Multivariable control of an industrial boiler-turbine unit with nonlinear model: A comparison between gain scheduling and feedback linearization approaches

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Abstract. Due to demands for the economical operations of power plants and environmental awareness, performance control of a boiler-turbine unit is of great importance. In this paper, a nonlinear Multi Input-Multi Output model (MIMO) of a utility boiler-turbine unit is considered. Drum pressure, generator electric output and drum water level (as the output variables) are controlled by manipulation of valves position for fuel, feed-water and steam flows. After state space representation of the problem, two controllers, based on gain scheduling and feedback linearization, are designed. Tracking performance of the system is investigated and discussed for three cases of 'near', 'far' and 'so far' set-points. According to the results obtained, using feedback linearization approach leads to more quick time responses with a bit more overshoots (in comparison with the gain scheduling method). In addition, in feedback linearization strategy, input control signals (valves position) are actuated in less time with less oscillations. It is observed that in the presence of an arbitrary random uncertainty in model parameters, the controller designed based on feedback linearization is more robust. Finally, according to the phase portraits of the problem, as the desired speed of tracking process is increased, dynamic system tends to demonstrate a chaotic behaviour.

1. Introduction

Industrial boiler-turbine units are extensively used for steam generation as a source of power or for achieving heating capabilities in thermal plants. Due to dynamic interaction between various components, such as furnace, evaporator, super-heaters, economizer, attemperator and drum, these units are inherently nonlinear systems. For the electricity generation, two configurations can be realized [1]. In the first

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configuration, as called boiler-turbine unit, the steam is produced by a single boiler and is fed to a single turbine, as shown in Figure 1 [2]. In the second one, several boilers generate total steam conducted to a collector and then distributed to several turbines. Since the boiler-turbine units show quick responses for the electricity demand from a power network, they are preferred to collector type systems.

Several dynamic models of the boiler system have been developed. In early works, dynamic modelling of a boiler-turbine unit based on data logs, parameter estimation [3-5], system identification [6] and simplification of nonlinear models [7] has been done. Also, several simulation packages such as SYNSIM for steam
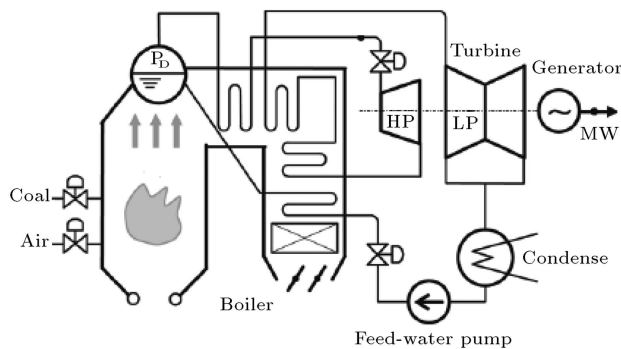
Figure 1. Schematic of a boiler-turbine unit [2].

plants [8], and simulation of large boilers with natural recirculation [9] have been carried out. Using basic conservation rules, a model for water level dynamics in natural circulation of drum-type boilers has been developed [10]. Using physical and neural networks principles, dynamic nonlinear modelling of power plant has been investigated [11]. In addition, other nonlinear models of the boiler-turbine units have been presented in [2,4,12-14].

In the case of large changes in operating conditions, effective control systems must be developed to have an appropriate performance of the boiler-turbine units. Various control methods have been used for boiler or boiler-turbine units. In the early works, linear optimal regulators [15,16] and decoupling controller [17] for performance control of boiler-turbine units have been designed. Also, multivariable predictive control based on local model networks [18], fuzzy-based control systems for thermal power plants [19,20], neuro-fuzzy network modelling and PI control of a steam-boiler system [21] have been presented. A loop-by-loop approach for water circulation control during once-through boiler start-up [22] and life extending control of boiler-turbine units by model predictive methods [23] have been investigated.

In some researches, linear controllers are designed for the nonlinear model of boiler-turbine units. For this purpose, nonlinearity is avoided by selecting the appropriate operating zones, such that the linear controllers perform effectively [1,2]. In other works, by constituting the linear parameter varying model of nonlinear boiler-turbine unit, gain scheduled optimal control [24] and approximate feedback linearization [25] have been applied. For robust performance of the boiler-turbine units, locally robust intelligent supervisory system [26], control design based on adaptive Grey predictor algorithm [27], backstepping-based nonlinear adaptive control [28], sliding mode and H_∞ robust controllers [29-33] have been designed.

For a wide range of operating conditions, conventional PID/PI type controllers and linear multivariable controllers based on LQG/LQR theory cannot result in a satisfactory performance. On the other hand

for nonlinear models of the boiler-turbine units, using fuzzy or robust control methods have the hindrance of disturbance estimation and rejection.

In this paper, unlike the previous works, gain scheduling and feedback linearization approaches are used for performance control of a multivariable nonlinear model of a boiler-turbine unit. By manipulation of valves position for fuel, feed-water and steam flows, tracking objective from an operating point to a 'near', 'far' and 'so far' operating points is achieved (for drum pressure, generator electric output and drum water level). Results are discussed and compared for both control approaches. According to the results, using feedback linearization method leads to more quick time responses of output variables, while input control signals associate with less oscillation. As a general inspection of the controller's robustness, an arbitrary uncertain model of the boiler-turbine unit is considered (while the controllers designed for the nominal plant are used). Results show that the desired tracking objectives are achieved for output variables in both methods, but electric output signal associates with some oscillations (especially in gain scheduling strategy). However, this problem can be solved by decreasing the speed of tracking objectives. Constructing the phase portraits of the problem, it is shown that by increasing the speed of tracking process, a chaotic behaviour of the dynamic system is occurred.

2. Performance and nonlinear dynamics of a boiler-turbine unit

Figure 1 shows a water-tube boiler in which preheated water is fed into the steam drum and flows through the down-comers into the mud drum [2]. Passing through the risers, water is heated and changes to the saturation condition. This saturated mix of steam and water enters the steam drum. There, steam is separated from water and flows into the primary and secondary super-heaters. Steam is more heated then, and is fed into the header. There is a spray attemperator between the two super-heaters that regulates the steam temperature by mixing low temperature water with the steam from the primary super-heater.

