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Investigating the role of ultrasonic wave on two-phase relative permeability in a free gravity drainage process

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Ultrasonic wave;
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Abstract. In this work, the process of free gravity drainage under the influence of ultrasonic waves was investigated. A glass bead pack porous medium was used to perform free fall gravity drainage experiments. The tests were performed in the presence and absence of ultrasonic waves, and the data of recovery were recorded versus time under both conditions. The wetting phase relative permeability curves were obtained using the data of recovery versus time, based on the Hagoort backward methodology. Subsequently, using the wetting phase relative permeability curve, the relative permeability of non-wetting phases were calculated by performing history matching to the experimental production data. The results revealed that ultrasound considerably increases the recovery factor of the free gravity drainage process. It was also observed that the relative permeability of both wetting and non-wetting phases increases under exposure to ultrasonic waves. The results of this work can be helpful in better understanding/evaluating the behavior of the relative permeability curves of both wetting and non-wetting phases during a free gravity drainage process under exposure to ultrasonic waves.

1. Introduction

Application of ultrasonic wave technology in enhancing fluid flow in porous media has been investigated experimentally by many researchers. Duhon and Campbel [1] conducted a series of flood tests to characterize the behavior of the system with and without addition of sonic energy. The results of their work serve to show that the addition of sonic energy increases displacement efficiency. Chen [2], as well as Fairbanks and Chen [3], reported an increase in oil percolation rate through porous medium due to wave radiation. Cherskiy et al. [4] described a sharp increase in the permeability of core samples saturated with fresh water in the presence of an acoustic field. Neretin and Yudin [5] observed an increase in the rate of oil displacement by water through loose sand under ultrasound. Beresnev and Johnson [6] provided a comprehensive review of methods using elastic wave stimulation of oil production, including both ultrasonic and seismic methods, and mentioned that the application of elastic waves improves the permeability and production rate in most cases. Numerical and experimental results of Aarts et al. [7] showed that, by increasing ultrasonic power, the velocity of fluid inside the porous media increases. Hamida and Babadagli [8] observed that ultrasonic waves can enhance capillary imbibition oil recovery, depending on the fluid, sound intensity, sound frequency, and the nature of the matrix. Naderi and Babadagli [9] also revealed the positive effect of ultrasonic energy on oil recovery for different wettability rocks. However, to the best of our knowledge, no attempt has been made to experimentally investigate flow enhancement during the gravity drainage process in the presence of ultrasonic radiation.

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Gravity drainage plays an important role in oil production from natural fractured reservoirs. This process will be highlighted, especially in cases where a gas cap (initially or secondary) is present in direct contact with the oil zone [10]. In some of Iran's giant fractured reservoirs, where oil has been produced for several decades and the reservoir has been subject to both water-drive from the bottom region and a gravity-drainage mechanism from the top region, material balance calculations indicate higher oil recoveries in the gravity drainage areas than the water displacement regions [11]. Since the relative permeability of present phases is the main parameter that has to be evaluated to complete the description of the gravity drainage process [12,13], in order to investigate the process of gravity drainage under the effect of an ultrasonic wave, it is critical to evaluate the wetting/non-wetting phase relative permeability curves under exposure to ultrasound.

In this study, the impact of ultrasonic waves on the relative permeabilities of both wetting and non-wetting phases in a drainage process is investigated through a series of drainage experiments. Each experiment is done in the presence and absence of ultrasonic wave radiation. Then, using the production data in each experiment, the wetting phase relative permeability curve is obtained through the Hagoort Backward Method [14,15]. Afterwards, the gas relative permeability is calculated using history matching to the production data. Finally, the relative permeabilities of the wetting and the non-wetting phases are plotted and compared in the presence and absence of ultrasonic radiation.

