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Research Note

Numerical solution of homogeneous double pipe heat exchanger: Dynamic modeling

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Abstract. Dynamic modeling of a double-pipe heat exchanger is the subject of the current study. The basis of this study is the same velocity of vapor and liquid phases or, in other words, homogeneous phase, in the annulus part of the exchanger. The model can predict the temperature and vapor quality along the axial pipe from the pipe inlet up to a distance where steady state conditions are achieved. The simulation is conducted for two modes of co- and counter-flow in a one dimensional transient system. The physical properties of water are estimated from empirical correlation and a saturated vapor table with cubic spline interpolation. The exchanger model, which is a set of Ordinary Differential Equations (ODEs), ODEs and algebraic equations, has been solved numerically. Modeling results have been investigated for different operating times and two modes of co- and counter-current.

1. Introduction

Heat exchangers are widely used in industrial application processes such as power plants, gas turbines, air conditioning, refrigeration, (domestic, urban, or central) heating, and cryogenic systems, among many others. Their universal application has led to research into a better comprehension of their dynamic behavior, modeling, simulation, identification, and control [1]. Many reports on numerical and experimental simulations for laminar or turbulent flow and heat transfer, concerning the employment of solid extended surfaces, such as fins and baffles, can be found in the literature [2,3]. Most of these works discuss the optimal spacing, shapes and orientations of these structures, which enhance the heat transfer performance for a given pumping power or flow rate [4]. The objective of any such equipment is to maximize the heat transferred between the two fluids. Therefore, a design which increases the heat transferred, but simultaneously keeps the pressure drop of the fluid flowing in the pipes to permissible limits, is very necessary. A common problem in industry is to extract maximum heat from a utility stream coming out of a particular process, and to heat a process stream [5]. The design of the heat exchanger might be developed by means of analytical methods. These methods give a quick and global approach to their behaviour. However, a large number of hypotheses and simplifications have to be assumed. Examples are the F$-$factor or $\varepsilon-$NTU methods [6]. More general and accurate approaches require the use of numerical methodologies, which subdivide the heat exchanger into many elemental volumes and solve the governing equations for each volume. In the two-phase flow region, the governing equations (mass, momentum

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and energy) can be formulated in different forms, depending on the model used. Homogeneous models [7], drift flux models [8] or two-fluid models [9,10] can be employed to solve the two-phase flow present in the condensation and evaporation process. To get a deep understanding of the mathematical model and the strategies for solving the governing equations, the double−pipe heat exchanger may be a good option for application, due to its relatively simple geometry for the secondary flow. For example, numerical and analytical results in steady and transient states have been compared in different studies on double−pipe heat exchangers [1,11,12]. A combination of analytical expressions with numerical methods is used in the double−pipe helical heat exchanger resolution [13], where a CFD modeling, together with a ε−NTU method, is used to solve the heat exchanger.

This paper shows a more general approach for simulation of the two-phase flow inside the annulus and liquid inside the center tube. The fluid region equations in the heat exchanger were discretized and a simulation was performed for one dimension. Moreover, variation of fluid properties in a radial direction was neglected. First, details of the mathematical formulation and the numerical techniques are shown. Then, numerical aspects are presented and, finally, the results obtained from comparisons of co− and counter−current operating conditions will be demonstrated.

2. Model description

Taking into account the following basic assumptions, the governing equations have been derived:

- Homogeneous phase assumes a slip ratio ($G = U_v/U_1$) equal to unity (both phases travel at the same velocity);
- One-dimensional flow is performed;
- The exchanger is horizontal, so the gravity effects are omitted;
- The wall of the heat exchanger is smooth and does not apply friction effects on flowing fluids;
- The velocity profile is only a function of fluid direction, and radial dependency is neglected;
- The system is adiabatic and insulated (i.e. heat transfer through outer surface is neglected);
- The fluid flowing in the inner tube is incompressible and single phase;
- The outlet vapor as well as the inlet vapor is in a saturated state;
- Pressure drops across vapor and liquid phases in the annulus tube are the same ($dP_v = dP_l = dP$);
- In the annulus tube, the temperatures of vapor and liquid at each node are the same and are equal to saturated temperature at node pressure ($T_v = T_l = T_{\text{sat}}$);
- Axial heat conduction inside the fluid is neglected;
- Working fluid in the tube and the annulus sections is water;
- The inlet vapor into the annulus tube is saturated with a quality of one and the inlet liquid in the inner tube is in a saturated condition.

