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Risk analysis of sourcing problem using stochastic programming

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Stochastic process;
Stochastic programming.

Abstract. Nowadays, the sourcing problem has become more challenging for supply chain members. Different types of sourcing for different market conditions are presented in the literature. In this paper, an option contract, as an efficient tool for sourcing, is developed in a multi-period setting in which the price and demand follow two stochastic processes. The sourcing decision is analyzed from a risk neutral and a risk averse decision-maker point of view. This paper applies the stochastic programming approach to model the presented option contract based on price and demand uncertainties. Next, using CVaR as a coherent risk measure, the effects of risk on sourcing problem are studied. By numerical example, using the presented efficient frontier, the simulation results of our developed models show that the decision maker can make a trade-off between risk and cost associated with the sourcing problem. The paper also performs a sensitivity analysis in order to demonstrate the effects of change in cost parameters on the results of our option model.

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1. Introduction

Globalization and an increase in outsourcing has made manufacturer-supplier interaction more significant. Outsourcing results in lower production costs and higher quality, but the more the system decentralizes, the more challenging becomes managing the supply chain [1,2]. The manufacturer, as the buyer of raw materials, has two main alternatives to provide the required materials: long-term or short-term sourcing.

Long-term sourcing will be established via supply contracts. Long-term contracts provide price stability, but much less flexibility, and can be known as operational risk hedging for high spot market price incidents. On the contrary, short-term sourcing provides more flexibility for the buyer, but more risk of price increase

and availability. Since each procurement option has a different cost and flexibility, the buyer should make a tradeoff by selecting long-term procurement contracts, short-term procurement or a combination of both [1]. Different supply contracts have been developed in uncertain markets, including wholesale contracts, buy back contracts, revenue sharing contracts, quantity flexibility contracts, and option contracts [3].

In a market suffering uncertainty, an option contract acts as a highly efficient risk hedging tool, by providing flexibility while decreasing price and availability risk [1]. These types of contracts are widely used in high-tech industries [2]. Two determinant parameters characterize the option contract; the option price and the exercise price. By paying the option price, the buyer possesses the right to exercise his option in the future up to the committed quantity by the pre-defined exercise price. In the presence of uncertain demand, a widely used option contract is the Capacity Reservation contract in which the buyer has only the long-term supplier and has no access to the spot

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market. Exercising option in the Capacity Reservation contract only depends on the demand situation. In the presence of a spot market with uncertain price, the buyer exercises the option if the inventory in hand does not satisfy the demand and, also, the spot market price is higher than the exercise price. This issue is considered in this paper as an “Option Contract”.

In this paper, an option contract in a multi-period sourcing problem is developed. Demand and price are assumed to be uncertain. Considering time dependencies, demand and price are assumed to follow two independent stochastic processes. In this article, *the periodic review base stock policy* is applied. Through this policy, the buyer increases the level of the inventory to S - the base stock level - at the beginning of each period. The buyer has three alternatives to satisfy the base stock: the exercising option, buying from the spot market or a combination of both.

Many studies in the literature of supply contracts rely on optimizing the expected value of the sourcing cost. This measure does not consider decision maker risk preference. In other words, this measure is risk neutral. Different measures are developed to assess the volatility of the objective function for a decision maker under different realization of uncertain variables. In order to provide a practical decision tool, in this paper, a multi objective model, considering the expected value and volatility of the decision maker’s objective function, is developed. In this paper, the volatility is measured by Conditional Value at Risk (CVaR) (see Section 3.4).

This paper is organized as follows: First, the related literature is explored, and the notations and stochastic models are presented as Mixed Integer Lin-

ear Programming (MILP) in the problem formulation section. Modeling uncertainties are described and a simulation based approach is used to verify the uncertainty modeling procedure in Section 4. Using numerical examples in Section 5, the cost performance of the presented sourcing contract and the effects of risk considerations are analyzed, and the sensitivity of the decision variables to problem parameters is investigated. Finally, the conclusion and suggestions for future research are presented.

2. Literature review

Although the majority of studies have focused on demand uncertainty, uncertainty of price has been less considered. Besides dividing the related literature regarding the considered uncertain parameter, one can make subgroups of one and multiple sourcing. In a multi-period supply problem, it is assumed that the buyer can buy and save inventory to use in other periods. Table 1 shows these aspects of related reviewed articles. It also shows the literature which has considered risk in decision making.