2. Experiments

2.1. Experimental setup and fluids

The experimental setup includes a cylinder made of Plexiglas with a length of 30 cm, an outer diameter of 4 cm and an inner diameter of 3 cm, a graduated cylinder to measure the amount of extracted liquid and an ultrasonic wave generator with an average effective output power of 80 watts, which produces waves with 22 kHz frequency. A schematic view of the experimental setup is shown in Figure 1. The average size of the beads was 150-180 μm. The porosity of the medium was calculated by measuring the volume of fluid entering the cell at each experiment. The value of porosity was $37 \pm 0.5\%$. The absolute permeability of the bead pack was also calculated by injecting fluid into the cell with a specific pressure gradient through the cell and measuring the flow rate. The amount of permeability was obtained to be 34 ± 1 Darcy. Air was used as the non-wetting phase, and water, kerosene, crude oil A and crude oil B (in different experiments) were the wetting phases. The properties of the wetting fluids can be seen in Table 1.

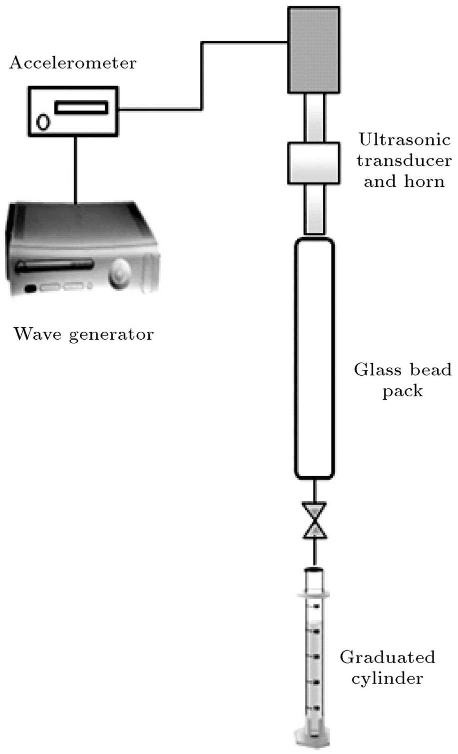

Figure 1. The experimental setup used in this study.

Table 1. Physical properties of wetting fluids under standard condition.

Fluid	Density (kg/m^3)	Viscosity (cp)	Asp. content
Dis. water	1000	1	-
Kerosene	739	0.9	-
Crude oil A	860	6.5	0.3%
Crude oil B	920	83	5%

Before beginning each experiment, the glass beads were packed in the Plexiglas cylinder and the apparatus was assembled. Unused dry glass beads were used in each experiment to avoid any error according to changes in wettability. The cell was firstly vacuumed to a desired pressure and then fully saturated with the liquid sample. Afterwards, the cell was placed in a vertical position, the faces of the cell were opened, and the liquid produced was measured versus time. The experiments were conducted under ambient laboratory temperature (25°C) under two different conditions; the presence and absence of ultrasonic waves, and the results were used for determination of relative permeability curves.

3. Relative permeability determination

Firstly, the Hagoort [15] backward methodology was used and wetting phase relative permeability curves were drawn using the data of recovery versus time.

Then, the results of wetting phase relative permeability and production data were used and relative permeability curves for the non-wetting phase were determined through history matching to the experimental production data.

3.1. Wetting phase relative permeability determination through Hagoort methodology

By considering the case of free gravity drainage at times beyond breakthrough time, one can consider gas velocity to be negligible [16]. Hagoort [15], by applying the Buckley-Leveret equation, assuming infinite gas mobility and negligible capillary pressure, proposed a backward methodology for the calculation of relative permeability based on Corey-type equations [15,17,18]. Derivation of the backward methodology is described in Appendix A. Due to Hagoort, after the time of breakthrough, the amount of normalized oil production can be calculated with Eq. (1):

$$N_p = 1 - \left(1 - \frac{1}{n}\right)\left(\frac{1}{nk_{ro}^0 t_D}\right)^{\frac{1}{n-1}}, \tag{1}$$

where n is the Corey equation exponent and k_{ro}^0 is the Corey constant. According to this equation, the plot of $1 - N_p$ versus t_D in a log-log paper results in a straight line, and the value of the Corey exponent (n) can be calculated from the slope of the straight line, as follows:

$$\frac{dln(1 - N_p)}{dlnt_D} = -(\frac{1}{n-1}). \tag{2}$$

Also, the amount of k_{ro}^0 can be obtained from Eq. (1), when $t_D = 1$. Derivations of Eqs. (1) and (2) are described in Appendix A. Now, by having Corey constants (n and k_{ro}^0), the values of relative permeability for the wetting phase after the time of breakthrough is determined. But, to obtain the relative permeabilities of the wetting phase before the time of breakthrough, an extrapolation of the values of relative permeability after breakthrough time has to be undertaken [14]. This process is illustrated by an example in the results and discussion section.