As noted in the above assumptions, the cooling liquid temperature increases and the steam temperature decreases. Consequently, the saturated vapor in the annulus part may provide two phases with the elevation of pressure drop. The vapor remains in a saturated condition through the exchanger, while its temperature, pressure and quality might vary along the exchanger and/or with time.

2.1. Governing equations in annulus tube

Usually, first law of thermodynamics for open systems, and considering the velocity term as $u = \dot{m}_t/(\rho_t A_t)$ in kinetic terms, gives:

$$\frac{\partial}{\partial t}(\rho_t C_p A_t T^{\text{sat}}) + \dot{m}_t \frac{d}{dz}(h_t + \frac{1}{2}u^2) + 2\pi R_o U_{\text{overall}}(T^{\text{sat}} - T) = 0. \tag{1}$$

Based upon the Maxwell's relations the following equations are obtained:

$$\begin{cases} dh_\nu = T_\nu dS_\nu + \nu_\nu dP_\nu \\ dh_l = T_l dS_l + \nu_l dP_l \end{cases}$$

$$\Rightarrow \frac{dh_t}{dz} = h_{l\nu}\frac{dx}{dz} + C_{pt}\frac{dT^{\text{sat}}}{dz} + \nu_t \frac{dP}{dz}, \tag{2}$$

$$h_{l\nu} = h_\nu - h_l. \tag{3}$$

The following equation, known as the Clapeyron equation, demonstrates the relation between saturated temperature and pressure:

$$\frac{dP}{dT^{\text{sat}}} = \frac{h_{l\nu}}{T^{\text{sat}}\nu_{l\nu}}, \tag{4}$$

$$\nu_{l\nu} = \nu_\nu - \nu_l. \tag{5}$$

Finally, the governing equation for the annulus pipe (Eq. (6)) would be obtained by combining Eqs. (2) to (5) with the initial and boundary conditions (Eq. (7)):

$$\frac{\partial}{\partial t}(\rho_t C_p A_t T^{\text{sat}}) + \dot{m}_t(h_{l\nu}\frac{dx}{dz} + C_{pt}\frac{dT^{\text{sat}}}{dz} + \nu_t \frac{h_{l\nu}}{T^{\text{sat}}\nu_{l\nu}}\frac{dT^{\text{sat}}}{dz} + \frac{1}{2}\frac{du^2}{dz}) + 2\pi R_o U_{\text{overall}}(T^{\text{sat}} - T) = 0, \tag{6}$$

$$\begin{cases} I.C. \quad T^{\text{sat}} = T^{\text{sat}}_{\text{in}} \quad \text{at} \quad (z, t = 0) \\ B.C. \quad T^{\text{sat}} = T^{\text{sat}}_{\text{in}} \quad \text{at} \quad (z = 0, t) \end{cases} \tag{7}$$

$$A_t = \pi(R_t^2 - R_o^2). \tag{8}$$

2.2. Governing equations in the center tube

2.2.1. Governing equations in center tube for co-current stream

The governing equations for the center pipe are valid, the same as for the annulus tube, however, the vapor quality is zero. Therefore, the equation might be expressed as follows:

$$\frac{\partial}{\partial t}(\rho_l C_{pl} A_i T) + \dot{m}_l \frac{dh_l}{dz} - 2\pi R_o U_{\text{overall}}(T^{\text{sat}} - T) = 0, \tag{9}$$

$$A_i = \pi R_i^2, \tag{10}$$

in which, the initial and boundary conditions are:

$$\begin{cases} I.C. \quad T = T^{c}_{\text{in}} \quad \text{at} \quad (z, t = 0) \\ B.C. \quad T = T^{c}_{\text{in}} \quad \text{at} \quad (z = 0, t) \end{cases} \tag{11}$$

