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Network design of a decentralized distribution supply chain: Analysis of non-cooperative equilibrium vs. coordination with discount or buyback mechanism

A. Hafezalkotob[a,*] and A. Makui[b]

a. *Department of Industrial Engineering, Islamic Azad University, South Tehran Branch, Tehran, P.O. Box 1151863411, Iran.*
b. *Department of Industrial Engineering, Iran University of Science and Technology, Tehran, P.O. Box 16846-13114, Iran.*

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Abstract. This paper develops a model for illustrating how a manufacturer can use his initiative to organize retailers when they make decisions as independent actors. The candidate retailers are able to distribute products over geographically dispersed markets with stochastic demands. Each manufacturer's decision about selecting a set of retailers results in a unique distribution network design. Taking transportation and inventory costs into account, each candidate retailer determines order quantity to satisfy market demand, while the manufacturer specifies the wholesale price, pursuing uniform or retailer-specific pricing policies, depending on trade legislation. In this single period problem and under mild assumptions on demand distribution, we show that a non-cooperative equilibrium exists for each distribution network design. We also propose distinctive coordination mechanisms corresponding to pricing policies. Using these mechanisms in each design of the distribution network, the profits of the manufacturer and retailer are better compared to those in non-cooperative situations. Lastly, numerical examples presented in this paper, comprised of the sensitivity analysis of some key parameters, seek to compare the results of different distribution network designs under various pricing policies, yielding some applicable managerial insights.

1. Introduction and literature review

The goal of a supply chain network design is to maximize the profit of a firm while customer needs, such as demand and responsiveness, are satisfied. A manager should take many trade-offs into account during network design, such as those between inventory and transportation costs, facility and processing costs, the costs of coordinating operations and responsiveness to customers. Generally, the holding inventory in some facilities enables firms to enjoy a reduction in inventory holding and facility costs, however, normally raising transportation costs. For example, the centralized distribution of PCs enables Dell to benefit from inventory aggregation, while it accepts an acceptable increase in transportation costs using package carrier customers [1]. If transportation costs rise dramatically, the company must either reduce the responsiveness to customers or use decentralized distribution schema to hold inventory closer to the customer.

Decentralized decision-making and decentralized distribution imply two disparate subjects. Supply Chains (SCs) are typically decentralized, which intimates that they are composed of independent firms, each with its own regularly conflicting goals [2]. Decen-

*. *Corresponding author. Tel.: +98 9127785681;*
E-mail addresses: a_hafez@azad.ac.ir and hafezalkotob@iust.ac.ir (A. Hafezalkotob); amakui@iust.ac.ir (A. Makui)

tralized decision-making models often apply the game theory approach to analyze the interactions of SC participants with regard to a single decision maker SC (a centralized SC). However, decentralized or centralized distribution schemas discuss how inventory spreads through a distribution network. In the decentralized distribution schema, a distinct inventory is kept to satisfy the demand at each source of demand, and, in centralized distribution, the inventories are aggregated in a central distribution center or warehouse and all demands are satisfied from this facility.

Transportation and inventory tradeoff is a critical analysis of the Distribution Network Design (DND) [3]. In the case of demand uncertainty, the centralized DND reduces inventory cost due to the impact of inventory aggregation [4]. Eppen [4] appears to be the earliest study to identify inventory cost saving from the centralization of multi-location newsvendor problems. He showed that inventory cost saving decreases as the correlation between demand increases. Chen and Lin [5] extended Eppen's results for the general distribution of demand uncertainty. Cherikh [6] and Lin et al. [7] developed the models considering a profit maximization approach. Chang and Lin [8] extended Eppen's model by adding the consideration of transportation costs. Chen et al. [9] is believed to be the first to take pricing, along with inventory control decisions, into account in a distribution system with one supplier and multiple retailers. These researchers analyzed the idea of risk pooling from stock centralization from a single decision maker prospective. In contrast, Anupindi and Bassok [10] investigated the inventory aggregation effect in multi-decision makers SC, which consists of one manufacturer and two retailers. They showed that although centralization (where the inventory at the central location is owned jointly by the retailers) is always advantageous to the retailers, it may not benefit the manufacturer.

If markets are geographically dispersed, the advantages of inventory aggregation in a centralized network diminish because of high transportation costs for serving faraway markets. Accordingly, the benefits of a decentralized DND, where each retailer serves nearby markets, may far outweigh those of a centralized one. For instance, with few facilities, Amazon.com incurs a lower inventory cost compared to Borders, which has about 450 stores. On the other hand, Borders has lower transportation costs [1]. We ask, what is the optimal design of a distribution network in geographically dispread markets which incurs minimum inventory and transportation costs for the manufacturer? How do the strategies of independent retailers affect the optimal design of the distribution network?

In the newsvendor context, besides stocking quantity, price is also an important business behavior affecting the profitability of the firms. Petruzzi and Dada [11] extended the newsvendor problem by incorporating selling price and stocking quantity from a single decision maker prospective. Numerous types of research investigated these factors in a distribution network as a multi-decision maker system. For instance, Lariviere and Porteus [12] gave a complete analysis of a wholesale price contract between a manufacturer and a newsvendor retailer, and they showed that the contract cannot result in SC coordination. Dong and Rudi [13] investigated wholesale pricing in a one-manufacturer and two-retailer distribution system, where transshipment was allowed between the retailers (see also [10]). Chen [14] showed that if the SC retailer is a newsvendor that makes the order quantity decision, his order quantity is contingent on the wholesale pricing decision of the manufacturer.

Generally, in one-manufacturer and multi-retailer systems, the manufacturer may adopt two different wholesale pricing policies: "retailer-specific" or "uniform". In the absence of explicit limitations, a profit maximizing manufacturer would normally prefer to set different wholesale prices for distinctive retailers. This may be a reasonable policy when the retailers are sufficiently geographically dispersed. Nevertheless, several problems may prevent a manufacturer from implementing a retailer-specific pricing policy. For example, when the local retailers are close together, in order to avoid conflict with retailers and to maintain equality, the manufacturer may set uniform wholesale prices for the retailers. Some trade legislations, such as the antitrust law (see e.g., [15,16]), may also prohibit the sale of identical goods, or services are transacted at different prices from the same provider, which is also called third degree price discrimination (see also [17-20] for further discussion about differences between price discrimination and uniform pricing policies). Additionally, the operational and supervisory costs of a retailer-specific pricing policy may be higher than a uniform policy. How does the pricing policy of a manufacturer affect the manufacturer's wholesale price(s) and retailers' order quantities? How does the profitability of each distribution design vary with regard to these pricing policies?

In recent years, several practitioners and academics have paid growing attention to the utilization of game theory in the coordination of SC participants. Coordination mechanisms are contractual agreements which enable a decentralized channel (including independent DMs) to operate in a centralized fashion (where the decisions are taken by a single DM) [21]. These contractual agreements also facilitate long-term partnerships and make the terms more explicit [22]. The favorite approach to reach coordination is often to retain decentralized decision-making but to organize the costs and revenues of all participants in order to align their objectives with the system wide objective. Cachon [22] reviewed the literature of coordination mechanisms and discussed corresponding side-payments. The coordination mechanisms are commonly categorized as buy-back [23-28], markdown money [29,30], revenue sharing [31-35] price discount [36-39], sale-rebate contracts [40-42], quantity flexibility and so on. A stream of literature has recently emerged that takes concurrent adoption of two mechanisms, for instance, returns with a wholesale-price-discount [14], sales-rebate and return [43], and a consignment contract with revenue sharing [44]. Chen [14] showed that the coordination mechanism composed of a wholesale-price-discount and return policy encourages the retailer to order more and ameliorates retailer loyalty. While the contract slightly lowers the manufacturer's profits, it can significantly enhance the profit of the retailer, as well as total SC efficiency.

Under the return policy, manufacturer commits to buying back the unsold inventory of goods at the end of their selling season. Therefore, such a policy can be employed in newsvendor-style products that have a short (limited) life cycle, such as newspapers or magazines, fashion apparel, holiday products, electrical products, records, etc. Tsay [45] argued that buy-back contracts are mechanisms by which a manufacturer can increase initial retail orders through converting his profit payoff from certain to uncertain. Therefore, the manufacturer accepts exposure to risk and popular vernacular tends to label this as a "sharing" or "transfer" of risk. Wang and Zipkin [46] investigated a two-stage supplier-retailer agent system, employing a buyback mechanism. Xiao et al. [28] integrated the buyback contract of the manufacturer with the customer return policy as a single framework. Since the buyback contract is a favorite solution to the double margination effect [47], we employ a buy back contract to coordinate each design of the distribution network under the retailer-specific pricing policy. As discussed by Pasternack [23], coordinating multiple distinctive retailers with a uniform coordination mechanism is not possible. Accordingly, under a uniform pricing policy, we propose a wholesale discount with the revenue-sharing contract, which partially coordinates each distribution design. While the wholesale price discount boosts SC profitability, the revenue-sharing contract based on side-payments makes all-members better off compared to a situation without SC coordination (see [21] for further discussion of side-payment contracts).

In many industries, manufacturers have the authority to select and organize a set of retailers in order to lower distribution costs. Each manufacturer's decision about candidate retailers results in a unique DND. The presence of distribution network decisions adds one new dimension to the relationship between manufacturers and retailers, and underscores the importance of manufacturer capability in contracting with independent retailers to serve geographically dispersed markets. To the best of the authors' knowledge, no research has been found in the context of multi-location newsvendors and a decentralized decision-making structure to investigate the manufacturer's authority to make distribution design decisions. According to this gap in the literature, there are three main contributions to this research: First, we study the optimal DND for a manufacturer whose retailers are independent DMs. In particular, the manufacturer, the SC leader, sets the wholesale price, and then the candidate retailers, the SC followers, as multi location newsvendors, determine optimal stoking quantities. Second, we put forward and compare two well-known pricing policies that may be pursued by the manufacturer in each DND. Third, we propose coordination mechanisms, corresponding to the two pricing policies, as solutions to the double marginalization problem. Table 1 demonstrates that the proposed approach covers new possible features in the multi-location newsvendor problem in comparison with other existing models.

The rest of the paper is organized as follows. In the next section, we first study equilibrium solutions for a given distribution design under uniform and retailer-specific pricing policies. Afterwards, optimal design of a distribution design is investigated. Section 3 gives numerical examples to demonstrate our results and insights. In Section 4, we provide managerial discussions about the model, with some directions for future research in this context. The paper concludes in Section 5. Proofs are given in the Appendix.

2. Non-cooperative equilibria and supply chain coordination for fixed distribution network

Consider a distribution SC network consisting of one manufacturer and a set of retailers which serve multiple markets. In this section, we take into account a given DND, i.e. the retailer channels are selected by the manufacturer to supply the markets. Each retailer, as an independent distribution channel, sells the products of the manufacturer to nearby markets. The manufacturer, as a leader, first declares the wholesale price and then his retailers, as followers, respond to the manufacturer's decision, which conforms to "leader and follower" game principles. The manufacturer may adopt a uniform or a retailer-specific pricing policy.

