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# Teaching-learning-based optimization for different economic dispatch problems

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## KEYWORDS

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Prohibited operating zone;  
Ramp rate limits;  
Teaching-learning optimization;  
Valve-point loading.

**Abstract.** This paper presents a Teaching-Learning-Based Algorithm (TLBO) to solve Economic Load Dispatch (ELD) problems involving different linear and non-linear constraints. The problem formulation also considers non-convex objective functions including the effect of valve-point loading and the multi-fuel option of large-scale thermal plants. Many difficulties, such as multimodality, dimensionality and differentiability, are associated with the optimization of large scale non-linear constraint based non-convex economic load dispatch problems. TLBO is a population-based technique which implements a group of solutions to proceed to the optimum solution. TLBO uses two different phases; ‘Teacher Phase’ and ‘Learner Phase’, and uses the mean value of the population to update the solution. Unlike other optimization techniques, TLBO does not require any parameter to be tuned, thus, making its implementation simpler. TLBO uses the best solution of the iteration to change the existing solution in the population, thereby increasing the convergence rate. In the present paper, Teaching-Learning-Based Optimization (TLBO) is applied to solve such types of complicated problems efficiently and effectively, in order to achieve a superior quality solution in a computationally efficient way. Simulation results show that the proposed approach outperforms several existing optimization techniques. Results also proved the robustness of the proposed methodology.

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## 1. Introduction

Economic load dispatch is the process of allocating generation among the available generating units, considering the most efficient, reliable and low cost operation of a power system, providing that load demand and other operational constraints are satisfied. Its main aim is to minimize the total cost of generations, while satisfying the operational constraints of the available thermal power generation resources. Initially, traditional tech-

niques [1] were applied to solve ELD problems. The linear programming method [2] is fast and reliable, but also has some drawbacks, and classical optimization techniques are excellent for uni-modal and continuous functions. In these methods, the essential assumption is that the incremental costs and emission curves of the generating units are monotonically increasing or piece-wise linear. A practical ELD problem sometimes takes the effect of valve-point loading, ramp-rate limits, prohibited operating zones, multi-fuel options etc. into consideration. Due to all these practical effects, the resulting ELD problems have become totally non-convex optimization problems. Therefore, in some cases, these methods converge to a locally, not globally, optimal solution. The Dynamic Programming (DP)

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approach was proposed by Wood and Wollenberg [3] to solve ELD problems. It imposes no restrictions on the characteristics of the generating units. However, it suffers from the curse of dimensionality and also increases execution time with the increase in system size.

Several attempts have been made to solve ELD problems using various soft computing techniques, such as Genetic Algorithms (GA) [4–5], Particle Swarm Optimization (PSO) [6], Ant Colony Optimization (ACO) [7], Evolutionary Programming (EP) [8], Simulated Annealing (SA) [9], Differential Evolution (DE) [10], Artificial Immune System (AIS) [11], Bacterial Foraging Algorithm (BFA) [12], and Biogeography-Based Optimization (BBO) [13] etc. The above-mentioned techniques have proven to be very fast and reasonably near a global optimal solution in solving nonlinear ELD problems, without any restriction on the shape of the cost curves. Recently, different hybridizations and modifications of GA, EP, PSO, DE and BBO have been adopted to solve different types of ELD problems, such as Improved GA with Multiplier Updating (IGA-MU) [14], hybrid Genetic Algorithm (GA)-Pattern Search (PS)-Sequential Quadratic Programming (SQP) (GA-PS-SQP) [15], Improved Fast Evolutionary Programming (IFEP) [16], New PSO with Local Random Search (NPSO-LRS) [17], Adaptive PSO (APSO) [18], Self-Organizing Hierarchical PSO (SOH-PSO) [19], Improved Coordinated Aggregation based PSO (ICA-PSO) [20], improved PSO [21], Combined Particle Swarm Optimization with Real-Valued Mutation (CBPSO-RVM) [22], DE with generation of chaos sequences and Sequential Quadratic Programming (DEC-SQP) [23], Variable Scaling Hybrid Differential Evolution (VSHDE) [24], hybrid Differential Evolution (DE) [25], Bacterial Foraging with Nelder-Mead algorithm (BF-NM) [26], and hybrid Differential Evolution with Biogeography-Based Optimization (DE/BBO) [27] etc.

Evolutionary algorithms, swarm intelligence and bacterial foraging all are population-based bio-inspired algorithms. However, the common disadvantages of these algorithms are their complicated computations, needing many parameters, and, therefore, for beginners they are difficult to understand. Moreover, all the nature-inspired algorithms, such as GA, EP, PSO, ACO, DE, BFA, AIS, BBO etc., require tuning of algorithm parameters for them to work properly. Proper selection of parameters is essential for the searching of the optimum solution by these algorithms, and a change in the algorithm parameters changes their effectiveness. To avoid this difficulty, an optimization method, Teaching-Learning-Based Optimization (TLBO), a parameter free algorithm, is implemented in this paper to solve complex ELD problems.

Teaching-Learning-Based Optimization (TLBO)

was proposed by Rao et al. in 2011 [28]. This method works like the effect of the influence of a teacher on learners. Like other nature-inspired algorithms, TLBO is also a population-based method, which uses a population of solutions to proceed to the global solution. For TLBO, the population is considered as a group or a class of learners. The process when using TLBO is divided into two parts. The first part consists of the ‘Teacher Phase’ and the second part consists of the ‘Learner Phase’. The ‘Teacher Phase’ means learning from the teacher and the ‘Learner Phase’ means learning through interaction between learners. The teacher is generally considered a highly learned person who shares his or her knowledge with the learners. The quality of a teacher affects the outcome of the learners. It is obvious that a good teacher trains learners such that they can have better results in terms of their marks or grades. Moreover, learners also learn from interaction between themselves, which also helps in their results. Like several other soft computing techniques, TLBO is also a population-based technique, which implements a group of solutions to proceed to the optimum solution. Many optimization methods require algorithm parameters that affect techniques, TLBO does not require any algorithm parameters to be tuned, thus making the implementation of TLBO simpler. TLBO uses the best solution of the iteration to change the existing solution in the population, thereby increasing the convergence rate. TLBO uses the mean value of the population to update the solution and, therefore, implements greediness to accept a good solution. It has been already observed that the performance of TLBO is quite satisfactory when applied to solving continuous benchmark optimization problems [28].

The improved performance of TLBO in solving continuous benchmark optimization problems has motivated the present authors to implement this newly developed algorithm to solve different complex ELD problems. This paper considers four types of ELD problem, namely (i) ELD with quadratic cost function, ramp rate limit, prohibited operating zone and transmission loss: –15 generators system, (ii) ELD with quadratic cost function without transmission loss: –38 generators system, (iii) ELD with valve-point effects, ramp rate limit, prohibited operating zone: –140 generators system, (iv) ELD having multiple fuels and valve-point effects: –160 generators system.