2.2.2. Governing equations in the center tube for counter-current stream

According to the system of equations presented, the counter current condition formulation is given by:

$$\frac{\partial}{\partial t}(\rho_l C_{pl} A_i T) - \dot{m}_l \frac{dh_l}{dz} - 2\pi R_o U_{\text{overall}}(T^{\text{sat}} - T) = 0, \tag{12}$$

whereas, boundary and initial conditions are:

$$\begin{cases} I.C. \quad T = T^{c}_{\text{in}} \quad \text{at} \quad (z, t = 0) \\ B.C. \quad T = T^{c}_{\text{in}} \quad \text{at} \quad (z = L, t) \end{cases} \tag{13}$$

In the aforementioned equations, the initial and boundary conditions are the same. This means that, initially, the annulus tube is filled with saturated vapor and in the inner tube, cooling liquid flows without heat transfer and, suddenly, the streams switch to exchange the heat. Utilization of the above initial conditions is guaranteed to avoid division by zero at the beginning of the solution. Besides, this technique leads to the exact modeling of the equations.

3. Physical properties

The physical properties of the heat exchanger, such as material selection, or fluid properties in the inner and outer tubes, and overall heat transfer coefficient at each point during the time, were calculated using steam tables by cubic spline interpolation or experimental relations that are listed in Table 1.

4. Numerical framework

According to the heat exchanger design, only axial heat transfer is considered and the system is meshed just in this direction. In order to simplify the calculations, the meshes are assumed uniform and the mesh centered method of calculation was investigated. The calculation path reported here has been performed with structured mesh. Within this scheme, the linearization of equations was done. The algorithm of solution is illustrated in Figure 1. The resulting ordinary differential equations were solved simultaneously using the finite difference technique under initial and boundary conditions. The discrete formulation of the equations was fully implicit. At each step, the resulting set of linear algebraic equations was solved by the iterative Gauss-Seidel method with the relaxation factor. Solution of the previous step was used as the initial guess for the iterative procedure. A computer program using MATLAB software (version 2010a) was developed to perform the above algorithm for numerical solutions.

The Clapeyron equation term in the annulus side equations causes nonlinearity. In order to obviate this restriction, the previous step time is used to calculate

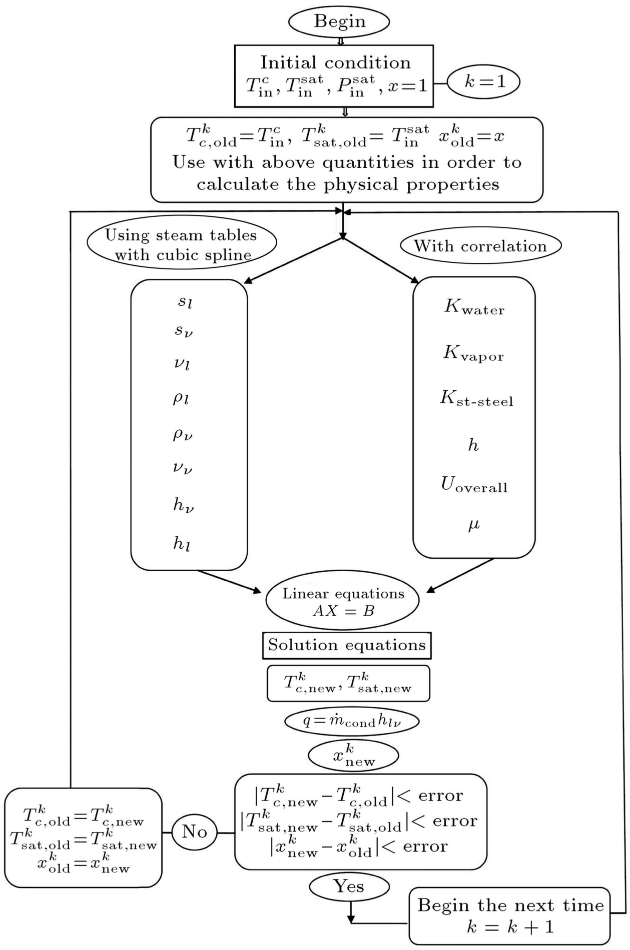

Figure 1. Algorithm of numerical solution.

