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Single-item lot-sizing and scheduling problem with deteriorating inventory and multiple warehouses

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Abstract. This paper introduces a Single-Item Lot-sizing and Scheduling Problem with Multiple Warehouses (SILSP-MW). In this problem, the inventory deteriorates over time, depending on the warehouse conditions, so multiple warehouses with different technologies are considered in this study. Each warehouse has a specified deterioration rate and holding cost. The purpose of the SILSP-MW is to determine production periods and quantities, and to select the appropriate warehouse to hold the inventory in each period, such that specified demand in each period is being satisfied while the total cost is minimized. We shall present a Mixed-Integer Linear Programming (MILP) formulation to model the problem. Moreover, a Simulated Annealing (SA) algorithm will be presented to solve this problem. We will evaluate the performance of the algorithm by computational experiments with small- and medium-sized examples. In addition, a full factorial design is developed to investigate the effect of the model parameters on the proposed SA algorithm.

1. Introduction

The Lot-sizing and Scheduling Problem (LSP) is a production-inventory planning problem that has been extensively studied in the literature. This problem includes n time periods and their specified demands. These demands should be satisfied by production runs in either the same period or the previous periods. There are three types of costs in this problem: setup costs, holding costs and production costs. Large lot sizes lead to lower setup costs and higher holding costs, and conversely, small lot sizes lead to lower holding costs and higher setup costs. The objective of the LSP is to determine the optimal lot size in each period, so that the demands are satisfied and the total cost is minimized.

Generally, inventories can be classified into four categories: (1) obsolescence inventory, (2) perishable inventory, (3) deteriorating inventory, and (4) none of the above. In the first category, inventories become obsolete after a certain time. For instance, newspapers, clothes and fashion goods become obsolete because of rapid change. In the second type, inventories have a maximum lifetime and afterwards become unusable (e.g., human blood, photographic film, drugs). The third category includes items that gradually degrade (e.g., they decay or vaporize). The profit and the volume of these items, such as alcohol, gasoline and radioactive substances, decrease with the passage of time. Finally, inventories in the fourth category are items that can be held indefinitely without a loss of their value. In the case addressed in this paper, it is assumed that items are from the third category (deteriorating inventory.)

Deteriorating and perishable inventories in lot-sizing problems have been considered in a large number of studies; we classified these studies into two groups.

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The first group is research, based on independent demand. The study conducted by Whitin [1] was the first work in this area. He assumed that the deterioration of the inventory occurred at the end of the prescribed storage periods. Ghare and Schrader [2] introduced a model for inventories that decrease exponentially. They observed that some items deteriorated with time, following an exponentially distributed function. Tadikamalla [3] presented an EOQ model for items with a Gamma-distributed deterioration that was applicable for items such as frozen food, roasted coffee, breakfast cereals, cottage cheese, ice cream, pasteurized milk or corn seeds. Khedhairi and Taj [4] also considered deteriorating inventory with a Gamma-distributed deterioration, and investigated optimal control for the production-inventory system. Nahmias and Wang [5] considered a lot-sizing model with a deteriorating inventory, and introduced a heuristic method to solve it. Hsu et al. [6] developed a deteriorating inventory policy when the retailer invests in preservation technology to reduce the rate of product deterioration. They presented a procedure to determine an optimal replenishment cycle. Shah [7], Cochen [8], Dave and Patel [9], Kang and Kim [10] and Wee [11] also worked in this area. Additional research and references in the field of deteriorating and perishable inventory can be found in the papers by Nahmias [12] and Raafat [13], Goyal and Giri[14] and Bakker et al. [15].