3.2. Non-wetting phase relative permeability determination

In this part, a free gravity drainage process is simulated. The relative permeability data of the wetting phase that has been calculated in the previous section is used. A Corey type relative permeability function is considered for the gas relative permeability and the constants of this function are determined using a history matching of the simulation results to the production data. The main assumptions here are as follows:

a) Capillary pressure at the interface is assumed constant in the whole porous media.
b) The Darcy flow equation and its conditions are valid.
c) The flow is assumed to be piston like and one-dimensional, while the lateral faces are sealed.

For a two phase gas and oil system the following equations are used:

$$Q_o = -\frac{k_o}{\mu_o}(A)(\frac{\Phi_{o2} - \Phi_{oz}}{H - z}), \tag{3}$$

$$Q_g = -\frac{k_g}{\mu_g}(A)(\frac{\Phi_{gz} - \Phi_{g1}}{z}), \tag{4}$$

where Q_o and Q_g are Darcy flow rates; k_o and k_g are effective permeabilities; μ_o and μ_g are viscosities; Φ_o and Φ_g are the hydraulic potentials of oil and gas, respectively; z is the distance of the oil and gas interface from the top of the column; A is the cross sectional area; and H is the height of the column. The schematic of the gravity drainage process is depicted in Figure 2. After integrating and using the concept of capillary pressure ($p_c = \Phi_{gz} - \Phi_{oz}$), the following equation could be generated:

$$\Phi_{g1} - \Phi_{o2} - P_c + \Delta\rho g(H - z) = \frac{Q}{\pi R^2}\left[\frac{\mu_g}{k_g}z + \frac{\mu_o}{k_o}(H - z)\right], \tag{5}$$

in which p_c is capillary pressure and subscripts 1 and 2 are introduced in Figure 2; $\Delta\rho$ is the difference between oil and gas (wetting and non-wetting) densities; constant R is the radius of the column; and g is the gravity acceleration. As the assumption is a piston-like displacement, both Q_o and Q_g are considered to be equal and are shown by Q in Eq. (5). By

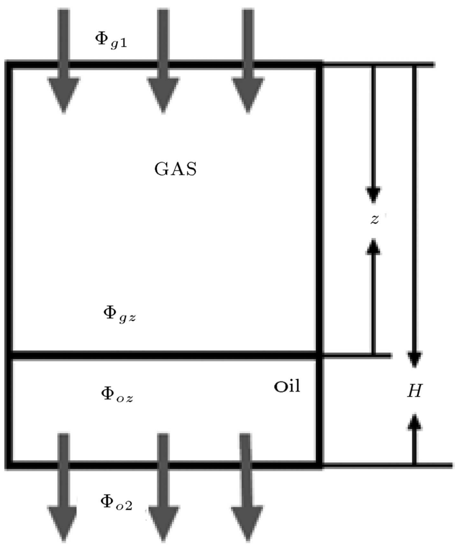

Figure 2. Schematic of gas oil drainage process.

considering negligible capillary pressure at the exit face ($\Phi_{g1} - \Phi_{o2} = 0$) [19], by replacing $\frac{Q}{\pi R^2}$ with u, by defining Mobility ratio, M, as $(\frac{k_g}{\mu_g})/(\frac{k_o}{\mu_o})$ and by involving Eq. (5) the following equation is obtained:

$$u_{\text{Darcy}} = \frac{k_g(-P_c + \Delta\rho g(H - z))}{\mu_g[z(1 - M) + MH]}. \tag{6}$$

The actual velocity is equal to the derivative of the interface location to time ($u_{\text{actual}} = \frac{dz}{dt}$). So, according to the definition of actual flow rate the following equation is obtained:

$$u_{\text{act}} = \frac{u_{\text{Darcy}}}{\oslash} = \frac{dz}{dt}, \tag{7}$$

in which t is time in seconds and $\oslash$ is the porosity of porous media [19]. Combining Eqs. (6) and (7), the location of the interface between two phases with time will be described in this form:

$$\frac{dz}{dt} = \frac{k_g(-P_c + \Delta\rho g(H - z))}{\oslash\mu_g\ [z(1 - M) + MH]}. \tag{8}$$