We first evaluate Stackelberg equilibrium under both pricing policies. Afterwards, we propose coordination mechanisms based on wholesale discount and buyback agreements to coordinate distribution networks under uniform and retailer-specific pricing policies, respectively. A proper allocation of the coordination surplus profit provides all members with

Table 1. Comparison of the features of the proposed model with the existing models.

Features	Eppen [4]	Chang and Lin [8]	Chen et al. [9]	Bakal et al. [48]	Kang and Kim [49]	Chen [14]	Proposed model
No. of retailers	FM[a]	FM	FM	FM	FM	S[b]	VM[c]
Multi-location newsvendor	Considered	Considered	Not considered	Considered	Not considered	Not considered	Considered
Geographically dispersed markets	Not considered	Not considered	Not considered	Considered	Considered	Not considered	Considered
Transportation costs	Not considered	Considered	Not considered	Not considered	Considered	Not considered	Considered
Manufacturer pricing authority	Not considered	Not considered	Considered	Not considered	Not considered	Considered	Considered
Pricing strategies of manufacturer	Not considered	Not considered	Not considered	Not considered	Not considered	Not considered	UP[d] & RSP[e]
Coordination mechanisms	Not considered	Not considered	Considered	Not considered	Not considered	Considered	Considered
Manufacturer authority to select retailers	Not considered	Not considered	Not considered	Not considered	Not considered	Not considered	Considered

[a]FM: Fixed and Multiple, [b]S: Single, [c]VM: Variable and Multiple, [d]UP: Uniform Pricing, [e]RSP: Retailer-Specific Pricing.

the opportunity to achieve more profitability than in the non-cooperative situation.

2.1. Manufacturer's pricing and retailers' ordering decisions with no coordination: Stackelberg equilibrium

The manufacturer's products have short life cycles such as regarding style or seasonal items. The retail-price, p, at the retailers' location is exogenously determined. Let sets of N and I denote, in turn, markets and candidate retailers that are geographically dispersed. Consider a given distribution network, k, with a set of selected retailers, $I_{[k]}(\subseteq I)$. Since the retail-price is identical, the customers in the markets choose the retailer with the lowest transportation cost (commonly the nearest retailer). The selected retailer, $i(\in I_{[k]})$, should pay extra transportation cost, TC_i, to receive products from the manufacturer. In the non-cooperative case, the manufacturer produces products with unit cost, c, and sets wholesale price(s), $w_{[k]}(< p - TC_i)$ (or $w_{i[k]}(< p - TC_i)$, in the retailer-specific pricing policy). Afterwards, the retailer decides to order quantity $q_{i[k]}$, with regard to the nearby market demands and manufacturer wholesale price(s).

The customers in market j bear transportation cost, TC_{ij}, to receive orders, where retailer i is the retailer with the lowest transportation cost. TC_i and TC_{ij} are cost parameters, which depend on the structures of the distribution network such as geographic location, distance, infrastructure, and transportation equipment. When customers in market j choose a nearby retailer, i, it is assumed that the market's demand, d_j, is random and depends on the retail-price in the market location, $p_{ij}(= p + TC_{ij})$. Following Petruzzi and Dada [11], the additive-demand, which is regularly used in pricing and newsvendor contexts, is defined as:

$$d_j(p_{ij}, \varepsilon_j) = y_j(p_{ij}) + \varepsilon_j, \tag{1}$$

where $y_j(p_{ij}) = a_j - b_j p_{ij}$ is a deterministic term, sensitive to price, and ε_j is a random term with Probability Density Function (PDF) $f_j(.)$, and Cumulative Distribution Function (CDF) $F_j(.)$. It is supposed that random terms of market demand have a similar type probability distribution. Since demand does not hold any negative value, we assume that random term ε_j has a lower bound, A_j. Therefore, to ensure that demand $d_j(p_{ij}, \varepsilon_j)$ is non-negative, the lower bound must be larger than $-a_j$, i.e. ε_j is well defined in the range $[A_j, \infty)$, where $A_j > -a_j$. Such assumptions are commonly used in an economic context with an additive demand curve (for instance, see [11]). We assume the PDF of random terms, transportation costs, and retail price, p, are known to the manufacturer and to his retailers, as well.

If the manufacturer chooses retailer i in the distribution design, k, the retailer satisfies the demands of nearby markets, $N_{i[k]}$, where $N_{i[k]} = \{j \in N : TC_{ij} < TC_{lj}, \forall l \in I_{[k]}\}$. Therefore, the retailer faces the demand, which is formulated as:

$$d_{i[k]} = \sum_{j \in N_{i[k]}} d_j(p_{ij}, \varepsilon_j) = \sum_{j \in N_{i[k]}} y_j(p_{ij}) + \xi_{i[k]}, \quad (2)$$

where $\xi_{i[k]} = \sum_{j \in N_{i[k]}} \varepsilon_j$ is the sum of the random terms with PDF $g_{i[k]}(.)$, and CDF $G_{i[k]}(.)$. We assume that computation of PDF $g_{i[k]}(.)(\forall i \in I_{[k]})$ is possible through the probability properties of random variables, $\varepsilon = \{\varepsilon_1, ..., \varepsilon_n\}$. When the random variables are independent and identically distributed with PDF $f(.)$, $\xi_{i[k]}$ has the density $g_{i[k]}(.)$, if $g_{i[k]}(.)$ has an infinity divisible property [50]. Several distributions, such as exponential, gamma, normal, lognormal, chi-squared and Cauchy distributions, have this property [51,52]. For instance, the sum of independent exponentially distributed random variables with identical PDF is a random variable with a gamma density. Generally, if the distribution of random terms, ε, belong to the class of stable distributions, the shape of distribution retains after summation [50]. For independent and identical random terms, ε, with general distributions, $\xi_{i[k]}$ can be approximated by normal distribution according to the central limit theorem, if the number of markets, $(|N_{i[k]}|)$, is sufficiently large [53]. Additionally, in the case of non-identical and dependent normal random variables, $\xi_{i[k]}(\forall i \in I_{[k]})$ is normally distributed according to multivariate normal distribution properties [54] (see, [4,8] for further discussion of correlated demands). Random variable $\xi_{i[k]}$ is well defined in the range $[B_{i[k]}, \infty)$, where $B_{i[k]} = \sum_{j \in N_{i[k]}} A_j \geq -\sum_{j \in N_{i[k]}} a_j$ assures that retailer i deals with the non-negative demand from the markets in the distribution network design, k.

2.1.1. Retailer best response order decisions

In the given distribution network design, k, we assume that a subset of retailers, $I_{[k]}$, is selected by the manufacturer. Consistent with Petruzzi and Dada [11], we apply the transformation of variable $Z_{i[k]} = q_{i[k]} - \sum_{j \in N_{i[k]}} y_j(p_{ij})$ as retailer i's safety stock level. If the demands of the markets during the selling period do not exceed $q_{i[k]}$, i.e., $z_{i[k]} \geq \xi_{i[k]}$, then, the revenue of the retailer is $pd_{i[k]}$, and each of the leftovers $(q_{i[k]} - d_{i[k]})$ is disposed at the unit salvage value, $v_i (< c)$. On the other hand, if the demands exceed $q_{i[k]}$, i.e. $z_{i[k]} < \xi_{i[k]}$, then, the retailer's revenue is $pq_{i[k]}$ and each of the $d_{i[k]} - q_{i[k]}$ of unmet demands incurs a shortage cost of u_i. Let subscripts M and R_i denote the manufacturer and retailer, i, respectively. The random profit of retailer i is obtained as sales revenue plus the salvage value, minus shortage and procurement costs. Thus, when the manufacturer adopts a uniform pricing policy, $w_{[k]}$, the retailer i's random profit under distribution design k, $\Pi_{R_i[k]}$, is given as:

$$\Pi_{R_i[k]}(w_{[k]}, z_{i[k]}) = \begin{cases} p\left(\sum_{j \in N_{i[k]}} y_j(p_{ij}) + \xi_{i[k]}\right) \\ \quad + v_i(z_{i[k]} - \xi_{i[k]}) - (w_{[k]} \\ \quad + TC_i)\left(\sum_{j \in N_{i[k]}} y_j(p_{ij}) + z_{i[k]}\right), \\ \quad z_{i[k]} \geq \xi_{i[k]}, \quad \forall i \in I_{[k]} \\ (p - w_{[k]} - TC_i)\left(\sum_{j \in N_{i[k]}} y_j(p_{ij}) \right. \\ \quad \left. + z_{i[k]}\right) - u_i(\xi_{i[k]} - z_{i[k]}), \\ \quad z_{i[k]} < \xi_{i[k]}, \end{cases} \quad (3)$$

Defining $\Lambda_{i[k]}(z_{i[k]}) = \int_{B_{i[k]}}^{z_{i[k]}} (z_{i[k]} - x) g_{i[k]}(x) dx$ and $\Theta_{i[k]}(z_{i[k]}) = \int_{z_{i[k]}}^{\infty} (x - z_{i[k]}) g_{i[k]}(x) dx$, we have:

$$E\left(\Pi_{R_i[k]}(w_{[k]}, z_{i[k]})\right) = \left(p - w_{[k]} - TC_i\right) \sum_{j \in N_{i[k]}} y_j\left(p_{ij}\right) - \left(w_{[k]} + TC_i - v_i\right) \Lambda_{i[k]}(z_{i[k]}) - \left(p + u_i - w_{[k]} - TC_i\right) \Theta_{i[k]}(z_{i[k]}), \quad \forall i \in I_{[k]}. \quad (4)$$

$\Lambda_{i[k]}(z_{i[k]})$ and $\Theta_{i[k]}(z_{i[k]})$ represent the expected leftover and the expected storage of retailer i in distribution design k, respectively. Likewise, if the manufacturer pursues the retailer-specific pricing policy, the expected profit of retailer i can be computed as follows:

$$E\left(\Pi_{R_i[k]}(w_{i[k]}, z_{i[k]})\right) = \left(p - w_{i[k]} - TC_i\right) \sum_{j \in N_{i[k]}} y_j\left(p_{ij}\right) - \left(w_{i[k]} + TC_i - v_i\right) \Lambda_{i[k]}(z_{i[k]}) - \left(p + u_i - w_{i[k]} - TC_i\right) \Theta_{i[k]}(z_{i[k]}), \quad \forall i \in I_{[k]}. \quad (5)$$

The first and second partial derivatives of $E_{\xi_{i[k]}}\left(\Pi_{i[k]}(w_{[k]}, z_{i[k]})\right)$ w.r.t. $z_{i[k]}$ are:

$$\partial E\left(\Pi_{R_i[k]}\right) / \partial z_{i[k]} = -\left(w_{[k]} + TC_i - v_i\right) G_{i[k]}(z_{i[k]}) + \left(p + u_i - w_{[k]} - TC_i\right) \left(1 - G_{i[k]}(z_{i[k]})\right),$$

$$\partial^2 E\left(\Pi_{R_i[k]}\right)/\partial z_{i[k]}^2 = -\left(p+u_i-w_{[k]}-TC_i\right) g_{i[k]}(z_{i[k]}).$$

Since $E\left(\Pi_{R_i[k]}(w_{[k]}, z_{i[k]})\right)$ is a concave function of $z_{i[k]}$, we conclude that for a given wholesale price, $w_{[k]}$ (similarly, for a given $w_{i[k]}$ in Eq. (5)), the first order condition leads to the optimum order quantity:

$$q_{i[k]}^* = \sum_{j\in N_{i[k]}} y_j(p_{ij}) + z_{i[k]}^* = \sum_{j\in N_{i[k]}} \left(a_j - b_j(p+TC_{ij})\right) + G_{i[k]}^{-1}\left(\frac{p+u_i-w_{[k]}-TC_i}{p+u_i-v_i}\right). \quad (6)$$

Regarding the risk of shortage of retailer i in distribution design, k, we define the probability, $P(z_{i[k]} < \varepsilon_{i[k]}) = 1-\alpha_{i[k]}$, where $\alpha_{i[k]}$ is referred to as the service level. Thus, $\alpha_{i[k]}^* = G_{i[k]}(z_{i[k]}^*) = (p+u_i-w_{[k]}-TC_i)/(p+u_i-v_i)$ and $\alpha_{i[k]}^* = G_{i[k]}(z_{i[k]}^*) = (p+u_i-w_{i[k]}-TC_i)/(p+u_i-v_i)$ indicate the optimum service level of the retailer under the uniform and retailer-specific pricing policies, respectively. In the case of a uniform pricing policy, the following corollary sums up the effects of wholesale price and transportation costs on order quantity, service level, and the optimal expected profit of the retailers.