Section 2 of the paper provides mathematical formulation of different types of ELD problems. Section 3 describes the proposed TLBO algorithm, along with a short description of the algorithm used in these test systems. Simulation studies are presented and discussed in Section 4 and the conclusion is drawn in Section 5.

## 2. Mathematical modeling of the ELD problem

The ELD may be formulated as both convex and non-convex nonlinear constrained optimization problems. Four different types of ELD problem have been formulated and solved using the TLBO approach. These are presented below.

### 2.1. ELD with quadratic cost function, ramp rate limit, prohibited operating zone and transmission loss

The overall objective function,  $F_T$ , of the ELD problem may be written as:

$$F_T = \min \sum_{i=1}^N F_i(P_i) \\ = \min \sum_{i=1}^N (a_i + b_i P_i + c_i P_i^2), \quad (1)$$

where  $F_i(P_i)$  is the cost function of the  $i$ th generator and is usually expressed as a quadratic polynomial;  $N$  is the number of committed generators;  $a_i$ ,  $b_i$  and  $c_i$  are the cost coefficients of the  $i$ th generator;  $P_i$  is the power output of the  $i$ th generator. The ELD problem consists in minimizing the  $F_T$  subject to the following constraints:

- 1) Real power balance constraint:

$$\sum_{i=1}^N P_i - (P_D + P_L) = 0, \quad (2)$$

where  $P_D$  is the total system active power demand, and  $P_L$  is the total transmission loss. Calculation of  $P_L$  using the B-coefficients matrix is expressed as:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{0i} P_i + B_{00}. \quad (3)$$

- 2) The generating capacity constraint: The power must be generated by each generator within their lower limit,  $P_i^{\min}$ , and upper limit,  $P_i^{\max}$ , so that:

$$P_i^{\min} \leq P_i \leq P_i^{\max}, \quad (4)$$

where  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and the maximum power outputs of the  $i$ th unit.

- 3) Ramp rate limit constraint: The power,  $P_i$ , generated by the  $i$ th generator at certain intervals neither should exceed that of the previous interval,  $P_{i0}$ , by more than a certain amount,  $UR_i$ , the up-ramp limit, nor should it be less than that of the previous interval by more than some amount,  $DR_i$ , the down-ramp limit of the generator. These give rise to the following constraints:

As generation increases:

$$P_i - P_{i0} \leq UR_i. \quad (5)$$

As generation decreases:

$$P_{i0} - P_i \leq DR_i, \quad (6)$$

$$\max(P_i^{\min}, P_{i0} - DR_i) \leq (P_i^{\max}, P_{i0} + UR_i). \quad (7)$$

- 4) Prohibited operating zone: Mathematically, the feasible operating zones of a unit can be described as follows:

$$P_i^{\min} \leq P_i \leq P_{i,1}^l,$$

$$P_{i,j-1}^u \leq P_i \leq P_{i,j}^l, \quad j = 2, 3, \dots, n_i,$$

$$P_{i,n_i}^u \leq P_i \leq P_i^{\max}, \quad (8)$$

where  $j$  represents the number of prohibited operating zones of unit  $i$ .  $P_{i,j}^u$  is the upper limit and  $P_{i,j}^l$  is the lower limit of the  $j$ th prohibited operating zone of the  $i$ th unit. The total number of prohibited operating zones of the  $i$ th unit is  $n_i$ .

### 2.2. ELD with quadratic cost function

In this type of ELD problem, the overall objective function is the same as mentioned in Eq. (1). Here, the objective function,  $F_T$ , is to be minimized, subject to the constraints of Eqs. (2) and (4). Here,  $P_L$  is zero.

### 2.3. ELD with valve-point effects, ramp rate limit, prohibited operating zone

The fuel cost function,  $F_T$ , in the ELD problem with valve point loading changes the simple cost function in Eq. (1). It becomes more complex and is represented below:

$$F_T = \left( \sum_{i=1}^N F_i(P_i) \right) = \left( \sum_{i=1}^N a_i + b_i P_i + c_i P_i^2 \right. \\ \left. + \left| e_i \times \sin \left\{ f_i \times (P_i^{\min} - P_i) \right\} \right| \right), \quad (9)$$

where  $e_i$  and  $f_i$ , the coefficients of the  $i$ th generator, reflect the valve-point effects. The objective function in Eq. (9) is to be minimized, subject to the same set of constraints given in Eqs. (4), (7) and (8).

### 2.4. ELD with non-smooth cost functions with multiple fuels and valve-point effects

For a power system with  $N$  generators and  $n_F$  fuel options for each unit, the cost function of the generator with valve-point loading is expressed as:

$$F_i(P_i) = a_{ip} + b_{ip}P_i + c_{ip}P_i^2 + \left| e_{ip} \times \sin \left\{ f_{ip} \times (P_{ip}^{\min} - P_{ip}) \right\} \right|, \quad (10)$$

if  $P_{ip}^{\min} \leq P_i \leq P_{ip}^{\max}$  for fuel option  $p$ ,

$p = 1, 2, \dots, n_F$ ,

where  $P_{ip}^{\min}$  and  $P_{ip}^{\max}$  are the minimum and maximum power generation limits of the  $i$ th generator with fuel option,  $p$ , respectively;  $a_{ip}$ ,  $b_{ip}$ ,  $c_{ip}$ ,  $e_{ip}$  and  $f_{ip}$  are the fuel-cost coefficients of the  $i$ th generator for fuel option  $p$ .

Considering  $N$  number of generators, the above-mentioned objective function is to be minimized subject to the constraints of Eqs. (2) and (4), without considering transmission loss. Therefore, the  $P_L$  term in Eq. (2) becomes zero.

### 2.5. Calculation for slack generator

Let  $N$  committed generating units deliver their power output, subject to the power balance constraint in Eq. (2) and the respective capacity constraints of Eqs. (4) and/or (7), and (8). Assuming the power loadings of the first  $(N - 1)$  generators are known, the power level of the  $N$ th generator (called the Slack Generator) is given by:

2.5.1 Without transmission loss:

$$P_N = P_D - \sum_{i=1}^{(N-1)} P_i. \quad (11)$$

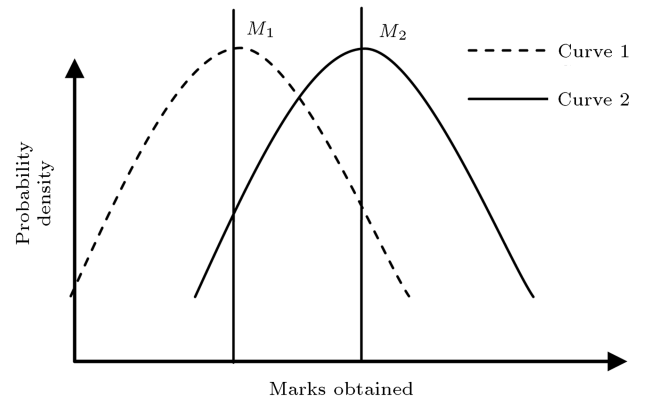
2.5.2 With transmission loss:

$$P_N = P_D + P_L - \sum_{i=1}^{(N-1)} P_i. \quad (12)$$

Using Eqs. (3) and (12), the modified form of the equation is:

$$\begin{aligned} B_{NN}P_N^2 + P_N \left( 2 \sum_{i=1}^{N-1} B_{Ni}P_i + \sum_{i=1}^{N-1} B_{ON} - 1 \right) \\ + \left( PD + \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} P_i P_{ij} P_j \right. \\ \left. + \sum_{i=1}^{N-1} B_{Oi}P_i - \sum_{i=1}^{N-1} P_i + B_{OO} \right) = 0. \end{aligned} \quad (13)$$

The solution procedure of Eq. (13) to calculate slack generator output,  $P_N$ , is the same as mentioned in [19]. To avoid repetition, it is not presented here.