In this research, nonlinear dynamic model of a boiler-turbine unit presented by Bell and Astrom is used [4]. Parameters of this model were estimated by data measurement from the Synvendska Kraft AB Plant in Malmo, Sweden. As shown in Figure 2 [24], output variables are denoted by y_1 for drum pressure (kgf/cm^2), y_2 for electric output (MW) and y_3 for drum water level (m). Input variables are denoted by u_1, u_2 and u_3 for valves position of fuel flow, steam flow and feed-water flow, respectively. Dynamics of this 160 MW oil-fired unit is given in state space representation
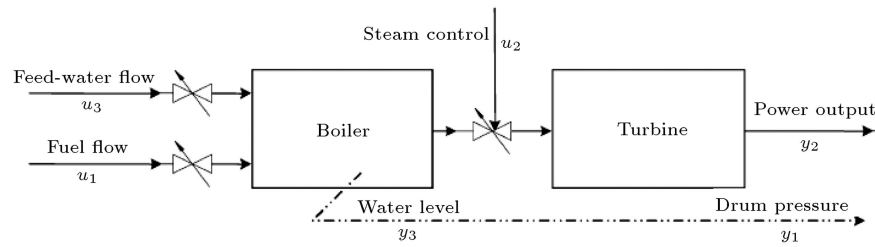
Figure 2. Multi input-output variables of the boiler-turbine unit.

as [4]:

$$\dot{x}_1 = -0.0018u_2 x_1^{9/8} + 0.9u_1 - 0.15u_3,$$

$$\dot{x}_2 = (0.073u_2 - 0.016)x_1^{9/8} - 0.1x_2,$$

$$\dot{x}_3 = [141u_3 - (1.1u_2 - 0.19)x_1]/85,$$

$$y_1 = x_1,$$

$$y_2 = x_2,$$

$$y_3 = 0.05(0.13073x_3 + 100a_{cs} + q_e/9 - 67.975), \quad (1)$$

where x_3 denotes fluid density (kg/m^3), a_{cs} and q_e are the steam quality and evaporation rate (kg/s), respectively, and are given by:

$$a_{cs} = \frac{(1 - 0.001538x_3)(0.8x_1 - 25.6)}{x_3(1.0394 - 0.00123404x_1)},$$

$$q_e = (0.854u_2 - 0.147)x_1 + 45.59u_1 - 2.514u_3 - 2.096. \quad (2)$$

Due to actuator limitations, control inputs and their rates are limited to:

$$0 \leq u_i \leq 1,$$

$$-0.007 \leq \dot{u}_1 \leq 0.007,$$

$$-2 \leq \dot{u}_2 \leq 0.02,$$

$$-0.05 \leq \dot{u}_3 \leq 0.05, \qquad (i = 1, 2, 3). \quad (3)$$

Table 1 gives some typical operating points of the Bell and Astrom model where the nominal system is working at operating point # 4 [1,4].

3. Control of the nonlinear boiler-turbine unit: Results and discussion

For proper performance of the boiler- turbine unit, control system must satisfy some requirements according to the varying operating conditions and load demands. Electricity output must be followed by the variation in demands from a power network. Steam pressure of

Table 1. Typical operating points of Bell and Astrom model [1,4].

	# 1	# 2	# 3	# 4	# 5	# 6	# 7
x_1^0	75.6	86.4	97.2	108	118.8	129.6	140.4
x_2^0	15.27	36.65	50.52	66.65	85.06	105.8	128.9
x_3^0	299.6	342.4	385.2	428	470.8	513.6	556.4
u_1^0	0.156	0.209	0.271	0.34	0.418	0.505	0.6
u_2^0	0.483	0.552	0.621	0.69	0.759	0.828	0.897
u_3^0	0.183	0.256	0.34	0.433	0.543	0.663	0.793
y_3^0	-0.97	-0.65	-0.32	0	0.32	0.64	0.98

the collector must be maintained constant, despite the variations in the network load. Also, to prevent overheating of drum components or flooding of steam lines, water level of the steam drum must be kept at the desired value [2]. In addition, the physical constraints exerted on the actuators must be satisfied by the control signals. These constraints are the magnitude and saturation rate for the control valves of the fuel, steam and feed-water flows [24].

3.1. Controller design based on gain scheduling approach

Controllers designed via linearization approach have this limitation that work properly in the neighbourhood of a single operating point (equilibrium point). Gain scheduling technique can guarantee the validity of linearization approach to a range of operating points. Usually, it is possible to found how the dynamics of a system vary with its operating point. It may even be possible to parameterize the operating points by one or more variables which are called scheduling variables. Under this condition, system is linearized at several operating points, and a linear feedback controller is designed at each point. This family of linear controllers can be implemented as a single controller whose parameters change by monitoring the scheduling variables [34]. As a result, better performance with robustness is achieved for a large range of operating zones.

Consider again the dynamic model given by Eq. (1). To maintain the system around each operating point of Table 1 at state vector $\bar{x}^0 = \begin{bmatrix} x_1^0 & x_2^0 & x_3^0 \end{bmatrix}$, a constant input vector $\bar{u}^0 = \begin{bmatrix} u_1^0 & u_2^0 & u_3^0 \end{bmatrix}$ must be imposed. To have simpler math equations, let define

the new variables as:

$$\varphi_1 = x_1^0, \qquad \varphi_2 = x_2^0, \qquad \varphi_3 = x_3^0,$$

$$\psi_1 = u_1^0, \qquad \psi_2 = u_2^0, \qquad \psi_3 = u_3^0. \qquad (4)$$

Linearizing Eq. (1) around any operating points of Table 1, yields:

$$\dot{\bar{x}}_\delta = A(\varphi_i, \psi_i)\bar{x}_\delta + B(\varphi_i, \psi_i)\bar{u}_\delta, \qquad i = 1, 2, 3,$$

$$\bar{x}_\delta = \bar{x} - \bar{x}^0, \bar{u}_\delta = \bar{u} - \bar{u}^0, \qquad (5)$$

where:

$$A(\varphi_i, \psi_i) = \begin{bmatrix} -0.00202\psi_2\varphi_1^{1/8} & 0 & 0 \\ 1.125(0.073\psi_2 - 0.016)\varphi_1^{1/8} & -0.1 & 0 \\ -\frac{1}{85}(1.1\psi_2 - 0.19) & 0 & 0 \end{bmatrix},$$

$$B(\varphi_i, \psi_i) = \begin{bmatrix} 0.9 & -0.0018\varphi_1^{9/8} & -0.15 \\ 0 & 0.073\varphi_1^{9/8} & 0 \\ 0 & -\frac{1.1}{85}\varphi_1 & \frac{141}{85} \end{bmatrix}. \qquad (6)$$

In state feedback control scheme, to achieve the desired locations of closed-loop control system and consequently the desired performance of the system, the control vector $\bar{u}_\delta$, is constructed as:

$$\bar{u}_\delta = -K(\varphi_i, \psi_i)\bar{e},$$

$$\bar{e} = \bar{x}_\delta - \bar{r}_\delta, \bar{r}_\delta = \bar{y}_R - \bar{y}^0, \qquad (7)$$

where $K(\varphi_i, \psi_i)$ is the variable gain matrix adjusted according to the monitored scheduling variables; $\bar{e}$ is the error vector, $\bar{y}_R$ is the command vector signal that must be tracked, and $\bar{y}^0 = \begin{bmatrix} y_1^0 & y_2^0 & y_3^0 \end{bmatrix}$ is the output vector defined in terms of state variables by Eq. (1) at each operating point of Table 1. Substituting Eqs. (6) and (7) in first derivative of Eq. (5), yields:

$$\dot{\bar{x}}_\delta = [A(\varphi_i, \psi_i) - B(\varphi_i, \psi_i)K(\varphi_i, \psi_i)]\bar{x}_\delta$$

$$+ B(\varphi_i, \psi_i)K(\varphi_i, \psi_i)\bar{r}_\delta. \qquad (8)$$

A schematic of the proposed control approach is shown in Figure 3. The procedure of designing the feedback gain matrix for the MIMO system is given in Appendix A. It is assumed that a maximum overshoot of $M_p = $