The second group is the research that addresses dependent demand or research that considers deteriorating inventory in Material Requirement Planning (MRP) systems. This area was originally investigated by Wee and Shum [16]. They noted that deteriorating inventory in MRP systems is practically non-existent. They investigated the influence of deteriorating inventory in MRP systems, and observed its effects on the total cost, and found that it may affect ordering policies. Furthermore, they modified the Least Period Cost (LPC) and the Least Unit Cost (LUC) heuristics for a deteriorating inventory. Ho et al. [17] also considered the lot-sizing problem with a deteriorating inventory in MRP systems. They modified the net Least Period Cost (nLPC) heuristic and a variant of the Least Total Cost (LTC) heuristic, denoted by LTC(-), and also a variant of the Part Period Cost (PPC), denoted by PPC(-), for deteriorating inventory. In addition, some works relevant to the Economic Lot-Sizing Problem (ELSP) with consideration of deteriorating inventory fall into this area. Chu and Shen [18] contemplated an economic lot-sizing problem with a perishable inventory. They supposed that an item's deterioration rate and inventory holding cost in each period depend on the age of the item. Hsu [19] investigated the economic lot-sizing problem with an economy-of-scale cost function in which the total cost is non-decreasing in the total volume and the average unit cost is non-increasing. Chu et al. [20] suggested an economic lot-sizing problem with a perishable inventory that is similar to Hsu's problem, except that the ordering and inventory cost functions are assumed to be concave. They presented approximate solutions and a worst-cost analysis. Bai et al. [21] investigated the economic lot-sizing problem with a deteriorating inventory in which backlogging is allowable, and demand in each period can be satisfied by subsequent periods. Pahl et al. [22] modeled the Discrete Lot-sizing and Scheduling Problem (DLSP) that consist of perishability and deterioration constraints. In addition, Pahl et al. [23] integrated the deterioration and perishability constraints, together with sequence-dependent setup costs and times in the Capacitated Lot-Sizing and Scheduling Problem (CLSP). They studied the effects of such constraints on the behavior mechanisms and the solutions of these models.

Recently, Bakker et al. [15] presented an excellent comprehensive review of the advances made in the field of inventory control of perishable items (deteriorating inventory). They followed the review conducted by Goyal and Giri [14], and used their classification according to the shelf-life and demand characteristics. Bakker et al. [15] search papers on deterioration inventory control that have been published between January 2001 (Goyal and Giri's review date) and December 2011, based on keyword search in a selection of a major journals. They found 227 relevant papers and classified them according to the modeling of deterioration (life time) and of demands. Furthermore, they discussed a number of key modeling elements consist of price increase/discount, treatment of stock-outs, single or multi item, number of warehouses, single vs. multi echelon, cost accounting aspects and permissible delay in payment.

There are several papers that considered two separate warehouses for the storage of the deteriorating inventory because of the limited storage capacity [24-36]; one warehouse is owned and the other is rented. In this paper, we consider multiple warehouses with different deterioration rate and holding cost too, but none of them is rented; in other words, in our model, warehouses are alternatives.

Deteriorating inventories are considered, because they have practical applications in production systems. However, in a factory, the manager tries to minimize the deterioration costs, and multiple warehouses with different holding technologies can be used as a solution.

In this paper, we investigate the LSP with a deteriorating inventory in MRP systems with multiple warehouses to hold the inventory in each period; we call this problem the SILSP-MW. We use the assumption in the work performed by Chu et al. [20]; that is, the deterioration rate and the holding cost for each warehouse in each period are time-dependent. In other words, they depend on the number of periods since their production period. Each warehouse has a distinct deterioration rate and holding cost. Note that alternative warehouses can be considered as different maintenance facilities or maintenance conditions, so whatever maintenance condition is better, the unit holding cost is higher, and the deterioration rate is lower. For example, if refrigeration equipment is considered as a maintenance strategy, then for the lower temperature, deterioration is lower but with a higher holding cost. In this example, each temperature is considered as an individual warehouse. The goal is to determine the optimal lot sizes and select one of the available warehouses to hold the inventory (if there is any) in each period. The remainder of the paper is organized as follows: Section 2 presents a mathematical formulation for the problem. In Section 3, we develop an SA algorithm. A computational study is presented in Section 4. Finally, we present a conclusion and propose future research directions in Section 5.

2. Mathematical formulation

In this section, a mathematical formulation of the SILSP-MW is presented. First, we present some notation and assumptions:

Parameters:

T number of periods in a time horizon;

N number of warehouses.

For $1 \leq i \leq t \leq T$ and $1 \leq k \leq N$, the followings are defined:

d_t level of demand in period t;

S_t setup cost in period t;

p_t unit production cost in period t;

h_{itk} unit inventory holding cost for warehouse k at period t for the items that are produced in period i;

α_{itk} deterioration rate for inventory stocked in warehouse k at period t that is produced in period i.

