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Mathematical modeling and decomposition of hydrodynamics-acoustic fields using perturbation methods

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Abstract. Conservation of mass, momentum, energy and state equations are recognized as basic mathematical models in analysis of the acoustic behavior of cavitation, as well as supercavitation. Also, it is known that the order of acoustic effects is not as high as that of hydrodynamics. Therefore, in this paper, initially, for comparing different terms of equations, using scale analysis, conservation equations are converted into dimensionless ones. Then, by comparing all conditions, coupled with weighting terms available in those equations, groups of parameters most appropriate with the hydrodynamics and hydroacoustics of the cavitating flow, are selected. By regarding acoustics as lower order phenomena, compared to the hydrodynamics of flow, and simultaneously using the perturbation method, two equations containing leading and first orders and different terms can be attained. Obtained results indicate that leading order equations represent the hydrodynamics of the cavitating flow, and first order equations indicate the acoustics of cavitation or supercavitation. Acoustic equations of the present study contain terms related to fluid viscosity, density and pressure changes, and background flow velocity. As acoustic equations are coupled with leading order equations, in order to find the noise of cavitation, equations of fluid flow for compressible flow should be resolved.

1. Introduction

Evaporation of liquid due to pressure decrease down to less than the pressure of the saturated vapor of that liquid, is called cavitation. Cavitation occurs when the local pressure of a liquid fluid suddenly reaches below a critical value. This critical value is related to the pressure of the vapor of the liquid fluid. By placing fluid in this area, small bubbles made of vapor and other gases begin to form. These bubbles move with the current of fluid and when they reach areas with higher pressure, will dissolve and dissipate. In general, there are two methods to realize cavitation phenomenon. First is visual observation, and second is acoustic observation [1]. As a result, noise generated by cavitation, although recognized as an undesired effect, can be viewed as a means for recognition of cavitation occurrence. Noise is a mechanical perturbation that propagates in an elastic region. In fluids, noise is generated when there is a relative motion between two fluids and/or between a fluid and a surface. Noise is always regarded as a sound or an undesired sound that has an effect on the normal performance of a system. Noise can appear in a fluid flow due to turbulence, because of chaos in the fluid or on the basis of cavitation occurrence. Turbulence and vertex usually lead to low frequency pressure fluctuations, whereas, noise caused

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by cavitation typically has an acoustic frequency range from 1 to 100 kHz [2]. Brennen's investigation [3] indicates that bursting bubbles at high frequencies occurs more often than similar occurrences without cavitation. The acoustic effect of cavitation is more in the range of ultrasonic wide-band waves (in excess of 20 kHz) [3]. In general, sound is a perturbation term in steady condition [4]. Pierce [5] and Goldstein [6] expressed means of finding general forms of wave equations in detail. Morch [7] and Chahine [8] concentrated their work on the dynamics of some fixed cavitation known as fixed unsteady cavitation, otherwise, recognized cloud cavitation. Experimental studies of Reisman et al. [9] showed that noise caused by bursting cloud cavitation exceeds by far that of collective noise generated by summation of all single bubbles existing in a cloud cavitation. Wang [10] investigated generated shock waves in cloud cavitation in his doctoral dissertation. The growth and dissolving of a spherical cloud made from cavitation bubbles were modeled in a nonlinear form. Because of that, he was able to find pressure pulses, as well as produced momentum, causing noise and erosion. Wang combined a continuum mixture model and Rayleigh-Plesset equation and solved it using the Lagrangian integration method. In following years, Brennen et al. [11,12] developed their studies in recognizing acoustic noise caused by cloud cavitation in numerical form. Reisman studied the acoustics of cloud cavitation, numerically and experimentally. By processing received signals from his experiments, he showed that the frequency range of signals belonging to noise of this type of cavitation wave was almost between 10 Hz to 100 kHz [13]. Levy et al. [14] developed his research on noise generated by cloud cavitation in numerical as well as experimental form in a water tunnel with high speed and on a NACA0015 hydrofoil. Seo et al. [15] estimated cavitation flow noise around a two-dimensional cylinder with circular cross-section and by using Direct Numerical Simulation (DNS). In recent years; extensive studies on supercavitation have taken place in the hydrodynamics laboratory of Iran University of Science and Technology. Nouri et al. [16] and Moghimi [17] studied the steady condition of a cavity boundary while supercavitation. Howe and Foley [18] and Foley et al. [19,20] have conducted vast investigations into finding propagated sounds from ventilated supercavitation. Formation of ventilated supercavitation around underwater vehicles causes vehicles to reach high speed and generates low and high frequency sound noises [21]. High frequency noises can also interfere with the guiding and controlling system of the vehicle, whereas, low frequency noises have a tendency to propagate to the field far from the supercavity [22]. As acoustic propeller noise is the most important noise source of underwater vehicles, some numerical [23,24] and experimental [25,26] investigations have been studied recently. Salvator et al. [27] used the Ffowcs Williams and Hawkings (FWH) model in investigation of underwater propeller noise. This type of noise estimation methodology presents many assumptions, such as those considered in linear acoustics, low Mach number and compressed sound sources [28]. Figure 1 depicts formation of supercavitation in a water tunnel.

