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Finite element simulation of the warm deep drawing process in forming a circular cup from magnesium alloy sheet

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Abstract. In recent years, due to its low density and high strength, the use of magnesium alloy in sheet metal forming has been of interest to many researchers. Because of the low formability of magnesium alloy at room temperature, warm forming has been developed in order to produce parts of a magnesium alloy sheet. In this paper, the warm deep drawing process of a circular cup from a magnesium alloy AZ31B sheet was simulated using the finite element software ABAQUS/Standard. Simulation results obtained from this study agree well with experimental results and show that the limit drawing ratio increases with an increase in forming temperature. Maximum LDR was obtained as 2.5 at the forming temperature of 200°C. Simulation results indicate that heat transfer plays an important role in increasing formability. By comparing isothermal and non-isothermal warm deep drawing, the heat transfer effect and location of sheet failure have been investigated. The effects of friction coefficient and punch speed on the limit drawing ratio were also studied. As a result, increase in the friction coefficient leads to an increase in the punch force and a decrease in the limit drawing ratio. Results also show that the maximum friction coefficient to produce acceptable parts increases with an increase in the forming temperature. In addition, results indicate that an increase in punch speed leads to a decrease in the drawing ratio limit.

1. Introduction

Lightening of components is of particular importance in different industries in order to reduce weight, which leads to reduced fuel consumption, energy use and cost. Magnesium alloy is a light alloy that has received much attention in industrial applications in recent years, due to its low density and high strength. Die casting of magnesium alloys is a popular method for manufacturing parts. But, parts which are produced by sheet metal forming processes have good mechanical properties and surface quality in comparison with die casting. For these reasons, sheet forming of magnesium alloys has been recommended for future development of parts, and because of the poor formability of magnesium alloy sheeting at room temperature, warm forming has been developed in order to produce parts of the magnesium alloy sheet. The warm forming of magnesium alloy sheet is done for two reasons. Firstly, warm working leads to the increased ductility and formability of the sheet, and secondly, the yield point and forming force decrease at elevated temperatures. Droder [1] studied the warm deep drawing of magnesium alloy sheets in producing circular and rectangular parts experimentally, and obtained significant temperature limitations for this process. He performed a uniaxial tensile test at

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different temperatures to obtain the stress-strain curve of magnesium alloy sheet. Doege and Droder [2] formed round cups of magnesium AZ31, AZ61 and M1 sheets by experiment. They showed that these alloys have good formability in ranges between 200°C and 250°C. In addition, they investigated the influence of punch velocity on the limit drawing ratio and reported that an increase in punch velocity leads to a decrease in the limit drawing ratio. Yoshihara et al. [3] used localized heating and cooling for increased formability of magnesium alloy sheeting to produce a circular cup. They obtained the limit drawing ratio, 5.0, by local heating and cooling using the variable blankholder pressure technique. Yoshihara et al. [4] also investigated optimization of magnesium alloy stamping using the finite element method and compared the numerical and experimental results. Palaniswamy et al. [5] studied the formability of magnesium alloy AZ31B sheet at elevated temperatures using the commercial FE code DEFORM 2D for round cups and DEFORM 3D for rectangular pans. They concluded that the limit drawing ratio increases with an increase in the forming temperature. Neugebauer et al. [6] explained the development of forming processes of magnesium and aluminum alloy sheets at elevated temperatures, and described warm forming and hot stamping that are being developed to produce light weight materials. El-Morsy and Manabe, [7] and also Huang et al. [8] employed finite element analysis for simulation of non-isothermal deep drawing. Palumbo et al. [9] conducted finite element and experimental studies on the warm deep drawing of magnesium AZ31 alloy sheet. They investigated the effect of heating and punch speed on the limit drawing ratio. Zhang et al. [10] also simulated the warm deep drawing of magnesium alloy sheets, numerically, and compared the results with experiments. Chang et al. [11] compared the results of experimental and numerical studies on the warm deep drawing of magnesium alloy AZ31 sheet. Kaya et al. [12] performed, experimentally, the warm deep drawing of magnesium and aluminum alloy round cups at elevated temperatures using a servo motor driven press.

In this paper, the warm deep drawing process of magnesium alloy AZ31B sheet in a circular die using the implicit finite element software ABAQUS/Standard was simulated. In order to validate the numerical results, the curves of the punch force, thickness distribution and the limit drawing ratio to the forming temperature were compared with the experimental study of Droder [1] and the numerical investigation of Palaniswamy et al. [5]. After validation, some parameters in warm deep drawing, such as temperature, location of thinning, friction coefficient and punch speed, have been studied.

2. Numerical simulation

2.1. Governing equations

The finite element is very effective method for analysis of metal forming processes and for optimizing their parameters. The finite element analysis of the warm deep drawing process is a thermo-mechanical analysis for mechanical and thermal loads. For the mechanical field, the updated Lagrangian formulation is used because of the large deformation of the warm deep drawing process. The equilibrium statement, written as the virtual work equation, and based on the updated Lagrangian formulation, is as follows [13]:

$$\int_{^tV} {}^{t+\Delta t}_{t}S_{ij}\delta^{t+\Delta t}_{t}\varepsilon_{ij}d^tV = \int_{^tS} {}^{t+\Delta t}_{t}f_i^S\delta u_i d^tS + \int_{^tV} {}^{t+\Delta t}_{t}f_i^B\delta u_i d^tV, \quad (1)$$

where S_{ij} is the second Piola-Kirchhof stress tensor, ε_{ij} is the Green-Lagrange strain tensor, δu_i is virtual displacement vector, ${}^tV$ is volume, ${}^tS$ is surface at time, t, and f_i^S and f_i^B are the components of surface and body forces.