10% and settling time of about $t_s = 150$ s in tracking behaviour of all output variables is desired. To achieve this, closed-loop poles of the system (including a far non-dominant pole, $\mu_3 = -0.15$) must be assigned as:

$$\mu_{1,2} = -0.03 \pm 0.04j, \qquad \mu_3 = -0.15.$$

Finding transformation $\tilde{T}$ and matrices A_d, F, H and Γ (as given in Appendix A), and using Eq. (A.8), feedback gain matrix, $K(\varphi_i, \psi_i)$, is found. Considering the operating points given in Table 1, results are presented for three case studies of tracking objective as:

1. From the nominal operating point # 4 to a 'near' operating point # 5;

2. From the nominal operating point # 4 to the 'far' operating point # 7;

3. From the operating point # 1 to a 'so far' operating point #7.

Figure 4 shows the time responses of state and output variable defined by Eq. (1), where $y_1 = x_1$ is the drum pressure (kg/cm^2), $y_2 = x_2$ is the generator electric output (MW), x_3 is the fluid density (kg/m^3) and y_3 is the drum water level (m). It must be noticed that direct control is provided for state variables, while drum water level is obtained from Eq. (1), without direct control. Overshot and settling time parameters of these three cases for state variables are given in Table 2.

According to Figure 4 and Table 2, for all three cases, electric output signal shows more quick response (less settling time) with more overshot, with respect to the drum pressure and fluid density. For drum pressure and fluid density responses, tracking from operating point # 1 to the 'so far' point # 7 associates with more overshoot. This behaviour physically indicates that dynamic system is more sensitive in tracking large values of drum pressure or fluid density (and consequently drum water level). However, for the electric output, more overshoot is seen in tracking from point # 4 to the 'near' point # 5. Therefore, following a near operating point, based on gain scheduling approach, has minor negative effects on power grid, due to the more oscillatory behaviour.

Figure 5 shows time responses of the required input control signals (where u_1, u_2 and u_3 stand for
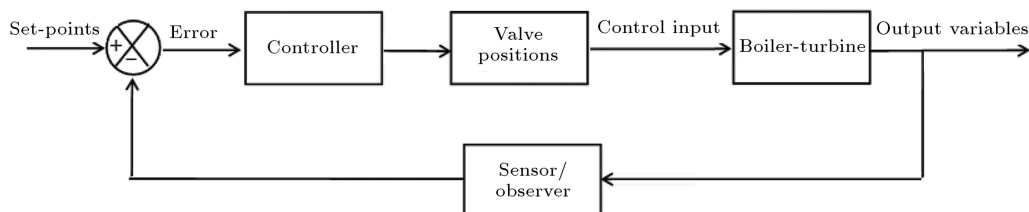

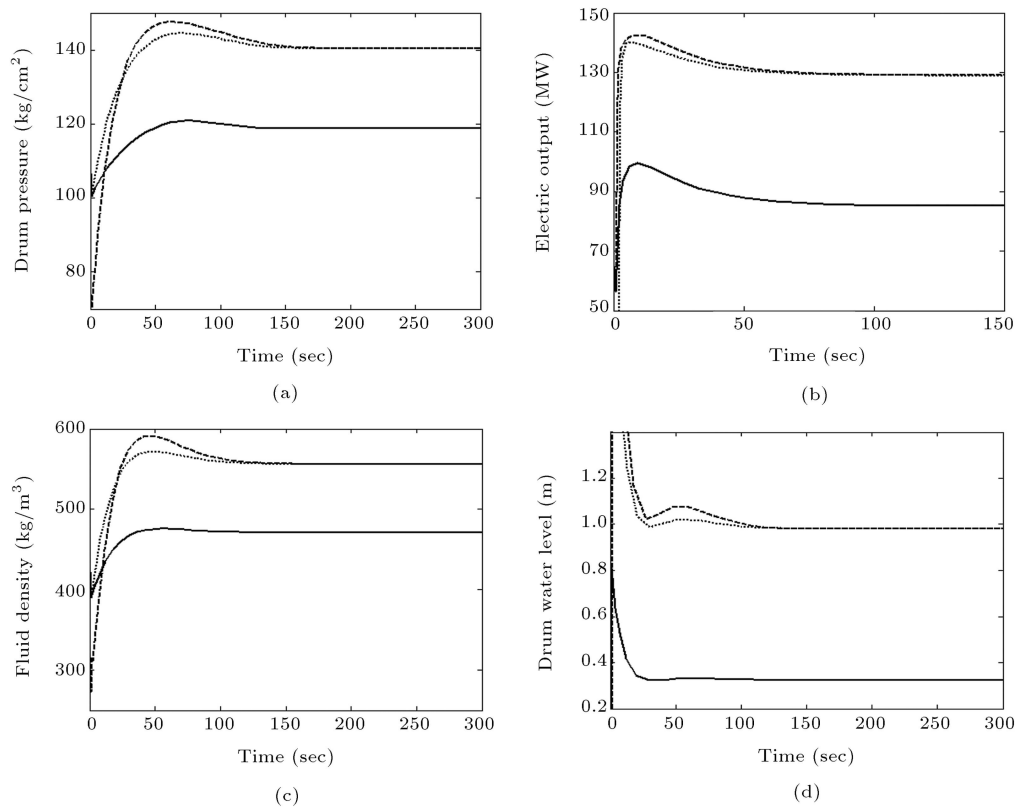**Figure 3.** Closed-loop control system for the boiler-turbine unit.

Figure 4. Time response of state and output variables, using gain scheduling approach for three cases: from operating point # 4 to 5 (solid lines), # 4 to 7 (dots) and #1 to 7 (dashed lines).

Table 2. Performance parameters in tracking behavior of 'near', 'far' and 'so far' set-points.

		Gain scheduling			Feedback linearization		
	Set-points path	# 4 to 5	# 4 to 7	# 1 to 7	# 4 to 5	# 4 to 7	# 1 to 7
x_1	t_s	170 s	170 s	170 s	100 s	100 s	100 s
	M_p	2%	3%	5%	2%	4%	7%
x_2	t_s	100 s	100 s	100 s	90 s	90 s	90 s
	M_p	15%	8%	9%	17%	13%	15%
x_3	t_s	180 s	180 s	180 s	180 s	180 s	180 s
	M_p	2%	3%	6%	3%	5%	11%

valves position of fuel flow, steam control and feed-water flow, respectively). According to Figure 5, valves position for fuel and feed-water flows in tracking from operating point # 1 to the 'so far' point # 7 are stronger in magnitude with more oscillation, with respect to the same signals for 'near' and 'far' set-point cases. This result is physically expected because the fuel and feed-water flows play a direct role in dynamic behaviour of the system. Therefore, for further tracking objectives, greater amounts of fuel and feed-water flow rates are required. But valves position for steam control shows more oscillation in the tracking objective from operating point # 4 to the 'near' point # 5 (with respect to the same signal for 'far' and 'so far' set-point cases). This result is in correlation with what was observed for the electric

output signal (Figure 4). This is because the electric output is essentially affected by the amount of valve position for steam flow.