Table 1. Estimation of physical properties of fluids.

Properties	Reference
s_l, s_ν, ν_l, ρ_l, ρ_ν, ν_ν, h_ν, h_l	[10]
$\mu[\text{Pa.s}] = A \times 10^{\left(\frac{B}{T-C}\right)}$ $A = 2.414 \times 10^{-5}$ [Pa.s] $B = 247.8$ [K] $C = 140$ [K]	[12]
$K_{\text{water}}\left[\frac{W}{m.K}\right] = 1 \times 10^{-5}T^2 - 0.0042T + 0.7222$ $K_{\text{vapor}}\left[\frac{W}{m.K}\right] = 8.3154 \times 10^{-5}T - 7.4556 \times 10^{-3}$	[11]
$K_{\text{st.steel}}\left[\frac{W}{m.K}\right] = 0.0667T - 10.2 \quad \text{for} \quad T \succ 273K$	[11]
$h = \frac{K_{\text{fluid}}\text{Nu}}{D_H}$ $\text{Nu} = \begin{cases} 0.023\text{Re}^{0.8}\text{Pr}^n, & 0.6 \prec \text{Pr} \prec 100 \\ 0.0214(\text{Re}^{0.8} - 100)\text{Pr}^{0.4}, & 0.5 \prec \text{Pr} \prec 1.5 \quad \text{and} \quad 10^4 \prec \text{Re} \prec 5 \times 10^6 \\ 0.0120(\text{Re}^{0.87} - 280)\text{Pr}^{0.4}, & 1.5 \prec \text{Pr} \prec 500 \quad \text{and} \quad 3 \times 10^3 \prec \text{Re} \prec 10^6 \end{cases}$ $\text{Pr} = \frac{C_p\mu}{K}$ $\text{Re} = \frac{\rho u D_H}{\mu}$ $\text{DH} = \begin{cases} 2R_i, & \text{for inner tube} \\ 2(R_t - R_o), & \text{for annulus} \end{cases}$	[11]
$U_{\text{overall}} = \left(\frac{A_o}{A_i h_l} + \frac{A_o \ln\left(\frac{R_o}{R_i}\right)}{2\pi K_{\text{wall}}} + \frac{1}{h_t}\right)^{-1}$	[11]

the temperature. Thereby, a set of linear equations with constant coefficients is obtained that might be solved by the iterative Gauss-Seidel method. In order to minimize errors caused by the above simplifications, at each step, the equations are solved using the physical properties of the previous step (lagging). Then, obtained solutions are used for calculation of new physical properties and this repetitious loop, at each step, would be iterated over and over again up to when the property approaches a fixed amount. Based on the energy balance at each node, the vapor quality at each step, due to the formation of liquid from vapor, is computed as follows:

$$q = \dot{m}_t C_{pt}(T_{\text{sat}}^k - T_{\text{sat}}^{k+1}) = \dot{m}_{\text{cond.}} h_{lv}, \tag{14}$$

$$\dot{m}_{\text{cond.}} = \frac{\dot{m}_t C_{pt}(T_{\text{sat}}^k - T_{\text{sat}}^{k+1})}{h_{lv}}, \tag{15}$$

$$x = \frac{\dot{m}_t - \dot{m}_{\text{cond.}}}{\dot{m}_t}. \tag{16}$$

Reduction of temperature during time steps causes the condensation of the vapor. On the other hand, considering the same velocity assumption for vapor and liquid phases at each node, the total flow rate is considered constant. However, the velocity at each node varies from others, due to different densities. Time steps in this method should be selected small enough to obtain reasonable solutions.

5. Results and discussion

In this paper, two scenarios of exchanger characteristics were used as the boundary conditions, as presented in Table 2. The scenarios are the same, except the mass flow rate of vapor. Variations of quality and temperature along the inner and annulus axial pipe, with two scenarios for co- and counter-current streams, are depicted in Figures 2-5.