Demand uncertainty is one of the main aspects of the present paper. Some papers consider only this parameter to be uncertain. In line with this assumption, and in a multi-period sourcing problem, Serel et al. [9] investigated the reactions of buyer and supplier to different inventory review policies. They considered demand to be Independent Identically Distributed (IID), while in Delft and Vial [5], the demand distribution can vary during time. As in this paper, they use the stochastic programming approach to model the presented capacity reservation problem.

Table 1. Overview of related literature.

	Demand uncertainty	Price uncertainty	Time correlation	Multi-period	Risk consideration
Inderfurth and Kelle [1]	•	•		•	
Arshinder et al. [4]	•				
Delf and Vial [5]	•		•	•	•
Li et al. [6]	•	•			
Zhao et al. [7]	•				•
Xu [2]	•	•			
Wang et al. [8]	•				•
Serel et al. [9]	•			•	
Jin and Wu [10]	•				
Mathur and Shah [11]	•				
Chen et al. [12]	•				
Boulaksil et al. [13]	•				
Padilla and Mishina [14]	•		•	•	
Our study	•	•	•	•	•

They also consider risk as one of the decision criteria. Gomez Padilla and Mishina [14] studied the effect of option contract on the performance of two parties in a supply chain with a non-stationary distributed demand in periods. In the presence of demand uncertainties, there are many papers that study the option contract in a one period problem, known as the newsvendor problem. These include Arshinder et al. [4], Zhao et al. [7], Boulaksil et al. [13] and Wang et al. [8].

Despite demand uncertainty, not much work has been carried out in the area of uncertain prices, while, in our paper, both demand and price are uncertain. Considering a multi-period problem, Inderfurth and Kelle [1] assumed both demand and price to be uncertain. They investigated the cost effectiveness of combining two sourcing types: capacity reservation contract and spot market sourcing. They considered both demand and price to be IID, when, in the present paper, this assumption is released. It is assumed that demand and price distribution can vary during time and is time correlated. Many others, in an uncertain price market, studied the one-period problem, such as Li et al. [6] and Xu [2].

Decision making based on only expected values of the objective function is risk neutral. This type of decision making may not be fit for practical decision making [5]. Few papers consider this fact and consider the risk measure in addition to optimizing the expected value [5,7,8].

According to the literature review, in a multi-period sourcing problem, few papers assume both demand and price to be uncertain. Besides which, none of them consider the time dependency of the demands and the prices simultaneously. To cover this gap in real world problems, in the present paper, the option contract is studied in a multi-period problem, where demand and price are uncertain, and follow two time correlated stochastic processes. In much research, the expected value of sourcing cost during the planning horizon is the only criteria to make decisions. Considering a multi objective problem, this paper utilizes the risk measures according to the risk preferences of the decision maker. The scenario based approach is used to model uncertainties, and a Mixed Integer Linear Stochastic Programming (MILSP) model is developed. The efficient frontier is also developed to show the trade-off between risk and the expected value of the sourcing cost.

3. Problem formulation

The main assumptions of the studied problem are as follows:

- The supply chain consists of one supplier and one buyer.

- The supplier sells one commodity to the buyer.
- Price and demand are independent and follow two Geometric Brownian stochastic processes with drift.
- The studied problem is multi-periodic and the periodic-review base stock inventory policy is applied.
- The shortage is allowed and will be backlogged.

3.1. Notations

T	Set of periods, $t \in T$;
S	Set of scenarios, $s \in S$;
d_s^t	Realization of demand in t th period under scenario s ;
p_t^s	Market price in t th period under scenario s ;
v	Shortage cost per unit and period;
h	Holding cost per unit and period;
e	Exercise price;
o	Option price;
L_t^s	Exercised option quantity in t th period under scenario s ;
K_t^s	Spot sourcing quantity in t th period under scenario s ;
R	Option quantity bought for each period;
S_U	The inventory base stock level;
I_t^{s+}	Inventory level at the end of t th period under scenario s ;
I_t^{s-}	Shortage level at the end of t th period under scenario s .