3.2. Controller design based on feedback linearization approach

The central concept of feedback linearization is to transform dynamics of a nonlinear system into a fully or partly linear one. Then, various powerful linear control techniques can be applied to complete the control design process. In this approach, the nonlinear terms of the dynamic system are eliminated by means of state variables feedback. Finally, an appropriate controller is designed to stabilize the desired trajectories of the system [35].
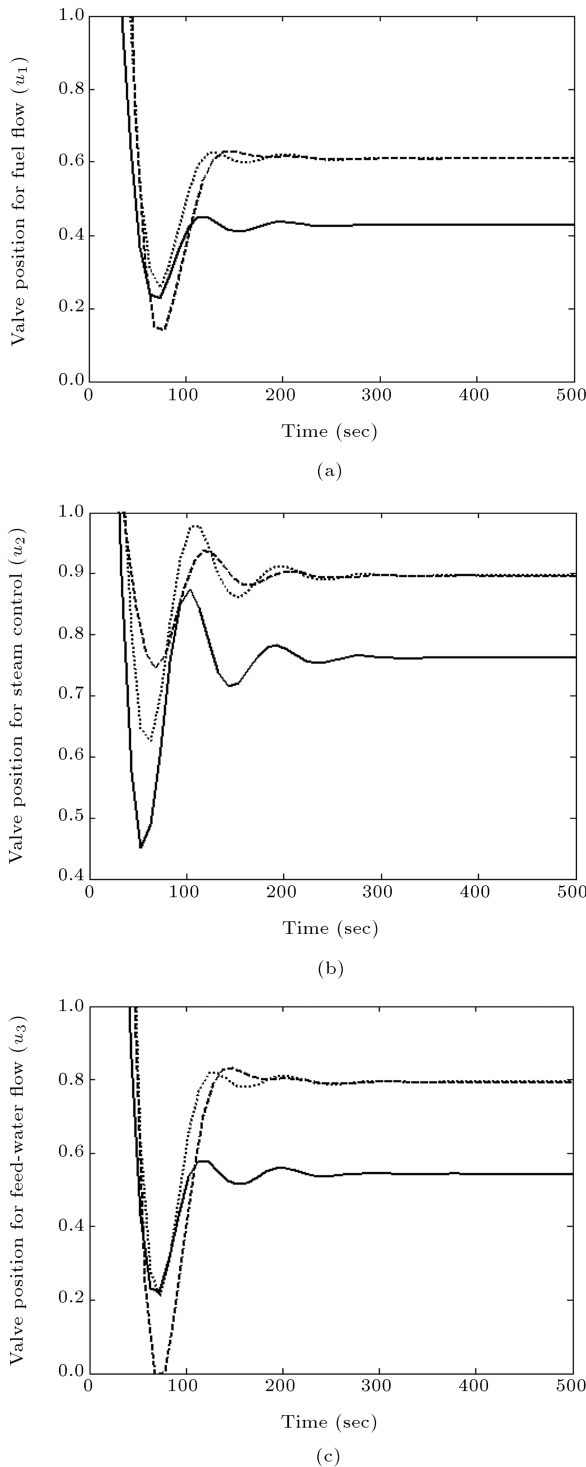
Consider a square MIMO system (multi input-

Figure 5. Time response of the required input control signals, using gain scheduling approach for three cases: from operating point # 4 to 5 (solid lines), # 4 to 7 (dots) and #1 to 7 (dashed lines).

multi output system with the same number of inputs and outputs) in the neighbourhood of a the operating point $\bar{x}^0$ as [35]:

$$\dot{\bar{x}} = f(\bar{x}) + G(\bar{x})\bar{n}, \qquad \bar{y} = h(\bar{x}), \tag{9}$$

where, $\bar{x}$ is $n \times 1$ state vector, $\bar{u}$ is $r \times 1$ control input vector, $\bar{y}$ is $m \times 1$ output vector, f and h are smooth vector fields and G is a $n \times r$ matrix whose columns (g_i) are smooth vector fields (in this paper, $m = r = 3$). Similar to the SISO systems, input-output linearization of MIMO cases is obtained by differentiating the outputs y_i until the inputs appear. In this paper, y_i^j represents output y_i at operating point j, while $y_i^{(j)}$ represents the differentiation of y_i of order j. Assume that λ_i is the smallest integer that at least one of the inputs appears in $y_i^{(\lambda_i)}$, then:

$$y_i^{(\lambda_i)} = L_f^{\lambda_i} h_i + \sum_{j=1}^{r} L_{g_j} L_f^{\lambda_i - 1} h_i u_j, \tag{10}$$

with $L_{g_j} L_f^{\lambda_i - 1} h_i(x) \neq 0$ for at least one j in a neighbourhood Ω_i of the operating point $\bar{x}^0$ (operations $L_f h$, $L_{f^i} h$ and $L_g L_{f^i} h$ are defined in Appendix B). Applying the same procedure for each output, y_i, yields:

$$\begin{bmatrix} y_1^{(\lambda_1)} \\ \cdots \\ \cdots \\ y_m^{(\lambda_m)} \end{bmatrix} = \begin{bmatrix} L_f^{\lambda_1} h_1(\bar{x}) \\ \cdots \\ \cdots \\ L_f^{\lambda_m} h_m(\bar{x}) \end{bmatrix} + E(\bar{x})\bar{u}, \tag{11}$$

where $r \times r$ matrix $E(\bar{x})$ is defined. The system of Eq. (9) is said to have relative degree $(\lambda_1, \lambda_2, \cdots, \lambda_m)$ at $\bar{x}^0$, and the scalar $\lambda = \lambda_1 + \cdots + \lambda_m$ is called the total relative degree of the system. Let Ω represents the intersection of Ω_i. If all the partial relative degrees, λ_i, are well defined, then Ω is a finite neighbourhood of $\bar{x}^0$. In addition, if $E(\bar{x})$ is invertible over the region Ω, the input transformation:

$$\bar{u} = E^{-1} \begin{bmatrix} v_1 - L_f^{\lambda_1} h_1 & \cdots & \cdots & v_m - L_f^{\lambda_m} h_m \end{bmatrix}^T, \tag{12}$$

yields a simpler form of m equations as:

$$y_i^{(\lambda_i)} = v_i. \tag{13}$$

Eq. (12) is called a decoupling control law, because the output y_i is only affected by the input v_i, and the invertible matrix $E(\bar{x})$ is called the decoupling matrix of the system.

In this section, to avoid tedious computation caused by differentiation of y_3, as given in Eq. (1), third state variable is chosen as the third output (instead of water level of drum, the fluid density is considered as the third output, $y_3 = x_3$). Through presented results, it will be shown that this definition of y_3 will not affect the control of real output (drum water level) represented by Eq. (1). By differentiating from y_i of Eq. (1), inputs will appear after first differentiation, so for all outputs $\lambda_i = 1$. Substituting $y_3 = x_3$ into Eq.

