - 1\right)^2, \tag{19}$$

$$H = \left(\bar{H} + 1\right)\left(1 + \frac{\mathrm{Ma}^2}{5}\right) - 1, \tag{20}$$

$$\frac{d\bar{H}}{dH_1} = -\frac{\left(\bar{H} - 1\right)^2}{1.72 + 0.02\left(\bar{H} - 1\right)^3}. \tag{21}$$

The above equations show the relationship between H, H_1 and $\bar{H}$. The value of F is obtained by the following equation:

$$F = \frac{0.02C_E + C_E^2 + \frac{0.8C_{f0}}{3}}{0.01 + C_E}. \tag{22}$$

C_{f0} is the skin friction coefficient for low gradient pressure, which is calculated by the following equation:

$$C_{f0} = \frac{1}{F_C}\left\{\frac{0.01013}{\log(F_R R_\theta) - 1.02} - 0.00075\right\}. \tag{23}$$

In the above equation, F_C and F_R are the temperature ratios, which are defined as follows:

$$F_C = (1 + 0.2\mathrm{Ma}^2)^{0.5}, \tag{24}$$

$$F_R = 1 + 0.056\mathrm{Ma}^2. \tag{25}$$

Also, R_θ is the flow Reynolds number based on momentum thickness, which is calculated by:

$$R_\theta = \frac{\rho_e U_e \theta}{\mu_e}. \tag{26}$$

The skin friction coefficient is calculated from:

$$C_f = C_{f0}\left\{-0.5 + 0.9\left(\frac{\bar{H}}{\bar{H}_0} - 0.4\right)^{-1}\right\}. \tag{27}$$

The relationship between $\bar{H}_0$ and C_{f0} is defined by:

$$\bar{H}_0 = \left\{1 - 6.55\sqrt{\frac{C_{f0}}{2}(1 + 0.04\mathrm{Ma}^2)}\right\}^{-1}. \tag{28}$$

Also, the shear stress factor is calculated using:

$$C_\tau = \left(0.024C_E + 1.2C_E^2 + 0.32C_{f0}\right)\left(1 + 0.1\mathrm{Ma}^2\right). \tag{29}$$

To complete the equations, a number of terms regarding the equilibrium conditions are required. Equilibrium flows are defined as flows in which the shape of velocity and shear stress profiles in the boundary layer do not vary with distance, x. Therefore:

$$\frac{dH}{dx} = \frac{(dC_\tau)_M}{dx} = 0, \tag{30}$$

where subscript M refers to the maximum value. Based on this description, the following equation is obtained for calculating the needed values under equilibrium conditions:

$$\left(\frac{\theta}{U_e}\frac{dU_e}{dx}\right)_{EQ_o} = \frac{1.25}{H}\left\{\frac{C_f}{2} - \left(\frac{\bar{H} - 1}{6.432\bar{H}}\right)^2(1 + 0.04\mathrm{Ma}^2)^{-1}\right\}. \tag{31}$$

Substituting $\frac{dH}{dx} = 0$ and replacing the above equation in the entrainment equation, the following equation is obtained:

$$(C_E)_{EQ_o} = H_1\left\{\frac{C_f}{2} - (H + 1)\left(\frac{\theta}{U_e}\frac{dU_e}{dx}\right)_{EQ_o}\right\}. \tag{32}$$

Also, the shear stress coefficient under equilibrium condition is obtained by:

$$(C_r)_{EQ_o} = \left[0.024(C_E)_{EQ_o} + 1.2(C_E)^2_{EQ_o} + 0.32C_{fo}\right](1 + 0.1\mathrm{Ma}^2). \quad (33)$$

At this stage, with the help of the proposed auxiliary equations, three differential equations of momentum, entrainment and lag-entrainment can be solved simultaneously using the fourth order Runge-Kutta method to obtain the main parameters of the boundary layer, including momentum thickness, displacement thickness and skin friction coefficient.