Corollary 1.

(i) The closeness of a retailer to markets, which reduce the dispatching costs, causes an increase in the quantity that the retailer orders and his service level. Likewise, the closeness of a retailer to the manufacturer boosts his service level and order quantity, as well. Moreover, the greater the wholesale price, the lower all retailers' order quantity and service level will be.

(ii) The nearness of a retailer to his markets and the manufacturer boosts the optimal expected profit of the retailer. Additionally, an increase in wholesale price deteriorates the optimal expected profit of all retailers.

A similar corollary can be drawn for the retailer-specific pricing policy. Several researchers showed that the centralized distribution schema, which integrates inventory in a single location, reduces inventory costs [4-8]. However, we conclude from Corollary 1 that if markets are geographically dispersed, the high dispatching costs of serving them from a single retailer shrink the retailer's order quantity and service level, as well as the expected profit. Therefore, a distribution network design should select a minimum number of retailers that are as close as possible to the markets in order to balance inventory cost savings with dispatching cost increases.

2.1.2. Manufacturer's best response pricing decision: Non-cooperative Stackelberg equilibria

This section considers the relationship between the manufacturer and a set of retailers in a given distribution design using a non-cooperative Stackelberg structure. Specifically, we suppose that the manufacturer, as a Stackelberg leader, has the initiative and can enforce his wholesale strategy on the retailers. Under a uniform pricing policy, the manufacturer sets the uniform wholesale price, $w_{[k]}$, for all retailers in distribution design, k. For a given $w_{[k]}$ of the manufacturer, each retailer, as a Stackelberg follower, obtains the best order quantity according to Eq. (6). The manufacturer can anticipate retailer order quantities for any wholesale price. Therefore, he determines the uniform wholesale price to maximize his expected profit, based on the best order quantities of the retailers. Thus, the problem reduces to:

$$\max E\left(\Pi_{M[k]}(w_{[k]})\right) = \left(w_{[k]}-c\right)\sum_{i\in I_{[k]}} \left(\sum_{j\in N_{i[k]}} y_j(p_{ij}) + \Lambda_{i[k]}(z_{i[k]}^*) - \Theta_{i[k]}(z_{i[k]}^*)\right), \quad (7)$$

$$\text{s.t.}: G_{i[k]}(z_{i[k]}^*) = (p+u_i-w_{[k]}-TC_i)/(p+u_i-v_i), \quad \forall i\in I_{[k]}. \quad (8)$$

Since $z_{i[k]}^* = \Lambda_{i[k]}\left(z_{i[k]}^*\right) - \Theta_{i[k]}\left(z_{i[k]^*}\right)$ and $q_{i[k]}^* = \sum_{j\in N_{i[k]}} y_j(p_{ij}) + z_{i[k]}^*$, objective function (7) represents the manufacturer's expected profit gained from his retailers in distribution network, k. Following Ross [53], let us define $r_{i[k]}(.) = g_{i[k]}(.)/\left(1-G_{i[k]}(.)\right)$ as the hazard (or failure) rate of random variable, $\xi_{i[k]}$. The hazard rate, $r_{i[k]}(.)$, gives an estimation of percentage decrease in the retailer's i stock out probability by increasing the stocking quantity by one unit [12]. In Proposition 2, we show that the existence of Stackelberg equilibrium, $(w_{[k]}^*, q_{i[k]}^*)$, $\forall i \in I_{[k]}$ depends on the non-decreasing property of hazard rate, $r_{i[k]}(.)$. Several of the commonly applied demand distributions have non-decreasing hazard rates, such as normal, uniform, logistic, extreme value, chi-squared, exponential, and the special case of gamma and beta [55].

Proposition 1. Under the uniform pricing policy, if the demand uncertainty of each selected retailer is in the class of non-decreasing hazard rate distribution, there exists a unique Stackelberg equilibrium, $(w^*_{[k]}, q^*_{i[k]})$, $\forall i \in I_{[k]}$ for given distribution design, k, which satisfies:

$$\left(w^*_{[k]} - c\right) \sum_{i\in I_{[k]}} \frac{1}{\left(w^*_{[k]} + TC_i - v_i\right) r_{i[k]}(z^*_{i[k]})} = \sum_{i\in I_{[k]}} \left[\sum_{j\in N_{i[k]}} y_j\left(p_{ij}\right) + z^*_{i[k]}\right], \quad (9)$$

$$q^*_{i[k]} = \sum_{j\in N_{i[k]}} y_j(p_{ij}) + z^*_{i[k]}, \quad (10)$$

where:

$$z^*_{i[k]} = G^{-1}_{i[k]}\left((p + u_i - w^*_{[k]} - TC_i)/(p + u_i - v_i)\right).$$

When different wholesale prices are allowed, the manufacturer pursues a retailer-specific pricing policy. Let us denote wholesale prices offered to the retailers in distribution design, k, by $\mathbf{w_{[k]}} = \{w_{i[k]} : i \in I_{[k]}\}$. Under this circumstance, the manufacturer problem is transformed into:

$$\max E\left(\Pi_{M[k]}(\mathbf{w_{[k]}})\right) = \sum_{i\in I_{[k]}} E\left(\Pi_{M_i[k]}(w_{i[k]})\right) = \sum_{i\in I[k]} (w_{i[k]} - c)\left(\sum_{j\in N_{i[k]}} y_j(p_{ij}) + \Lambda_{i[k]}(z^*_{i[k]}) - \Theta_{i[k]}(z^*_{i[k]})\right), \quad (11)$$

$$\text{s.t.}: G_{i[k]}(z^*_{i[k]}) = (p + u_i - w_{i[k]} - TC_i)/(p + u_i - v_i), \quad \forall i \in I_{[k]}, \quad (12)$$

where $E\left(\Pi_{Mi[k]}(w_{i[k]})\right)$ represents the manufacturer's profit from retailer, i, in distribution design, k.

Proposition 2. Under the retailer-specific pricing policy, if the demand uncertainty of each selected retailer is in the class of non-decreasing hazard rate distribution, there exists a unique Stackelberg equilibrium, $(w^*_{i[k]}, q^*_{i[k]})$, $\forall i \in I_{[k]}$, for given distribution design, k, which satisfies:

$$w_{i[k]^*} = \frac{c + (TC_i - v_i) r_{i[k]}(z^*_{i[k]})\left[\sum_{j\in N_{i[k]}} y_j(p_{ij}) + z^*_{i[k]}\right]}{1 - r_{i[k]}(z^*_{i[k]})\left[\sum_{j\in N_{i[k]}} y_j(p_{ij}) + z^*_{i[k]}\right]}, \quad (13)$$

$$G_{i[k]}(z^*_{i[k]}) = (p + u_i - w_{i[k]} - TC_i)/(p + u_i - v_i). \quad (14)$$

A retailer-specific pricing policy implies that the manufacturer employs independent strategies for the retailers, which can be interpreted as distribution networks comprised of a set of independent distribution channels. Thus, the optimum strategy for each retailer (in each channel) in Proposition 2 is consistent with a single channel distribution, such as the one proposed by Chen [14] (when $TC_i = v_i = 0$).

2.2. Supply chain coordination in a given distribution design

Under non-cooperative SC, the retailers shoulder the responsibility of unsold products and the manufacturer maximizes his profit, regardless. There are two margins, and neither the manufacturer nor the retailers consider the entire distribution network margin at the time of decision-making. Thus, the double marginalization effect prevents the SC from reaching a system-wide profit. To coordinate the SC, we design an appropriate side-payment contract, which enables the decentralized channels to perform as if they were operating in a centralized fashion. Such a contractual mechanism should properly allocate the surplus profit (obtained from coordination) between channel members to make them better off compared to the situation without contract (we refer the reader to [21] for further discussion).

2.2.1. System-wide optimum for a given distribution network design

To provide a benchmark, we first find the optimal ordering decision of retailers that maximizes the total profit for a given DND, as if its members are performing in a centralized fashion. For such a system, let $q^o_{i[k]}$, $z^o_{i[k]}$ and $\alpha^o_{i[k]}$ denote the globally optimal order quantity, safety stock level, and service level of retailer, i, in distribution design, k, respectively. The system-wide profit in distribution design, k, is:

$$\Pi_{[k]} = \Pi_{M[k]} + \sum_{i\in I_{[k]}} \Pi_{R_i[k]}. \quad (15)$$

Let $\mathbf{z_{[k]}} = \{z_{i[k]} : i \in I_{[k]}\}$ be the safety stock level of retailers in distribution design, k. The expected value of the system-wide profit is then the sum of Eqs. (4) and (7), i.e:

$$E\left(\Pi_{[k]}(\mathbf{z_{[k]}})\right) = \sum_{i\in I_{[k]}} E\left(\Pi_{i[k]}(z_{i[k]})\right) = \sum_{i\in I_{[k]}} (p - c - TC_i) \sum_{j\in N_{i[k]}} y_j(p_{ij}) - (c + TC_i - v_i)\Lambda_{i[k]}(z_{i[k]}) - (p + u_i - c - TC_i)\Theta_{i[k]}(z_{i[k]}), \quad (16)$$

where $E\left(\Pi_{i[k]}(z_{i[k]})\right)$ represents the expected channel i's profit. The expected system-wide profit under the retailer-specific pricing is also obtained from the sum of Eqs. (5) and (11), which is equal to Eq (16), as well. Therefore, the system-wide optimal solution is not contingent upon the pricing policy and wholesale price of the manufacturer, and only depends on the safety stock levels of retailers.