**Figure 1.** Marks distribution by learners taught by  $T_1$  and  $T_2$ .

### 3. Teaching-learning-based algorithm

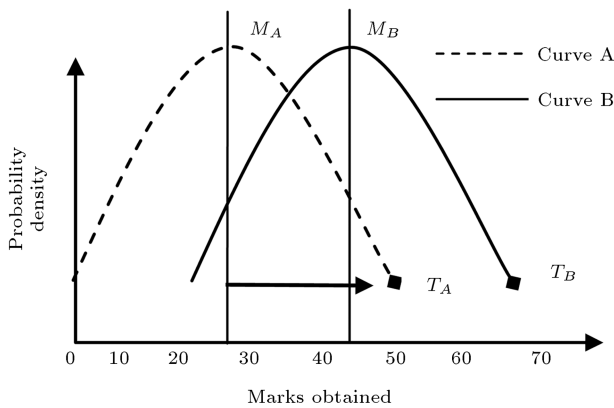
This section presents an interesting new optimization algorithm called Teaching-Learning-Based Optimization (TLBO), which has been recently proposed in [28]. The TLBO method works on the philosophy of the effect of manipulation of a teacher on the output of learners in a class, and, consequently, learning by interaction between class members, which helps in their grades. Therefore, the TLBO method works on the philosophy of teaching and learning.

Consider two different teachers,  $T_1$  and  $T_2$ , teaching a topic to the same merit level learners in two different classes. The distribution of marks obtained by the learners for these two varying classes is evaluated by the teachers and is illustrated in Figure 1. Curves 1 and 2 represent the evaluated marks obtained by the learners taught by teacher  $T_1$  and  $T_2$ , respectively. Normal distribution for the goal achieved by the learners is defined as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{\sigma^2}}, \quad (14)$$

where  $\sigma^2$  is the variance,  $\mu$  is the mean, and  $x$  is any value of whichever normal distribution function is required. Comparing the mean value of Curves 1 and 2 of Figure 1, it is seen that the learners from Curve 2 get better results than the learners from Curve 1. So, it can be said that teacher  $T_2$  is better than teacher  $T_1$  in terms of teaching. Learners also learn from interaction between themselves, which promotes their results.

Figure 2 shows a model for the marks obtained by the learners in a class having mean  $M_A$  in Curve A. Teachers are considered the most intelligent members of society and, therefore, the best learner is considered to be the teacher here. This is shown by  $T_A$  in Figure 2. The teacher tries to spread knowledge among the learners, which, in turn, increases the knowledge level of the whole class and help learners to get good marks or grades. Teacher  $T_A$  puts maximum effort



**Figure 2.** Distribution of score for learners.

into teaching his or her students and tries to move class mean from  $M_A$  towards a new mean,  $M_B$ , by means of increasing the learners' knowledge level. At that stage, the learners require a new teacher,  $T_B$ , of superior quality than themselves, which is shown by Curve B in Figure 2.

TLBO is also a population-based algorithm, whose population is described as a class of learners. In any nature-inspired based optimization algorithms, the population consists of different design variables. In TLBO, different design variables are the different subjects offered to the learners, and the learners' outcome corresponds to the 'fitness function'. The teacher is considered the best solution obtained so far. The process of TLBO is divided into two parts. 'Teacher Phase' and 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through interaction between learners. These two parts of TLBO are described below.

### 3.1. Teacher phase

A good teacher always tries to improve the quality of learners in terms of knowledge, i.e. a teacher tries to increase the mean value of the class from  $M_A$  to  $M_B$ , as seen in Figure 2. But, in real practice, this is not possible and a teacher can only move the average quality of a class up to some limit, depending on the quality of the class.

Let  $M_k$  be the mean and  $T_k$  be the teacher at any iteration,  $k$ .  $T_k$  tries to move mean  $M_k$  towards its own level. The solution is updated according to the difference between the existing and the new mean. It is given by:

$$X_{\text{diff}} = \text{rand}()x(T_k - R_t M_k), \quad (15)$$

where,  $\text{rand}()$  is a random number in the range  $[0,1]$ ; the value of  $R_t$  can be either 1 or 2, which can be decided randomly with equal probability.

This difference modifies the existing solution according to the following expression:

$$X_{\text{new}} = X_{\text{old}} + X_{\text{diff}}. \quad (16)$$

### 3.2. Learner phase

In the learner phase, the learners increase their knowledge by two different methods. The first is through input from the teacher and the other through some interaction between themselves. A learner interacts with other randomly selected learners by participation in formal communication, group discussion and presentations. By interaction, a learner learns something new if the other learners have more knowledge than the corresponding learner [28]. In order to design the mathematical model, two learners,  $X_i$  and  $X_j$ , are randomly chosen, where  $i \neq j$ . Objective functions for the learners,  $X_i$  and  $X_j$ , are evaluated. The achieved objective functions of  $X_i$  and  $X_j$  are compared. If the achieved objective function of  $X_i$  is less than the achieved objective function of  $X_j$ , then:

$$X_{\text{new}} = X_{\text{old}} + \text{rand}()x(X_i - X_j). \quad (17)$$

Otherwise:

$$X_{\text{new}} = X_{\text{old}} + \text{rand}()x(X_j - X_i). \quad (18)$$

If the new solution is better than the existing one, then it is accepted. The pseudo codes and flow chart for all steps are available in [28].

### 3.3. Sequential steps of TLBO algorithm

There are two stages in TLBO: teacher phase and learner phase. All the steps are mentioned below:

- 1) At the initialization stage, read in the initial number of learners (PopSize) (equivalent to population size of many heuristic algorithms); maximum iteration number ( $\text{Iter}_{\text{max}}$ ). Specify the number of design variables ( $D$ ), in this case, assigned as the number of subjects offered. Mention the lower and upper limits of design variables.
- 2) Generate the learner matrix ( $X_{ij}$ ) randomly, according to population size, number of design variables and limits of the variables (where  $i = 1, 2, \dots, \text{PopSize}$ , and  $j = 1, 2, \dots, D$  and total matrix size is  $\text{PopSize} \times D$ ).
- 3) Determine objective function values for each learner set. The size of the objective function matrix is, therefore  $\text{PopSize} \times D$ . The minimum value to come out of these objective function values is the local optimum value, and the corresponding value of  $X_{ij}$  is set as the teacher ( $X_{\text{teacher}}$ ). So,  $X_{\text{teacher}} = T_k$  in Eq. (15).
- 4) Calculate the mean value of each design variable column-wise. So, the size of the mean value is  $1 \times D$ , and is used in Eq. (15) as  $M_k$ .
- 5) Modify each learner by Eqs. (15) and (16). The value of  $R_t$  is randomly selected as 1 or 2. Calculate

the objective function values for each modified learner. If the new value of the objective function of any learner is better than the previous one, then accept a new learner and replace the corresponding old one. Otherwise, keep the old learner without any modification.