In light of host applications concerning noise from cavitation and supercavitation, mainly from environmental and marine aspects, the degree of its importance becomes obvious. In this research, in order to estimate noise caused by the occurrence of cavitation and supercavitation in fluid flow, governing conservation equations of fluid flow need to be derived. So, in this paper, using scale analysis, conservation equations including differential terms and weighting terms, are converted into dimensionless form. Hydrodynamic studies of cavitating flow are not in the same order as a hydroacoustic study of it. So, for decomposing these studies into two different orders, a perturbation method should be used. By selecting a suitable weighting term as a perturbed term and applying the perturbation method, conservation equations are decomposed into two orders containing leading and first orders. Leading order equations display the hydrodynamics of fluid flow with cavitation (or supercavitation), and first order equations explain the acoustics of cavitation. Finally, the validity of these two different models is investigated by comparing them with other study results.

2. Scale analysis

Scale analysis is one of the most useful methods in classical fluid mechanics. Using scale analysis and making equations non-dimensional causes all terms of equations to have appropriate weight. Consequently, comparing the weight of each term with each other specifies the importance of each term or variable parameter, and the lower weight can be neglected. So,

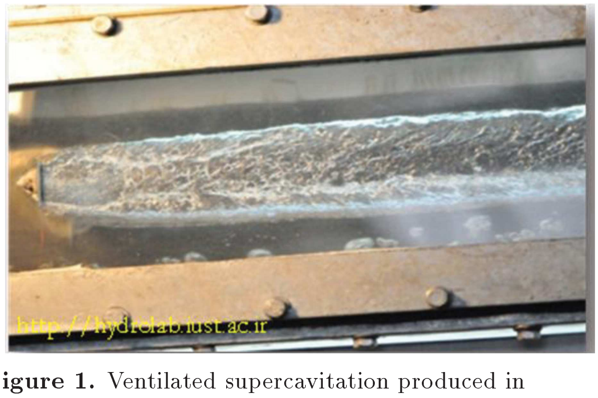

Figure 1. Ventilated supercavitation produced in hydrodynamic laboratory at Iran University of Science and Technology.

applying scale analysis can be helpful for using simpler models for investigation of a problem. Acoustic noise is part of fluid dynamics phenomena which occurs in a cavitating flow. Hence, using governing conservation laws of mass, momentum and energy for a fundamental element of that fluid, in addition to the hydrodynamics of fluid flow, acoustics generated through cavitation or supercavitation can also be described and modeled. Moreover, since the objective of this investigation is to present a model for an unknown phenomenon, it is necessary to consider the governing equations be analyzed in a dimensionless condition. Using scale analysis to produce governing dimensionless equations leads to the comparison among the weight of present terms, which yields recognition of the importance of those terms. Also, gaining more knowledge about the importance of unknown terms causes the solution process to be simple and, consequently, reduces the dependability of the solution on physical quantities for presenting a dimensionless model. Eqs. ((1) to (3)) represent a dimension form of continuity (conservation of mass), momentum and combined equations of state and energy for compressible liquid fluid in a reversible process [29,30].

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{u}) = \rho Q, \tag{1}$$

$$\frac{\partial(\rho \vec{u})}{\partial t} + (\vec{u}.\nabla)(\rho \vec{u}) = -\nabla p + \mu\big[\nabla^2 \vec{u} + \left(\left(\frac{1}{3} + \frac{\mu_v}{\mu}\right)\nabla(\mathrm{div}(\vec{u}))\right)\big] + \rho \vec{u} Q, \tag{2}$$

$$[\frac{\partial p}{\partial t}] + [(\vec{u}.\nabla)p] = c^2[-\rho \mathrm{div}(\vec{u})] + \left(\frac{c^2\beta}{c_p}\right)\left[\mu_v(\nabla.\vec{u})^2 + 2\mu\{e_{ij}e_{ji} - \frac{1}{3}e_{ii}^2\}\right]. \tag{3}$$