(1), and differentiating from it, yields:

$$
\begin{bmatrix} y_1^{(1)} \\ y_2^{(1)} \\ y_3^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1x_2 - 0.016x_1^{9/8} \\ \frac{0.19}{85}x_1 \end{bmatrix}
$$

$$
+ \begin{bmatrix} 0.9 & -0.0018x_1^{9/8} & -0.15 \\ 0 & 0.073x_1^{9/8} & 0 \\ 0 & -\frac{1.1}{85}x_1 & \frac{141}{85} \end{bmatrix} \bar{u}. \qquad (14)
$$

According to Eq. (12), control signal $\bar{u}$ is constructed as:

$$
\bar{u} = E^{-1} \begin{bmatrix} v_1 \\ v_2 + 0.1x_2 + 0.016x_1^{9/8} \\ v_3 - \frac{0.19}{85}x_1 \end{bmatrix}, \qquad (15)
$$

where:

$$
E = \begin{bmatrix} 0.9 & -0.0018x_1^{9/8} & -0.15 \\ 0 & 0.073x_1^{9/8} & 0 \\ 0 & -\frac{1.1}{85}x_1 & \frac{141}{85} \end{bmatrix}. \qquad (16)
$$

Using this control law results in three separate dynamics for three outputs as:

$$
y_i^{(1)} = v_i, \qquad i = 1, 2, , 3. \qquad (17)
$$

After decoupling the outputs dynamics, a PI controller is designed as:

$$
v_i = -K_{1i}e_i - K_{2i}\sigma_i, \qquad \dot{\sigma}_i = e_i = y_i - r_i, \qquad (18)
$$

where r_i is the command input signal that is desired to be tracked. Differentiating from Eq. (17) yields:

$$
\ddot{y}_i + K_{1i}\dot{y}_i + K_{2i}y_i = K_{1i}\dot{r}_i + K_{2i}r_i. \qquad (19)
$$

Transforming this equation into the Laplace domain, yields:

$$
\frac{Y_i(s)}{R_i(s)} = \frac{K_{1i}s + K_{2i}}{s^2 + K_{1i}s + K_{2i}}. \qquad (20)
$$

If the closed loop system is expected to have a behaviour similar to the system with the following characteristic equation:

$$
s^2 + 2\zeta\omega_n s + \omega_n^2 = 0, \qquad \omega_n > 0, \qquad 0 < \xi < 1, \qquad (21)
$$

control gains must be adjusted as:

$$
K_{1i} = 2\xi_i\omega_i,
$$

$$
K_{2i} = \omega_i^2. \qquad (22)
$$

Again, to have a maximum overshoot of $M_p = 10\%$ and settling time of about $t_s = 150$ s in tracking behaviour

of all output variables, parameters of Eq. (22) must be selected as $\omega_i = 0.05$, $\zeta_i = 0.6$, $i = 1, 2, 3$.

Figure 6 shows time responses of state and output variables after applying the designed controller based on feedback linearization approach (drum water level
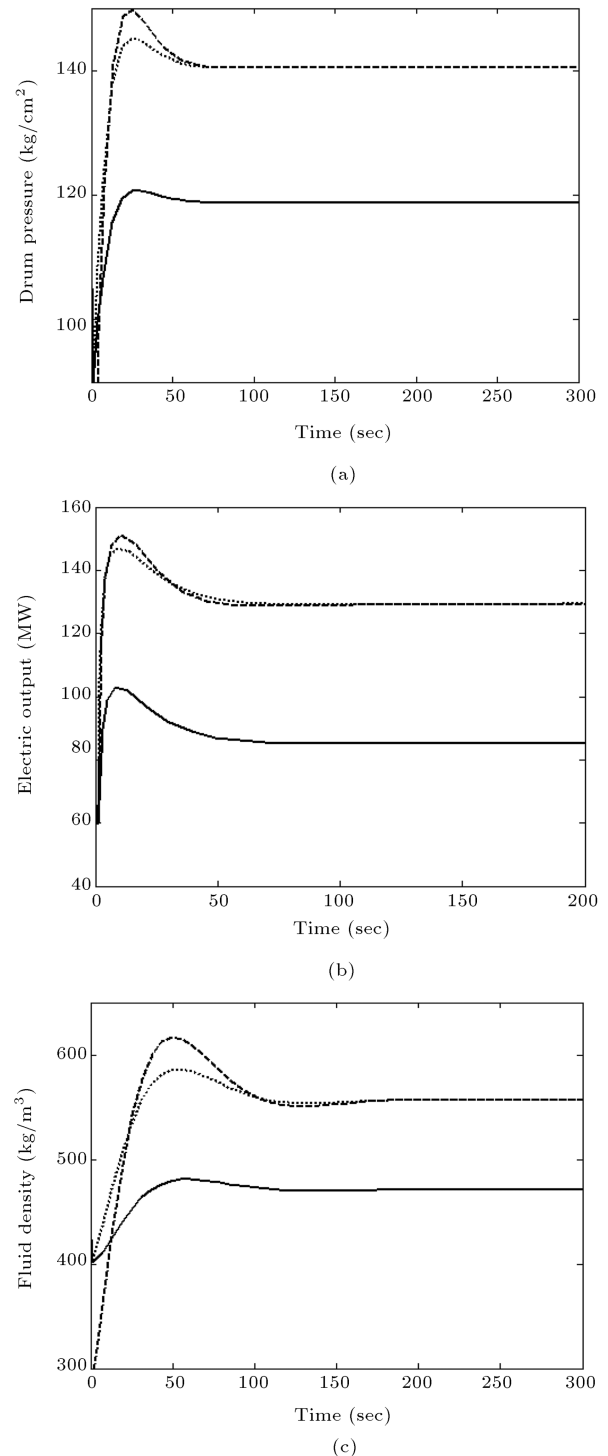

(a)

(b)

(c)

Figure 6. Time response of state and output variables, using feedback linearization approach for three cases: from operating point # 4 to 5 (solid lines), # 4 to 7 (dots) and #1 to 7 (dashed lines).

shows a similar behaviour as given in Figure 4(d). Overshot and settling time parameters of these three cases for state variables is given in Table 2. According to Figure 6 and Table 2, electric output behaviour shows more quick response (less settling time) with more overshoot, with respect to the drum pressure and fluid density for all three cases. For electric output response, tracking from operating point # 4 to the 'near' point # 5 associates with more overshoot, while for the drum pressure and fluid density, more overshoot is seen in tracking from point # 1 to the 'so far' point # 7. Therefore, for both control approaches (Figures 4 and 6), dynamic system is more sensitive in tracking of larger values of drum pressure or fluid density, and in tracking of closer values of electric output.

Figure 7 shows time responses of the required input control signals after applying the controller. According to Figure 7, in tracking from operating point # 1 to the 'so far' point # 7, valves position for fuel and feed-water flows are stronger in magnitude with more oscillation, with respect to the same signals for 'near' and 'far' set-point cases, as physically discussed in Figure 5. Valves position for steam control is less in magnitude with less oscillation in the tracking objective from operating point # 4 to the 'near' point # 5 (with respect to the same signal for 'far' and 'so far' set-point cases). In tracking a 'near' operating point, although the large overshoot of electric output is a minor negative aspect for the power grid, less oscillatory behaviour of control signals is an advantage for the actuators, when the controller based on feedback linearization is used.

Finally, it is assumed that the dynamic model given by Eqs. (1) and (2) is associated with a random uncertainty less than 10% in its constant coefficients. Control of the nonlinear uncertain system, based on gain scheduling and feedback linearization approaches, is simulated by SIMULINK Toolbox of MATLAB. Controllers designed in the previous section, for the plant with no uncertainty, are used. In both control approaches and for all three cases of 'near', 'far' and 'so far' set-points, the desired tracking behaviour for drum pressure and fluid density are obtained, similar to those shown in Figures 4 and 6 (for the sake of brevity, they are not shown for the uncertain plant). However, as shown in Figure 8, electric output behaviour is associated with small chatters during the tracking path (e.g., from point # 1 to # 7). As it is shown, the uncertain system with the controller designed, based on feedback linearization method, is more robust to the model uncertainties (its time response shows less overshoot with smaller chatter amplitudes, with respect to that of gain scheduling approach).