- 6) Learner phase: Learners increase their knowledge with the help of mutual interaction. For each learner  $X_i (i = 1, 2, \dots, D)$ , arbitrarily choose any learner,  $X_j$ , from the learner matrix. Compare the objective function corresponding to  $X_i$  and  $X_j$ . If the value of the objective function of  $X_i$  is lower than the objective function value of  $X_j$ , then modify the  $i$ th learner using Eq. (17), otherwise, modify the  $i$ th learner using Eq. (18).
- 7) If the maximum number of iterations is reached or the specified accuracy level is achieved, terminate the iterative process, otherwise, go to step 3 for continuation. Interested readers may refer to [28] which contains detailed steps of the TLBO Algorithm.

### 3.4. TLBO algorithm for economic load dispatch problem

In this subsection, the procedure to implement the TLBO algorithm for solving ELD problems has been described. This algorithm is also used to deal with the equality and inequality constraints of ELD problems. The sequential steps of the TLBO algorithm applied to solve the ELD problem are:

- 1) Representation of the learner matrix,  $X$ : Since the assessment variables for the ELD problem are the real power output of the generators, they are together used to represent the individual learner. Each individual element of a learner is the subject studied by the corresponding learner, and it is same as the real power outputs of the generators in ELD. For initializations, choose the number of generator units,  $m$ , as a design variable,  $D$ . The total number of the learner structure is population size, which is denoted as 'PopSize'.

The complete learner matrix is represented in the form of the following matrix:

$$X = X_i = [X_1, X_2, X_3, \dots, X_{\text{PopSize}}] \quad (19)$$

where  $i = 1, 2, \dots, \text{Popsize}$ .

In the case of the ELD problem, each learner is presented as:

$$\begin{aligned} X_i &= [X_{i1}, X_{i2}, \dots, X_{im}] = [Pg_{ij}] \\ &= [Pg_{i1}, Pg_{i2}, \dots, Pg_{im}], \end{aligned}$$

where,  $j = 1, 2, \dots, m$ . Each learner is one of the possible solutions for the ELD problem. The

element,  $X_{ij}$ , of  $X_i$  is the  $j$ th position component of learner,  $i$ .

- 2) Initialization of the learner: Each individual element of the learner matrix ( $X$ ), i.e. each element of a given learner, is initialized randomly within the effective real power operating limits. The initialization is based on Eq. (4) for generators without ramp rate limits, on Eqs. (4) and (7) for generators with ramp rate limits and on Eqs. (4), (7) and (8) for generators with ramp rate limits, prohibited operating zone.
- 3) Evaluation of objective functions: In the case of ELD problems, the objective function of each learner is represented by the total fuel cost of generation for all the generators of that given learner. It is calculated using Eq. (1) for the system having quadratic fuel cost characteristics, Eq. (9) for the system having valve-point effects, and Eq. (10) for the system having multi-fuel type fuel cost characteristics.

Now, the steps of the algorithm to solve ELD problems are given below:

- **Step 1.** For initialization, choose the number of generator units,  $m$ , i.e. number of design variables,  $D$ , and number of learners, PopSize. Specify the maximum and minimum capacity of each generator, the power demand, the B-coefficient matrix for calculation of transmission loss and other input data. Set the maximum number of iterations,  $\text{Iter}_{\max}$ .
- **Step 2.** Each learner of the  $X$  matrix should satisfy the equality constraint of Eq. (2) using the concept of slack generator, as mentioned in Section 2.5.
- **Step 3.** Calculate the objective function value for each learner following the procedure mentioned in "Evaluation of objective functions".
- **Step 4.** Based on objective function values, identify the elite learner, which is assigned as the teacher of the learner matrix. Here, the elite term is used to indicate the learner that gives the best fuel cost. The elite learner is taken as  $T_k$  in Eq. (15).
- **Step 5.** From the learner matrix ( $X$ ), calculate the mean value of each design variable, i.e. the mean value of the individual generator power output column wise. The mean value is assigned as  $M_k$  in Eq. (15).
- **Step 6.** Modify each learner, i.e. the power output of the generators, using Eqs. (15) and (16). Verify the feasibility of each newly generated learner of the modified  $X$  matrix. Individual elements of each modified learner must satisfy the generator operating limit constraint of Eq. (4). If any element of a learner violates either upper or lower operating limits, then

fix the values of those elements of the corresponding learner at the limit reached by them. Again, satisfy the constraint of Eq. (2) using the concept of slack generator, as presented in Section 2.5 ( $P_L = 0$  in Eq. (12) if loss is not considered). If the output of the slack generator does not meet generator operating limit constraint, as in Eq. (4), or some generators do not satisfy the prohibited operating zone or ramp rate limit constraints, where applicable, then reject that new learner and reapply Step 6 on the old one, until all constraints are satisfied.

- **Step 7.** Calculate the values of the objective function of each modified learner of the learner matrix. If the new value of the objective function of any learner is better than the previous one, then accept the new learner and replace the corresponding old one. Otherwise, keep the old learner without any modification.
- **Step 8.** For each learner,  $X_i (i = 1, 2, \dots, D)$ , arbitrarily choose any learner,  $X_j$ , from the learner matrix. Compare the objective function corresponding to  $X_i$  and  $X_j$ . If the value of the objective function of  $X_i$  is lower than the objective function value of  $X_j$ , then modify the  $i$ th learner using Eq. (17). Otherwise, modify the  $i$ th learner using Eq. (18).
- **Step 9.** Individual elements of each modified learner must satisfy their generator constraints. If any element of a modified learner violates either upper or lower operating limits, then fix the values of those elements of the corresponding learner at the limit reached by them. Again, satisfy the constraint of Eq. (2) using the concept of the slack generator, as presented in Section 2.5 ( $P_L = 0$  in Eq. (12) if loss is not considered). If the output of the slack generator does not meet the generator operating limit constraint, as in Eq. (4), or some generators do not satisfy the prohibited operating zone or ramp rate limit constraints, where applicable, reject that modified learner and reapply Step 8 on the old one, until all the constraints are satisfied.
- **Step 10.** As individual learners of the learner matrix change, the values of their objective function also change. Calculate the objective function of each newly generated learner. If the new value of the objective function of a given learner is better than its previous value, then accept the new learner and replace the corresponding old one. Otherwise, keep the old learner without any modification.
- **Step 11.** If the maximum number of iterations is reached or specified accuracy level is achieved, terminate the iterative process. Otherwise, go to Step 4 for continuation.