3.2. Basic stochastic programming formulation

Stochastic programming is one of the most powerful analytical tools to support sequential decision-making under uncertainty. In this paper, we face sequential decisions: tactical and operational decision variables. The option contract quantity for each period as a tactical decision should be defined at the beginning of the planning horizon. Operational decisions, including exercising options and buying from the spot market, are made in the second stage. There are two main components in this type of programming: the underlying discrete stochastic process and a deterministic discrete time dynamic linear optimization model [6]. Here, we make the fundamental assumption that the decision variables have no impact on the underlying stochastic processes. To achieve a good modeling of the aforementioned sourcing problem, a two stage stochastic programming is used. The classic model is

as follows [15]:

$$\begin{aligned} \min z &= c^T x + E_{\zeta}[\min q^T y], \\ \text{s.t.:} \\ Ax &= b, \\ Tx + Wy &= h, \\ x \geq 0, y &\geq 0, \end{aligned} \quad (1)$$

where x includes the first stage decision variables (R and S_U in this paper), which are independent from Scenarios (S), c is a constant multiplier, ζ is the set of stochastic variables (D^t and P^t), where ζ^s is its realization in each scenario, and y is the scenario dependent second stage decision variable ($L_t^s, K_t^s, I_t^{s+}, I_t^{s-}$). q, T, h are scenario based multipliers. By using the following equation:

$$Q(x, \zeta^s) = \min_y \{q^T y \mid Wy = h - Tx, y \geq 0\}. \quad (2)$$

The model can be written as:

$$\begin{aligned} \min z &= c^T x + E_{\zeta}Q(x, \zeta^s), \\ \text{s.t.:} \\ Ax &= b, \\ x &\geq 0. \end{aligned} \quad (3)$$

$Q(x, \zeta^s)$ is the optimal decision for the second stage decision variables. The expected value of $Q(x, \zeta^s)$ is known as the recourse function.

It is worth mentioning that the developed two stage stochastic model for the problem is a complete fixed recourse, as, for each positive first stage decision variable, the recourse problem is feasible [16].

As mentioned in [16], the value of uncertain parameters will be realized at the second stage in which the operational decisions will be made. Therefore, the first stage tactical decision in all models should be taken into account by considering future uncertain effects measured by the recourse function introduced as an expected value of making first stage decisions.

3.3. Option contract MILP model

As mentioned in the introduction, signing an option contract gives the buyer the right to exercise his option in each period. At the beginning of the planning horizon, the buyer purchases R options to exercise each period and pays option price, o , for each reserved unit. Regarding the base stock policy, the buyer should increase the inventory level up to S_U . After realizing the market price and demand, the buyer decides about

ordering from sourcing alternatives: exercising option by predetermined exercise price, e , or buying from the spot market with uncertain price, p_s^t . The MILP model of this type of sourcing is as follows:

$$\begin{aligned} \min Z &= R \times o \times N_T \\ &+ \sum_{s \in S} \sum_{t \in T} \Pr(L_t^s e + K_t^s p_t^s + I_t^{s-} v + I_t^{s+} h), \end{aligned} \quad (4)$$

s.t. :

$$S_U = d_t^s + I_t^{s+} - I_t^{s-} \quad \forall t \in T, \quad s \in S, \quad (5)$$

$$L_t^s \leq R \quad \forall t \in T, \quad s \in S, \quad (6)$$

$$L_t^s + K_t^s = S_U - I_{t-1}^{s+} + I_{t-1}^{s-} \quad \forall t \in T, \quad s \in S, \quad (7)$$

$$\{L_t^s, K_t^s, S_U, I_t^{s+}, I_t^{s-}, R\} \geq 0 \quad \forall t \in T, \quad s \in S. \quad (8)$$

Eq. (4) is the objective function that minimizes the sum of the first-stage costs and the expected value of second-stage costs. The first-stage costs represent the reservation cost. The objective function of the second-stage includes four types of cost: Cost of exercising options, buying from spot market, holding cost and shortage cost. Constraint (5) guarantees that the base stock level of inventory at the beginning of each period is balanced by periodic demand, inventory and shortage of the end of the period in each scenario. As the buyer has reserved R units at the beginning of the planning horizon for each period, the maximum purchase of the long-term supplier in each period is R (Eq. (6)). By Eq. (7), the total number of purchased units from sourcing alternatives is determined to reach the inventory to base stock, S , in the beginning of each period. The last constraint indicates the positivity of the decision variables.