Decision variables:

x_t production quantity in period t;

I_{it} the fraction of goods produced at period i that remain at the end of period t;

I_{itk} amount of inventory stocked at warehouse k at the end of period t that is produced in period i;

z_{it} amount of demand in period t that is satisfied by the production in period i;

y_t binary variable that indicates whether a setup cost is incurred ($y_t = 1$), or not ($y_t = 0$) for period t.

Assumptions:

For a deteriorating inventory in each warehouse, the longer they have been held, the faster they deteriorate and the higher the holding cost will be. That is,

$$\alpha_{itk} \geq \alpha_{jtk}, \quad 1 \leq i \leq j \leq t, \quad k = 1, ..., N,$$

$$h_{itk} \geq h_{jtk}, \quad 1 \leq i \leq j \leq t, \quad k = 1, ..., N.$$

Naturally, we can assume that the warehouse with a higher level of technology, which has a lower deterioration rate, will have the higher holding cost, or:

$$\text{If}\, \alpha_{itk} \leq \alpha_{itk'},\ \text{then}\ h_{itk} \geq h_{itk'},\ 1 \leq i \leq t \leq T,$$

$$1 \leq k \leq N,\ 1 \leq k' \leq N.$$

Furthermore, we assume that the inventory at the beginning of the first period is equal to zero, and also that no inventory is required at the end of the time horizon.

The mixed-integer linear programming formulation of this problem is:

$$\text{Minimize} \sum_{t=1}^{T} (P_t.x_t + S_t.y_t + \sum_{i=1}^{t} \sum_{k=1}^{N} h_{itk}.I_{itk}), \quad (1)$$

S.t :

$$x_t - z_{tt} = I_{tt}, \quad 1 \leq t \leq T, \quad (2)$$

$$\sum_{k=1}^{N} (1 - \alpha_{i(t-1)k}).I_{i(t-1)k} - z_{it} = I_{it},$$

$$1 \leq i < t \leq T, \quad (3)$$

$$I_{it} = \sum_{k=1}^{N} I_{itk}, \quad 1 \leq i \leq t \leq T, \quad (4)$$

$$\sum_{i=1}^{t} z_{it} = d_t, \quad 1 \leq t \leq T, \quad (5)$$

$$x_t \leq y_t.M, \quad 1 \leq t \leq T, \quad (6)$$

$$x_t,\ z_{it},\ I_{it},\ I_{itk} \geq 0,$$

$$1 \leq i \leq t \leq T,\ 1 \leq k \leq N, \quad (7)$$

$$y_t \in \{0, 1\}, \quad 1 \leq t \leq T. \quad (8)$$

The objective function (1) to be minimized is the sum of the production, setup and inventory holding costs. Constraints (2) and (3) express the inventory balance, subject to the deterioration rate. Constraints (4) state that the amount of inventory at the end of each period is equal to the sum of the inventory that is being held at the available warehouses. Constraints (5) make sure that the demand in each period has been satisfied. Constraints (6), where M is a large number, guarantee that a setup cost will be paid if there is any production in period t. Constraints (7) state that variables x_t, z_t, I_{it} and I_{itk} are continuous, and constraints (8) state that the setup variables y_t are binary.

3. Proposed simulated annealing

Proofs from complexity theory, as well as computational experiments, indicate that most of lot-sizing problems are difficult to solve [37]. Hence, meta-heuristic approaches are appropriate tools for solving these problems, because they can reduce the computational time and obtain near-optimal solutions. Among existing meta-heuristics, Tabu Search (TS), Genetic Algorithms (GA) and Simulated Annealing (SA) have received considerable attention in this field. For example, applications of these approaches appear in [37-42].

In this section, we develop an SA algorithm to solve the problem, including the solution representation scheme and the steps of the algorithm.