## 4. Examples and simulation result

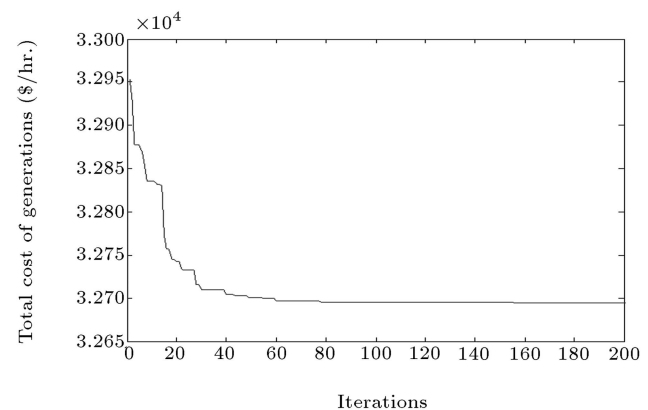
The proposed TLBO algorithm has been applied to solve ELD problems in four different test cases, and its performance has been compared to several other optimization techniques, like GA [7], DE/BBO [7,27], and PSO [7,21] etc., for verifying its feasibility. The necessary codes have been written in MATLAB-7 language and executed on a 2.0-GHz Intel Pentium (R) Dual Core personal computer with 1-GB RAM.

### 4.1. Description of the test systems

**Test system 1:** In this example, 15 generating units with ramp rate limit and prohibited zone constraints have been considered. Transmission loss has been included in the problem. Power demand is 2630 MW and system data have been taken from [7]. Results obtained from the proposed TLBO, PSO [7], different versions of PSO [21] and other method, have been presented here, and their best solutions are shown in Table 1. The convergence characteristics of the 15-generator system in the case of TLBO are shown in Figure 3. Minimum, average and maximum fuel costs obtained by TLBO and different versions of PSO [21], over 50 trials, are presented in Table 2.

**Test system 2:** A 38-generator system with quadratic fuel cost characteristics is used here. The input data are taken from [29]. The load demand is 6000 MW. Transmission loss has not been considered here. The result obtained using the proposed TLBO method has been compared with BBO [27], DE/BBO [27], PSO-TVAC [27] and New-PSO [27], whose best solutions are shown in Table 3. A convergence characteristic of the 38-generator system in the case of TLBO are shown in Figure 4. Minimum, average and maximum fuel costs obtained by TLBO over 50 trials are shown in Table 4.

**Test system 3:** A 140-generator system having ramp rate limit and prohibited zone constraints is considered. The effect of valve-point loading has



**Figure 3.** Convergence characteristic of 15-generator systems, obtained by TLBO.

**Table 1.** Best power output for 15-generator systems ( $P_D = 2630$  MW).

Unit	TLBO	GA [7]	PSO [7]	CTPSO [21]	CSPSO [21]	COPSO [21]	CCPSO [21]
1	455.000000	415.3108	439.1162	455.0000	455.0000	455.0000	455.0000
2	380.000000	359.7206	407.9727	380.0000	380.0000	380.0000	380.0000
3	130.000000	104.4250	119.6324	130.0000	130.0000	130.0000	130.0000
4	130.000000	74.9853	129.9925	130.0000	130.0000	130.0000	130.0000
5	170.000000	380.2844	151.0681	170.0000	170.0000	170.0000	170.0000
6	460.000000	426.7902	459.9978	460.0000	460.0000	460.0000	460.0000
7	430.000000	341.3164	425.5601	430.0000	430.0000	430.0000	430.0000
8	73.081166	124.7867	98.5699	71.7430	71.7408	71.7427	71.7526
9	51.646599	133.1445	113.4936	58.9186	58.9207	58.9189	58.9090
10	160.000000	89.2567	101.1142	160.0000	160.0000	160.0000	160.0000
11	80.000000	60.0572	33.9116	80.0000	80.0000	80.0000	80.0000
12	80.000000	49.9998	79.9583	80.0000	80.0000	80.0000	80.0000
13	26.577183	38.7713	25.0042	25.0000	25.0000	25.0000	25.0000
14	17.150894	41.9425	41.4140	15.0000	15.0000	15.0000	15.0000
15	16.033243	22.6445	35.6140	15.0000	15.0000	15.0000	15.0000
Total (MW)	2659.489085	2668.4	2662.4	2660.6615	2660.6615	2660.6615	2660.6616
Loss (MW)	29.489085	38.2782	32.4306	30.6615	30.6615	30.6615	30.6616
Fuel cost (\$/hr.)	<b>32697.215085</b>	33113	32858	32704	32704	32704	32704

**Table 2.** Comparison between different methods taken after 50 trials (15-generator systems).

Methods	Generation cost (\$/hr.)				Time/iteration (sec)	No. of hits to minimum solution
	Max.	Min.	Average	Standard deviation		
<b>TLBO</b>	<b>32697.215085</b>	<b>32697.215085</b>	<b>32697.215085</b>	<b>0.00</b>	<b>4.0</b>	<b>50</b>
CTPSO [21]	32704.4514	32704.4514	32704.4514	-	22.5	NA*
CSPSO [21]	32704.4514	32704.4514	32704.4514	-	16.1	NA
COPSO [21]	32704.4514	32704.4514	32704.4514	-	85.1	NA
CCPSO [21]	32704.4514	32704.4514	32704.4514	-	16.2	NA

\*NA: Data not available.

been incorporated within the generator fuel cost characteristics of unit numbers 5, 10, 15, 22, 33, 40, 52, 70, 72, 84, 119 and 121. The input data of this system are taken from [21]. The load demand is 49342 MW. The best results obtained by the proposed TLBO are shown in Table 5. Out of 50 trials, minimum, maximum and average fuel cost obtained using TLBO algorithm, different versions of PSO [21] and Modified Teaching-Learning Algorithm (MTLA) [30] are shown in Table 6. Its convergence characteristic is presented in Figure 5.

**Test system 4:** A complex system with 160 thermal units is considered here. The input data are available in [31]. The system demand is 43200 MW. Transmission loss has not been included. The best result obtained using the proposed TLBO algorithm is shown in Table 7. Minimum, average and maximum fuel costs obtained by TLBO, ED-DE [31], and different GA [31] methods over 50 trials are presented in Table 8. The convergence characteristic of the 160-generator systems obtained by TLBO is shown in Figure 6.