Defining the amount of recovery as $N_p = \frac{z}{H}$, the following equation can be derived:

$$\frac{dN_p}{dt} = \frac{k_g(-\frac{P_c}{H} + \Delta\rho g(1 - N_p))}{H \oslash \mu_g[N_p(1 - M) + M]}. \tag{9}$$

To predict the relative permeabilities of gas, a free gravity drainage process is simulated by solving Eq. (9) using the Runge-Kutta method of order 4, knowing that N_p is equal to zero at the initial time. The values of the wetting phase relative permeability from Hagoort backward methodology [15] are used, together with estimates for the relative permeability of the non-wetting phase. The gas relative permeability parameters are changed through an optimization technique until a reasonable match between the simulation calculated recoveries and experimental production data is achieved.

4. Results and discussion

The results of free fall gravity drainage experiments can be seen in Figure 3. The relative error for the oil recovery data plotted in this figure is in the range of 2-4%.

Radiation of the ultrasonic wave could highly increase the amount of ultimate recovery, as well as increasing the rate of drainage recovery. Figure 3(a) demonstrates that kerosene ultimate recovery has increased from 51% to 62%. In the case of crude sample A (Figure 3(b)) and water (Figure 3(c)), the values of final recovery were increased from 35% and 33% to 49% and 45%, respectively.

However, an unexpected result was observed for gravity drainage in the case with crude oil B during exposure to the ultrasonic wave, which is an asphaltenic crude oil. As can be observed in Figure 3(d), the values of oil recovery have been decreased after expo-

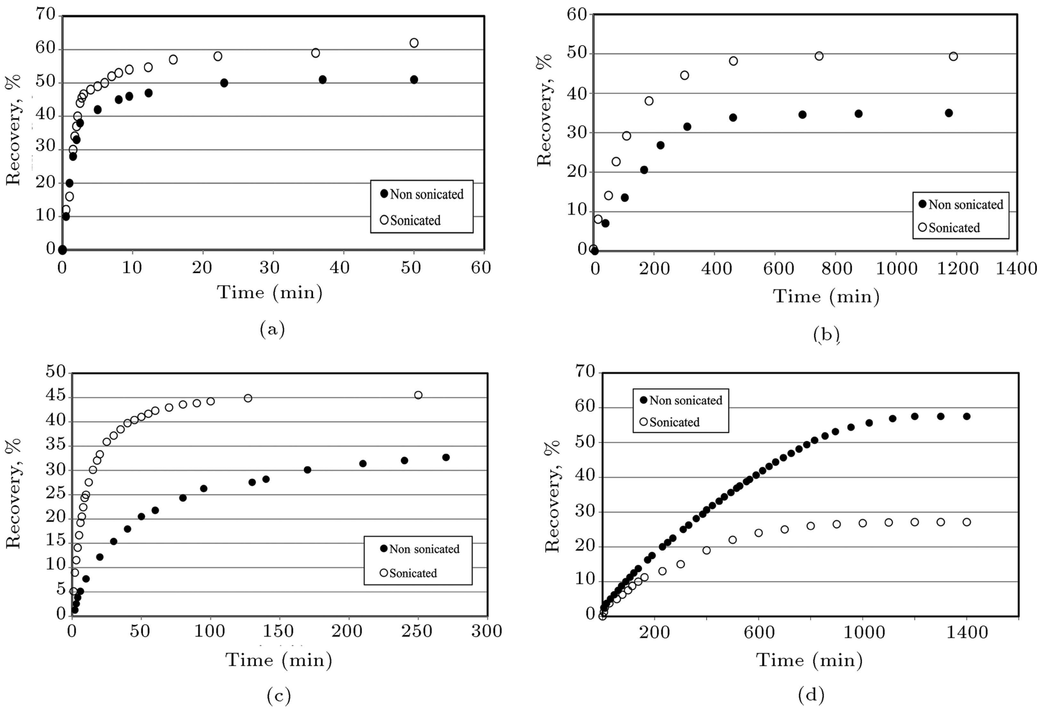

Figure 3. Results of sonication on total recovery and rate of fluids drainage: (a) Kerosene; (b) crude oil A; (c) water; and (d) crude oil B.

sure to ultrasonic waves. According to the previous studies on the effects of ultrasonic waves on asphaltenic oil properties [20], this observation can be described as a result of the breakdown of asphaltene micelle structures in the oil and their dissolution in the fluid, which increases the viscosity of the fluid after long exposure to radiation. The increment in the viscosity of fluid leads to slowing down the rate of drainage recovery.