3.4. Risk consideration model

As mentioned before, the classical risk neutral objective function is:

$$\min z = c^T x + E_{\zeta}Q(x, \zeta^s), \quad (9)$$

which we can write as:

$$\min_x E(f(x, \zeta)) = c^T x + E_{\zeta}Q(x, \zeta^s). \quad (10)$$

Since we focus on the total cost of sourcing, smaller values of the objective function are preferred. In order to make decisions in an uncertain environment, it is crucial to consider the effect of variability, and specify the preference relations among random variables using risk measures. One of the main approaches in the practice of decision making under risk is using mean-risk models. In these models, the decision maker

minimizes the mean-risk function, which involves a specified risk measure, $\rho : \mathbb{Z} \rightarrow \mathbb{R}$, where ρ is a functional and $\mathbb{Z}$ is a linear space of F -measurable functions on the probability space (Ω, F, P) :

$$\min_x \{E(f(x, \zeta)) + \gamma \rho(f(x, \zeta))\}. \quad (11)$$

In this approach, γ is a non-negative trade-off coefficient representing the exchange rate of expected values of cost and risk. We also refer to it as the risk coefficient, which is specified by decision makers according to their risk preferences [17].

In this paper, CVaR is applied as the risk measure. This risk measure was introduced by Artzner et al. [18] to improve the shortcomings of more familiar values of risk. This risk measure provides an interesting concept of risk and is very computationally tractable in the concept of stochastic programming [19,20].

In our developed MILP optimization model, CVaR is used to produce robust first-stage decisions in which the loss cost in the second-stage is to be minimized. Rockafellar and Uryasev [19] show CVaR can enter into the objective or the constraints of optimization problems under uncertainty, using linear programming techniques. Noyan [17] proposes a formulation for CVaR in two-stage stochastic programming using the formulation developed by [19]. In this paper, we apply Noyan's approach to our addressed problem.

In this regard, the objective function is changed into Eq. (12), and Constraints (13) and (14) are added to the mathematical formulation:

$$\begin{aligned} \min Z' = & (1 + \gamma)(R \times o \times N_T) \\ & + \sum_{s \in S} \sum_{t \in T} \Pr(L_t^s e + K_t^s p_t^s + I_t^{s-} v + I_t^{s+} h) \\ & + \gamma \left(\xi + \frac{1}{1 - \alpha} \sum_{s \in S} \Pr \mu_s \right), \end{aligned} \quad (12)$$

$$\begin{aligned} \sum_{t \in T} L_t^s e + K_t^s p_t^s + I_t^{s-} v + I_t^{s+} h - \xi - \mu_s & \leq 0 \\ \forall s \in S, \end{aligned} \quad (13)$$

$$\xi, \mu_s \quad \forall s \in S, \quad (14)$$

where the possible loss for each scenario and the confidence level are denoted by μ_s and α respectively, ξ is the Value-at-Risk (VaR) and γ is the risk-aversion factor.

$\text{CVaR}_\alpha(Q(x, \zeta))$ can be computed by the expected value of costs that exceed $\text{VaR}_\alpha(Q(x, \zeta))$. Therefore, $\text{CVaR}_\alpha(Q(x, \zeta))$ can be calculated as the following relation:

$$\text{CVaR}_\alpha(Q(x, \zeta)) = \xi + \frac{1}{1 - \alpha} \sum_{s \in S} \Pr \mu_s. \quad (15)$$

Consequently, the CVaR of the objective function, Z , is defined as follows:

$$\text{CVaR}_\alpha(Z) = \xi + \frac{1}{1 - \alpha} \sum_{s \in S} \Pr \mu_s + R \times o \times N_T. \quad (16)$$

4. Modeling uncertainties

This paper studies a manufacturer's decision to buy one of his main raw materials in an uncertain environment. In the real world, the demand and price of raw materials vary during time with an uncertain behavior. This uncertainty in price can be a result of changes in exchange rate, uncertainty in supply, lack of future market, and so on. The uncertainty in the demand of raw material can be related to uncertainty associated with the uncertainty of finished goods or the random yield of production [21].

In order to adapt more to real world problems, it is assumed that demand and price distribution can vary during time. In order to apply the time dependency of these uncertain variables, the widely used Geometric Brownian Motion (GBM) stochastic process is used. This type of stochastic process is frequently invoked as a model for such diverse quantities as stock prices, natural resource prices, and the growth in demand for products or services [22]. The basic concept of GBM with drift is as follows:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad (17)$$

where $W_t \sim N(0, t)$ is a Wiener process (also called Brownian motion process), t is the length of each period, μ is the percentage drift, and σ shows the percentage volatility in the time horizon [23].