Proposition 3. For a given distribution design, k, the unique optimal order quantity of $q^o_{i[k]}$ that maximizes channel-wide profit is:

$$q^o_{i[k]} = \sum_{j\in N_{i[k]}} y_j(p_{ij}) + z^o_{i[k]} = \sum_{j\in N_{i[k]}} y_j(p_{ij}) + G^{-1}_{i[k]}\left(\frac{p+u_i-c_i-TC_i}{p+u_i-v_i}\right), \quad \forall i \in I_{[k]}. \qquad (17)$$

The service level of retailer, i, under cooperation (centralized decision-making) is $\alpha^o_{i[k]} = G_{i[k]}(z^o_{i[k]}) = (p + u_i - c_i - TC_i)/(p+u_i-v_i)$. Taking $w^*_{[k]} > c$ (or $w^*_{i[k]} > c$) into account, it is straightforward that retailers' service levels are boosted due to cooperation through a distribution network, i.e., $\alpha^o_{i[k]} \geq \alpha^*_{i[k]}, \forall i \in I_{[k]}$. Likewise, we can conclude that $q^o_{i[k]} \geq q^*_{i[k]}, \forall i \in I'_{[k]}$ (or equivalently $z^o_{i[k]} \geq z^*_{i[k]}$) under both pricing policies of the manufacturer. By comparing cooperative SC with a non-cooperative one, the following corollary can be drawn.

Corollary 2. Regardless of the pricing policy of the manufacturer and the structure of the distribution design, the expected system-wide profit of the distribution network is constantly not smaller than the expected total profit of the distribution network design under non-coordination.

To enjoy the surplus profit of a centralized decision-making structure and increments in retailer service levels, the manufacturer should offer appropriate incentives to his retailers in each design of decentralized distribution (decentralized decision-making). Now, let $\gamma_{[k]} \geq 0$ (according to Corollary 2) denote this system-wide surplus profit, i.e.:

$$\gamma_{[k]} = E\left(\Pi_{[k]}(\mathbf{z^o_{[k]}})\right) - E\left(\Pi_{[k]}(\mathbf{z^*_{[k]}})\right), \qquad (18)$$

where $\mathbf{z^o_{[k]}} = \{z^o_{i[k]}, \forall i \in I_{[k]}\}$ and $\mathbf{z^*_{[k]}} = \{z^*_{i[k]}, \forall i \in I_{[k]}\}$ are optimal safety stock levels of retailers in cooperative SC and non-cooperative SC, respectively.

Coordination mechanisms are distinctive under uniform and retailer-specific pricing policies. When the manufacturer pursues a retailer-specific pricing policy, he will be able to employ a unique coordination mechanism for each retailer. In this situation, all traditional coordination mechanisms (mentioned in [22]) may yield the completely coordinated distribution network. In Section 2.2.2., we propose the buy-back contract for coordinating each design of the distribution network. Nevertheless, when it comes to a uniform pricing policy, the manufacturer should use a uniform mechanism to coordinate all retailers in a distribution network. Pasternack [23] discussed that a uniform buyback mechanism would not coordinate multiple distinctive retailers. We found that not only buyback contracts but also no other simple traditional coordination mechanisms, mentioned by Cachon [22], may simultaneously coordinate all distinctive retailers, because the resulting effect on retailer profitability will not be consistent. That is, some retailers may not profit from coordinating a distribution network with a uniform buyback mechanism, whereas other retailers may excessively benefit from the contract. In Section 2.2.1, we investigate a partial coordination mechanism, where the manufacturer ignores some part of his initiative in wholesale pricing, and retailers commit the manufacturer to paying some part of the channels' surplus profits, instead.

2.2.2. Partial coordination mechanism for uniform pricing policy under retailer commitments

From Corollary 2, we found that the incentive mechanism that induces retailers to order more, boosts SC's profit, i.e. SC profit is an increasing function in $q_{i[k]}$ for $q_{i[k]} \in [q^*_{i[k]}, q^o_{i[k]}]$. However, when the manufacturer exploits his dominant position in setting wholesale price, he sets $w^*_{[k]}$ according to Proposition 1. The retailer, i, then orders $q^*_{i[k]}$, inevitably; thus, SC performance reaches the minimum. In this situation, Cachon [22] concluded that an increase in retailer power can actually improve SC performance. We investigate a revenue sharing contract in which the manufacturer, by ignoring some part of his dominant position in pricing, enables SC to achieve a part of coordination surplus profit, $\gamma_{[k]}$.

We now describe the coordination mechanism in which the manufacturer offers a contract of a wholesale price discount (represented by $\hat{w}_{[k]}$, where $\hat{w}_{[k]} \in [c, w^*_{[k]}]$) and the retailers consent to transfer a part of the channels' profits in return. Let $TR_{i[k]}$ be the transfer (side) payment given by retailer, i, to the manufacturer under distribution design, k. Moreover, let $\hat{z}_{i[k]} = G^{-1}_{i[k]}\left[(p+u_i-\hat{w}_{[k]}-TC_i)/(p+u_i-v_i)\right]$ denote the optimum safety stock level of retailer, i, under the wholesale price discount, $\hat{w}_{[k]}$. We refer to such a contract as a discount-revenue sharing contract, $(\hat{w}_{[k]}, \hat{\mathbf{z}}_{[\mathbf{k}]}, \mathbf{TR}_{[\mathbf{k}]})$, where $\hat{\mathbf{z}}_{[\mathbf{k}]} = \{z_{i[k]}, i \in I_{[k]}\}$ and $\mathbf{TR}_{[\mathbf{k}]} = \{TR_{i[k]}, i \in I_{[k]}\}$. Thus, under the contract, the expected profit of retailer, i, in distribution design, k, can be found as follows:

$$E\left(\Pi_{R_i[k]}(\hat{w}_{[k]},\hat{z}_{i[k]},TR_{i[k]})\right)$$
$$=(p-\hat{w}_{[k]}-TC_i)\sum_{j\in N_{i[k]}}y_j(p_{ij})$$
$$-(\hat{w}_{[k]}+TC_i-v_i)\Lambda_{i[k]}(\hat{z}_{i[k]})$$
$$-(p+u_i-\hat{w}_{[k]}-TC_i)\Theta_{i[k]}(\hat{z}_{i[k]})TR_{i[k]},$$
$$\forall i\in I_{[k]}. \tag{19}$$

The manufacturer's expected profit can be written conveniently as:

$$E\left(\Pi_{M[k]}(\hat{w}_{[k]},\hat{\mathbf{z}}_{[\mathbf{k}]},\mathbf{TR}_{[\mathbf{k}]})\right)=$$
$$\sum_{i\in I_{[k]}}\Bigg[(\hat{w}_{[k]}-c)\Bigg(\sum_{j\in N_{i[k]}}y_j(p_{ij})$$
$$+\Lambda_{i[k]}(\hat{z}_{i[k]})-\Theta_{i[k]}(\hat{z}_{i[k]})\Bigg)+TR_{i[k]}\Bigg]. \tag{20}$$

The expected value of the system-wide profit is then the sum of Eqs. (19) and (20), i.e.:

$$E\left(\Pi_{[k]}(\hat{w}_{[k]},\hat{\mathbf{z}}_{[\mathbf{k}]})\right)$$
$$=\sum_{i\in I_{[k]}}\Bigg((p-c-TC_i)\sum_{j\in N_{i[k]}}y_j(p_{ij})$$
$$-(c+TC_i-v_i)\Lambda_{i[k]}(\hat{z}_{i[k]})$$
$$-(p+u_i-c-TC_i)\Theta_{i[k]}(\hat{z}_{i[k]})\Bigg). \tag{21}$$

Now, we can compute the surplus profit of the partial coordination for the SC as:

$$\hat{\gamma}_{[k]}=E\left(\Pi_{[k]}(\hat{w}_{[k]},\hat{\mathbf{z}}_{[\mathbf{k}]})\right)-E\left(\Pi_{[k]}(\mathbf{z}^*_{[\mathbf{k}]})\right).$$

It is noteworthy that only if the manufacturer makes no profit from the sale, i.e. $\hat{w}_{[k]}=c$, can distribution network become completely coordinated, and we have $\hat{z}_{i[k]}=z^o_{i[k]},\forall i\in I_{[k]}$ and $\hat{\gamma}_{[k]}=\gamma_{[k]}$. Complete coordination needs a binding commitment and honest relationship among members. To be more specific, by setting appropriate side payments, the retailers should undertake that the manufacturer becomes more profitable, as compared to the case of non-cooperative wholesale pricing (in Proposition 1). However, if the manufacturer has some doubt about retailer commitment, the partial coordination may come about. In this case, the manufacturer retains the profit margin, $\hat{w}_{[k]}-c(>0)$. Thus, channels fail to reach complete coordination, i.e. $\hat{z}_{[k]}<z^o_{i[k]},\forall i\in I_{[k]}$ and $\hat{\gamma}_{[k]}<\gamma_{[k]}$. The manufacturer tends to use his pricing initiative when profound mistrust exists, that is, he sets $\hat{w}_{[k]}=w^*_{[k]}$, and the non-cooperation case takes effect, i.e. $\hat{z}_{i[k]}=z^*_{i[k]},\forall i\in I_{[k]}$ and $\hat{\gamma}_{[k]}=0$. Corollary 3 summarizes the effect of wholesale discount on profit surplus under a partial coordination contract.

Corollary 3.

(i) In each distribution design, a lower wholesale discount (higher $\hat{w}_{[k]}$) will shrink the possible surplus profit of coordination ($\hat{\gamma}_{[k]}$).

(ii) If the demand uncertainty of each selected retailer is in the class of non-decreasing hazard rate distribution, as the wholesale price ($\hat{w}_{[k]}$) increases towards its non-cooperative optimum ($w^*_{[k]}$), the surplus profit of coordination ($\hat{\gamma}_{[k]}$) becomes more sensitive to slight changes in the wholesale price ($\hat{w}_{[k]}$).