4.1. Wetting and non-wetting phase relative permeability results

It was shown in the previous section that ultrasonic energy could considerably enhance the recovery of a drainage process. Here, it is aimed to calculate the changes in the relative permeability of both wetting and non-wetting phases in the drainage process under exposure to ultrasonic waves.

Here, the procedure for calculations of relative permeability for the experiment, in which kerosene was the wetting phase, is presented as a sample of the calculations. As depicted in Figure 4(a), breakthrough occurs at $t_D = 0.15$ for non-sonicated and at $t_D = 0.14$ for sonicated kerosene drainage, which shows a slightly sooner breakthrough under ultrasound.

To calculate the value of n, it is expected to use the experimental data after the breakthrough time by crossing a line through the data, as shown in Figure 4(b). So, n is obtained according to Eq. (2), and the value of N_p acquired from the straight line at $t_D = 1$ is also used for determination of k_{ro}^0 through Eq. (1). The result of the backward methodology for calculation of kerosene relative permeability is demonstrated in Figure 4(c). n and k_{ro}^0 were found to be 5.926 and 1.87 in the case of non-sonicated and 4.03 and 1.96 in the case of sonicated kerosene, respectively. Values of Corey's constants for each experiment could be seen in Table 2. Ultrasound decreased the value of n for all cases and increased k_{ro}^0 for water and crude oil, A. It should be noted that the Hagoort methodology can only be applied to situations in which there are no changes in the physical properties of the fluid. So, we cannot apply this method to the case of crude oil B because there are some variations in oil viscosity as the

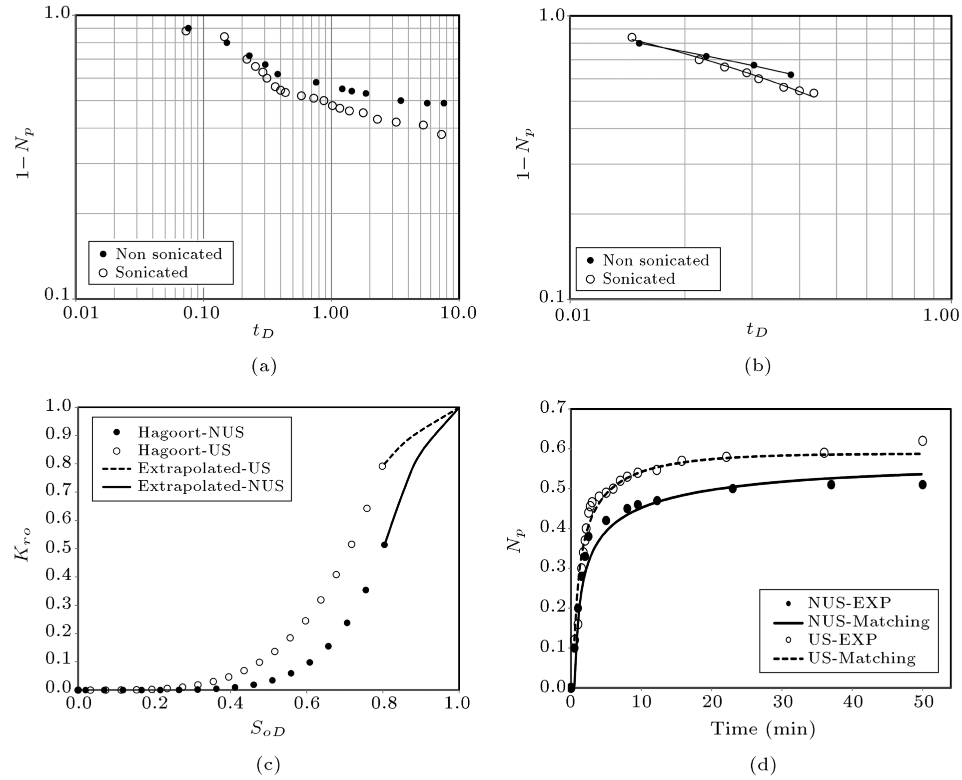

Figure 4. Step by step calculations of Corey type equation constants for oil relative permeability.