3.1. Problem representation

The first issue in designing the SA algorithm is to define a proper encoding. Therefore, we use this fundamental property: Concavity implies that there is an optimal solution in which each period's demand must be satisfied, whether by production in the same period or by inventory carried from the previous periods. In other words, production and inventory costs carried from the previous periods are not incurred simultaneously in any period ($x_t.I_{t-1}$=0, for all t) [42]. This property is true for the SILSP-MW, because costs are typically concave, whereas the non-zero fixed setup cost is paid whenever we have production. Therefore, we can encode a problem solution with T binary variables instead of integers for setups and continuous variables for production quantities. This leads to an encoding in which its entries indicate whether a setup cost is incurred or not; using a vector consisting of binary entries, we can write:

$$Y = [y_1, y_2, ..., y_T],$$

where y_t=1 if setup is required in period t, and y_t=0 otherwise. Furthermore, we define a vector K whose ith entry ($1 \leq i \leq T-1$) denotes the selected warehouse for holding the inventory (if there is any) in the ith period. The entries in vector K are integers and can be between 1 and N.

$$K = [k_1, k_2, ..., k_{T-1}],$$

Using these vectors, the production quantity in each period can be obtained. If $y_t = 1$ $(1 \leq t \leq T)$, the production quantity is equal to the sum of the inflated demands from period t to period $t' - 1$, where $y_{t'} = 1$ and $y_j = 0$ $(t \leq j \leq t')$. If $y_t = 0$, the production quantity is equal to zero. Inflated demand is included because of the consideration of deteriorating inventory. For example, to satisfy the demands from period c to period e by production in period c, the lot size is equal to:

$$Q_{ce} = d_c + \sum\nolimits_{t=c+1}^{e} \frac{d_t}{\prod_{l=c}^{t-1} (1-\alpha_{clk})}. \tag{9}$$

Briefly, each solution of the problem is given by the two vectors Y and K in the SA algorithm.

3.2. Solution generation

According to the aforementioned fundamental property, we can restrict the solution space regarding each specified vector, K. We develop the matrix M in Eq. (10) to illustrate the solution space in which entry $M_{i,j}$ indicates the production quantity in period i to satisfy the demands in periods $i, i+1..., j$:

$$M = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & & M_{1,T} \\ 0 & M_{2,1} & M_{2,2} & M_{2,3}\dots & M_{2,T} \\ 0 & 0 & M_{3,3} & M_{3,4}\dots & M_{3,T} \\ 0 & 0 & 0 & M_{4,3}\dots & M_{4,T} \\ \vdots & & & & \\ 0 & 0 & 0\dots & & M_{T,T} \end{bmatrix} \tag{10}$$

in which:

$$M_{i,j} = d_i + \sum\nolimits_{t=i+1}^{j} \frac{d_t}{\prod_{l=i}^{t-1} (1-\alpha_{ilk})}. \tag{11}$$

Therefore, we can calculate x_t for specified vectors Y and K as follows:

$$x_t = \begin{cases} M_{t,t'-1}, & \text{if } y_t = 1 \\ 0, & \text{if } y_t = 0 \end{cases} \tag{12}$$

where $y_{t'}$=1 and y_j=0 $(t \leq j \leq t')$.

The solution generation is better illustrated by using the following example.

Suppose that we have 10 periods and 2 alternative warehouses. In one step, vectors K and Y are obtained as [1 2 2 2 1 2 1 2 1] and [1 1 0 1 0 0 1 0 0 0], respectively. Thus, vector K specifies the values of α_{itk} in Eq. (11), and subsequently the matrix M can be constructed. From vector Y, it can be observed that setup is required in periods 1, 2, 4 and 7. Hence, the production quantities are computed as follows:

$$x_1 = M_{1,1} = d_1,$$

$$x_2 = M_{2,3} = d_2 + \frac{d_3}{(1-\alpha_{222})},$$

$$x_4 = M_{4,6} = d_4 + \frac{d_5}{(1-\alpha_{442})} + \frac{d_6}{(1-\alpha_{442}).(1-\alpha_{451})},$$

$$x_7 = M_{7,10}(7,10) = d_7 + \frac{d_8}{(1-\alpha_{771})} * \frac{d_9}{(1-\alpha_{771}).(1-\alpha_{782})} + \frac{d_{10}}{(1-\alpha_{771}).(1-\alpha_{782}).(1-\alpha_{791})},$$

and:

$$x_3 = x_5 = x_6 = x_8 = x_9 = x_{10} = 0.$$

3.3. Steps of the proposed solution based on SA

Using the above definitions and notation, the proposed algorithm is developed as follows.