6. Results

In this research, the inviscid-viscous interaction method is used to calculate the parameters of the boundary layer in a supersonic two-phase flow inside an adiabatic converging-diverging nozzle. Considering that, basically, in these types of flow, attention is focused on the diverging section, i.e. the location of condensation and nucleation, analysis of the boundary layer results is undertaken only in this region. It is to be noted that in the converging section of the channel, the vapor remains subsonic and single phase. The results of comparing the analysis of pressure ratio variation under inviscid conditions to experimental results are shown in Figure 2, which validates the model.

Furthermore, the theoretical results associated with the nucleation rate, based on Hall's comparative model and the wetness percentage of the two-phase inviscid flow, are shown in Figures 3 and 4. The results are consistent with Figure 2, regarding the condensation shock region.

On the basis of this analysis, the effects of viscosity on the mentioned flow is investigated, first, using laminar boundary layer equations and, then, by Green's three differential equations. The calculated values for the boundary layer thickness, displacement thickness and momentum thickness are shown in Figures 5-7. As the figures show, the variations of boundary layer parameters are considerable in the condensation shock region.

Also Figure 8 illustrates the calculated skin friction coefficient using Green's three differential equations. In all these figures, a relative drastic variation is observed at the beginning of the diverging section, which is a result of the condensation shock at that region. Furthermore, as expected, the values of the boundary layer parameters increase along the channel length.

To better show the variation of the boundary layer thickness and displacement thickness in the condensation shock region, magnification is performed and the result is shown in Figure 9.

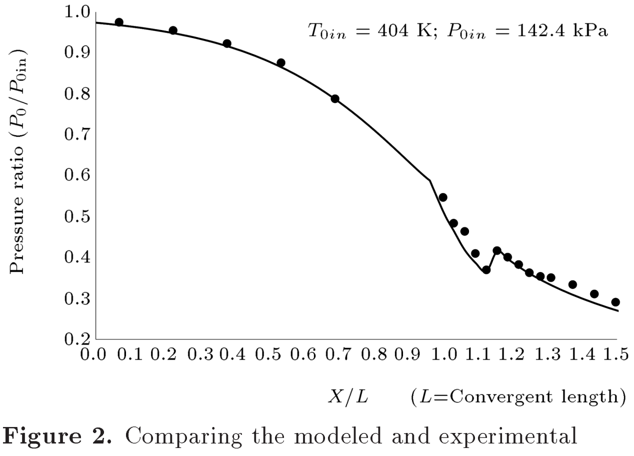

Figure 2. Comparing the modeled and experimental pressure ratios.

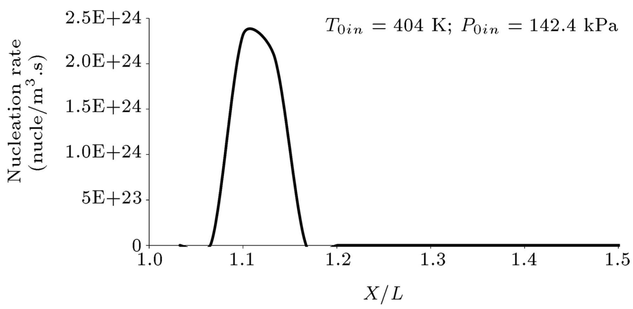

Figure 3. Theoretical results of nucleation rate for inviscid vapor.

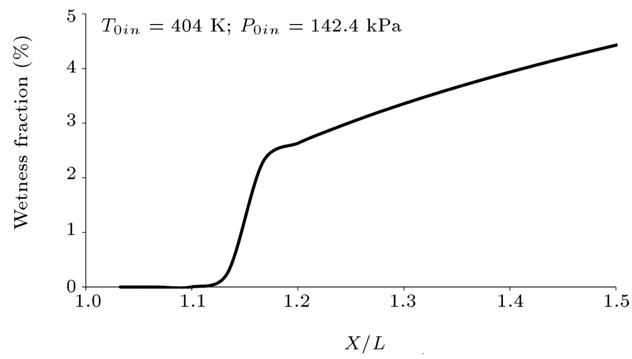

Figure 4. Theoretical results of wetness fraction percentage for inviscid vapor.