In these equations $\rho(X,t)$ is density, $\vec{u}(X,t)$ is velocity, $Q(X,t))$ is the volume source of the expansion of fluid mass, $p(X,t)$ is fluid pressure, μ is viscosity, μ_v is bulk viscosity, e_{ij} is the tensor of the strain rate, c is sound speed, c_p is heat capacity at constant pressure and $\beta = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{p_0}$ is constant. In order to dimensionally analyze the above equations, the following presumptions are considered:

$$p = \tilde{P}p^* \quad \text{for example} \quad \Rightarrow \quad \tilde{P} = P_\infty = \rho_0 U_0^2,$$

$$u = \tilde{U}u^* \quad \text{for example} \quad \Rightarrow \quad \tilde{U} = U_0,$$

$$\rho = \tilde{\rho}\rho^* \quad \text{for example} \quad \Rightarrow \quad \tilde{\rho} = \rho_0 = \rho_{\text{water}},$$

$$Q = \tilde{Q}Q^*,$$

$$t = \frac{t^*}{\tilde{\omega}} \quad \Rightarrow \quad \tilde{\omega} = \frac{\tilde{U}}{\lambda} \quad \Rightarrow \lambda = \frac{\tilde{U}}{\tilde{\omega}},$$

$$X, Y, Z = Lx^*, Ly^*, Lz^* \quad \Rightarrow \quad \nabla = \frac{\partial}{\partial x_i} = \frac{\partial}{L\partial x_i^*}. \tag{4}$$

In the above relations, terms that have the upper case index (*) are dimensionless in order of one [$O(1)$]. So, ρ^* is dimensionless density, $\vec{u}^*$ is dimensionless velocity, Q^* is the dimensionless volume source for expansion in the mass of fluid, p^* is the dimensionless pressure of the fluid and t^* is the dimensionless time ([$O(1)$]). Also, terms containing upper case index (∼) are scale parameters that are related to the same physical parameters in the problem. In case of pressure and density, examples are mentioned therein. By placing the above relations in Eqs. (1) to (3), the following equations will be produced:

$$\{\tilde{\rho}\tilde{\omega}\}\frac{\partial \rho^*}{\partial t^*} + \{\frac{\tilde{\rho}\tilde{U}}{L}\}\nabla(\rho^* \vec{u}^*) = \{\tilde{\rho}\tilde{Q}\}\rho^* Q^*, \tag{5}$$

$$\{\tilde{\rho}\tilde{U}\tilde{\omega}\}\left[\frac{\partial(\rho^* \vec{u}^*)}{\partial t^*}\right] + \left\{\frac{\tilde{\rho}\tilde{U}^2)}{L}\right\}[(\vec{u}^*.\nabla)(\rho^* \vec{u}^*)] = \left\{\frac{\tilde{p}}{L}\right\}[-\nabla p^*] + \left\{\frac{\tilde{U}}{L^2}\right\}\mu\left[\nabla^2 \vec{u}^* + \left(\frac{1}{3} + \frac{\mu_v}{\mu}\right)(\nabla \mathrm{div}(\vec{u}^*))\right] + \{\tilde{\rho}\tilde{U}\tilde{Q}\}\rho^* \vec{u}^* Q^*, \tag{6}$$

$$\{\tilde{p}\tilde{\omega}\}[\frac{\partial p^*}{\partial t^*}] + \left\{\frac{\tilde{U}\tilde{p}}{L}\right\}[(\vec{u}^*.\nabla)p^*] = \left\{\frac{\tilde{\rho}\tilde{U}}{L}\right\}c^2[-\rho^* \mathrm{div}(\vec{u}^*)] + \left(\frac{c^2\beta}{c_p}\right)\left[\mu_v\left\{\frac{\tilde{U}^2}{L^2}\right\}(\nabla.\vec{u}^*)^2 + \mu\left\{\frac{\tilde{U}^2}{L^2}\right\}2\{e_{ij}^* e_{ji}^* - \frac{1}{3}e_{ii}^{*2}\}\right]. \tag{7}$$

As evident, the presence of terms with (*) index, has caused differential terms to be in one order [$O(1)$]. Also, the presence of parameters having sign (∼) as the coefficients of differential terms causes the weight of differential terms to become comparable, despite being dimensional. This weight, in fact, could represent the degree importance of different terms compared with each other. In order to better compare, the weight of differential terms, coefficients also need to become dimensionless. Eventually, by multiplying suitable parameters, and simplification, along with the definition of dimensionless numbers, in sequence, the non-dimensional form of continuity equations (conservation of mass) and momentum, as well as combined equations of state and energy, are found in the following forms:

$$\mathrm{St}\frac{\partial \rho^*}{\partial t^*} + \nabla.(\rho^* \vec{u}^*) = \left(\frac{L\tilde{Q}}{\tilde{U}}\right)\rho^* Q^*, \tag{8}$$

$$\mathrm{St}\frac{\partial(\rho^* \vec{u}^*)}{\partial t^*} + (\vec{u}^*.\nabla)(\rho^* \vec{u}^*) = \mathrm{Eu}[-\nabla p^*] + \frac{1}{\mathrm{Re}}\left[\nabla^2 \vec{u}^* + \left(\frac{1}{3} + \frac{\mu_v}{\mu}\right)(\nabla \mathrm{div}(\vec{u}^*)\right] + \left(\frac{L\tilde{Q}}{\tilde{U}}\right)\rho^* \vec{u}^* Q^*, \tag{9}$$

$$(\mathrm{St})(\mathrm{Eu})[\frac{\partial p^*}{\partial t^*}] + \mathrm{Eu}[(\vec{u}^*.\nabla)(p^*)] = (\frac{c}{\tilde{U}})^2[-\rho^* \mathrm{div}(\vec{u}^*)] + \left(\frac{c^2\beta}{c_p}\right)\left[\frac{l}{\mathrm{Re}_v}(\nabla.\vec{u}^*)^2 + \frac{l}{\mathrm{Re}}2\{e^*_{ij}e^*_{ji} - \frac{1}{3}e^{*2}_{ii}\}\right]. \tag{10}$$

In the above equations, dimensionless parameters of Reynolds number, Strouhal number and Euler number are defined as follows. Also, it is noteworthy that in this condition, differential terms, as well as coefficients, have become dimensionless.

$$\mathrm{Sr} = \frac{\tilde{\omega}L}{\tilde{U}}, \quad \frac{1}{\mathrm{Re}} = \frac{\mu}{\tilde{\rho}\tilde{U}L}, \quad \mathrm{Eu} = \frac{\tilde{p}}{\tilde{\rho}\tilde{U}^2}. \tag{11}$$

Eqs. (8) to (10) are dimensionless forms of equations describing the dynamics of fluid flow. Since terms having an asterisk as their upper-case are in the order of one, it could be noticed that terms of differential order are also in the order of one. Moreover, dimensionless numbers are, in fact, the weight of differential terms, depicting the importance of each term. By comparing the weight of differential terms (dimensionless numbers), being 3 in number, it becomes evident that for the above three equations, 27 different conditions are feasible. This is reflective of the fact that there are 3 different conditions for every dimensionless number: A) The order of the dimensionless number is far greater than the order of one; B) It is equal to the order of one; and C) It is much smaller than the order of one. Different conditions of governing equations are indicative of different problems. Favorable conditions for the occurrence of cavitation (or supercavitation) in a flow is when Euler and Strouhal numbers are in the order of one and the Reynolds number is not much smaller than the order of one. The reason for this is that under other conditions, the needed terms for the modeling of cavitation flow, as well as supercavitation, are eliminated. As an example, in Reynolds numbers much smaller than the order of one, the effects of the convective term are eliminated. This is where, in the modeling of bubble movement by fluid flow, the calculation of this term is necessary. Also, in conditions related to very small Strouhal number, steady flow will form, which is not an objective of this study.

3. Utilization of perturbation method

In general sense, in a physical problem when value of a term is extremely minute, whereas its effect is significant, the perturbation method becomes rather important. As an example, viscosity in a viscose flow is considered a turbulent term in a non-viscous flow. This fact indicates that although viscosity is a small parameter, its effect in a flow sometimes is very noticeable. As mentioned earlier, acoustic noise is a part of fluid flow dynamics. However, the entity of acoustic noise and its behavior in propagation are different with fluid flow. This could be justified by the positioning of noise from flow and flow dynamics in different orders of basic equations. In cases where cavitation is observed through perturbation, equations used in the modeling of flow will be placed in an order higher than the acoustic equations of the flow. It is because, when the sound waves propagate through a fluid, such small changes occur in pressure, density and velocity vector components, that no one can see them. On the other hand, the occurrence of hydrodynamic fluctuations of pressure, density and velocity vector components during cavitation (or supercavitation) is much higher than for acoustic ones.

In the light of everything mentioned, it will be attempted to separate continuity equations (conservation of mass), momentum and the combined equations of state and energy from the point of view of order. This way, the necessary equations for the recognition and simulation of the acoustics generated by cavitation and supercavitation, will be presented. Also, the relation of this type of equation with the dynamics of fluid flow could be shown.