**Table 3.** Best power output for 38-generator systems ( $P_D = 6000$  MW).

Output (MW)	TLBO	DE/BBO [27]	BBO [27]	PSO_TVAC [27]	NEW_PSO [27]
$P_1$	425.891375	426.606060	422.230586	443.659	550.000
$P_2$	426.828618	426.606054	422.117933	342.956	512.263
$P_3$	430.318693	429.663164	435.779411	433.117	485.733
$P_4$	429.480487	429.663181	445.481950	500.00	391.083
$P_5$	429.996241	429.663193	428.475752	410.539	443.846
$P_6$	430.036039	429.663164	428.649254	492.864	358.398
$P_7$	429.142948	429.663185	428.119288	409.483	415.729
$P_8$	428.764849	429.663168	429.900663	446.079	320.816
$P_9$	114.000000	114.000000	115.904947	119.566	115.347
$P_{10}$	114.000000	114.000000	114.115368	137.274	204.422
$P_{11}$	119.373112	119.768032	115.418662	138.933	114.000
$P_{12}$	127.864848	127.072817	127.511404	155.401	249.197
$P_{13}$	110.000000	110.000000	110.000948	121.719	118.886
$P_{14}$	90.000000	90.000000	90.0217671	90.924	102.802
$P_{15}$	82.000000	82.000000	82.0000000	97.941	89.0390
$P_{16}$	120.000000	120.000000	120.038496	128.106	120.000
$P_{17}$	159.332636	159.598036	160.303835	189.108	156.562
$P_{18}$	65.000000	65.000000	65.0001141	65.0000	84.265
$P_{19}$	65.000000	65.000000	65.0001370	65.0000	65.041
$P_{20}$	271.994045	272.000000	271.999591	267.422	151.104
$P_{21}$	271.999334	272.000000	271.872680	221.383	226.344
$P_{22}$	259.997110	260.000000	259.732054	130.804	209.298
$P_{23}$	130.995978	130.648618	125.993076	124.269	85.719
$P_{24}$	10.000001	10.000000	10.4134771	11.535	10.000
$P_{25}$	113.306372	113.305034	109.417723	77.103	60.000
$P_{26}$	88.045293	88.0669159	89.3772664	55.018	90.489
$P_{27}$	37.532207	37.5051018	36.4110655	75.000	39.670
$P_{28}$	20.000000	20.000000	20.0098880	21.628	20.000
$P_{29}$	20.000000	20.000000	20.0089554	29.829	20.995
$P_{30}$	20.000000	20.000000	20.0000000	20.326	22.810
$P_{31}$	20.000000	20.000000	20.0000000	20.000	20.000
$P_{32}$	20.000000	20.000000	20.0033959	21.840	20.416
$P_{33}$	25.000000	25.000000	25.0066586	25.620	25.000
$P_{34}$	18.000000	18.000000	18.0222107	24.261	21.319
$P_{35}$	8.000000	8.000000	8.00004260	9.6670	9.1220
$P_{36}$	25.000000	25.000000	25.0060660	25.000	25.184
$P_{37}$	21.907418	21.7820891	22.0005641	31.642	20.000
$P_{38}$	21.192396	21.0621792	20.6076309	29.935	25.104
Fuel cost (\$/hr.)	<b>9411938.5572307333</b>	9417235.786391673	9417633.6376443729	9500448.307	9516448.312

**Table 4.** Comparison between maximum, minimum and average value taken after 50 trials (38-generator systems).

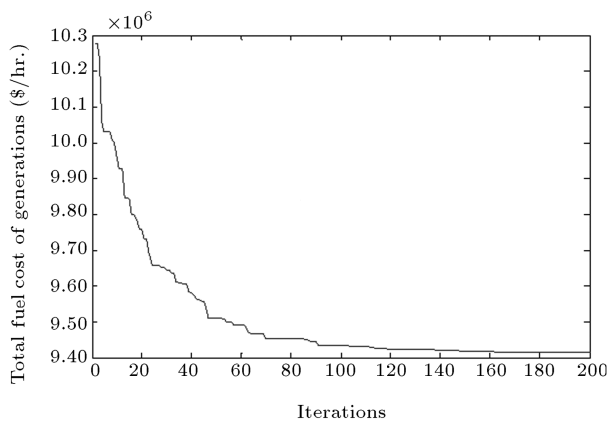
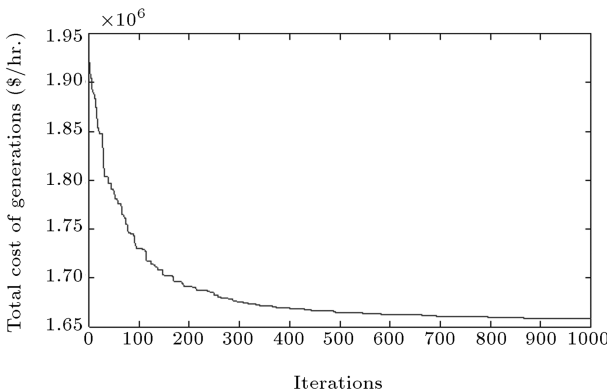
Methods	Generation cost (\$/hr.)				Time/iteration (sec)	No. of hits to minimum solution
	Max.	Min.	Average	Standard deviation		
TLBO	9411938.5572307333	9411938.5572307333	9411938.5572307333	0.00	0.50	50