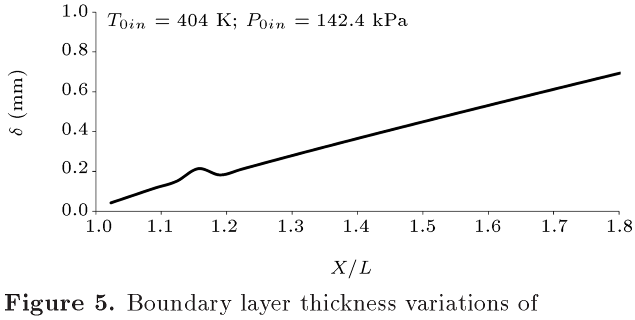

Figure 5. Boundary layer thickness variations of turbulent flow in the converging section.

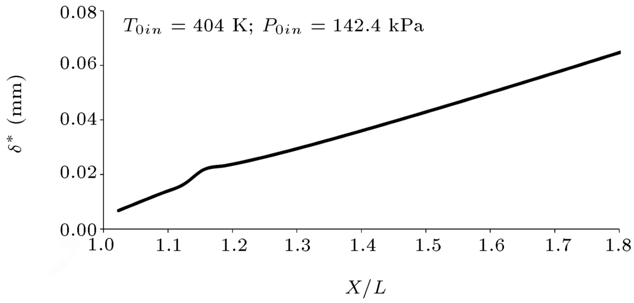

Figure 6. Displacement thickness variations of turbulent flow in the converging section.

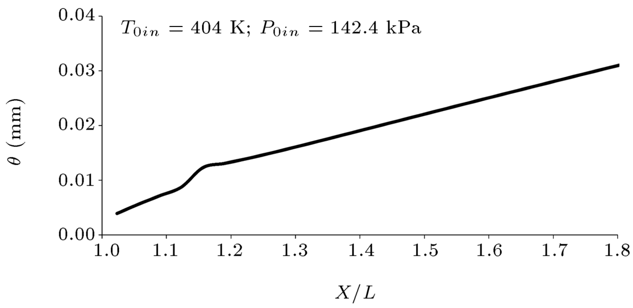

Figure 7. Momentum thickness variations of turbulent flow in the converging section.

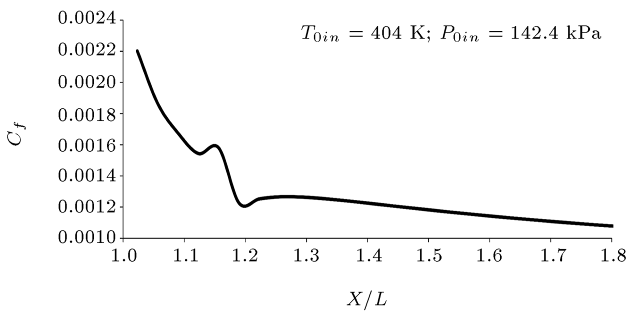

Figure 8. Skin friction coefficient variations of turbulent flow in the converging section.

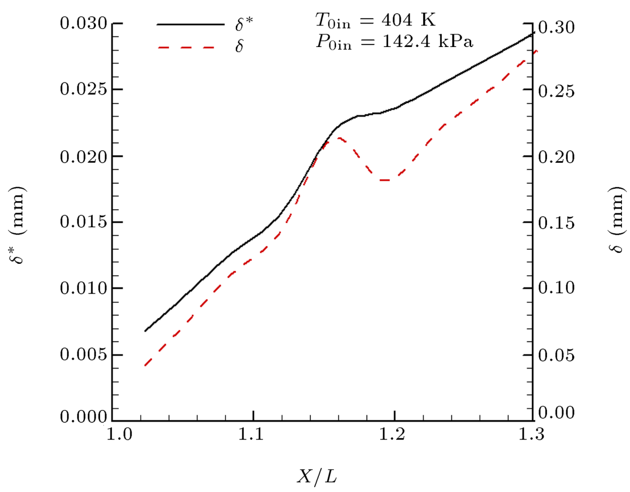

Figure 9. Variations of the boundary layer thickness and displacement thickness in the condensation shock region.