4.1. Scenario generation phase

In this phase, using Eq. (17), the uncertain value for a particular period under each scenario can be generated using Eq. (18):

$$\hat{x}_t^s = \hat{x}_{t-1}^s e^{\left(\left(\mu - \frac{\sigma^2}{2} \right) t - \sigma \hat{W}_t^s \right)}. \quad (18)$$

W_t^s is the generated random term for each period and scenario, and $\hat{x}_t^s$ is the predicted value for stochastic parameter, x , under scenario, s .

The scenario generation methodology used in this study is schematically shown in Figure 1.

The stochastic parameter behaviors, μ and σ , that were previously introduced, are determined from historical data. Using the generated random term, W_t^s , for each period and scenario, predicted stochastic parameters can be constructed by the following formulation:

$$\hat{x}_t^s = \hat{x}_{t-1}^s e^{\left(\left(\mu - \frac{\sigma^2}{2} \right) t - \sigma \hat{W}_t^s \right)}, \quad (19)$$

where $\hat{x}_t^s$ is the predicted value for stochastic parameter, x , under scenario, s , and W_t^s is generated, as mentioned before.

Since a large number of scenarios makes the offering of models computationally intractable, to deal with this issue, the initial set of scenarios is reduced using a fast-forward reduction algorithm [24]. This reduction should have in-sample stability, i.e. the reduction technique should be good enough to represent basic system behavior.

4.2. Simulation phase

Using simulation techniques, the out-of-sample stability will be proved. Out-of-sample stability reveals that the true objective function values are also the same for those decisions obtained by different scenario trees or a large number of scenarios. The out-of-sample performance is measured normally using some type of simulation. The simulation phase pseudo code is presented in the Appendix.

5. Numerical results

To show how the presented option contract model provides optimal sourcing decisions for manufacturers, a typical numerical example is used (Table 2).

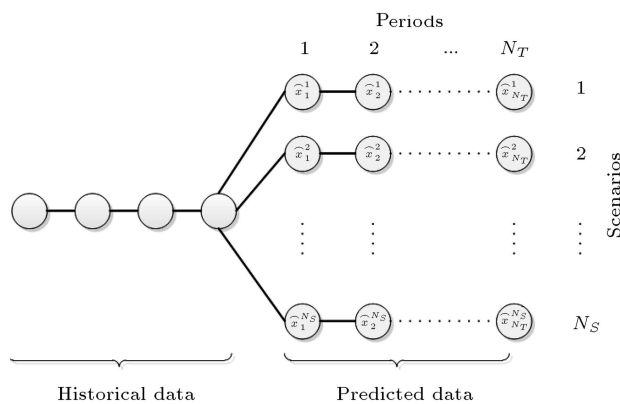


Figure 1. Scenario generation methodology.

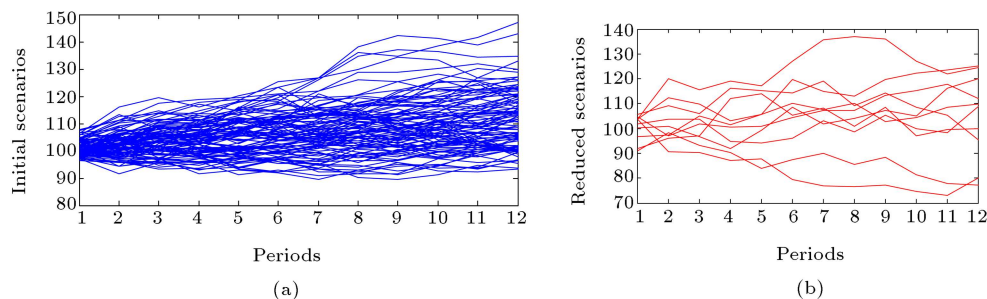


Figure 2. a) Initial scenarios for demand. b) Reduced scenarios for demand.

Table 2. List of parameters for numerical example.

Parameter	p_0	d_0	p^l	r	h	v	ini	N_T	μ_p	σ_p	μ_d	σ_d
Value	100	100	100	5	10	30	20	12	0.1	0.2	0.1	0.1

As mentioned in the modeling uncertainties section, a scenario generation and reduction technique is applied to discretize stochastic variables. The planning horizon is divided into 12 periods. Price and demand scenarios are generated based on Geometric Brownian Motion, with annual volatility and drift, presented in Table 3. As a sample, the demand initial and reduced scenarios are illustrated in Figure 2.