If a properly-developed side-payment scheme makes the (partial) coordination stable, each member must be better off than in the non-cooperative situation. To be more specific, if $\hat{\gamma}_{[k]}>0$, the transfer payments should satisfy the following conditions:

$$E\big(\Pi_{M[k]}(\hat{w}_{[k]},\hat{\mathbf{z}}_{[k]},\mathbf{TR}_{[k]})\big)>E\left(\Pi_{M[k]}(w^*_{[k]},\mathbf{z}^*_{[k]})\right), \tag{22}$$

$$E\big(\Pi_{R_i[k]}(\hat{w}_{[k]},\hat{z}_{i[k]},TR_{i[k]})\big)>E\left(\Pi_{R_i[k]}(w^*_{[k]},z^*_{i[k]})\right). \tag{23}$$

2.2.3. Coordination mechanism for retailer-specific pricing policy

We now investigate buy-back contracts between the retailers and the manufacturer to coordinate each distribution design network. Under the price-discrimination policy, the manufacturer can use a retailer-specific buy-back contract to induce retailer, i, to order quantity, $q^o_{i[k]}$. We refer to such a contract as buy-back $(w_{i[k]},b_{i[k]},z^o_{i[k]})$ for retailer, i. According to the contract, if retailer, i, encounters an excess inventory (i.e. $q^o_{i[k]}>d_{i[k]}$), then, he will be able to return the unsold product to the manufacturer at the buy-back price, $b_{i[k]}$. The manufacturer then shoulders the transportation of the products and disposes of them at the unit salvage value v $(<c)$. It is assumed that $b_{i[k]}\leq w_{i[k]}+TC_i,\forall i\in I_{[k]}$, which implies that retailers do not directly benefit from ordering excessive quantities and returning them to the manufacturer. Moreover, we have $b_{i[k]}>v_{i[k]},\forall i\in I_{[k]}$; otherwise, the retailer prefers to dispose of the products himself. Let $\mathbf{w}_{[\mathbf{k}]}=\{w_{i[k]},\forall i\in I'_{[k]}\}$, $\mathbf{b}_{[\mathbf{k}]}=\{b_{i[k]},\forall i\in I'_{[k]}\}$, and $\mathbf{z}^{\mathbf{o}}_{[\mathbf{k}]}=\{z^o_{i[k]},\forall i\in I'_{[k]}\}$ denote, in turn, wholesale prices, buy-back prices, and optimal safety stock levels, corresponding to active retailers in distribution design, k. When the manufacturer considers a buyback contract $(\mathbf{w_{[k]}}, \mathbf{b_{[k]}}, \mathbf{z^o_{[k]}})$, the expected profit of retailer, i, under the coordination condition is:

$$E\left(\Pi_{R_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right) = (p - w_{i[k]} - TC_i) \sum_{j\in N_{i[k]}} y_j(p_{ij}) - (w_{i[k]} + TC_i - b_{i[k]})\Lambda_{i[k]}(z^o_{i[k]}) - (p + u_i - w_{i[k]} - TC_i)\Theta_{i[k]}(z^o_{i[k]}), \quad \forall i \in I_{[k]}. \tag{24}$$

The expected profit of the manufacturer under the coordination condition is

$$E\left(\Pi_{M[k]}(\mathbf{w_{[k]}}, \mathbf{b_{[k]}}, \mathbf{z^o_{[k]}})\right) = \sum_{i\in I'_{[k]}} \left[E\left(\Pi_{M_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right)\right] = \sum_{i\in I'_{[k]}} \left[(w_{i[k]} - c)\left(\sum_{j\in N_{i[k]}} y_j(p_{ij}) + \Lambda_{i[k]}(z^o_{i[k]}) - \Theta_{i[k]}(z^o_{i[k]})\right) - (b_{i[k]} + TC_i - v)\Lambda_{i[k]}(z^o_{i[k]})\right], \tag{25}$$

where $E\left(\Pi_{M_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right)$ represents the manufacturer's profit obtained from coordination with retailer, i, in distribution design, k. Moreover, the expected channel-wide profit under the contract is:

$$E\left(\hat{\Pi}_{i[k]}(z^o_{i[k]})\right) = E\left(\Pi_{M_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right) + E\left(\Pi_{R_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right) = (p - c - TC_i) \sum_{j\in N_{i[k]}} y_j(p_{ij}) - (c + 2TC_i - v_i)\Lambda_{i[k]}(z^o_{i[k]}) - (p + u_i - c - TC_i)\Theta_{i[k]}(z^o_{i[k]}) = E\left(\Pi_{i[k]}(z^o_{i[k]})\right) + (v - TC_i - v_i)\Lambda_{i[k]}(z^o_{i[k]}).$$

$E\left(\Pi_{i[k]}(z^o_{i[k]})\right)$ is the optimum expected profit of channel, i, when the manufacturer and retailer, i, work as an integrated channel (see Eq. (16)). It is worthy to note that when $v > TC_i + v_i$, the buy-back contract provides more benefit than the integrated channel. Since $E\left(\hat{\Pi}_{i[k]}(z^o_{i[k]})\right)$ is a decreasing function of TC_i, the financial attractiveness of the buyback contract diminishes when the physical returning of products is excessively costly (for example, due to long distances). However, a buyback contract may be wholly beneficial for channel i, if the salvage value of the specific product in the manufacturer plant is considerably higher than the salvage value in retailer i's location (for instance, because of advanced recycling machinery in manufacturing plants and the ability to reuse salvage). The case of $v < TC_i + v_i$ is also possible, which implies that physically returning products is not profitable as an integrated channel. The buy-back mechanism for channel i will be more beneficial, compared to non-cooperative pricing (in Proposition 2), if $v - TC_i - v_i$ is sufficiently high, that is, as in Eq. (26) shown in Box I.

We focus on the coordination mechanism based on a buyback contract for the manufacturer and retailer i in Proposition 4 and Corollary 4.

Proposition 4. Under distribution design k, channel i can be coordinated by the buy-back contract $(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})$ with $\underline{b_{i[k]}} < b_{i[k]} < \overline{b_{i[k]}}$, where:

$$w_{i[k]}(b_{i[k]}) = (p + u_i - TC_i) - (p + u_i - b_{i[k]})G_{i[k]}(z^o_{i[k]}), \tag{27}$$

$$\overline{b_{i[k]}} = (p + u_i) - \frac{u_i \sum_{j\in N_{i[k]}} y_j(p_{ij}) + E\left(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right)}{q^o_{i[k]}G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})}, \tag{28}$$

$$v - TC_i - v_i > \frac{E\left(\Pi_{i[k]}(z^o_{i[k]})\right) - E\left(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right) - E\left(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right)}{\Lambda_{i[k]}(z^o_{i[k]})}. \tag{26}$$

Box I

$$\underline{b_{i[k]}} = v_i - \frac{(v - TC_i - v_i)\Lambda_{i[k]}(z^o_{i[k]}) - E\Big(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\Big)}{q^o_{i[k]} G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})}. \quad (29)$$

Proposition 4 shows that both the manufacturer and retailer i obtain more profit than the non-cooperative Stackelberg equilibrium, when the manufacturer sets $b_{i[k]}$ in the range of $\underline{b_{i[k]}}$ to $\overline{b_{i[k]}}$. The range of buy-back price can be computed by Eq. (30) shown in Box II.

From Proposition 3 and Eq. (30), we can draw the following corollary.

Corollary 4.

(i) The range of $(\underline{b_{i[k]}}, \overline{b_{i[k]}})$ for coordination with retailer i exists (or equivalently the buyback contract is possible), if the buyback contract affords more profit, compared to non-cooperative pricing.

(ii) Retailer i withdraws from the buy-back contract if the manufacturer sets $b_{i[k]}$ too high (i.e. $b_{i[k]} > \overline{b_{i[k]}}$); on the other hand, if the manufacturer sets the $b_{i[k]}$ too low (i.e. $b_{i[k]} < \underline{b_{i[k]}}$), then, he will not profit, compared to non-cooperative pricing.

(iii) The range of $\underline{b_{i[k]}}, \overline{b_{i[k]}}$ shrinks as transportation cost (TC_i) increases, or when $v - v_i$ decreases.

Corollary 4 means that if $b_{i[k]}$ does not fall in the range of $(\underline{b_{i[k]}}, \overline{b_{i[k]}})$, one of the members leaves the coordination contract.

2.3. The distribution design decision

Now, we study the optimal distribution decision of the manufacturer. The manufacturer faces a set of candidate retailers I to distribute products throughout the markets. Each subset, $I_{[k]}$, of candidate retailers set I represents a possible design of distribution. Considering the profit of each distribution design, the manufacturer chooses the design that maximizes his expected profit. That is, under the non-cooperative situation, the optimum distribution designs in uniform and retailer-specific pricing policies are, in turn, $k^* = \underset{k}{\arg\max}\, E\left(\Pi_{M[k]}(w_{[k]})\right)$, and $k^* = \underset{k}{\arg\max}\, E\left(\Pi_{M[k]}(\mathbf{w}_{[k]})\right)$. In the case of cooperative strategies, optimum distribution designs in a discount-revenue sharing contract $(\hat{w}_{[k]}, \hat{\mathbf{z}}_{[\mathbf{k}]}, \mathbf{TR}_{[\mathbf{k}]})$ and a buy-back contract $(\mathbf{w}_{[\mathbf{k}]}, \mathbf{b}_{[\mathbf{k}]}, \mathbf{z}^{\mathbf{o}}_{[\mathbf{k}]})$, are, in turn:

$$k^* = \underset{k}{\arg\max}\, E\left(\Pi_{M[k]}(\hat{w}_{[k]}, \hat{\mathbf{z}}_{[k]}, \mathbf{TR}_{[k]})\right), \quad (31)$$

$$k^* = \underset{k}{\arg\max}\, E\Big(\Pi_{M[k]}(\mathbf{w}_{[\mathbf{k}]}, \mathbf{b}_{[\mathbf{k}]}, \mathbf{z}^{\mathbf{o}}_{[\mathbf{k}]})\Big). \quad (32)$$

When multiple distribution designs generate a similar profit for the manufacturer, other criteria, such as total sale quantity or (and) the service levels of retailers, can also be applied.

3. Numerical examples

The numerical example, depicted in Figure 1, is comprised of a manufacturer, two potential retailers, and four demand markets. The data for this example was constructed for easy interpretation purposes; thus, the retailers, as well as the markets, are considered identical (except transportation costs illustrated in Figure 1). Since normal distribution is a non-decreasing hazard rate distribution, which is most commonly employed

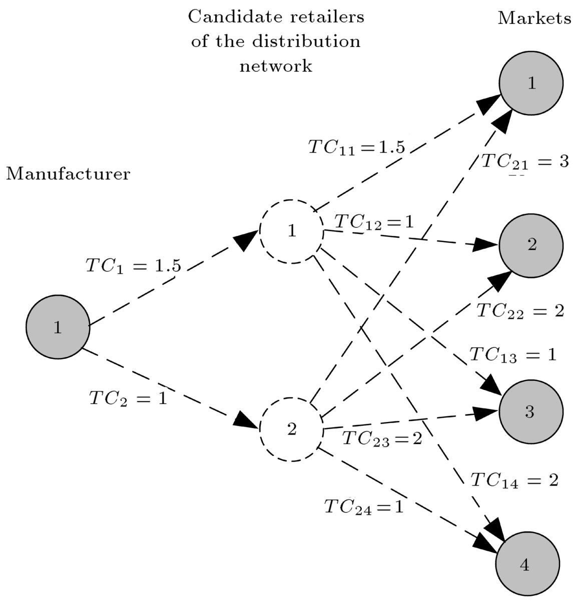

Figure 1. The SC network structure and transportation costs for the numerical example.

$$\overline{b_{i[k]}} - \underline{b_{i[k]}} = \frac{E\Big(\hat{\Pi}_{i[k]}(z^o_{i[k]})\Big) - E\Big(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\Big) - E\Big(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\Big)}{q^o_{i[k]} G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})}. \quad (30)$$

Box II

in modeling demand functions, let us consider that the random terms of market demand are normally distributed, i.e. $\varepsilon_j \sim N(0, \sigma_j^2) \forall j \in N$, and $\sigma_{jj'} = \mathrm{cov}(\varepsilon_j, \varepsilon_{j'}) = \sigma_j \sigma_{j'} \rho \forall j, j' \in N \& j \neq j'$, where ρ is the correlation coefficient of random terms. We assume that the default values of parameters are as follows:

$$p = 18, \quad c = 4, \quad N = \{1, 2, 3, 4\},$$

$$I = \{1, 2\}, \quad u_1 = u_2 = 1, \quad v_1 = v_2 = 2,$$

and:

$$\forall j \in N, \quad a_j = 100, \quad b_j = 2, \quad \sigma_j = 30.$$

With regard to the manufacturer's decision about the selection of retailer(s), there are three distribution network designs indicated as $I_{[1]} = \{1\}$, $I_{[2]} = \{2\}$, and $I_{[3]} = \{1, 2\}$. If only one retailer is selected in the distribution design, products are distributed among markets in a centralized manner, and the retailer will monopolize all markets, i.e. $N_{1[1]} = N_{2[2]} = \{1, 2, 3, 4, 5\}$ and $N_{1[2]} = N_{2[1]} = \{\}$. However, when the manufacturer chooses both retailers, we have $N_{1[3]} = \{1, 2, 3\}$ and $N_{2[3]} = \{4, 5\}$, regarding transportation costs. It is straightforward from the property of the sum of the dependent normal variables that $\xi_{1[1]} = \xi_{2[2]} \sim N(0, 120^2)$, $\xi_{2[1]} = \xi_{1[2]} = 0$, $\xi_{1[3]} \sim N(0, 90^2)$, and $\xi_{2[3]} \sim N(0, 30^2)$ (we refer the reader to Eppen [4] for further discussion).