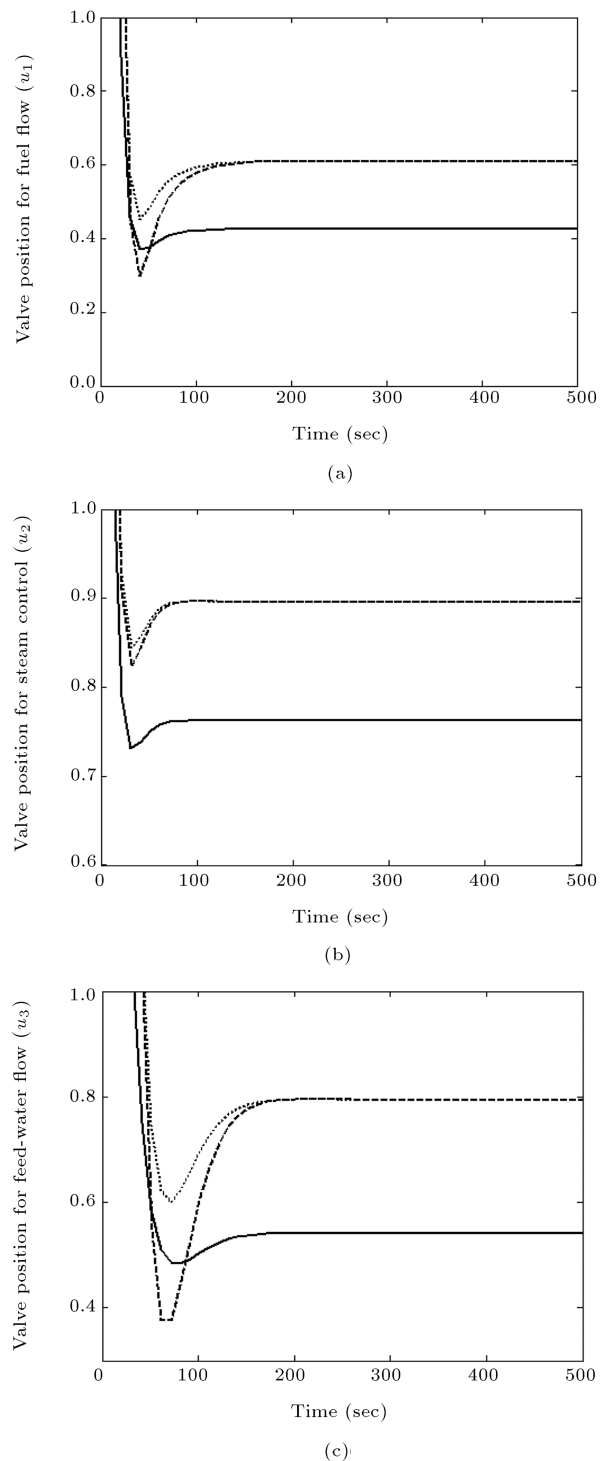For investigation of the effect of uncertainty amount, another arbitrary random uncertainty less

Figure 7. Time response of required input control signals using feedback linearization approach for three cases: from operating point # 4 to 5 (solid lines), # 4 to 7 (dots) and #1 to 7 (dashed lines).

than 25% in constant coefficients of the nominal model is considered (here, simulations are performed for the feedback linearization controller due to its better performance). Figure 9 shows the time response of electric output for the nominal model and uncertain
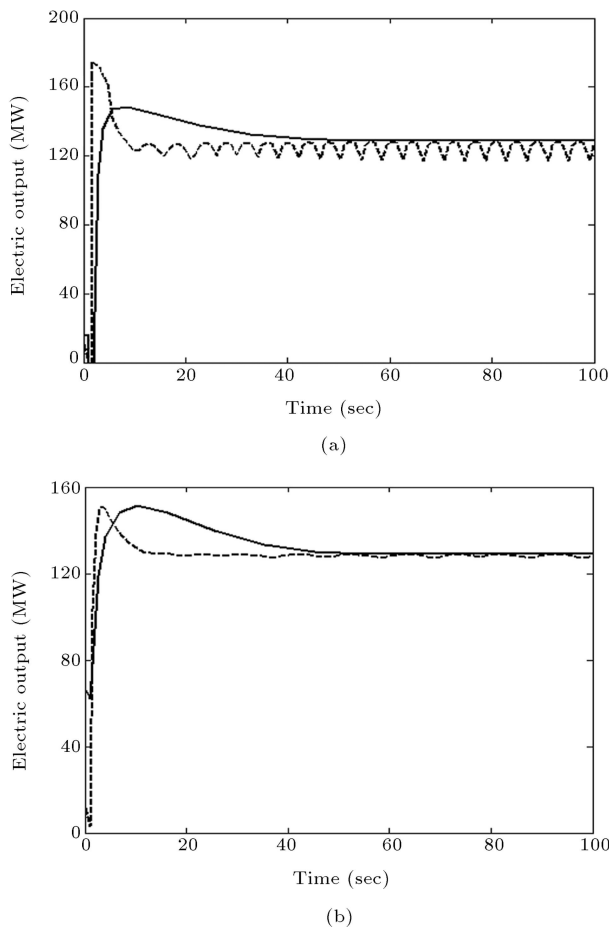
(a)

(b)

Figure 8. Time response of the electric output for the nominal (solid lines) and uncertain (dashed lines) model of boiler-turbine unit, using (a) gain scheduling and (b) feedback linearization controllers from operating point # 1 to 7.
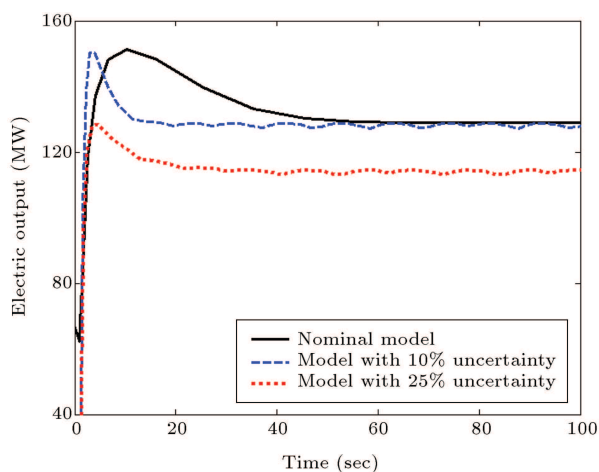

Figure 9. Time response of the electric output for the nominal (black solid line) and uncertain models with a random 10% uncertainty (blue dashed line) and 25% uncertainty (red dot line) when the feedback linearization controller is implemented for tracking from operating point # 1 to 7.

models with random 10% and 25% uncertainty. It is observed that as the amount of uncertainty increases, controller designed based on feedback linearization looses its ability in perfect tracking objective (steady error exists in Figure 9 for the case of 25% uncertainty). Under such conditions and in the presence of large uncertainties, design of a nonlinear based robust controller with modelling the details of uncertainty is necessary.