Always, in the perturbation method, the perturbing parameter is depicted by (ε). By glancing through presumed equation systems in Eqs. (8) to (10), it is evident that five dimensionless terms (coefficients of differential terms) can be very small. In other words, they could be $\varepsilon_1 \sim \varepsilon_5$. In this type of problem, a term is selected from all the perturbing parameters. Then, separation of the different order of equations takes place on the basis of that parameter. The point is that selection of that parameter has to be based on the type of investigated problem. As volume fluctuations in fluid lead to the propagation of acoustic noise, the order of governing equations in solving the acoustics of fluid flow could be chosen equal to the order of these volume fluctuations. In light of the points stated above, for the present problem in this study, in identification of acoustics stemming from supercavitation, the perturbing term will be $\varepsilon = \tilde{Q}L/\tilde{U}$. The reason for this selection is that, in supercavitation flow, the existence of very small bubbles inside the cavity is regarded as perturbation in a flow without cavitation. Hence, (Q) has to be viewed as a perturbing term in the equations. An interesting point is that one of the major sources of acoustic noise generation is fluctuations of bubbles inside the cavity. Results presented at the end of this section in the form of obtained acoustical equations, are indicative of the point stated above. Therefore, initially, the following assumptions will be entered in the equations:

$$\begin{aligned} p^*(X,t,\varepsilon) =& h_0(\varepsilon)p_0(X,t) + h_1(\varepsilon)p_1(X,t) \\ &+ o(h_1(\varepsilon)), \\ \vec{u}^*(X,t,\varepsilon) =& f_0(\varepsilon)\vec{u}_0(X,t) + f_1(\varepsilon)\vec{u}_1(X,t) \\ &+ o(f_1(\varepsilon)), \\ \rho^*(X,t,\varepsilon) =& g_0(\varepsilon)\rho_0(X,t) + g_1(\varepsilon)\rho_1(X,t) \\ &+ o(g_1(\varepsilon)), \\ Q^*(X,t,\varepsilon) =& m_0(\varepsilon)Q_0(X,t) + m_1(\varepsilon)Q_1(X,t) \\ &+ o(m_1(\varepsilon)). \end{aligned} \tag{12}$$

In these relations, parameters which have zero indexes, include $\vec{u}_0, \rho_0, p_0$ and Q_0, which are dimensionless velocity vector, dimensionless density, dimensionless pressure and dimensionless volume source fluctuations, respectively, in the order of hydrodynamic. Also, parameters which have one index, include $\vec{u}_1, \rho_1, p_1$ and Q_1, which are dimensionless velocity vector, dimensionless density, dimensionless pressure and dimensionless volume fluctuations, respectively, in the order of hydroacoustics. Terms $h_0, h_1, ..., m_0, m_1$ are the indicated weight of dimensionless parameters.

$$x_{n+1}(\varepsilon) = o(x_n(\varepsilon)), \quad \varepsilon = \frac{\tilde{Q}L}{\tilde{U}}.$$

After placing Eq. (12) in the dimensionless Eqs. (8) to (10), they need to be separated into two different orders. This requires that the relation of weighing terms with perturbation parameter (ε), in terms of order, is being determined. Also, according to the definition applied in this study, leading order equations must be satisfied conditions, which govern the hydrodynamics of fluid flow, and first order equations must be satisfied conditions governing hydroacoustics. In other words, the relation between weighing terms and perturbation terms has to be found, such that, after decomposition of equations into different orders, the obtained equations in the leading order must be able to display the flow field precisely. Moreover, the obtained equations of the first order have to be indicative of linear wave equations after applying linear acoustics assumptions (non-viscous and stationary background fluid flow). Beside these assumptions, the definition of $x_{n+1}(\varepsilon) = o(x_n(\varepsilon))$, and the principle of minimum possible conditions have been used. By doing so, in Eqs. (8) to (10), and then simplifying, by using the principle of minimum possible conditions, it becomes evident that the above parameters need to be presented in the following form:

$$p^* = p_0 + \varepsilon p_1, \quad \vec{u}^* = \vec{u}_0 + \varepsilon\vec{u}_1,$$

$$\rho^* = \rho_0 + \varepsilon\rho_1, \quad Q^* = Q_0 + \varepsilon Q_1. \tag{13}$$

Placing the parameters as Eq. (13) causes different orders of equations to be formed. By separation of all terms having one order, $O(1)$ as the terms of leading order equations and all terms having (ε) order $0(\varepsilon)$, as first order equations, it will be possible to recognize equations related to the dynamics of supercavitation flow (hydrodynamics of flow), as well as acoustics generated by it. The leading order of formed equations represents the modeling of cavitation and supercavitation flow, and first order equations present their acoustics.