By using the scenario reduction technique introduced in Section 4.1, the generated scenarios for price and demand are reduced and used to obtain the best policies in each type of sourcing. Best policies are obtained by solving the corresponding stochastic model using the CPLEX solver. The Monte Carlo simulation is then applied to estimate the total cost of each model by using the best policies obtained from model optimizations. The simulation process was presented in Section 4.2.

5.1. Risk neutral decision making

Considering the manufacturer as a risk neutral decision maker, the optimal sourcing policies are calculated and presented in Table 3. As mentioned before, the optimal first stage decisions obtained by optimization phase entered to the simulation phase. The sameness of the optimization and simulation objective functions indicate that the scenario reduction method is efficient and prove the in and out of sample stabilities.

Problem parameters and market conditions have significant effects on optimal decision making. These

Table 3. Model and simulation response for each sourcing type.

Optimal decisions	Optimization objective value	Simulation objective value	Gap
$S = 104$	132410	132910	0.3%
$R_e = 92$			

Table 4. The impact of holding cost on decision variables.

Holding cost	2	4	6	8	10	12	14	16	18	20
S_U	109	108	106	105	104	103	103	102	101	101
R	92	92	92	92	92	92	92	92	92	92
Total cost	132230	132420	132610	132770	132910	133050	133160	133280	133390	133480

Table 5. The impact of shortage cost on decision variables.

Shortage cost	5	10	15	20	25	30	35	40	45	50
S_U	20	87	96	99	102	104	106	107	109	110
R	92	92	92	92	92	92	92	92	92	92
Total cost	126580	130460	131360	131990	132480	132910	133280	133610	133900	134170

Table 6. The impact of option price on decision variables.

Option price	1	2	3	4	5	6	7	8	9	10
S	104	104	104	104	104	104	104	104	104	104
R_e	122	112	105	99	92	84	0	0	0	0
Total cost	127800	129200	130510	131750	132910	133990	134680	134680	134680	134680

Table 7. The impact of exercise price on decision variables.

Exercise price	88	90	92	94	96	98	100	102	104	106
S	105	105	105	105	105	104	104	104	104	104
R_e	111	109	107	105	103	99	92	79	0	
Total cost	119830	122210	124560	126850	129050	131100	132910	134390	134680	134680

Table 8. The impact of price fluctuation on decision variables.

Price fluctuation	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	104	104	104	104	104	104	104	104	104	104
R_e	92	101	107	110	114	114	115	115	116	117
Total cost	132910	129740	127600	123710	119380	115920	122260	110040	107930	101610

effects are investigated through sensitivity analysis (Tables 4 to 8).

Holding cost plays a significant role in the multi periodic sourcing problem. According to Table 4, by increasing the holding cost, the optimal decisions change and sourcing cost increases. The buyer reduces his base stock level in order to reduce cost associated with the risk of over stocking. An interesting result is that the numbers of options do not change by changing holding cost. This fact can be the result of using the option contract, as it does not increase holding cost, because it provides the buyer with the right to exercise buy only when the order is needed.

By increasing the shortage cost, the model increases the base stock inventory level to mitigate the shortage cost. Like increasing holding cost, the increase in shortage cost does not change option quan-

tity. The total cost increases when shortage cost increases.

As shown in Tables 6 and 7, the option price and exercise price have a significant impact on option quantity and total cost. As shown in Table 6, increasing option price may lead to avoid signing the option contract.

Signing the option contract will provide opportunities to benefit from low price incidents, where it prevents high price market conditions. In order to show how price fluctuation influences optimal decisions, the sensitivity of best policies for changing price behavior is presented in Table 8 and Figures 3 and 4.

The increase in R , with respect to price volatility, indicates that, as price volatility increases, the buyer can benefit from a low price opportunity in the future. This is the main advantage of the option contract,

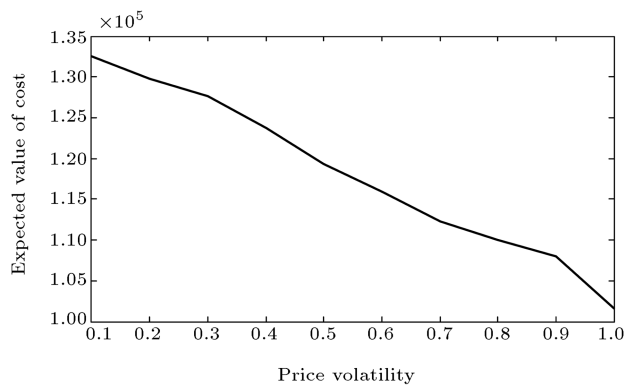


Figure 3. The impact of price volatility on total cost.