To investigate the nonlinear behaviour of the system, e.g. in tracking from set-point #1 to #7, phase portraits of the boiler unit after applying the designed gain scheduling and feedback linearization controllers are shown in Figures 10 and 11 (solid lines). To increase the speed of tracking objective two times (for instance), closed-loop poles of the system including gain scheduling controller are selected as $\mu_{1,2} = -0.06 \pm 0.08j$ and $\mu_3 = -0.3$. Also for the feedback linearization controller, parameters ω_i and ζ_i are chosen as $\omega_i = 0.1$, $\zeta_i = 0.6$ and $i = 1, 2, 3$, in Eq. (22). As shown in Figures 10 and 11, by increasing the speed of tracking, dynamic system tends to show a chaotic behaviour.
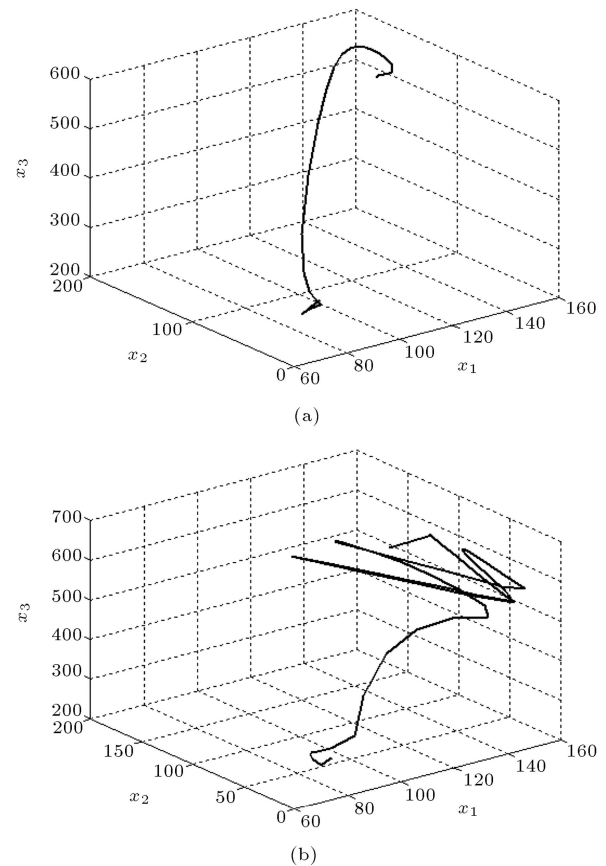

(a)

(b)

Figure 10. Phase portrait of the boiler-turbine unit, using gain scheduled controller for (a) a nominal plant with a desired common speed of tracking objective and (b) increasing the speed of tracking objective two times (in tracking from set-point # 1 to # 7).
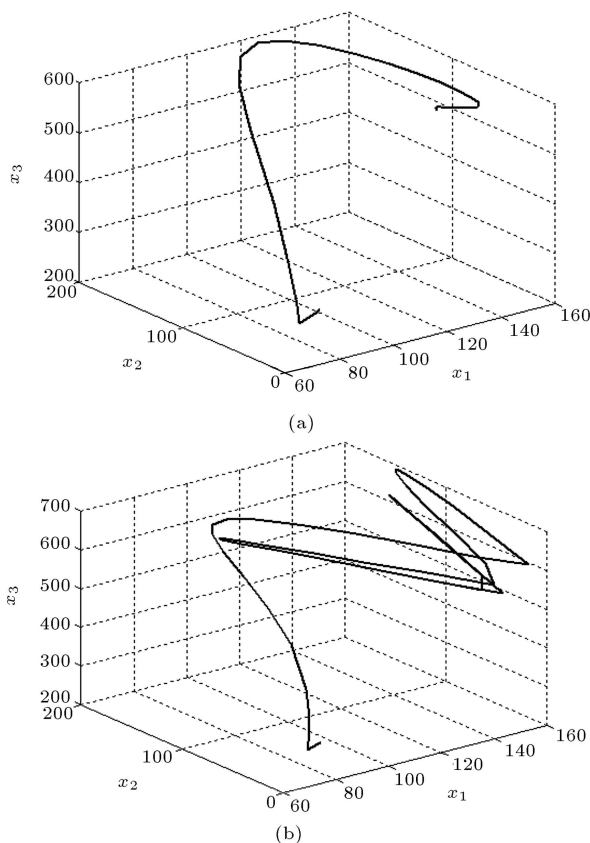
Figure 11. Phase portrait of the boiler-turbine unit, using the controller based on feedback linearization for (a) a nominal plant with a desired common speed of tracking objective and (b) increasing the speed of tracking objective two times.

4. Comparison between two control approaches

According to the presented results for two different control strategies, the following conclusions are obtained:

1. According to Figures 4 and 6 and Table 2, using the controller based on feedback linearization results in time responses with less settling time; more quick response, but more overshoot. So, when it is desired to achieve a quick response in tracking objectives, feedback linearization strategy is selected. However, it should be noticed that this strategy associates with high overshoot values in electric output response, which may cause problems for the power network.

2. Feedback linearization approach is suitable for tracking objective of a 'near' set-point, e.g. point # 4 to # 5. This is because according to Table 2, tracking objectives for state variables are satisfied in less time, while the overshoot is only a bit more than that of gain scheduling approach.

3. Gain scheduling approach is appropriate for the 'so far' tracking objectives, specifically for the electric

output which is the most important output of the problem. According to Table 2, using gain scheduling approach for a 'so far' tracking path, e.g. point # 1 to # 7, results in a considerable less overshoot (9%), with respect to that of feedback linearization (15%), while the settling time for both approaches is almost equal (100 s, 90 s).

4. According to Figures 5 and 7, during the transient conditions, valves position for 'near', 'far' and 'so far' tracking objectives associate with less oscillation in the case of feedback linearization strategy. In addition, valves positions reach their steady state values in less time. Therefore, control efforts act in less time with less oscillation, as the controller designed, based on feedback linearization strategy, is used.

5. According to Figure 8, in the presence of an arbitrary random uncertainty in model parameters, the controller designed, based on feedback linearization method, is more robust, showing less overshoot with less chatter under the steady state condition. However, as the amount of uncertainty increases, controller designed, based on feedback linearization, looses its ability in perfect tracking objective.

6. According to Figures 10 and 11, both controlled systems tend to show a chaotic behaviour as the performance speed in tracking objectives is increased. Considering conclusions given above, using feedback linearization approach introduces more advantages, with respect to the gain scheduling approach. The only disadvantage of the feedback linearization strategy is the considerable overshoot associated with tracking objectives of electric output.

5. Conclusions

In this paper, application of two control strategies for performance improvement in a nonlinear model of boiler-turbine unit is investigated. Drum pressure, electric output and drum water level are controlled by manipulation of valves position for fuel, steam and feed-water flows. Two controllers are designed using gain scheduling and feedback linearization approaches, based on pole placement. Results are presented and compared for tracking objectives from an operating point to the 'near', 'far' and 'so far' set-points. Advantages and disadvantages of both control strategies are discussed.

According to the results obtained, using feedback linearization strategy leads to more quick time responses of output variables with a bit more overshoots (with respect to the gain scheduling approach). In addition, valves position for fuel, steam and feed-water flows reach their final steady state values in less time with less oscillation.