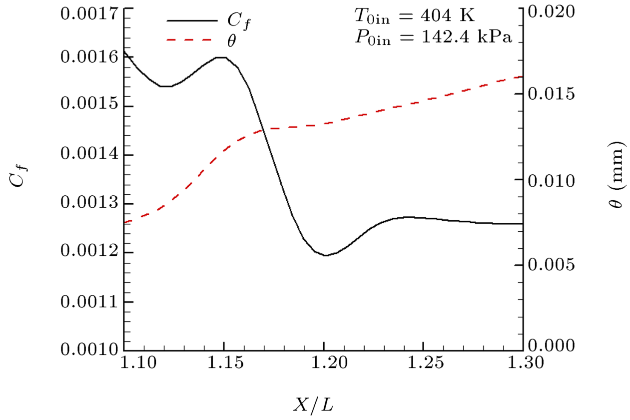

Figure 10. Variations of momentum thickness and skin friction coefficient in the condensation shock region.

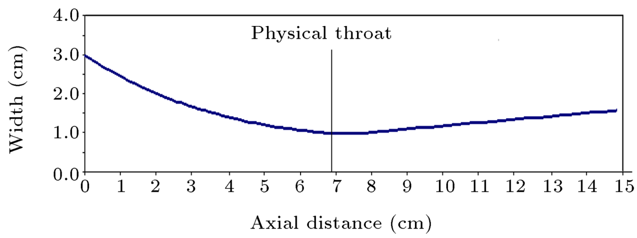

Figure 11. The geometry of convergent divergent nozzle.

As can be seen in this figure, the variations of the boundary layer thickness in the condensation shock region are considerable. For better presenting the variations of momentum thickness and skin friction coefficient in the condensation shock region, the magnified region is shown in Figure 10.

As can be observed, the variations of momentum thickness in this region are considerable, which shows the significance of condensation shock on boundary layer parameters and, particularly, momentum thickness. The channel shape is shown in Figure 11.

For more detailed investigation of the effects of considering boundary layer on two-phase adiabatic flow, the pressure ratio, wetness percentage and droplet radius are compared in tabular format. The obtained results show that these parameters have limited effects from the boundary layer due to the expanding flow. Based on this, a better comparison of the pressure ratio, wetness percentage and droplet radius is shown in Tables 1-3. The distinct sections of the tables show the considerable pressure increase or the condensation shock region.

Investigating the above tables, it is observed that for expansion flow, the boundary layer thickness is small. Consequently, its effects on the two-phase flow parameters, including the radius of the formed droplets, can be up to 3%. As a result, for more accurate modeling of supersonic two-phase flow, even the limited effects of the boundary layer can be incorporated in the study of such flows.

Table 1. Pressure ratio along the converging section and the percentage error from interaction method and inviscid solution.

X/L	$(P/P_{0\text{in}})$ from invicsid-viscous interaction method	$(P/P_{0\text{in}})$ from inviscid solution	Percentage of error
1.02	0.472	0.472	0.13
1.06	0.423	0.423	0.17
1.09	0.391	0.390	0.19
1.12	0.372	0.372	0.09
1.16	0.420	0.419	0.12
1.19	0.400	0.399	0.28
1.22	0.381	0.380	0.26
1.29	0.349	0.348	0.25
1.36	0.323	0.322	0.24
1.42	0.301	0.301	0.23
1.49	0.282	0.282	0.23
1.56	0.266	0.265	0.23
1.62	0.251	0.250	0.23
1.69	0.238	0.237	0.23
1.76	0.226	0.226	0.23
1.82	0.215	0.215	0.23
1.89	0.206	0.205	0.22
1.96	0.197	0.196	0.22

Table 2. Wetness percentage along the converging section and the percentage error from interaction method and inviscid solution.