As the work is preceded, initially, results obtained for different orders are presented and, then, while the validation of leading and first order equations is inspected, the interpretation of terms related to these equations is developed.

4. Results

Results indicate that acoustical equations are at a lower order compared to the dynamics of cavitation (or supercavitation) flow. In other words, acoustics as a "perturbation phenomena" have been imposed on the cavitation (or supercavitation) flow. Hence, in light of obtained results, two groups of leading and first order equations could be designated as a mathematical model of the fluid dynamics of cavitation and supercavitation, and as a mathematical model for its acoustics.

4.1. Mathematical hydrodynamic model for simulating of cavitation or supercavitation flow

On the basis of obtained results and by separating terms having the order of one, a group of leading order equations for simulation of cavitation (or supercavitation) flow is used. The leading order system related to continuity and momentum equations, as well as those of combined equations of state and energy, are presented in Eqs. (14) to (16):

$$(\mathrm{St})\frac{\partial \rho_0}{\partial t}+\nabla.(\rho_0\vec{u}_0)=0, \tag{14}$$

$$(\mathrm{St})\frac{\partial(\rho_0\vec{u}_0)}{\partial t}+(\vec{u}_0.\nabla)(\rho_0\vec{u}_0)=(\mathrm{Eu})[-\nabla p_0]$$
$$+\frac{1}{\mathrm{Re}}\left[\nabla^2\vec{u}_0+\left(\frac{1}{3}+\frac{\mu_v}{\mu}\right)(\nabla\mathrm{div}(\vec{u}_0))\right], \tag{15}$$

$$(\mathrm{St})(\mathrm{Eu})\frac{\partial p_0}{\partial t)}+(\mathrm{Eu})(\vec{u}_0.\nabla)p_0=(\frac{c}{\tilde{U}})^2\rho_0(\nabla.\vec{u}_0)$$
$$+\left(\frac{c^2\beta}{c_p}\right)\left[\frac{1}{\mathrm{Re}_v}(\nabla.\vec{u}_0)^2+\frac{1}{\mathrm{Re}}2\{e^0_{ij}e^0_{ji}-\frac{1}{3}(\nabla.\vec{u}_0)^2\}\right]. \tag{16}$$

Consequently, it becomes evident that obtained results for a leading order system, in fact, indicating that used equations for the simulation of cavitation or supercavitation flow are true, by the assumption of fluid flow being viscous and compressible.

4.2. Mathematical acoustics model for simulating of cavitation or supercavitation flow

Since acoustical equations are in a lower order compared with hydrodynamic equations, the order of these equations is assumed equal to that of the perturbation parameter mentioned in Eq. (12). Similarly, by reordering the available terms in the equations on the basis of a perturbation term, otherwise known as terms with an order of (ε), equations of the first order could be obtained. In this regard, systems of governing equations are presented in the following form:

$$(\mathrm{St})\frac{\partial \rho_1}{\partial t}+\nabla.(\rho_0\vec{u}_1)+\nabla.(\rho_1\vec{u}_0)=\rho_0 Q_0, \tag{17}$$

$$(\mathrm{St})\left[\frac{\partial(\rho_1\vec{u}_0)}{\partial t}+\frac{\partial(\rho_0\vec{u}_1)}{\partial t}\right]+(\vec{u}_0.\nabla)(\rho_1\vec{u}_0)$$
$$+(\vec{u}_0.\nabla)(\rho_0\vec{u}_1)+(\vec{u}_1.\nabla)(\rho_0\vec{u}_0)=(\mathrm{Eu})[-\nabla p_1]$$
$$+\frac{1}{\mathrm{Re}}\left[\nabla^2\vec{u}_1+\left(\frac{1}{3}+\frac{\mu_v}{\mu}\right)(\nabla\mathrm{div}(\vec{u}_1))\right]+\rho_0 u_0 Q_0, \tag{18}$$

$$(\mathrm{St})(\mathrm{Eu})\frac{\partial p_1}{\partial t}+(\mathrm{Eu})(\vec{u}_0.\nabla)p_1+(\mathrm{Eu})(\vec{u}_1.\nabla)p_0$$
$$=-(\frac{c}{\tilde{U}})^2\rho_0(\nabla.\vec{u}_1)-(\frac{c}{\tilde{U}})^2\rho_1(\nabla.\vec{u}_0)+(\frac{c^2\beta}{c_p})$$
$$\begin{bmatrix}\frac{2}{\mathrm{Re}_v}(\nabla.\vec{u}_0)(\nabla.\vec{u}_1)+\frac{2}{\mathrm{Re}}\{\frac{1}{4}\frac{\partial u_{oi}}{\partial x_j}\frac{\partial u_{1j}}{\partial x_i}+\frac{1}{4}\frac{\partial u_{oi}}{\partial x_j}\frac{\partial u_{1i}}{\partial x_j}\\+\frac{1}{4}\frac{\partial u_{1i}}{\partial x_j}\frac{\partial u_{oj}}{\partial x_i}+\frac{1}{4}\frac{\partial u_{oj}}{\partial x_i}\frac{\partial u_{1j}}{\partial x_i}-\frac{2}{3}(\nabla.\vec{u}_0)(\nabla.\vec{u}_1)\}\end{bmatrix}. \tag{19}$$