In the presence of an arbitrary parametric uncertainty in the nonlinear model, the desired tracking objectives are achieved for output variables in both methods. However, for the controller designed based on gain scheduling approach, electric output signal is associated with considerable oscillations. This problem can be solved by decreasing the speed of tracking setpoints. Finally, a chaotic behaviour of the boiler-turbine unit is seen when the speed of tracking process is increased. In future, to improve the robust performance of such MIMO systems against possible uncertainties, a nonlinear-based robust controller can be developed.

Acknowledgments

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Appendix A

Dynamic model of boiler-turbine unit is of rank $n = 3$. Since the controllability matrix:

$$\Phi_c = \begin{bmatrix} B & AB & A^2B & \cdots A^{n-1}B \end{bmatrix},$$

is of rank 3, dynamic system is completely state controllable. Using the similarity transformation $\tilde{T}$ as $\bar{x} = \tilde{T}z$, Eq. (5) is represented as:

$$\dot{\bar{z}}_\delta = \hat{A}_G \bar{z}_\delta + \hat{B}_G \bar{u}_\delta$$

$$\hat{A}_G = \tilde{T}^{-1}AT, \qquad \hat{B}_G = \tilde{T}^{-1}B, \qquad (A.1)$$

where $\bar{z}_\delta$ is the new introduced state vector. Also, using the following transformations:

$$\bar{u}_\delta = F\bar{w}_\delta; \qquad \bar{w}_\delta = \bar{v}_\delta - H\bar{z}_\delta. \qquad (A.2)$$

Eq. (A.1) is described as:

$$\dot{\bar{z}}_\delta = A_G \bar{z}_\delta + B_G \bar{v}_\delta,$$

$$A_G = \hat{A}_G - \hat{B}_G FH, \qquad B_G = \hat{B}_G F, \qquad (A.3)$$

where $\bar{v}_\delta$ is the new control input vector and A_G, B_G have the general canonical form with elements of $[A_i]_{\gamma_i \times \gamma_i}, [B_i]_{\gamma_i \times 1}, i = 1, 2, .., r$ and $\sum_{i=1}^{r} \gamma_i = n$ as [36]:

$$A_G = \begin{bmatrix} [A_1] & 0 & \cdots & 0 \\ 0 & [A_2] & \cdots & 0 \\ & & \cdot & \\ 0 & 0 & \cdots & [A_r] \end{bmatrix}_{n \times n},$$

$$B_G = \begin{bmatrix} [B_1] & 0 & \cdots & 0 \\ 0 & [B_2] & \cdots & 0 \\ & & \cdot & \\ 0 & 0 & \cdots & [B_r] \end{bmatrix}_{n \times r},$$

$$[A_i] = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & & \cdot & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{\gamma_i \times \gamma_i},$$

$$[B_i] = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}_{\gamma_i \times 1}, \qquad (A.4)$$

where r is the number of input variables (in this case, $r = 3$). Introducing the modified controllability matrix as:

$$\bar{\Phi}_c = \begin{bmatrix} b_1 & b_2 & \cdots & b_r & \vdots \end{bmatrix}$$

$$Ab_1 \quad Ab_2 \quad \cdots \quad Ab_r \quad \vdots \quad \cdots \quad \vdots$$

$$A^{n-r}b_1 \quad A^{n-r}b_2 \quad \cdots \quad A^{n-r}b_r \Big],$$

where b_i are the columns of matrix B given in Eq. (5). A regular basis of $\bar{\Phi}_c$ is developed as

$$\hat{\Phi}_c = \begin{bmatrix} b_1 & Ab_1 & \cdots & A^{\gamma_1 - 1}b_1 \vdots \end{bmatrix}$$

$$b_2 \quad Ab_2 \quad \cdots \quad A^{\gamma_2 - 1}b_2 \quad \vdots \quad \cdots \quad \vdots$$

$$b_r \quad Ab_r \quad \cdots \quad A^{\gamma_r - 1}b_r \Big], \qquad (A.5)$$

where each column, $A^j b_i$, $i = 1, \cdots, r$, $j = 0, \cdots, r$, is independent from its previous columns. Inverse of $\hat{\Phi}_c$ given by Eq. (A.5) is displayed as (all of the paper, [ ]$'$ stands for transpose of the [ ] quantity):

$$\hat{\Phi}_c^{-1} = \begin{bmatrix} e'_{11} & \cdots & e'_{1\gamma_1} & \vdots \\ \\ e'_{21} & \cdots & e'_{2\gamma_2} & \vdots & \cdots & \vdots \\ \\ e'_{r1} & \cdots & e'_{r\gamma_r} \end{bmatrix}'.$$

Similarity transformation $\tilde{T}$ is defined as [36]:

$$\tilde{T} = \left(\begin{bmatrix} e'_{1\gamma_1} & e'_{1\gamma_1}A & \cdots & e'_{1\gamma_1} & A^{\gamma_1-1} & \vdots \\ \\ e'_{2\gamma_2} & e'_{2\gamma_2}A & \cdots & e'_{2\gamma_2} & A^{\gamma_2-1} & \vdots & \cdots & \vdots \\ \\ e'_{r\gamma_r} & e'_{r\gamma_r}A & \cdots & e'_{r\gamma_r} & A^{\gamma_r-1} \end{bmatrix}' \right)^{-1}. \tag{A.6}$$

Considering again Eq. (A.3) and constructing the feedback control law, as $v_\delta = -\Gamma z_\delta$, yields:

$$\dot{\tilde{z}}_\delta = A_d \tilde{z}_\delta, \qquad A_d = A_G - B_G \Gamma, \tag{A.7}$$

where A_d is the desired state matrix including coefficients representing desired closed loop poles ($|sI - A_d| = (s - \mu_1)(s - \mu_2) \cdots (s - \mu_n)$), having the general form of A_G as given by Eq. (A.4). Considering Eqs. (7) and (A.2) and similarity transformation $\bar{x}_\delta = \tilde{T} \tilde{z}_\delta$, yields the feedback control law of the system as:

$$\bar{u}_\delta = -K(\varphi_i, \psi_i)\bar{x}_\delta,$$

$$K(\varphi_i, \psi_i) = F[\Gamma + H]\tilde{T}^{-1}, \tag{A.8}$$

where F, H and Γ are obtained using Eqs. (A.2), (A.3) and (A.7) as follows:

$$F = (B'_G \hat{B}_G)^{-1}, \qquad H = B'_G(A_G - \hat{A}_G),$$

$$\Gamma = B'_G(A_G - A_d). \tag{A9}$$

Appendix B

Lie derivative definition

Let $h : R^n \to R$ be a smooth scalar function and $f : R^n \to R$ be a smooth vector field on R^n. The Lie derivative of h with respect to f is a scalar function defined by [35]:

$$L_f h = \nabla h.f.$$

Repeated Lie derivatives can be defined recursively as:

$$L_{f^0} h = h, \qquad L_{f^i} h = L_f(L_{f^{i-1}} h) = \nabla(L_{f^{i-1}} h).f.$$

Similarly, if g is another vector field, then the scalar function $L_g L_f h(x)$ is:

$$L_g L_f h = \nabla(L_f h).g.$$

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