First, an initial solution should be determined. K is a $(T-1)$-tuple of random numbers between 1 to N. Y is a T-tuple of random binary numbers.

In the second step, a neighboring solution is constructed. A random entry (i) of the vector K is selected, and its value is changed, where the new value is also between 1 and N, randomly. Then, one period (j) is randomly selected, and the corresponding entry is switched from one to zero or zero to one in the vector Y. Notice that the first period should not be selected, because we assume that backlogging is not allowed in the problem; in the first period, the setup must occur as well. At this time, using the obtained vectors Y and K, we exploit the lot sizes in each period according to Eqs. (11) and (12), and then compute the fitness value.

This procedure is continued according to the simulated annealing algorithm, and the pseudo-code of the proposed SA for the SILSP-MW is presented in Figure 1.

4. Computational study

In this section, we present a numerical example for further exposition of the problem and to further illustrate the proposed SA algorithm. Next, we compare the

Begin

Set initial temperature $\text{Temper}_0 = 100$.

Select final temperature $\text{Temper}_f = 0.1$.

$\text{Temper} = \text{Temper}_0$

Set cooling rate $cr = 0.95$.

Initialize

Construct vector K by generating $T-1$ random numbers between 1 and N.

Construct matrix M according to the vector K.

Construct vector Y by generating T random binary numbers.

Calculate x_t for $t = 1$ to T.

Calculate fitness value (F_0).

$\text{Best\_}F = F_0$.

While $\text{Temper} > \text{Temper}_f$

Select one entry (i) of vector K randomly, and change its value, also between 1 and N, randomly.

Reconstruct vector M.

Select one entry (j) of vector Y randomly, and switch it from one to zero or zero to one.

Calculate x_t for $t = 1$ to T with respect to vector Y.

Calculate fitness value (F).

If $F \leq \text{Best\_}F$, set $\text{Best\_}F = F$.

Else with probability $\exp(-(F - \text{Best\_}F)/\text{Temper})$, $\text{Best\_}F = F$.

End if

$\text{Temper}=\text{Temper}^* cr$

End

Figure 1. Simulated annealing pseudo code for SILSP-MW.

performance of the proposed SA algorithm with that of the heuristics proposed by Ho et al. [17], where the number of warehouses is equal to 1. Finally, an experiment is designed to evaluate the performance of the SA algorithm, and to investigate the effect of the parameters on the SA algorithm.

4.1. Numerical example

The sample problem instances have been taken from Ho et al. [17], which were selected from the 20 benchmark problems that appeared in Kaimann [43], Berry [44] and Baker [45]. The time horizon consists of 12 periods (T=12), and the demands for the 12 periods are: 10, 10, 15, 20, 70,180, 250, 270, 230, 40, 0, and 10. For all periods, the setup cost and the production unit cost are equal to 92 (S=92) and 10 (P=10), respectively. Furthermore, we suppose that there are 2 warehouses such that for each period i and j and warehouse k, the deterioration rate is $\alpha_{ijk} = \beta_k.(j - i + 1)$, and the holding cost is $h_{ijk} = \lambda_k.(j - i + 1)$, where β_k and λ_k are the base deterioration rate and the base holding cost for warehouse k, respectively. It is clear that the deterioration rate and the holding cost for each warehouse directly depend on the holding periods. We solve this problem for 5 instances with $(\beta_1 = 0.005, \beta_2 = 0.01), (\beta_1 = 0.01, \beta_2 = 0.015), (\beta_1 = 0.015, \beta_2 = 0.02), (\beta_1 = 0.02, \beta_2 = 0.025)$ and $(\beta_1 = 0.025, \beta_2 = 0.03)$, using the SA algorithm, and compare the results with the optimal solution. The parameters λ_1 and λ_2 for all instances are fixed and equal to 3 and 2, respectively. To obtain the optimal solutions, we solved the mixed-integer linear model with the LINGO optimization software. Table 1 displays the results for each instance. As can be observed in Table 1, the proposed SA algorithm yields the optimal solution in all instances. Furthermore, detailed solution results for the first instance are shown in Table 2. We illustrate how the SA algorithm solves this instance. The final iteration of the SA algorithm is further illustrated. In the final iteration, the vectors K and Y are obtained as:

$$K = [21212121212],$$

and:

$$Y = [101011111101].$$

Table 1. Total cost of SILSP-MW for five instances, using an exact and SA algorithms.