These equations are indicative of the fact that a group of acoustic equations are coupled with equations of leading order and are affected by them. In other words, although acoustic equations of flow have no effect on the hydrodynamic equations of cavitation flow, as presented by Eqs. (14) to (16), the hydrodynamics equations of flow affect the acoustic equations. Then, the validity of the system of leading and first order equations is inspected regarding certain assumptions. After obtaining the equations from different orders, the validity of these equations needs to be verified.

4.3. Validity inspection of hydrodynamic model of supercavitation flow

In order to inspect the validity of equations of leading order, they should be compared with equations of momentum, continuity, energy and state used in the simulation of supercavitation flow. However, since equations were presented in previous studies of the simulation of supercavitation as incompressible, in this study, also, that assumption was held. Then, the equation systems of leading order were obtained and, subsequently, incompressible forms of the equations were compared with the results of other studies. Therefore, incompressible forms of governing equations in supercavitation flow are presented. Also, it is assumed in these equations that the Strouhal and Euler number are in the order of one. Initially, it is assumed that the flow is incompressible. So, in that case:

$$\nabla.(\vec{u}_0)=0. \tag{20}$$

By considering this assumption in the leading order equations, continuity equations will be presented in the following form:

$$\frac{\partial \rho_0}{\partial t}+(\vec{u}_0.\nabla)\rho_0=0,$$
$$\frac{D\rho_0}{Dt}=0. \tag{21}$$

And, similarly, for equations of momentum and combined state, as well as energy, Eqs. (22) and (23) will be:

$$\left[\rho_0\frac{\partial\vec{u}_0}{\partial t}+\vec{u}_0\frac{\partial\rho_0}{\partial t}\right]+(\vec{u}_0.\nabla)(\rho_0\vec{u}_0)$$
$$=-\nabla\rho_0+\frac{1}{Re}[\nabla^2\vec{u}_0], \tag{22}$$

$$\frac{\partial p_0}{\partial t}+(\vec{u}_0.\nabla)p_0=\left(\frac{c^2\beta}{c_p}\right)\left[\frac{1}{Re}2\{e^0_{ij}e^0_{ji}\}\right]. \tag{23}$$

Obtained results are the same equations as the governing equation of incompressible fluid flow in the Open Foam [16]. Also, these equations are those used in Moghimi's PhD dissertation [16] for simulation of artificial supercavitation assuming incompressible fluid. Consequently, in light of points discussed, the validity of leading order equations for compressible flow is realized.

4.4. Validity inspection of acoustic model generated by supercavitation flow.

In order to inspect the validity of first order equations, these equations are compared with those of momentum, continuity, energy and state, all presenting simulation of supercavitation flow acoustics. Since obtaining linear equations of acoustics is also done using equations of continuity, momentum, energy and state, utilizing this method to inspect the validity of obtained acoustics equations in the first order is suitable. To do this, initially, assumptions relating to linear acoustics are entered into the equations and, then, obtained results are compared with the results of linear acoustics. Also, it is assumed that Euler and Strouhal numbers are in the order of one. Assumptions used in obtaining linear acoustic equations are as follows:

$$\rho_0 = \text{cons}, \quad u_0 = 0,$$

$$\mu = 0, \quad Q = 0. \tag{24}$$

By exerting these assumptions in the equations of continuity, momentum, energy and state of first order, the following equations are obtained:

$$\frac{\partial \rho_1}{\partial t} + \rho_0(\nabla.\vec{u}_1) = 0, \tag{25}$$

$$\rho_0 \frac{\partial(\vec{u}_1)}{\partial t} = [-\nabla(p_1)], \tag{26}$$

$$\frac{\partial p_1}{\partial t} = -(\frac{c}{\vec{U}})^2 \rho_0(\nabla.\vec{u}_1). \tag{27}$$

It is evident that these equations are equal to those of linear acoustics equations [20]. Therefore, in the light of points presented in the preceding two sections, the validity of the obtained equations is realized from both leading order equations, as well as first order.