Table 2 demonstrates the differences and similarities between the optimal values of the pricing policies calculated according to Propositions 1 and 2. Firstly, in distribution designs 1 and 2, in which a retailer is a monopolist, the optimal values are identical under both pricing policies. Secondly, in both pricing policies, a monopolist retailer earns more profit compared with a two-retailer distribution, because markets are divided between retailers. Consequently, from a retailers' viewpoint, each retailer often has an incentive to persuade the manufacturer to exclusively distribute products throughout the markets. Thirdly, in two-retailer distribution ($k = 3$), the effect of a pricing policy on retailer profit varies from retailer to retailer. Specifically, retailer 1's profit under the uniform pricing policy is lower than the profit under the retailer-specific pricing policy; however, the situation is contrary for retailer 2. Finally, from the manufacturer's point of view, two-retailer distribution is preferred under both pricing policies.

Figures 2 and 3 illustrate how the non-cooperative

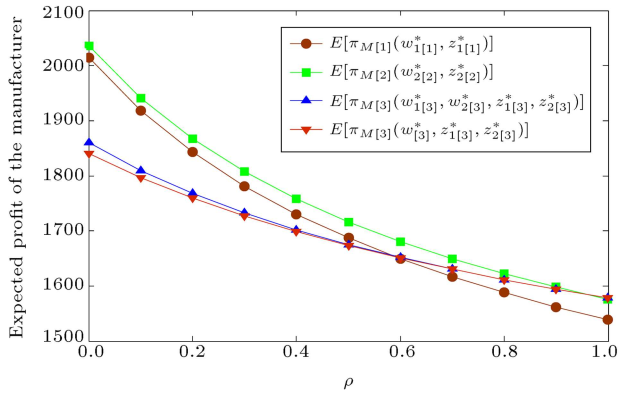

Figure 2. The manufacturer's expected profit in each distribution design versus the correlation coefficient under the pricing policies.

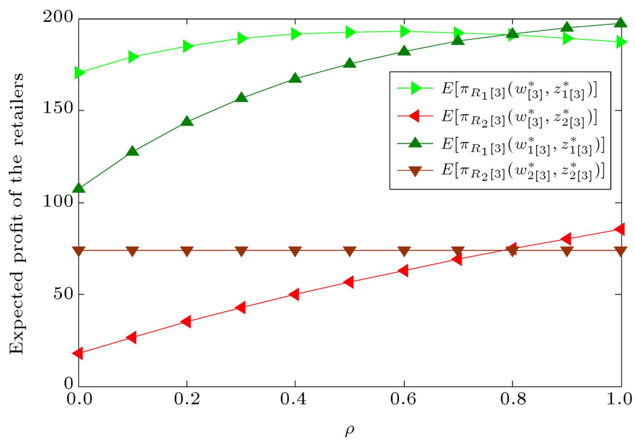

Figure 3. The retailers' expected profit in distribution design 3 versus the correlation coefficient under the pricing policies.

Table 2. Comparison of the optimal values between two pricing polices ($\rho = 1$).

	Uniform pricing policy			**Retailer-specific pricing policy**		
k	**1**	**2**	**3**	**1**	**2**	**3**
$w^*_{[k]}$	12.619	12.836	12.73	-	-	-
$(w^*_{1[k]}, w^*_{2[k]})$	-	-	-	(12.619,-)	(-,12.836)	(12.653,12.977)
$(\alpha^*_{1[k]}, \alpha^*_{2[k]})$	(0.287,-)	(-,0.304)	(0.281,0.31)	(0.287,-)	(-,0.304)	(0.285,0.295)
$(z^*_{1[k]}, z^*_{2[k]})$	(-67.42,-)	(-,-61.624)	(-52.298,-14.875)	(-67.42,-)	(-,-61.624)	(-51.094,-16.123)
$(q^*_{1[k]}, q^*_{2[k]})$	(178.58,-)	(-,178.374)	(133.702,47.125)	(178.58,-)	(-,178.374)	(134.906,45.877)
$(\Pi_{R_1[k]}, \Pi_{R_2[k]})$	(261.598,-)	(-,288.026)	(187.123,85.295)	(261.598,-)	(-,288.026)	(197.432,73.816)
$\mathbf{\Pi_{M[k]}}$	**1539.19**	**1576.056**	**1578.611**	**1539.19**	**1576.056**	**1579.174**

optimal profits of the manufacturer and retailers depend on the correlation of market demand under both pricing policies. From Figure 2, we know that the expected profit of the manufacturer in all distribution designs and under both pricing policies decreases with the dependency of the markets. The figure also implies that, as the dependency of the markets diminishes, the centralized distribution by retailer 2 generates more profit for the manufacturer than the other distribution designs. However, when the markets are excessively dependent, the manufacturer prefers two-retailer distribution under both pricing policies; see also Table 2. In two-retailer distribution, the retailer-specific policy yields more profit for the manufacturer compared with the uniform pricing policy, as the dependency of the markets decreases. Figure 3 illustrates the retailer's profit often increases with market dependency. Moreover, the figure demonstrates that the growth of retailer 1's profit in the retailer-specific pricing is higher than the uniform pricing policy. However, the situation is contrary for retailer 2.

We now compute globally optimal values as if the SC is operating in a centralized pattern and all decisions are made by the manufacturer. Since system-wide profit (15) does not depend on wholesale price, Table 3 gives the globally optimal values for both pricing policies, according to Proposition 3.

In the case of partial coordination under a uniform pricing policy, Figure 4 illustrates the change in the profit of the total SC with respect to the wholesale price. The figure depicts that the total (system-wide) profit of the SC, in distribution design k, is maximized when the wholesale price is set to be equal to the production cost (i.e. $\hat{w}_{[k]} = c$), and minimized when the wholesale price is set to be equal to the non-cooperative wholesale price (i.e. $\hat{w}_{[k]} = w^*_{[k]}$), which is consistent with Corollary 3. We also know from the figure that centralized distribution by retailer 2 is more profitable for the SC, compared to other distribution designs. Surplus profit, $\hat{\gamma}_{[k]}$, should be divided among parties by transfer payments, $\mathbf{TR}_{[k]}$, to insure Inequalities (22) and (23). The more the surplus profit, the higher the bargaining space for the parties will be. Since the retailers bargain with the manufacturer over transfer payments, $\mathbf{TR}_{[k]}$, a part of $\hat{\gamma}_{[k]}$ is commonly transferred to the manufacturer. Therefore, the final profit of the manufacturer, according to Eq. (31), is the selection criterion of the distribution design.

In Figures 5-8, we investigate how the buyback prices influence the profits of the manufacturer and the retailers under the retailer-specific pricing policy. It is obvious from the figures that an increase in buyback price offered to a retailer decreases retailer profit, however, it increases manufacturer profit, which is consistent with Corollary 4. Figure 5 demonstrates that for a given buyback price offered by the manufacturer, retailer 1 in design 1 gains more profit than retailer 2 in design 2.

The profits of the retailers in a two-retailer distribution are depicted in Figure 7, which imply that for a similar buyback price offered to the retailers, retailer 1

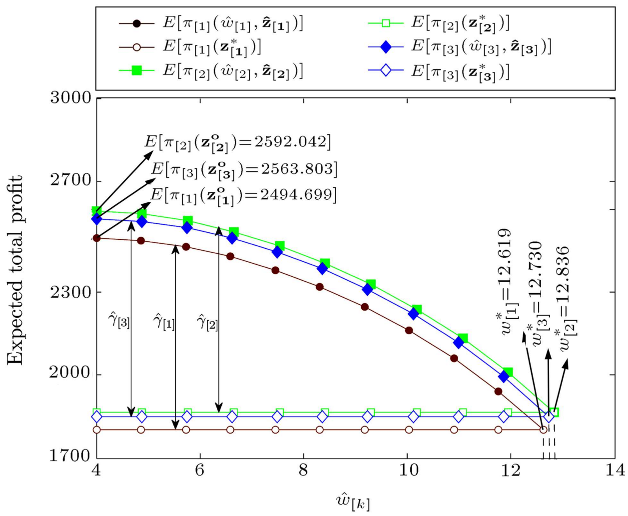

Figure 4. The expected profit of SC in each distribution design versus the wholesale discount ($\rho = 1$).

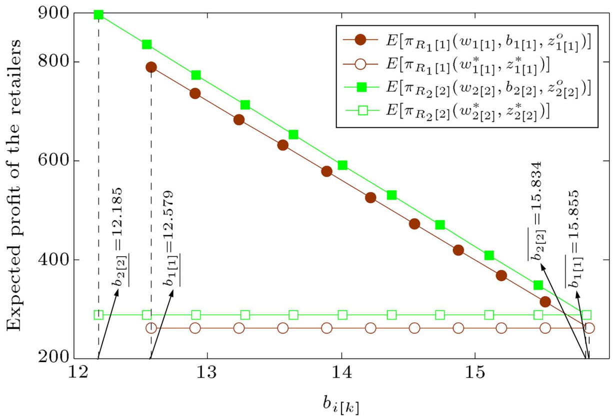

Figure 5. The retailers' expected profit in distribution designs 1 and 2 versus the buyback prices ($\rho = 1$).

Table 3. The globally optimal values of the network designs for the correlation coefficient $\rho = 1$.

k	$\alpha^o_{1[k]}$	$\alpha^o_{2[k]}$	$z^o_{1[k]}$	$z^o_{2[k]}$	$q^o_{1[k]}$	$q^o_{2[k]}$	$\Pi_{[k]}(\mathbf{z}^o_{[k]})$	$\gamma_{[k]}$
1	0.794	-	98.495	-	344.495	-	**2494.699**	**693.911**
2	-	0.824	-	111.468	-	351.468	**2592.042**	**727.96**
3	0.794	0.824	73.871	27.867	259.871	89.867	**2563.803**	**712.774**

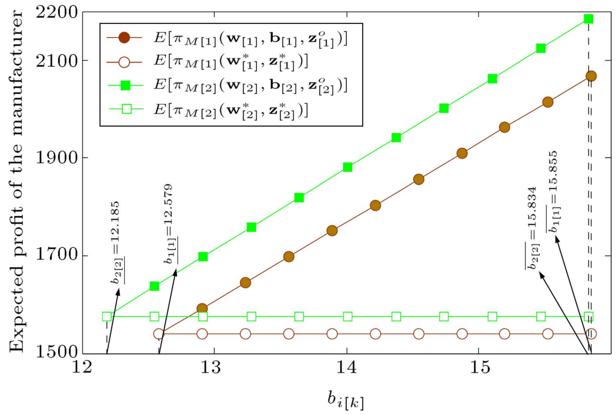

Figure 6. The manufacturer's expected profit in distribution designs 1 and 2 versus the buyback prices ($\rho = 1$).