**Table 5.** Best power output for 140-generator systems ( $P_D = 49342$  MW).

Unit	Power output (MW)	Unit	Power output (MW)	Unit	Power output (MW)
$P_1$	119.000000	$P_{48}$	249.994057	$P_{95}$	837.500000
$P_2$	163.992556	$P_{49}$	249.946191	$P_{96}$	682.000000
$P_3$	189.972341	$P_{50}$	249.929215	$P_{97}$	720.000000
$P_4$	189.998972	$P_{51}$	165.209529	$P_{98}$	718.000000
$P_5$	168.535362	$P_{52}$	165.011169	$P_{99}$	720.000000
$P_6$	189.997956	$P_{53}$	165.016223	$P_{100}$	964.000000
$P_7$	490.000000	$P_{54}$	165.451209	$P_{101}$	958.000000
$P_8$	490.000000	$P_{55}$	180.017382	$P_{102}$	947.900000
$P_9$	496.000000	$P_{56}$	180.022796	$P_{103}$	934.000000
$P_{10}$	496.000000	$P_{57}$	103.221141	$P_{104}$	935.000000
$P_{11}$	496.000000	$P_{58}$	198.019702	$P_{105}$	876.500000
$P_{12}$	496.000000	$P_{59}$	312.000000	$P_{106}$	880.900000
$P_{13}$	506.000000	$P_{60}$	310.335980	$P_{107}$	873.700000
$P_{14}$	509.000000	$P_{61}$	163.059478	$P_{108}$	877.400000
$P_{15}$	506.000000	$P_{62}$	95.011962	$P_{109}$	871.700000
$P_{16}$	505.000000	$P_{63}$	510.936198	$P_{110}$	864.800000
$P_{17}$	506.000000	$P_{64}$	510.798512	$P_{111}$	882.000000
$P_{18}$	506.000000	$P_{65}$	489.960051	$P_{112}$	94.008366
$P_{19}$	505.000000	$P_{66}$	255.973389	$P_{113}$	94.008341
$P_{20}$	505.000000	$P_{67}$	489.682262	$P_{114}$	94.002109
$P_{21}$	505.000000	$P_{68}$	490.000000	$P_{115}$	244.043393
$P_{22}$	505.000000	$P_{69}$	130.012045	$P_{116}$	244.017301
$P_{23}$	505.000000	$P_{70}$	339.411380	$P_{117}$	244.021535
$P_{24}$	505.000000	$P_{71}$	139.530668	$P_{118}$	95.016467
$P_{25}$	537.000000	$P_{72}$	388.321434	$P_{119}$	95.012018
$P_{26}$	537.000000	$P_{73}$	201.593238	$P_{120}$	116.010750
$P_{27}$	549.000000	$P_{74}$	175.736242	$P_{121}$	175.016446
$P_{28}$	549.000000	$P_{75}$	211.418208	$P_{122}$	2.000193
$P_{29}$	501.000000	$P_{76}$	274.267672	$P_{123}$	4.001186
$P_{30}$	499.000000	$P_{77}$	382.327348	$P_{124}$	15.012599
$P_{31}$	506.000000	$P_{78}$	330.234153	$P_{125}$	9.010491
$P_{32}$	506.000000	$P_{79}$	531.000000	$P_{126}$	12.001651
$P_{33}$	506.000000	$P_{80}$	531.000000	$P_{127}$	10.001491
$P_{34}$	506.000000	$P_{81}$	541.971416	$P_{128}$	112.019297
$P_{35}$	500.000000	$P_{82}$	56.003078	$P_{129}$	4.004812
$P_{36}$	500.000000	$P_{83}$	115.032582	$P_{130}$	5.034679
$P_{37}$	241.000000	$P_{84}$	115.003931	$P_{131}$	5.001229
$P_{38}$	241.000000	$P_{85}$	115.027600	$P_{132}$	50.000415
$P_{39}$	774.000000	$P_{86}$	207.012109	$P_{133}$	5.001042
$P_{40}$	769.000000	$P_{87}$	207.012532	$P_{134}$	42.021338
$P_{41}$	3.014093	$P_{88}$	175.000656	$P_{135}$	42.002799
$P_{42}$	3.001595	$P_{89}$	175.148390	$P_{136}$	41.005287
$P_{43}$	250.000000	$P_{90}$	182.053148	$P_{137}$	17.004924
$P_{44}$	249.166734	$P_{91}$	175.129746	$P_{138}$	7.018298
$P_{45}$	250.000000	$P_{92}$	575.400000	$P_{139}$	7.001898
$P_{46}$	249.803132	$P_{93}$	547.500000	$P_{140}$	26.291702
$P_{47}$	249.981180	$P_{94}$	836.800000	cost (\$/hr.): <b>-1657586.7157401750</b>	

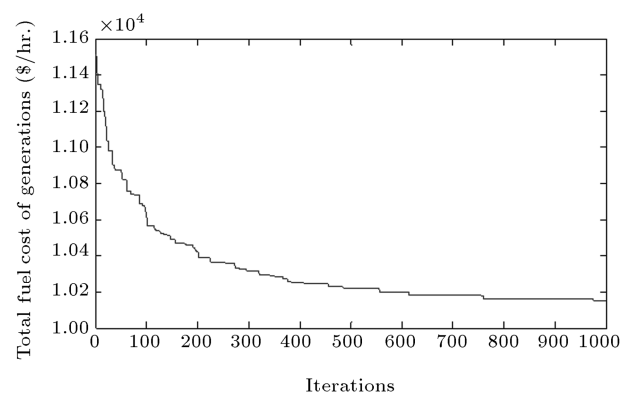
**Table 6.** Comparison between different methods taken after 50 trials (140-generator systems).

Methods	Generation cost (\$/hr.)				Time/iteration (sec)	No. of hits to minimum solution
	Max.	Min.	Average	Standard deviation		
TLBO	1657596.2512	1657586.7157	1657587.2878	2.2875	12.8	47
CTPSO [21]	1658002.7900	1657962.7300	1657964.0600	-	100	NA
CSPSO [21]	1657962.8500	1657962.7300	1657962.7400	-	99	NA
COPSO [21]	1657962.7300	1657962.7300	1657962.7300	-	150	NA
CCPSO [21]	1657962.7300	1657962.7300	1657962.7300	-	150	NA
MTLA [30]	1657951.9053	1657951.9053	1657951.9053	-	2.28	NA

**Figure 4.** Convergence characteristic of 38-generator systems, obtained by TLBO.**Figure 5.** Convergence characteristic of 140-generator systems, obtained by TLBO.

#### 4.2. Effect of learner size for TLBO algorithms

Very large or small values of learner size may not be capable of getting the minimum value of fuel costs. For each learner size of 20, 50, 100, 150 and 200, 50 trials have been run. Out of these, the learner size of 50 achieves the best fuel cost of generations for this system. For other learner sizes, no significant improvement of fuel cost has been observed. Moreover, beyond

**Figure 6.** Convergence characteristic of 160-generator systems obtained by TLBO.

learner size of 50, simulation time also increases. The best output obtained by the TLBO algorithm for each learner size is presented in Table 9.

##### 4.2.1. Comparative study

1. **Solution quality:** Tables 1, 3, 5, and 7 present the best fuel cost obtained by TLBO for 4 different test systems. The minimum costs obtained for the 4 test system are better, compared to the results obtained by many previously developed techniques, and are also shown in Tables 2, 4, 6 and 8. These tables also represent the comparative studies for maximum, minimum and average values obtained by different algorithms. From the results, it is clear that the performance of the TLBO algorithm is better, in terms of quality of solution, compared to many already existing techniques.
2. **Computational efficiency:** In Tables 2, 4, 6 and 8, it is shown that the time taken by TLBO to achieve minimum fuel costs is much less compared to many other techniques. These results prove the significantly better computational efficiency of TLBO.
3. **Robustness:** The performance of any heuristic algorithm cannot be judged by a single run. Nor-