Table 2. Constants of Corey's equation before and after wave application.

Experiment	Fluid	$n(NUS)/n(US)$	$k_{ro}^0(NUS)/k_{ro}^0(US)$
1	Dis. water	5.926/4.03	1.87/1.96
2	Kerosene	9.47/5.98	1.58/1.38
3	Crude oil B	5.184/5.09	1.43/1.76

result of exposure to ultrasonic waves. As mentioned earlier, it should be kept in mind that the values obtained for the constants only pertain to a limited range of saturation (i.e. saturations after breakthrough time). To have a thorough curve of relative permeability versus saturation, we are supposed to make an extrapolation between these determined values of relative permeability and the value with k_{ro} equal to 1 at $S_{oD} = 1$. The interpolation part of the curve is depicted in Figure 4(c).

Relative permeabilies of the non-wetting phase (air) in different experiments were also determined through the history matching of the results of solving Eq. (9) to the experimental production data (Figure 4(d)). Relative permeability of the non-wetting phase was increased in all the experiments. Relative permeabilities of both phases are plotted together in Figure 5. For each case, there is an increase for oil and gas relative permeabilities as the result of ultrasonic wave radiation. The amount of gas end point relative permeability has been enhanced and also there is a reduction in the value of critical gas saturation.

One point that should be considered here is that for all the experiments, except in the case of asphaltenic crude oil, in which changes in the physical properties of the fluid are observed, recovery of the drainage process was increased, which is somehow in contrast with the fact that the gas breakthrough occurs sooner under exposure to ultrasound. On the other hand, most of the recovery is drained after the time of breakthrough. So, it should be mentioned that ultrasound stimulates both the wetting and non-wetting phases, which leads to a quicker breakthrough, but, ultimately, the value of final recovery will be augmented. It should be mentioned that it would be very complicated if one were to consider an extra term for the contribution of ultrasonic waves in the gravity drainage process. But, calculated values of the relative permeability of wetting and non-wetting phases discussed in this work, in the presence and absence of ultrasonic waves, respectively, could reveal the enhancement effect of ultrasound on two-phase gravity drainage flow behavior.