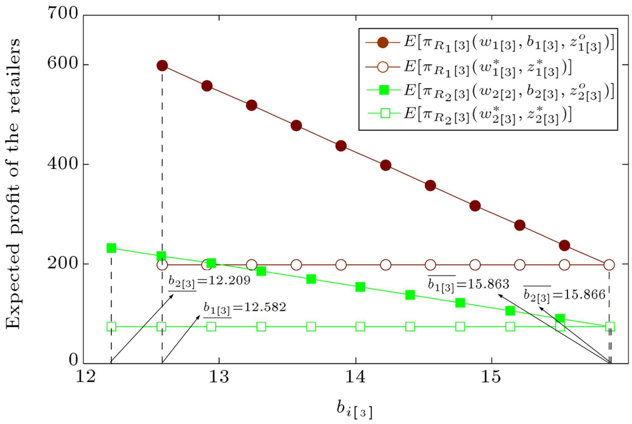

Figure 7. The retailers' expected profit in distribution design 3 versus the buyback prices ($\rho = 1$).

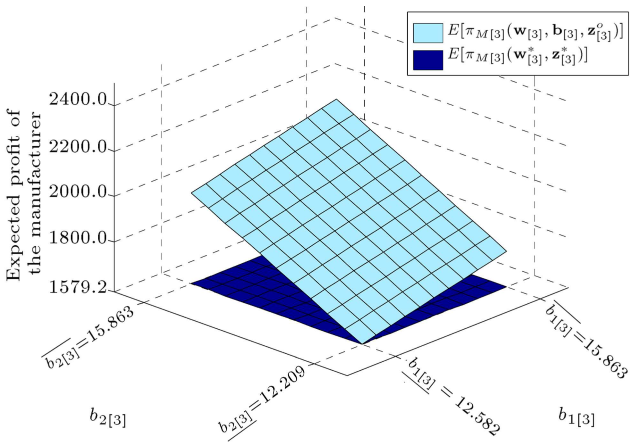

Figure 8. The manufacturer's expected profit in distribution design 3 versus the buyback prices ($\rho = 1$).

obtains further profit, compared to retailer 2. With being given buyback prices in each distribution design, Figures 6 and 8 provide the manufacturer's profit, and the manufacturer is able to choose the most profitable design, according to Eq. (32).

4. Managerial insights and extensions

We derive the following managerial implications from the corollaries and numerical examples:

- Although integrating inventory in a single location reduces inventory costs, it may result in high transportation costs in geographically dispersed markets that should be borne by the retailers. Transportation costs also affect manufacturer profits because an inappropriate distribution network increases the final product price in the markets, on the one hand, and shrinks retailer order quantities on the other. Therefore, in selecting the optimal distribution design, a careful analysis between inventory cost savings and transportation cost factors should be carried out.
- Pricing policies exert great influence on the profits of the manufacturer and retailer. To be more specific, manufacturer profits under retailer-specific pricing are always higher than the profits made under a uniform pricing policy. However, some trade legislations, such as antitrust laws or the Robinson act, may strictly forbid the manufacturer from discriminating in price.
- Increasing market demand dependency leads to higher retailer profits but lower manufacturer profits. Furthermore, when market demands are extremely dependent, both uniform and retailer-specific pricing policies yield similar profits for the manufacturer.
- In the partial coordination under the uniform pricing policy, the advantages of coordination diminish when wholesale price increases. Moreover, the impact of wholesale price discount on the growth of SC surplus profit shrinks when wholesale price decreases. Therefore, the manufacturer can considerably raise SC profit surplus by setting a slight discount, which provides open space for bargaining with the retailers.
- In the buyback coordination mechanism for retailer-specific pricing, if the buyback contract is more profitable than in the non-cooperative case, there is a win-win contract for the retailer and manufacturer. However, retailer willingness for coordination declines as the buyback price increases. Moreover, the advantages of a buyback contract diminish when the geographical distance between a retailer and manufacturer increases. Therefore, other mechanisms, such as "markdown money" (see Tsay [29]), where the retailer disposes of the remaining products and the manufacturer charges the retailer for the products, may be more beneficial.
- This research can be extended in several directions. First, we assume that the retail price is the same for

all retailers, which is exogenously given. In the real world, the retailers may jointly set order quantities and retail prices. Therefore, the price competition of retailers for markets in each DND will be very interesting. Second, in the real world application of the model, when the manufacturer faces many candidate retailers, metaheuristic algorithms for DND could also be an interesting extension. Lastly, we proposed a wholesale-discount with revenue-sharing and buyback contracts for coordinating each distribution network under the uniform and retailer-specific pricing policies, respectively. One can develop our model utilizing other well-known coordination mechanisms.

5. Conclusions

We considered a distribution network problem in geographically dispersed markets under demand uncertainty environments. Each candidate retailer, as an independent decision maker, determines order quantity, and the manufacturer dictates the wholesale price by adopting either the uniform or retailer-specific pricing policy. For each design of the distribution network, the non-cooperative equilibrium was computed as a benchmark. We showed that coordination mechanisms, wholesale-price discounts with revenue sharing and buy-back contracts ensure a win-win situation for parties compared to non-cooperative situations. The effects of market demand dependency, wholesale price discount, and buyback prices on player profits and manufacturer distribution design decisions were mainly discussed. We found that the optimal distribution design is contingent upon the dependency of market demands. When the demands are independent, the centralized distributions are more profitable than the decentralized ones for the manufacturer. This paper also suggests that in design k, coordination quality under the uniform pricing policy is partial, when $c < \hat{w}_{[k]} < w^*_{[k]}$, and in the cases of $\hat{w}_{[k]} = c$ and $\hat{w}_{[k]} = w^*_{[k]}$, in turn, pure coordination and non-coordination situations take place. Furthermore, the advantages of a buyback contract, where unsold products are physically returned to the manufacturer, are highly affected by the geographical location of retailers, and salvage values of products in the manufacturer and retailer locations.

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Appendix

Proof of Corollary 1.

(i) Differentiating optimal order quantity (Eq. (6)) w.r.t. TC_{ij}, TC_i and $w_{[k]}$, respectively, results in:

$$\partial \quad q^*_{i[k]}/\partial \mathrm{TC}_{ij} = -b_j and \partial q^*_{i[k]}/\partial \mathrm{TC}_i$$

$$= \partial q^*_{i[k]}/\partial w_{[k]} = -\left((p+u_i-v_i)g(z^*_{i[k]})\right)^{-1},$$

The optimum expected profit of the retailer is:

$$E\left(\Pi_{R_i[k]}(w_{[k]}, z^*_{i[k]})\right) = (p - w_{[k]} - \mathrm{TC}_i) \times \sum_{j\in N_{i[k]}} y_j(p_{ij}) - (w_{[k]} + \mathrm{TC}_i - v_i)\Lambda_{i[k]}(z^*_{i[k]}) - (p + u_i - w_{[k]} - \mathrm{TC}_i)\Theta_{i[k]}(z^*_{i[k]}), \quad \forall i \in I_{[k]}, \tag{A.1}$$

where $G_{i[k]}(z^*_{i[k]}) = (p+u_i - w_{[k]} - \mathrm{TC}_i)/(p+u_i - v_i), \forall i \in I_{[k]}$.

(ii) Taking derivative of Eq. (6), w.r.t. $w_{[k]}$ by using the chain rule, we have:

$$\partial E\left(\Pi_{R_i[k]}(w_{[k]}, z^*_{i[k]})\right)/\partial w_{[k]} = -\sum_{j\in N_{i[k]}} y_j(p_{ij}) - \Lambda_{i[k]}(z^*_{i[k]}) + \Theta_{i[k]}(z^*_{i[k]}) = -\sum_{j\in N_{i[k]}} y_j(p_{ij}) - z^*_{i[k]}.$$

It is straightforward to show that:

$$\partial E\left(\Pi_{R_i[k]}(w_{[k]}, z^*_{i[k]})\right)/\partial w_{[k]} = \partial E\left(\Pi_{R_i[k]}(w_{[k]}, z^*_{i[k]})\right)/\partial \mathrm{TC}_i < 0.$$

Additionally, we have:

$$\partial E\left(\Pi_{R_i[k]}(w_{[k]}, z^*_{i[k]})\right)/\partial \mathrm{TC}_{ij} = -b_j(p - w_{[k]} - \mathrm{TC}_i) < 0$$

Proof of Proposition 1. Taking $r_{i[k]}(z^*_{i[k]}) = g_{i[k]}(z_{i[k]})/[1 - G_{i[k]}(z^*_{i[k]})]$ into account, the first- and the second-order derivatives of Eq. (7) w.r.t. $w_{[k]}$ are calculated by Eqs. (A.2) and (A.3) as shown in Box III If we have:

$$(w_{[k]} - c)\left[dr_{i[k]}(z^*_{i[k]})/dz^*_{i[k]} + r^2_{i[k]}(z^*_{i[k]})\right] + 2(c + \mathrm{TC}_i - v_i)r^2_{i[k]}(z^*_{i[k]}) > 0,$$

for each $i \in I_{[k]}$, then $E\left(\Pi_{M[k]}(w_{[k]})\right)$ is a concave function of $w_{[k]}$. Since we know $c \geq v_i$, $\forall i \in I_{[k]}$, the conditions $dr_{i[k]}(z^*_{i[k]})/dz^*_{i[k]} + r^2_{i[k]}(z^*_{i[k]}) > 0$, $\forall i \in I_{[k]}$, which hold for all non-decreasing hazard rate distributions.

Barlow and Proschan (Barlow, R.E. and Proschan, F., *Statistical Theory of Reliability and Life Testing: Probability Models*, Florida state Univ., Tallahassee (1975).) assure the concavity of the function. Under such conditions, a unique Stackelberg equilibrium $(w^*_{[k]}, q^*_{i[k]})$, $\forall i \in I_{[k]}$ exists which satisfies Eqs. (A.2) and (8).