**Table 7.** Best power output for 160-generator systems ( $P_D = 43200$  MW).

Unit	Power	Unit	Power output (MW)	Unit	Power output (MW)
$P_1$	230.231072	$P_{55}$	268.702738	$P_{109}$	420.285071
$P_2$	210.446980	$P_{56}$	235.502986	$P_{110}$	274.018137
$P_3$	286.038870	$P_{57}$	299.035088	$P_{111}$	223.101446
$P_4$	242.966508	$P_{58}$	243.769370	$P_{112}$	211.644783
$P_5$	282.709600	$P_{59}$	435.661539	$P_{113}$	287.251212
$P_6$	241.572630	$P_{60}$	275.710290	$P_{114}$	237.024233
$P_7$	293.534605	$P_{61}$	236.103212	$P_{115}$	276.677853
$P_8$	241.844669	$P_{62}$	211.015683	$P_{116}$	242.713774
$P_9$	428.520764	$P_{63}$	269.917872	$P_{117}$	298.850305
$P_{10}$	273.970461	$P_{64}$	240.104662	$P_{118}$	240.329015
$P_{11}$	223.822498	$P_{65}$	289.996904	$P_{119}$	409.123323
$P_{12}$	213.659852	$P_{66}$	247.063654	$P_{120}$	268.489742
$P_{13}$	296.793002	$P_{67}$	297.981577	$P_{121}$	216.378669
$P_{14}$	243.099952	$P_{68}$	235.432930	$P_{122}$	223.185522
$P_{15}$	283.978652	$P_{69}$	436.397367	$P_{123}$	282.345249
$P_{16}$	241.597356	$P_{70}$	273.077310	$P_{124}$	244.262927
$P_{17}$	284.202003	$P_{71}$	231.023042	$P_{125}$	278.861687
$P_{18}$	243.441124	$P_{72}$	211.442586	$P_{126}$	242.752663
$P_{19}$	430.831438	$P_{73}$	263.863789	$P_{127}$	274.027211
$P_{20}$	283.002119	$P_{74}$	245.211579	$P_{128}$	240.527749
$P_{21}$	217.450883	$P_{75}$	262.494238	$P_{129}$	436.881594
$P_{22}$	213.075368	$P_{76}$	237.375342	$P_{130}$	275.124501
$P_{23}$	279.454877	$P_{77}$	278.695112	$P_{131}$	222.507071
$P_{24}$	238.652449	$P_{78}$	243.530392	$P_{132}$	210.184492
$P_{25}$	267.130395	$P_{79}$	438.426795	$P_{133}$	279.589430
$P_{26}$	238.526165	$P_{80}$	270.445893	$P_{134}$	232.842543
$P_{27}$	274.066249	$P_{81}$	221.562195	$P_{135}$	274.300277
$P_{28}$	242.075343	$P_{82}$	210.474538	$P_{136}$	235.855180
$P_{29}$	427.901473	$P_{83}$	293.338588	$P_{137}$	291.097887
$P_{30}$	264.284943	$P_{84}$	241.945638	$P_{138}$	236.748676
$P_{31}$	219.466474	$P_{85}$	301.572104	$P_{139}$	435.188836
$P_{32}$	209.112710	$P_{86}$	241.132843	$P_{140}$	258.139680
$P_{33}$	287.864658	$P_{87}$	289.654387	$P_{141}$	203.969339
$P_{34}$	241.574369	$P_{88}$	234.692550	$P_{142}$	208.977942
$P_{35}$	272.641652	$P_{89}$	431.272142	$P_{143}$	283.658807
$P_{36}$	234.826416	$P_{90}$	273.957457	$P_{144}$	238.575237
$P_{37}$	292.822639	$P_{91}$	219.095369	$P_{145}$	280.256373
$P_{38}$	237.978690	$P_{92}$	214.723938	$P_{146}$	241.034880
$P_{39}$	436.636667	$P_{93}$	283.451750	$P_{147}$	289.328078
$P_{40}$	265.432210	$P_{94}$	245.506570	$P_{148}$	241.582038
$P_{41}$	217.930519	$P_{95}$	273.004206	$P_{149}$	432.032684
$P_{42}$	222.583499	$P_{96}$	236.794502	$P_{150}$	273.777239
$P_{43}$	290.494600	$P_{97}$	291.482917	$P_{151}$	216.757453
$P_{44}$	233.438274	$P_{98}$	235.155228	$P_{152}$	225.888284
$P_{45}$	295.299022	$P_{99}$	416.363970	$P_{153}$	271.727563
$P_{46}$	237.854959	$P_{100}$	255.410343	$P_{154}$	234.249233
$P_{47}$	278.510221	$P_{101}$	216.223183	$P_{155}$	276.486019
$P_{48}$	248.035703	$P_{102}$	209.982441	$P_{156}$	236.439473
$P_{49}$	424.865371	$P_{103}$	256.746663	$P_{157}$	281.837647
$P_{50}$	275.625373	$P_{104}$	238.692105	$P_{158}$	238.199988
$P_{51}$	211.497807	$P_{105}$	276.972440	$P_{159}$	438.866929
$P_{52}$	205.196578	$P_{106}$	241.383638	$P_{160}$	267.589374
$P_{53}$	284.858260	$P_{107}$	270.763647		
$P_{54}$	236.131977	$P_{108}$	239.556436	cost (\$/hr.)	<b>10005.9944539382</b>

**Table 8.** Comparison between different methods taken after 50 trials (160-generator systems).

Methods	Generation cost (\$/hr.)				Time/iteration (Sec)	No. of hits to minimum solution
	Max.	Min.	Average	Standard deviation		
TLBO	10006.28210000	10005.9944539382	10006.01170000	0.0690	48.216	47
ED-DE [31]	NA	10012.68	NA	-	NA	NA
CGA-MU [31]	NA	10143.73	NA	-	NA	NA
IGA-MU [31]	NA	10042.47	NA	-	NA	NA

**Table 9.** Effect of learner size on 160-generator systems.

Learner size	No. of hits to best solution	Simulation time (sec.)	Max. cost (\$/hr.)	Min. cost (\$/hr.)	Average cost (\$/hr.)
20	23	47.765	10006.8320	10006.5210	10006.6890
<b>50</b>	<b>47</b>	<b>48.216</b>	<b>10006.2821</b>	<b>10005.9944</b>	<b>10006.011</b>
100	20	53.233	10006.7609	10006.5274	10006.6675
150	12	58.610	10006.9919	10006.5751	10006.8919
200	10	64.702	10007.2527	10006.5962	10007.1214

mally, their performance is judged after running the programs for a certain number of trials. A great number of trials should be made to obtain a useful conclusion about the performance of the algorithm. An algorithm is said to be robust if it gives consistent results during these trial runs. Tables 2, 4, 6 and 8 show that out of 50 trials for four different test systems, TLBO reaches minimum costs 50, 50, 47 and 47 times, respectively. The efficiency of the TLBO algorithm to reach minimum solution is 100% and 94%, respectively. This performance is far superior to many other algorithms presented in different literature. Therefore, the above results establish the enhanced ability of TLBO to achieve superior quality solutions, in a computationally efficient and robust way.

## 5. Conclusion

In the present paper, a newly developed TLBO algorithm has been successfully implemented in the field of power systems to solve different convex and non-convex ELD problems. The simulation results show that the performance of TLBO is better compared to that of several previously developed optimization techniques. The TLBO has obtained superior quality solutions with high convergence speed in a very robust way. The results also show the advantage of TLBO, compared to many previously developed optimization techniques, in term of computational time, as the proposed algorithm is parameter free. Therefore, TLBO can be considered to be a strong tool for solving complex ELD problems. Moreover, the successful implementation and superior performance of TLBO to solve ELD problems has

created a new path in the field of power systems, which may encourage the researcher to apply this newly developed algorithm to solve different, greatly complex power system optimization problems, like optimal power flow, hydro thermal scheduling, loss minimization, optimal placement of distributed generators, and FACTS devices etc. Therefore, it may finally be concluded that the proposed TLBO algorithm is able to solve any complex constrained optimization problem with a faster convergence rate, irrespective of the nature of the objective function.

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