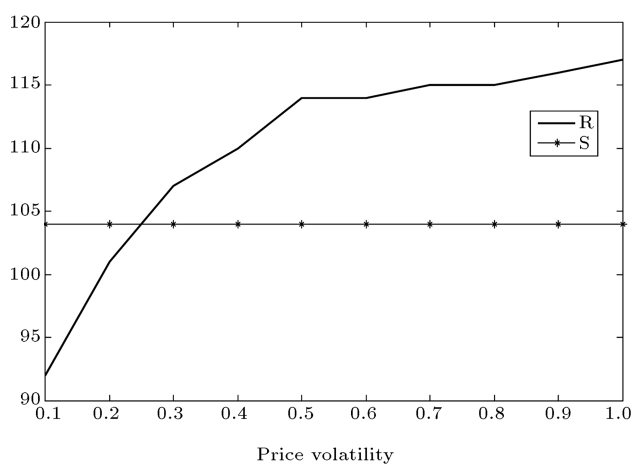


Figure 4. The impact of price volatility on sourcing policies.

which hedges the risks associated with a higher price, as well as providing an opportunity to buy from the spot market when the price is low. As expected, the base stock level (S) does not change as the price volatility varies. As the option contract provides opportunities for using lower prices, the total cost of sourcing will decrease with respect to price volatility increase.

5.2. Risk consideration decision making

Decision making based only on the expected value of the objective function is risk neutral. This type of decision making may not be fit for practical decision making [5]. A decision maker makes a decision with respect to the expected value and the risk of each alternative.

Figure 5 shows the distribution of the total cost of sourcing under different scenarios of demand and price for a risk neutral decision maker. Figure 6 is the same distribution for a risk averse decision maker. A risk averse decision maker avoids high cost incidences, so the skewness of the distribution will decrease.

The more risk averse the decision maker is, the lower the optimal risk measure and the higher the cost of sourcing. This fact is reflected in Table 9 and Figure 7.

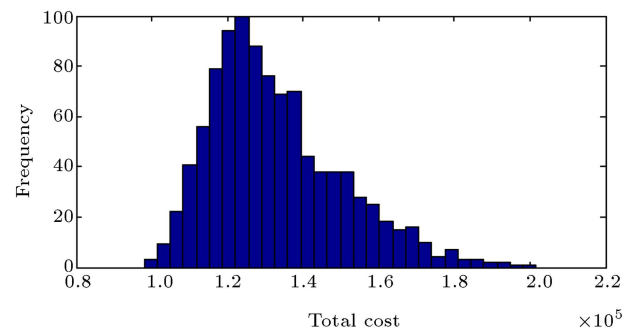


Figure 5. Risk neutral decision maker. Expected value: 132910, Variance: 304400000, Skewness: 0.85.

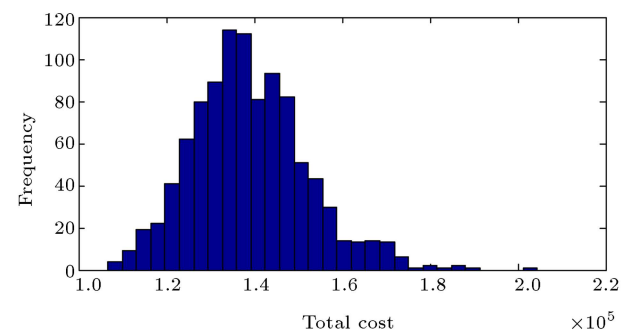


Figure 6. Risk averse decision maker: $\gamma = 4$, $\alpha = 0.95$, Expected value: 138280, Variance: 172750000, Skewness: 0.59.