Proof of Proposition 2. Considering the negative sign of second-order derivative of $E\left(\Pi_{[k]}(\mathbf{z_{[k]}})\right)$ w.r.t. $z_{i[k]}, \forall i \in I_{[k]}$, and $\partial^2 E\left(\Pi_{[k]}(\mathbf{z_{[k]}})\right)/\partial z_{i[k]}\partial z_{l[k]} = 0$, $\forall i, l \in I_{[k]}$, $i \neq l$, Hessian matrix of the expected

$$\frac{\partial E\left(\Pi_{M[k]}(w_{[k]})\right)}{\partial w_{[k]}} = \sum_{i\in I_{[k]}}\left[\sum_{j\in N_{i[k]}} y_j(p_{ij}) + z^*_{i[k]} - \frac{w_{[k]} - c}{(w_{[k]} + \mathrm{TC}_i - v_i)r_{i[k]}(z^*_{i[k]})}\right], \tag{A.2}$$

$$\frac{\partial^2 E(\Pi_{M[k]}(w_{[k]}))}{\partial w^2_{[k]}} = -\sum_{i\in I_{[k]}}\left[\frac{(w_{[k]} - c)\left[\frac{dr_{i[k]}(z^*_{i[k]})}{dz^*_{i[k]}} + r^2_{i[k]}(z^*_{i[k]})\right] + 2(c + \mathrm{TC}_i - v_i)r^2_{i[k]}(z^*_{i[k]})}{(w_{[k]} + \mathrm{TC}_i - v_i)^2 r^3_{i[k]}(z^*_{i[k]})}\right]. \tag{A.3}$$

Box III

profit (Eq. (16)) is negative definite at $\mathbf{z_{[k]}}$ (Bazaraa, M.S., Sherali, H.D. and Shetty, C.M., *Nonlinear Programming: Theory and Algorithms*, John Wiley & Sons (2006).)

Bazaraa et al., 2006). Accordingly, the expected profit is jointly concave on $\mathbf{z_{[k]}}$ and the first order condition $\partial E\left(\Pi_{[k]}(\mathbf{z_{[k]}})\right)/\partial z_{i[k]} = 0$, $\forall i \in I'_{[k]}$ results in Eq. (17).

Proof of Corollary 2. Total profit of a decentralized distribution network is independent of manufacturer's wholesale price (prices) and equals to Eq. (16). Since $q^o_{i[k]}, \forall i \in I_{[k]}$ are unique optimal order quantities of system-wide profit (Eq. (16)), and taking $q^o_{i[k]} > q^*_{i[k]}, \forall i \in I_{[k]}$ into account, we conclude that $E\left(\Pi_{[k]}(\mathbf{z^o_{[k]}})\right) > E\left(\Pi_{[k]}(\mathbf{z^*_{[k]}})\right)$, where $\mathbf{z^o_{[k]}} = \{z^o_{i[k]}, \forall i \in I_{[k]}\}$ and $\mathbf{z^*_{[k]}} = \{z^*_{i[k]}, \forall i \in I_{[k]}\}$.

Proof of Corollary 3. Considering $r_{i[k]}(\hat{z}_{i[k]}) = g_{i[k]}(\hat{z}_{i[k]})/[1 - G_{i[k]}(\hat{z}_{i[k]})]$, the first- and second-order derivatives of system-wide profit (Eq. (21)) w.r.t. $\hat{w}_{[k]}$ are $\partial E\left(\Pi_{[k]}(\hat{w}_{[k]}, \hat{\mathbf{z}}_{\mathbf{[k]}})\right)/\partial \hat{w}_{[k]} = -\sum_{i\in I'}(\hat{w}_{[k]} - c)\left[(\hat{w}_{[k]} + \mathrm{TC}_i - v_i)r_{i[k]}(\hat{z}_{i[k]})\right]$ and:

$$\frac{\partial^2 E\left(\Pi_{[k]}(\hat{w}_{[k]}, \hat{\mathbf{z}}_{\mathbf{[k]}})\right)}{\partial \hat{w}^2_{[k]}} = -\sum_{i\in I'} \left[\frac{(\hat{w}_{[k]} - c)\left[\frac{dr_{i[k]}(\hat{z}_{i[k]})}{d\hat{z}_{i[k]}}\right] + (c + \mathrm{TC}_i - v_i)r_{i[k]}(\hat{z}_{i[k]})}{(\hat{w}_{[k]} + \mathrm{TC}_i - v_i)^2 r^2_{i[k]}(\hat{z}_{i[k]})}\right] \tag{A.4}$$

Since $\hat{w}_{[k]} \geq c > v_i$, it follows that $\partial E\left(\Pi_{[k]}(\hat{w}_{[k]}, \hat{\mathbf{z}}_{\mathbf{[k]}})\right)/\partial \hat{w}_{[k]} \leq 0$. Furthermore, $\partial^2 E\left(\Pi_{[k]}(\hat{w}_{[k]}, \hat{\mathbf{z}}_{\mathbf{[k]}})\right)/\partial \hat{w}^2_{[k]} < 0$ is satisfied by all non-decreasing hazard rate distribution.

Proof of Proposition 3. The expected profit of retailer i under buy-back contract $(w_{i[k]}, b_{i[k]}, z_{i[k]})$ becomes:

$$E\left(\Pi_{R_i[k]}(w_{i[k]}, b_{i[k]}, z_{i[k]})\right) = (p - w_{i[k]} - \mathrm{TC}_i) \times \sum_{j\in N_{i[k]}} y_j(p_{ij}) - (w_{i[k]} + \mathrm{TC}_i - b_{i[k]}) \times \Lambda_{i[k]}(z_{i[k]}) - (p + u_i - w_{i[k]} - \mathrm{TC}_i) \times \Theta_{i[k]}(z_{i[k]}), \forall i \in I'. \tag{A.5}$$

Since the $E\left(\Pi_{R_i[k]}(w_{i[k]}, b_{i[k]}, z_{i[k]})\right)$ is a concave function of $z_{i[k]}$, solving the first order condition results in:

$$G_{i[k]}(\hat{z}_{i[k]}) = \frac{p + u_i - w_{i[k]} - \mathrm{TC}_i}{p + u_i - b_{i[k]}}. \tag{A.6}$$

When channel i is completely coordinated, the optimal safety stock level of the retailer should satisfy $\hat{z}_{i[k]} = z^o_{i[k]}$ (which is also equal to $\hat{q}_{i[k]} = q^o_{i[k]}$). Regarding Eqs. (17) and (A.6), by solving $\hat{z}_{i[k]} = z^o_{i[k]}$ for w, we have:

$$w_{i[k]}(b_{i[k]}) = (p + u_i - \mathrm{TC}_i) - (p + u_i - b_{i[k]}) G_{i[k]}(z^o_{i[k]}) = (p + u_i - \mathrm{TC}_i) - \frac{(p + u_i - b_{i[k]})(p + u_i - c - \mathrm{TC}_i)}{(p + u_i - v_i)}. \tag{A.7}$$

Both the manufacturer and retailer i accept buy-back contract $(w_{i[k]}, b_{i[k]}, z^o_{i[k]})$, if such a contract ensures they will be more profitable relative to the case of non-cooperative wholesale pricing (Proposition 2). Consequently, the buy-back contract should satisfy the following conditions:

$$E\left(\Pi_{R_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right) > E\left(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right), \tag{A.8}$$

$$E\left(\Pi_{M_i[k]}(w_{i[k]}, b_{i[k]}, z^o_{i[k]})\right) \geq E\left(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right). \tag{A.9}$$

Substituting Eq. (A.7) into Inequalities (A.8) and (A.9) (see Eqs. (22) and (23)), and then solving outcome inequalities for $b_{i[k]}$, we obtain Limitations (26) and (27) after some mathematic manipulation.

Proof of Corollary 4.

(i) The denominator of the fraction in Eq. (28) is positive, due to the fact that:

$$q^o_{i[k]} G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]}) = G_{i[k]}(z^o_{i[k]}) \times \sum_{j\in N_{i[k]}} y_j(p_{ij}) + \int_{B_{i[k]}}^{z_{i[k]}} \xi_{i[k]} g_{i[k]}(x)dx > 0. \tag{A.10}$$

Thus, $\overline{b_{i[k]}} - \underline{b_{i[k]}}$ has the same sign as the numerator of the fraction, i.e. $\overline{b_{i[k]}} > \underline{b_{i[k]}}$, if we have:

$$E\left(\hat{\Pi}_{i[k]}(z^o_{i[k]})\right) > E\left(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right) + E\left(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right),$$

which is also equal to Condition (24).

(ii) Differentiating Relation (22) w.r.t. $b_{i[k]}$, and by employing the chain rule (regarding Eq. (25)), we have:

$$\partial E\left(\Pi_{R_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})\right) / \partial b_{i[k]} = -\left(q^o_{i[k]} G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})\right).$$

According to Eq. (A.10), we conclude that $\Pi_{R_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})$ is a decreasing function of $b_{i[k]}$. Since:

$$E\left(\hat{\Pi}_{i[k]}(z^o_{i[k]})\right) = E\left(\Pi_{M_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})\right) + E\left(\Pi_{R_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})\right)$$

is independent of $b_{i[k]}$, it is straightforward that:

$$\partial E\left(\Pi_{M_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})\right) / \partial b_{i[k]} = q^o_{i[k]} G_{i[k]}(z_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})$$

Considering Eq. (A.10), it follows that $E\left(\Pi_{M_i[k]}(w_{i[k]}(b_{i[k]}), b_{i[k]}, z^o_{i[k]})\right)$ is a increasing function of $b_{i[k]}$.

Consequently, if $b_{i[k]}$ is too high ($b_{i[k]} > \overline{b_{i[k]}}$), the expected profit of retailer i will be less than $E\left(\Pi_{R_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right)$. On the other hand, when $b_{i[k]}$ is too low ($b_{i[k]} < \underline{b_{i[k]}}$), the expected profit of the manufacturer will be less than $E\left(\Pi_{M_i[k]}(w^*_{i[k]}, z^*_{i[k]})\right)$.

(iii) Differentiating Eq. (28) w.r.t. TC_i, we have:

$$\partial(\overline{b_{i[k]}} - \underline{b_{i[k]}})/\partial \mathrm{TC}_i = -1/\left(q^o_{i[k]} G_{i[k]}(z^o_{i[k]}) - \Lambda_{i[k]}(z^o_{i[k]})\right).$$

From Eq. (A.10), it is straightforward that $\overline{b_{i[k]}} - \underline{b_{i[k]}}$ is a decreasing function of TC_i. Similarly, it is unequivocal to show that $\overline{b_{i[k]}} - \underline{b_{i[k]}}$ is an increasing function of $(v - v_i)$.

Biographies

Ashkan Hafezalkotob received his BS degree in Industrial Engineering, in 2004, an MS degree in Industrial Engineering, in 2007, and a PhD degree in Industrial Engineering, in 2012, from Iran University of Science and Technology, Tehran, Iran. He is currently Associate Professor in the Industrial Engineering Department of the South Tehran Branch of the Islamic Azad University, Iran. His research interests include supply chain, decision making techniques, game theory, and mathematical modeling. He has authored numerous papers presented at conferences and published in journals, including MPE, JMSY, IJE and IJIEC.

Ahmad Makui received his BS degree in Industrial Engineering, in 1985, an MS degree in Industrial Engineering, in 1991, and a PhD degree in Industrial Engineering (Operations Research), in 2000. He is currently Associate Professor in the Industrial Engineering Department of the Iran University of Science and Technology, Tehran, Iran. His research interests include production planning, supply chain, decision making techniques and mathematical modeling. He has authored numerous papers presented at conferences and published in journals, including JORS, EJOR, PPC, IJAMT, JOMS, ESWA and MPE.