Instance	β_1	β_2	Optimal	SA	Deviation percentage
1	0.005	0.01	112418.9	112418.9	0
2	0.01	0.015	12434.6	12434.6	0
3	0.015	0.02	12450.4	12450.4	0
4	0.02	0.025	112466.15	112466.15	0
5	0.025	0.03	112471.5	112471.5	0

Table 2. Detailed solution results for the first instance.

Period	d_i	SA solution			Optimal solution	
		Selected warehouse(k)	Y	x	x	Selected warehouse(k)
1	10	2	1	20.10101	20.10101	2
2	10	1	0	0	0	-
3	15	2	1	35.20202	35.20202	2
4	20	1	0	0	0	-
5	70	2	1	70	70	-
6	180	1	1	180	180	-
7	250	2	1	250	250	-
8	270	1	1	270	270	-
9	230	2	1	240	240	-
10	40	1	1	40	40	-
11	0	2	0	0	0	-
12	10	-	1	10	10	-
Total cost	-	112418.9			112418.9	

$$M=\begin{pmatrix}
10 & 20.10 & 35.40 & 56.44 & 131.57 & 334.94 & 626.13 & 964.28 & 1291 & 1350.84 & 1350.84 & 1368.54\\
0 & 10 & 25.1 & 45.6 & 118.5 & 313.7 & 591.8 & 911.3 & 1216.7 & 1272 & 1272 & 1288.1\\
0 & 0 & 15 & 35.2 & 106.6 & 296 & 564.3 & 869.3 & 1157.8 & 1209.5 & 1209.5 & 1224.3\\
0 & 0 & 0 & 20 & 90.4 & 274.9 & 535.2 & 828.1 & 1102 & 1150.6 & 1150.6 & 1164.3\\
0 & 0 & & 0 & 70 & 251.8 & 506.9 & 790.9 & 1053.9 & 1100 & 1100 & 1112.8\\
0 & 0 & 0 & 0 & 0 & 180 & 431.3 & 708.2 & 961.9 & 1005.9 & 1005.9 & 10018\\
0 & 0 & 0 & 0 & 0 & 0 & 250 & 522.7 & 770.1 & 812.6 & 812.6 & 824\\
0 & 0 & 0 & 0 & 0 & 0 & 270 & 512.4 & 553.7 & 553.7 & 564.6 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 240 & 280.4 & 280.4 & 290.9\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 40 & 40 & 50.3\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10
\end{pmatrix}$$

Box I.

Using vector K, a 12×12 matrix M is calculated as summarized in the following in Box I.

Then, using vector Y, the production quantity in each period is computed as follows:

$$x_1 = M_{1,2} = 20.10, \qquad x_2 = 0,$$

$$x_3 = M_{3,4} = 35.2, \qquad x_4 = 0,$$

$$x_5 = M_{5,5} = 70, \qquad x_6 = M_{6,6} = 180,$$

$$x_7 = M_{7,7} = 250, \qquad x_8 = M_{8,8} = 270,$$

$$x_9 = M_{9,9} = 240, \qquad x_{10} = M_{10,10} = 40,$$

$$x_{11} = 0, \qquad x_{12} = M_{12,12} = 10.$$

4.2. Comparative study

In this section, we apply the proposed SA algorithm for the problem reported by Ho et al. [17], and compare the results with the results obtained from their heuristics. We assume that the number of warehouses is equal to 1 and suppose that the deterioration rate and the holding cost are not time-dependent ($\alpha_{ijk} = \alpha$ and $h_{ijk} = h$) to have the same conditions. Ho et al. [17] considered the Single Item Lot-Sizing Problem (SILSP) with a deteriorating inventory and modified certain heuristics, consisting of nLPC, LTC and PPA(-), to solve the problem. They calculated the total relevant costs according to Eq. (13), and compared the modified heuristics with the modified LUC and LPC heuristics. They observed that nLPC has the best performance among these heuristics. We apply our algorithm to their examples, and compute the total relevant cost instead of the total cost. Then, we compare the results with those of nLPC. Computational testing was performed on several instances of the problem. The number of periods and the demand in each period are the same as in the above numerical example, and the values $h = 2$, $p = 100$ and $S = 92$ are assumed. Furthermore, the problem is solved with $\alpha = 0, 0.005, 0.01, 0.015, 0.02$ and 0.025. The results in Table 3 show that the SA algorithm has yielded the optimal solution for each of the six instances, whereas nLPC yields the optimal solution in five of the six instances. Therefore, it is concluded that the proposed SA algorithm outperforms nLPC.

$$TRC(i,j) = S + h.\sum_{x=i+1}^{j}\sum_{y=i}^{x-1} d(x)/(1-\alpha)^{y-i+1} + P.\sum_{x=i+1}^{j}(d(x)/(1-\alpha)^{x-i} - d(x), \quad \text{for} \quad j > i, \tag{13}$$

and:

$$TRC(i,j) = S, \quad \text{for} \quad j = i.$$

Table 3. The total relevant cost for comparative evaluation between the SA algorithm and the nLPC heuristic.

Instance	α	Optimal	nLPC	SA
1	0.000	836.00	844.00	836.00
2	0.005	861.75	861.75	861.75
3	0.010	887.82	887.82	887.82
4	0.015	914.21	914.21	914.21
5	0.020	940.91	940.91	940.91
6	0.025	966.15	966.15	966.15
	Mean	901.14	902.47	901.47

4.3. An experimental design for additional analysis

An experiment was designed and performed for an optimality sensitivity analysis of the proposed SA for small- and medium-sized problems. A full factorial experiment is conducted by varying the problem parameters consisting of T (the number of periods), N (the number of warehouses), r (the ratio of the setup cost to the base holding costs mean; that is $S/(\sum_{i=1}^{N}\lambda_i/N)$), and $\bar{\beta}$(the mean of the base deterioration rate for the warehouses.)In this analysis, the number of periods was set at either of the two values: $T = 10$ and 30. Demand in each period is generated by a discrete uniform distribution between 0 and 100; that is DU(0, 100). The number of warehouses is also set at one of three values: $N = 2, 5$ and 10. The ratio r is set to one of three values: 10, 40 and 100, and $\bar{\beta}$ is set to one of three values: 0.003, 0.007 and 0.01. The problems with 10 and 30 periods were considered small- and medium-sized problems, respectively. Table 4 shows the selected parameters and their values. Combinations of these four parameters produced a total of 54 (2×3×3×3) experiments. Each experiment was repeated 10 times. The proposed SA algorithm was coded in the MATLAB programming environment.

4.4. Computational results

For the designed experiments, the SA performance can be analyzed by comparing the results with the optimal solution. Tables 5 and 6 show the average differences from the optimal solution for each conducted experiment. On average, there is only a 0.0314% difference from the optimum for small-sized problems, and the average difference from the optimum is 0.0786% for medium-sized problems. This verifies the algorithm performance. Notably, the optimal solution was obtained using the LINGO optimization package.

An analysis of variance (ANOVA) was used to identify the parameters with a significant effect on the performance of the proposed SA algorithm. The results of the ANOVA at a 95% confidence level are shown in Figure 2. As shown in Table 7, T (the number of periods), N (the number of warehouses) and r(the ratio of the setup cost to the mean base holding costs) have a significant effect on the SA performance, whereas $\bar{\beta}$ (the mean base deterioration rate for warehouses) has no significant effect.

Table 4. Selected parameters and their values for the designed experiment.