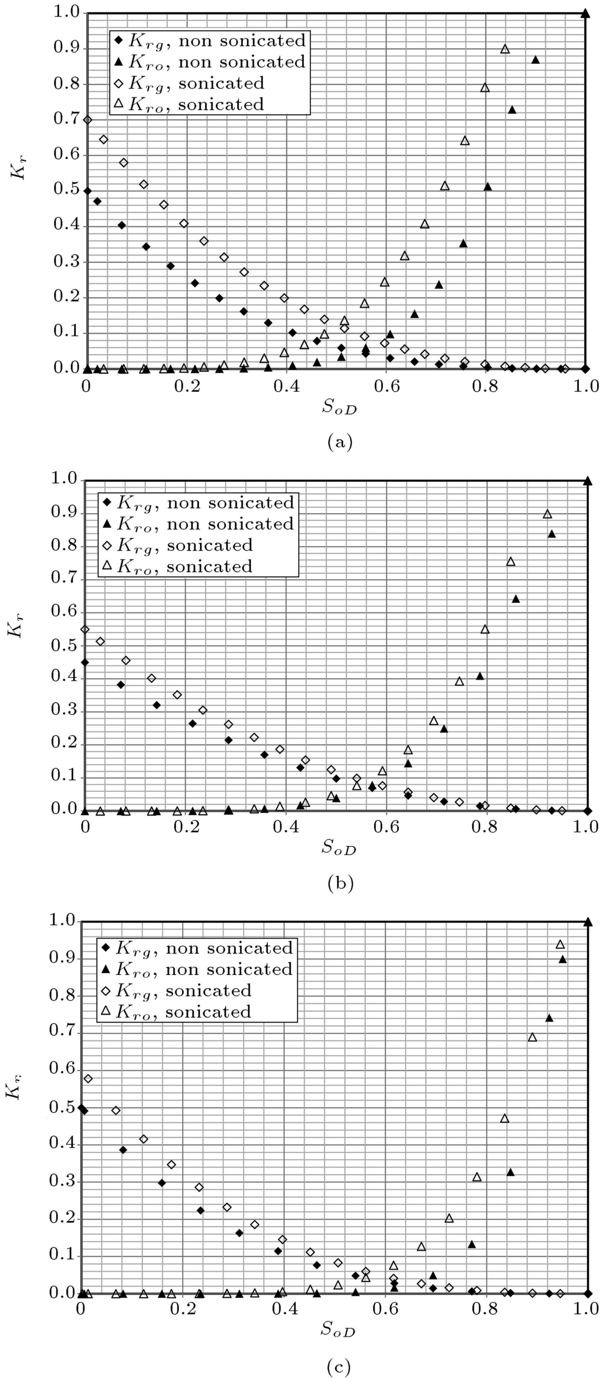

Figure 5. Obtained relative permeability curves for wetting and non wetting phases: (a) Kerosene-air; (b) crude oil A-air; and c) water-air.

5. Conclusions

In this paper, the process of free-fall gravity drainage under the influence of ultrasonic wave radiation was studied. The Hagoort (1980) backward methodology was applied and the relative permeabilities of wetting phases (kerosene, water and crude oil A) were obtained from unsteady state displacement data. Then, the obtained wetting-phase relative permeability and production data were used for history matching to determine the relative permeability of the non-wetting phase, air. Based on the obtained results in this work, the following conclusions can be drawn,

- This work illustrates successful application of the Hagoort backward methodology for evaluating wetting phase relative permeability from unsteady displacement data of the free-fall gravity drainage process under the influence of ultrasound wave radiation.

- The ultrasonic wave increased the recovery of the gravity drainage process for non-asphaltenic fluid samples, while an adverse result was observed in the case of asphaltenic crude oil. This could be justified as the result of increments in viscosity under long time ultrasonic exposure.
- It was shown that wave radiation increases the relative permeability of both wetting and non-wetting phases. It also increased the end point relative permeability of the non-wetting phase and decreased the value of critical gas saturation.
- The results of this work can be helpful to better understand the role of ultrasonic waves on the relative permeability curves of the two-phase gravity drainage process.

Nomenclature

A	Cross sectional area of porous media (L)
f_{oD}	Dimensionless fractional flow function
g	Gravity acceleration (L/T^2)
H	Height of the porous media (L)
J	Leverett function
k	Absolute permeability (L^2)
k_r	Relative permeability
k_{ro}^0	Corey equation constant
M	Mobility ratio
n	Corey equation exponent
N_{cg}	Capillary to gravity forces
N_{gv}	Gravity to viscous forces
N_p	Fractional produced oil
PV	Pore volume (L^3)
p_c	Capillary pressure (M/LT^2)
Q	Flow rate (L^3/T)
q_t	Production rate (L^3/T)
R	Radius of the sand pack (L)
S_{wi}	Initial water saturation
S_{org}	Residual oil saturation after gas injection
S_{oD}	Outlet normalized oil saturation
S_{oDe}	Outlet normalized oil saturation at end point
t	Time (T)
t_D	Dimensionless time
u	Fluid velocity (L/T)
u_{Darcy}	Darcy fluid velocity (L/T)
z_D	Dimensionless displacement

Greeks

μ	Viscosity (M/LT)
ρ	Density (M/L^3)
Φ	Hydraulic potential (M/LT^2)
$\oslash$	Porosity
σ	Interfacial tension (M/T^2)