5. Discussions

As mentioned earlier, equations of leading order are indicative of a system of equations for determination of fluid flow conditions. These equations introduce a general form of principle equations in fluids. Lack of thermal charge existence, otherwise known as non-occurrence of work on the basis of heat transfer, is the only assumption in this form of equations. On the other hand, this assumption, in light of the problem under consideration, meaning simulation of supercavitation flow, is a suitable and logical point. Considering the obtained leading order Eqs. (14) to (16), as well as the dimensionless weight of their different terms, it becomes possible to decide about problem formation in different conditions. In simple terms, this means that in a given problem, the effect of a term with lower weight could be neglected against other terms and vice versa. At any rate, having this type of equation (leading order) at hand enables researchers to simulate supercavitation flow in a compressible viscous flow in a designated time.

Regarding acoustic equations of the first order, it is also fair to state that these obtained equations are a general form of those equations used in the analysis of linear acoustics. Differences between the obtained first order equations and linear acoustics equations are:

a) Equations of linear acoustic are connected to stationary fluid flow. Whereas, obtained equations in this study include the velocity of fluid flow as well. The term expressing the fluids velocity is depicted by $\vec{u}_0$.

b) In equations of linear acoustics, the fluid is assumed to be incompressible with fixed density. In conditions where obtained equations from a perturbation method have regarded the fluid as compressible, then, density in these acoustic equations could not be constant.

c) Contrary to linear acoustic equations, in these equations, the term of the acoustics source exists and is shown by (Q), which is present in both continuity equations as well as momentum. Nevertheless, the importance of this term becomes evident when large supercavity occurs in the fluid flow comprised of many smaller bubbles. These small bubbles are transferred from low pressure areas to high pressure areas via the movement of flow. This causes considerable changes in the volume and radii of each bubble. Fluctuations of each bubble, in addition to being observed experimentally, are monitored through numerical analysis by utilizing the equation of Rayleigh–Plesset [13]. Since noise is generated by the fluctuation of each bubble, it can be regarded as the source of acoustic noise production. Therefore, this term (Q) has to be considered. Another interesting point is that the term of the volume source in equations of leading order is not evident. The reason for this is the small order of these fluctuations compared with those of leading order, despite being in equations of the first order. So, even though leading order equations are producers of supercavitation flow, it is expected that by using equations of leading order, the fluctuations of each bubble become recognized. Then, by using equations of the first order, the acoustic noise of each fluctuation could be obtained.

d) Equations of linear acoustics are for non-viscous fluid flow, whereas the acoustic equations of the present study contain terms related to fluid viscosity. Viscosity terms could be observed in the form of Reynolds number in the equations. The left side of the combined equation of state and energy are representative of acoustic pressure distribution, and its right side presents two effective mechanisms on the distribution of acoustic pressure. Those terms are like dilatation terms, having Mach number as their coefficient, and another term related to viscosity could be obtained via equations, in which, by comparing the weight of Reynolds number and Mach number, each mechanism's contribution to acoustic pressure could be determined.

6. Conclusions

As the acoustic modeling of cavitation and supercavitation are instrumental in recognition of these hydrodynamic phenomena, presenting a precise and valid model for estimating propagated acoustic noise is of great importance. Consequently, as presented earlier in this study,

1. In order to provide a more precise numerical and mathematical model of the noise produced by supercavitation, initially, governing equations of the fluid containing equations of state, continuity, momentum and energy were divided into different orders. This was done by dimensional analysis and the perturbation method. Obtained results from different orders of the equations were indicative of acoustic noise present as a perturbation term in the governing equations of fluid hydrodynamics. It is noteworthy that the terms of the equations were scaled and had become unity ordered using dimensional analysis and gauge functions. The degree of scaled leading equations is in the order of the unit. Also, the order of the acoustic equations is from the order of volume source fluctuations. These volume sources in supercavitation flow are bubbles inside a large cavity.

2. According to the conducted analysis, in order to analyze the acoustic behavior of supercavitation, first, equations of the leading order need to be solved. Under most general conditions, in order to estimate propagated noise as precisely as possible from cavitation and supercavitation, first, equations of continuity, Navier stokes, energy and state for compressible fluid (with or without viscosity) have to be solved.

3. By solving this system of equations, a supercavity and bubbles inside it will be formed. Then, by considering the conditions of flow (location of each bubble, pressure and pressure gradient distribution, velocity distribution, density and gradient of velocity and density) as boundary conditions, as well as initial conditions, equations of first order, otherwise known as acoustic equations (coupled with equations of hydrodynamic of flow), will be solved. Through solving these equations, acoustic pressure distribution, velocity and the density induced by noise can be calculated.

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