X/L	$(P/P_{0\text{in}})$ from invicsid-viscous interaction method	$(P/P_{0\text{in}})$ from inviscid solution	Percentage of error
1.02	0.00	0.00	0.00
1.06	0.00	0.00	0.00
1.09	0.00	0.00	0.00
1.12	0.21	0.25	15.76
1.16	2.26	2.29	1.39
1.19	2.62	2.64	0.71
1.22	2.88	2.90	0.66
1.29	3.35	3.37	0.63
1.36	3.74	3.77	0.63
1.42	4.10	4.12	0.65
1.49	4.41	4.44	0.67
1.56	4.71	4.74	0.69
1.62	4.98	5.01	0.68
1.69	5.23	5.27	0.72
1.76	5.46	5.50	0.74
1.82	5.69	5.73	0.76
1.89	5.90	5.95	0.78
1.96	6.10	6.15	0.80

Table 3. Droplet radius along the converging section and the percentage error from interaction method and inviscid solution.

X/L	$(P/P_{0\text{in}})$ from invicsid-viscous interaction method	$(P/P_{0\text{in}})$ from inviscid solution	Percentage of error
1.02	0.0008	0.0008	0.00
1.06	0.0007	0.0007	0.00
1.09	0.0008	0.0008	0.10
1.12	0.0020	0.0020	2.38
1.16	0.0043	0.0044	2.96
1.19	0.0045	0.0046	3.19
1.22	0.0046	0.0048	3.21
1.29	0.0048	0.0050	3.22
1.36	0.0050	0.0052	3.21
1.42	0.0052	0.0053	3.21
1.49	0.0053	0.0055	3.20
1.56	0.0054	0.0056	3.19
1.62	0.0055	0.0057	3.19
1.69	0.0056	0.0058	3.18
1.76	0.0057	0.0059	3.17
1.82	0.0058	0.0060	3.17
1.89	0.0059	0.0060	3.16
1.96	0.0059	0.0061	3.15

7. Conclusions

In most analytical or mathematical models proposed for supersonic two-phase flow, usually, due to the complexity of the solution method and the expansive nature of the flow, in order to decrease the computational burden, the effects of viscosity are ignored. In this research, for the first time, the effects of boundary layer on two-phase liquid vapor flow were investigated using mathematical models and the inviscid-viscous interaction method. The results show that despite the limited effects of the boundary layer on this expanding flow, the radius of the droplets increases by 3%.

The other innovation of this research is determining the amount of effect of condensation shock on the important parameters of the boundary layer. As the results show, in the condensation shock region, which occurs in a limited region, variations of the boundary layer parameters, including boundary layer thickness, displacement thickness, momentum thickness and skin friction coefficient, are 15%, 13%, 5% and 55%, respectively.

Nomenclature

A	Area
C_p	Specific heat at constant pressure

D_e	Equivalent diameter
f	Friction factor
ΔG	Change in Gibbs free energy
J	Rate of formation of critical droplets per unit volume and time
Kn	Knudsen number
L	Latent heat
Ma	Mach number
m_r	Mass of droplet
P	Vapor pressure
$P_{0\text{in}}$	Inlet total pressure
f	Friction factor
$\rho_s(T_L, r)$	Density corresponding to saturation pressure at temperature T_L over a surface of curvature r
Sc	Schmidt number
$P_s(T_G)$	Saturation pressure at T_G
q	Condensation coefficient
R	Gas constant for water vapor
r	Radius of droplet
$S\&T$	Entropy & temperature
$T_s(P)$	Saturation temperature at P
ΔT	Degree of supercooling $[T_s(P) - T_G]$
t	time
U	Velocity
M	Total mass flow rate
x	Distance along duct axis
L	Nozzle throat length
X, Y	Functions of temperature and density in equation of state
α_r	Coefficient of heat transfer
γ	Isentropic component
μ_G	Dynamic viscosity of vapor
ρ	Density of mixture
λ	Overall comparison factor
σ	Surface tension
H	Shape factor
$\bar{H}$	Modified shape factor
F_C, F_R	Temperature ratios
C_f	Skin friction coefficient
δ^*	Displacement thickness
C_E	Entrainment coefficient
R_θ	Reynolds number
C_τ	Shear stress factor
θ	Momentum thickness
δ	Boundary layer thickness

Subscripts

G	Vapor phase
L	Liquid phase
O	Stagnation condition
S	Saturation
EQ	Equilibrium condition

Superscript

$*$	Critical condition

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