The quality of vapor and temperature along the heat exchanger with a counter flow arrangement, under conditions of scenario I, is shown in Figure 2(a) and (b), respectively. As assumed, the exchanger is filled with pure and saturated vapor and a cooling liquid, which suddenly comes to exchange heat transfer. The saturated vapor is brought to a heat saturated liquid with a great initial heat transfer rate, leading to an extreme reduction in vapor temperature and quality. Thereafter, steady state conditions are achieved continuously. As results show, the steady state conditions

Table 2. The scenarios of double pipe heat exchanger.

Properties	Scenario I	Scenario II
Outer tube inner diameter (cm)	13	13
Inner tube inner diameter (cm)	5.4	5.4
Inner tube thickness (mm)	1	1
Heat exchanger length (cm)	300	300
Cold water mass flow rate (kg/s)	1	1
Vapor mass flow rate (kg/s)	0.1	0.5
Inlet water temperature (K)	300.15	300.15
Inlet vapor temperature (K)	373.15	373.15

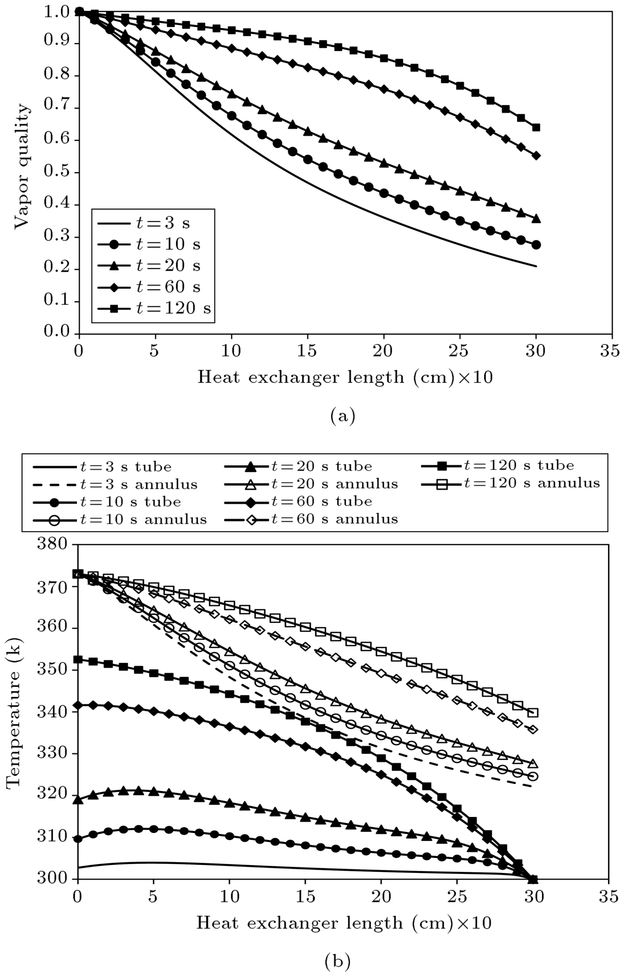

Figure 2. Vapor quality and temperature for counter-current versus length of double pipe heat exchanger under scenario I with time step of 0.01 seconds.

in the heat exchanger would be obtained after 120 seconds.

The temperature and vapor quality for the counter flow arrangement under scenario II is shown in Figure 3(a) and (b). The difference between scenarios I and II is just the mass flow rate of saturated vapor. As Figure 3(a) clearly indicates, the reduction in vapor