Parameters	Values
T	10, 30
N	2, 5, 10
R	10, 40, 100
$\bar{\beta}$	0.003 ,0.007 ,0.01

Table 5. Computational results for small-sized ($T = 10$) problems

N	r	$\bar{\beta}$	% Mean deviation from optimum (gap)
2	10	0.003	0
		0.007	0
		0.01	0
	40	0.003	0
		0.007	0
		0.01	0
	100	0.003	0
		0.007	0.0064
		0.01	0.017
5	10	0.003	0
		0.007	0.0031
		0.01	0.0037
	40	0.003	0.027
		0.007	0.0009
		0.01	0.0067
	100	0.003	0.098
		0.007	0.0102
		0.01	0.08
10	10	0.003	0.002
		0.007	0.0014
		0.01	0.0015
	40	0.003	0.027
		0.007	0.014
		0.01	0.01
	100	0.003	0.173
		0.007	0.152
		0.01	0.12
Mean			0.0314

Source	DF	Seq SS	Adj SS	Adj MS	F	P
T	1	0.30109	0.30109	0.30109	49.70	0.000
N	2	0.54005	0.54005	0.27003	44.58	0.000
r	2	1.76961	1.76961	0.88481	146.07	0.000
$\bar{\beta}$	2	0.02289	0.02289	0.01145	1.89	0.152
Error	532	3.22265	3.22265	0.00606		
Total	539	5.85630				

$R-Sq = 74.97$ $\quad R-Sq(adj)$=64.25

Figure 2. Results of ANOVA at 95% confidence level for selected parameters.

Furthermore, Figure 3 illustrates the effect of each parameter on the percent difference from the optimum.

5. Conclusions

In this paper, we presented a mixed integer linear programming model for the single-item, single-level

Table 6. Computational results for medium-sized ($T = 30$) problems.

N	r	$\bar{\beta}$	% Mean deviation from optimum (gap)
2	10	0.003	0
		0.007	0
		0.01	0
	40	0.003	0.0019
		0.007	0.0024
		0.01	0.0041
	100	0.003	0.0835
		0.007	0.0640
		0.01	0.0656
5	10	0.003	0.0222
		0.007	0.1089
		0.01	0.0138
	40	0.003	0.036
		0.007	0.0332
		0.01	0.326
	100	0.003	0.2325
		0.007	0.1012
		0.01	0.1962
10	10	0.003	0.0068
		0.007	0.0032
		0.01	0.0015
	40	0.003	0.0681
		0.007	0.0507
		0.01	0.0439
	100	0.003	0.3616
		0.007	0.3304
		0.01	0.2583
Mean			0.0786

Table 7. Significant and insignificant parameters.

Parameters	Responses (% deviation from optimum)
T	√
N	√
r	√
$\bar{\beta}$	*

√: Significant parameter;

*: Insignificant parameter.

lot-sizing problem with a deteriorating inventory and multiple warehouses. Because this problem can be solved optimally with exact solutions in a reasonable time only for small-sized instances, an SA algorithm was developed to solve the problem. The performance of this algorithm was investigated in comparison with

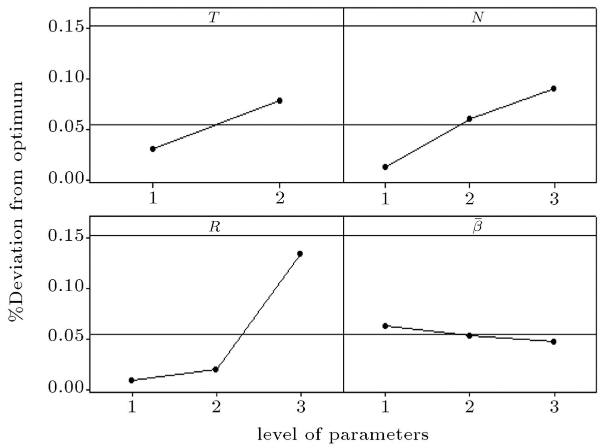

Figure 3. Parameter effects on the % difference from the optimum.

the optimal solution for small- and medium-sized problems. The results confirmed the improved performance of the proposed Simulated Annealing algorithm. In addition, the effects of four parameters, T (the number of periods), N (the number of warehouses), r (the ratio of the setup cost to the mean base holding costs) and β (the mean base deterioration rate for warehouses), on the SA performance, were investigated. For future research, the model can be extended to a multi-level lot-sizing problem. It would also be interesting to consider both a deteriorating inventory and disposal costs in lot-sizing with multiple warehouses in MRP systems.

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