Subscripts

D	Dimensionless
g	Gas
o	Oil

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Appendix A

In the presence of viscous, capillary and gravity forces, the following equation can present the amount of oil fractional flow:

$$\frac{\partial S_{oD}}{\partial t_D}+\frac{\partial f_{oD}}{\partial z_D}=0, \tag{A.1}$$

where dimensionless time, outlet normalized oil saturation, dimensionless displacement and dimensionless fractional flow function are presented as follows:

$$t_D=\frac{\Delta\rho_{og}gkt}{\mu_o\varphi(1-S_{org}-S_{wi})}, \tag{A.2}$$

$$S_{oD}=\frac{S_o-S_{org}}{1-S_{org}-S_{iw}}, \tag{A.3}$$

$$z_D=\frac{z}{L}, \tag{A.4}$$

$$f_{oD}=\frac{\frac{1}{N_{gv}}\frac{\mu_g k_{ro}}{k_{rg}\mu_o}+k_{ro}+N_{cg}\frac{\partial J}{\partial S_{oD}}\frac{\partial S_{oD}}{\partial z_D}}{1+\frac{\mu_g k_{ro}}{k_{rg}\mu_o}}, \tag{A.5}$$

where N_{gv}, N_{cg} and $J(S_{oD})$ could be obtained by the following equations:

$$N_{gv}=\frac{\Delta\rho_{og}gk}{\mu_o u}, \tag{A.6}$$

$$N_{cg}=\frac{\sigma\sqrt{\frac{\varphi}{k}}}{\Delta\rho_{og}gl}, \tag{A.7}$$

$$J(S_{oD})=\frac{\sigma\sqrt{\frac{\varphi}{k}}}{p_c}. \tag{A.8}$$

By considering Hagoort assumptions for capillary pressure and gas mobility, the fractional flow will be only dependent on S_{oD}:

$$f_{oD}=k_{ro}(S_{oD}). \tag{A.9}$$

So, the fractional flow equation could be written as:

$$\frac{\partial S_{oD}}{\partial t_D}+\frac{\partial k_{ro}}{\partial S_{oD}}\frac{\partial S_{oD}}{\partial z_D}=0. \tag{A.10}$$

This equation has been solved by the characterization method of the Buckley-Leverett solution:

$$z_D(S_{oD})=\frac{\partial k_{ro}}{\partial S_{oD}}t_D(S_{oD}). \tag{A.11}$$

Before the gas break-through, the amount of entered gas is equal to the amount of oil produced:

$$N_p=\frac{\int_0^t q_t dt}{PV}=\frac{t_D}{N_{gv}}. \tag{A.12}$$

After gas break-through:

$$N_p=1-\int_0^{S_{oD}}(1-z_{S_{oD}})dS_{oD}=1-S_{oDe}+k_{ro}(S_{oDe})t_D. \tag{A.13}$$

$z_{S_{oD}}$ is the vertical distance at the specific saturation, and S_{oDe} is also the outflow normalized oil saturation, which can be expressed as:

$$S_{oDe}=\left(\frac{1}{nk_{ro}^0 t_D}\right)^{\frac{1}{n-1}}, \tag{A.14}$$

where k_{ro}^0 is considered to be a curve fitting parameter. By assuming a Corey-type function for relative permeability, $k_{ro}=k_{ro}^0 S_{oD}^n, N_p$ can be calculated as:

$$N_p = 1 - S_{oDe} + k_{ro}^0 S_{oD}^0 t_D. \tag{A.15}$$

At the outflow end, $z_D = 1$

$$\frac{\partial k_{ro}}{\partial S_{oD}} t_D(S_{oD}) = 1. \tag{A.16}$$

So:

$$N_p = 1 - \left(1 - \frac{1}{n}\right)\left(\frac{1}{nk_{ro}^0 t_D}\right)^{\frac{1}{n-1}}. \tag{A.17}$$

The amount of k_{ro}^0 can be obtained from this equation, when $t_D = 1$. The value of n is calculated from the following equation:

$$\frac{dln(1-N_p)}{dlnt_D} = -\left(\frac{1}{n-1}\right). \tag{A.18}$$

Biographies

Behnam Keshavarzi holds an MS degree in Petroleum Engineering from Sharif University of Technology, Tehran, Iran. He has authored/co-authored a number of publications in the area of ultrasonic applications in the oil industry, asphaltene damage removal and fluid percolation in fractured porous medium under elastic waves. His research interests include modeling asphaltene damage stimulation by ultrasonic wave, well testing technology and minimum miscibility pressure calculation methods.

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