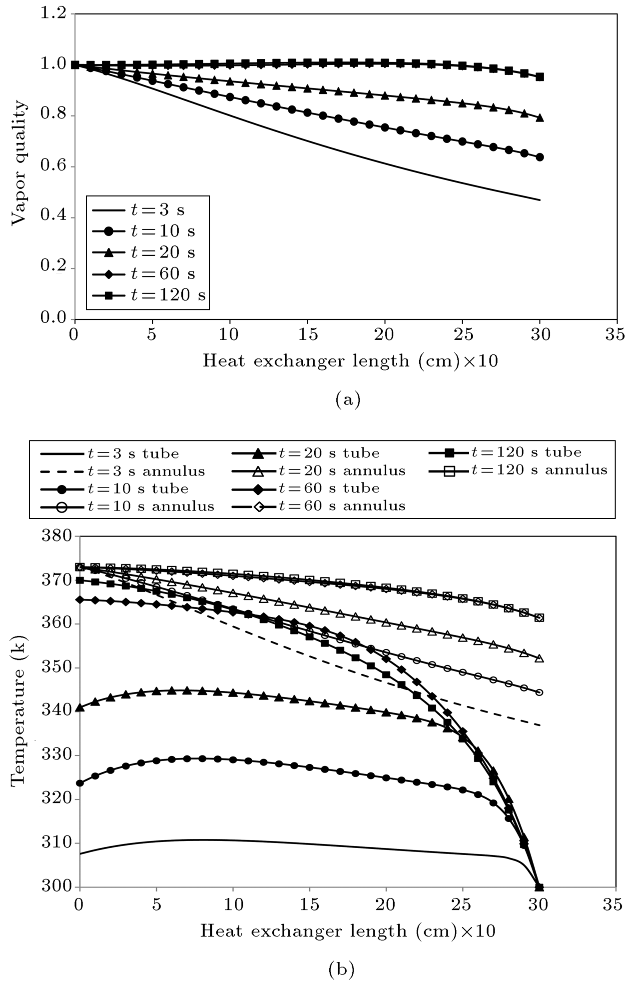

Figure 3. Vapor quality and temperature for counter-current versus length of double pipe heat exchanger under scenario II with time step of 0.01 seconds.

quality in scenario II is lower than in scenario I. From Figure 3(b), in scenario II, the fluid temperature of the inner tube increases further and, also, the vapor temperature reduction is less; in other words, the efficiency is lower.

The quality of vapor and temperature along the exchanger, with a co-current flow arrangement under conditions of scenario I, are shown in Figure 4(a) and (b). The saturated vapor is brought to a heat saturated liquid with a great initial heat transfer rate, leading to an extreme reduction in vapor temperature and quality. Thereafter, steady state conditions are achieved continuously. As the results show, the steady state conditions in the heat exchanger will be obtained after 120 seconds. In co-current flow close to the outlet end of the exchanger, the temperature difference between inner and annulus tubes tends to zero. Therefore, the exchanger length could be decreased for economical considerations.

Figure 5(a) and (b) depict the vapor quality and temperature along the double pipe heat exchanger for