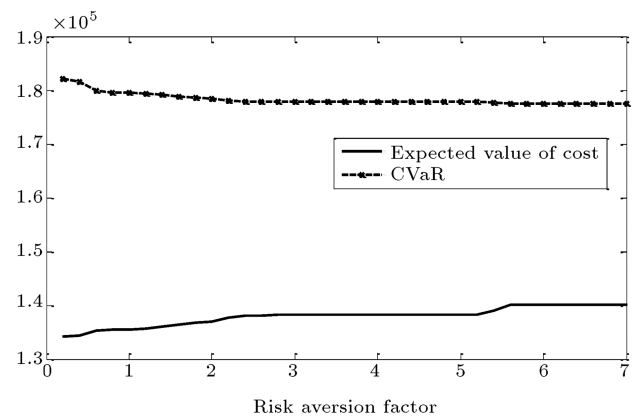


Figure 7. The impact of risk aversity factor on risk and expected value of cost.

Table 9. The impact of risk aversity factor on decision variables.

γ	Expected	CVaR	R_e	S
	value of cost			
0	132910	2568400	92	104
2	136910	178500	53	127
4	138280	177800	58	135
6	140160	177500	66	145
8	140160	177500	66	145

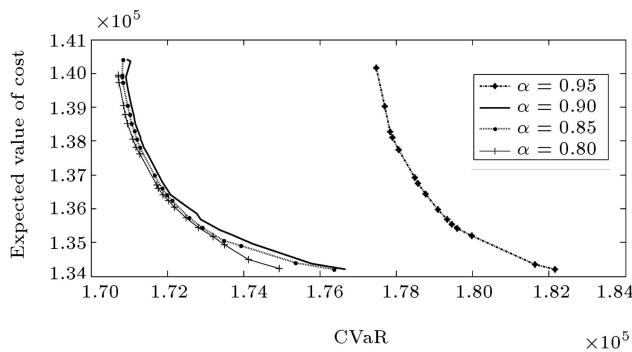


Figure 8. The Efficient fronttier in different confidence levels.

The efficient frontier that shows the trade-off between risk and expected value of cost is presented in Figure 8.

It can be inferred from Figure 8 that while the decision maker aims to have a lower risk level, he should expect a higher cost of sourcing. This trade-off in an optimal combination is called an efficient frontier. The efficient frontier for different confidence levels is also illustrated in Figure 8.

6. Conclusion

Regarding the importance of sourcing as a strategic decision for supply chain members, in this article, an option contract is formulated as a MILP model. A stochastic programming approach is used to analyze the demand and price stochasticity in a two stage supply chain, including one buyer and one supplier, and its effect on sourcing decisions. Price and demand are considered a GBM stochastic process and represented as scenarios using a two-step scenario generation technique. Using CVaR as an efficient risk measure, the impact of risk consideration on the decision variable is studied.

The computational results show how different problem parameters affect the best sourcing policies, cost and risk associated. These findings are gathered through sensitivity analyses. The problem is studied from risk neutral and risk averse decision maker viewpoints. The impact of the risk aversity factor on sourcing decisions is investigated, and an efficient frontier is presented to show a guideline for making a trade-off between risk and expected value of the cost of sourcing.

There are some useful future research topics. First, it is suggested to take into account the supplier parameters. Another suggestion is to calculate the option price, which is assumed to be an exogenous parameter in this study. Considering other risk measures and comparing them, as another extension, will help decision makers to select the best from amongst them.

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Appendix

Simulation phase pseudo code:

Step 1 **Load** best policy for each sourcing type, obtained from optimization phase (S_U, R),

Step 2 **Load** Initial scenarios,

Step 3 **For** each scenario $s \in S$
For each period $t \in T$

$$L_t^s = \begin{cases} 0 & \text{if } e > p_t^s, \\ \max(0, \min(R, S_U - I_t^{s+} + I_t^{s-})) & \text{otherwise} \end{cases}$$

$$\begin{aligned} K_t^s &= \max(0, S_U - I_{t-1}^{s+} + I_{t-1}^{s-} - L_t^s) \\ I_t^{s+} &= \max(0, d_t^s - L_t^s - K_t^s - I_{t-1}^{s+} + I_{t-1}^{s-}) \\ I_t^{s-} &= \max(0, L_t^s + K_t^s + I_{t-1}^{s+} - I_{t-1}^{s-} - d_t^s) \\ \text{Cost}_t^s &= K_t^s p_{t,s}^k + L_t^s e + I_t^{s+} h + I_t^{s-} v \end{aligned}$$

End

$$\text{Cost}_s = \sum_{t=1}^{N_t} \text{Cost}_{t,s}$$

End

Biographies

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