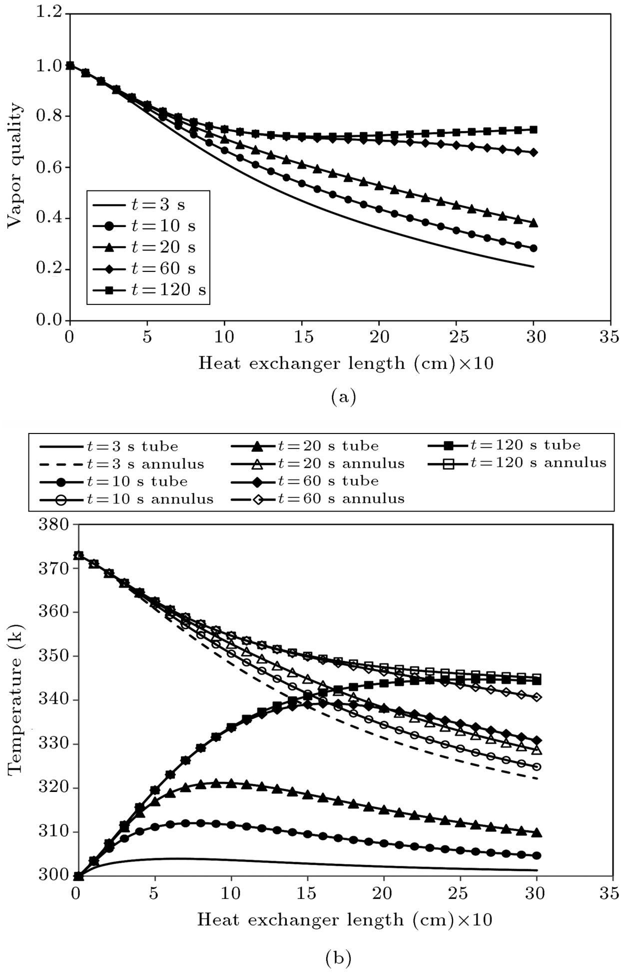

Figure 4. Vapor quality and temperature for co-current versus length of double pipe heat exchanger under scenario I with time step of 0.01 second.

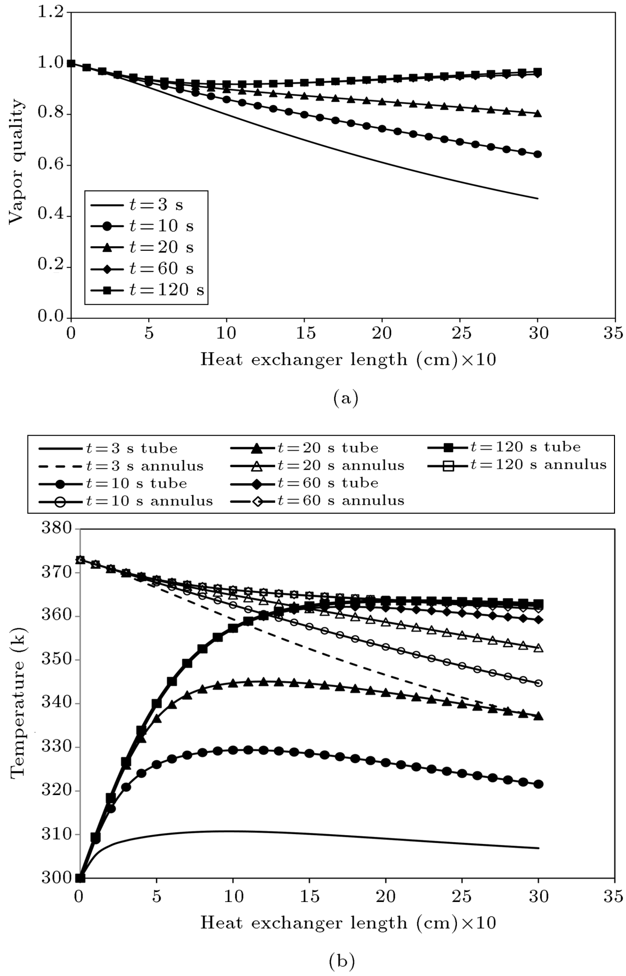

Figure 5. Vapor quality and temperature for co-current versus length of double pipe heat exchanger under scenario II with time step of 0.01 second.

co-current flow under scenario II. Trends similar to those of Figure 3(a) and (b) are observed. It is seen that the temperature increased sharply along the pipe close to the inlet end and, finally, reached a nearly constant value due to a reduction in heat transfer rate. Furthermore, it is observed that the vapor quality decreases through the exchanger.

Since inlet vapor is saturated, pressure distribution at each node can be obtained simply by using steam tables. The reduction of vapor quality and elevation of inlet liquid temperature in the co-current configuration is less than in the counter current. This result shows that heat transfer efficiency in the counter current arrangement is higher than in the co-current arrangement.

6. Conclusions

The present study is a numerical analysis of heat transfer in a double pipe heat exchanger with two configurations. The obtained results are exploited by highlighting the derived equations at the inner and annulus pipes. Physical parameters were predicted from correlations on the structure of the fields. The temperature and vapor quality along the axial pipe from the inlet, ending at a distance where steady state conditions are achieved, is predicted. The reduction of vapor quality in the co-current condition is less than in the counter current, which shows that the heat transfer efficiency of the counter current is more than that of the co-current. When the mass flow rate of the vapor is increased, with the constant mass flow rate of coolant, heat transfer efficiency decreases, as expected.

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