The full Newton-Raphson iterative method, as a numerical technique, is used to solve the nonlinear equilibrium equations. Because of appearing the Cauchy stress after linearization of the equation of motion, and its change under rigid motion, the Jaumann stress rate is also used in the updated Lagrangian formulation.

For the numerical solution of heat transfer in the warm deep drawing process, the principle of virtual temperatures is as follows [14]:

$$\int_V \delta\theta'^T k\theta' dV = \int_V \delta\theta q^B dV + \int_S \delta\theta^S q^S dS, \quad (2)$$

where:

$$k = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}, \qquad \theta'^T = \begin{bmatrix} \frac{\partial\theta}{\partial x} & \frac{\partial\theta}{\partial y} & \frac{\partial\theta}{\partial z} \end{bmatrix}. \quad (3)$$

In the heat transfer equation, $\delta\theta$ is the virtual displacement vector, θ' is the temperature gradient vector, k is thermal conductivity, q^B is the rate of heat generation per unit volume, θ^S is the known surface temperature on S, and q^S is the heat flux input on this surface.

Fully coupled thermo-mechanical analysis is used to obtain the nodal displacement and temperatures in this research, because of the inelastic deformation of the material and the contact condition in the mechanical field that should be coupled to the thermal field. Therefore, mechanical and thermal solutions affect each other strongly and must be solved simultaneously. An exact implementation of Newton's method is used to solve the nonlinear coupled system. The coupled equations are as follows [15]:

$$\begin{bmatrix} K_{uu} & K_{u\theta} \\ K_{\theta u} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} R_u \\ R_\theta \end{bmatrix}, \tag{4}$$

where Δu and $\Delta\theta$ are the incremental displacement and temperature, K_{ij} are submatrices of the fully coupled Jacobian matrix, and R_u and R_θ are the mechanical and thermal residual vectors.

2.2. Finite element modeling

A 2D axial-symmetric model is used in the simulation because of the axial symmetry of the process. The geometry of the sheet and the tools, based on Droder's study [1], are shown in Table 1. Tools were modeled as rigid bodies by performing constraints on the displacement of all nodes by a reference node. The mechanical properties of the magnesium alloy AZ31B sheet is shown in Table 2. Figure 1 shows the flow stress-strain curve of the magnesium alloy AZ31B sheet at different temperatures used by Palaniswamy et al. [5] in their simulation. The thermal properties of the sheet and the tools are also shown in Table 3. The sheet is modeled as a body with elastic-plastic and thermal properties. The von Mises yield function is applied for the material hardening behavior which is isotropic hardening.

Table 1. The geometry of the sheet and the tools [1].

Punch Diameter	100 mm
Punch and die corner radius	12 mm
Initial punch temperature	25°C
Sheet thickness	1.3 mm

Table 2. Mechanical properties of magnesium alloy AZ31B sheet [1,2].

Module of elasticity	45 (GPa)
Poisson ratio	0.35
Flow stress curve	Figure 1

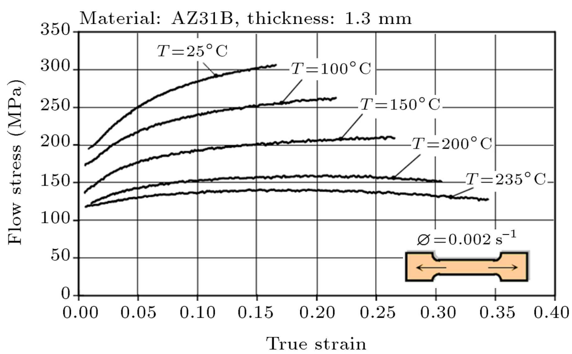

Figure 1. Flow stress of magnesium alloy AZ31B sheet at different temperatures [1,2].

Table 3. Thermal properties of sheet and tools [1,5].

Thermal expansion coefficient (sheet)	25e-6 (1/°C)
Thermal conductivity (sheet)	159 (W/m°C)
Heat capacity (sheet)	1.7675 (N/mm^2°C)
Thermal conductivity (tools)	60.5 (W/m°C)
Heat capacity (tools)	3.41 (N/mm^2°C)
Convection coefficient	30 (W/m^2°C)
Interface heat transfer coefficient	4500 (W/m^2°C)

For creating contact between the sheet and the tools, the penalty method, as a contact algorithm with Coulomb friction law, is used. The friction coefficient is considered 0.1, from the strip draw test done by Droder [1]. In this study, the friction coefficients for the punch, the die and the blankholder in contact with the sheet are the same. According to Table 2, the interface heat transfer coefficient is considered to be 4500 W/m^2°C for contact regions [5,15]. For the heating of the sheet inside the tool, the sheet should be fixed between the die and the blankholder for a few seconds until the temperature of the sheet reaches the temperature of the die and the blankholder before the drawing process [1,2]. Therefore, for analysis of the warm deep drawing process, the initial temperatures of the sheet, the die and the blankholder are considered the forming temperatures. The initial temperature of the punch is also considered as room temperature (25°C). In coupled thermo-mechanical analysis, an element should be employed which has both displacement and temperature in its degrees of freedom. As a result, in this study, the solid 8-node axisemmetric quadrilateral element, CAX8RT, with a reduced integration technique is used for meshing the sheet and the tools. The shape function of this element for interpolation of displacement is biquadratic and for interpolation of temperature is bilinear. Figure 2 shows the finite element model of the warm

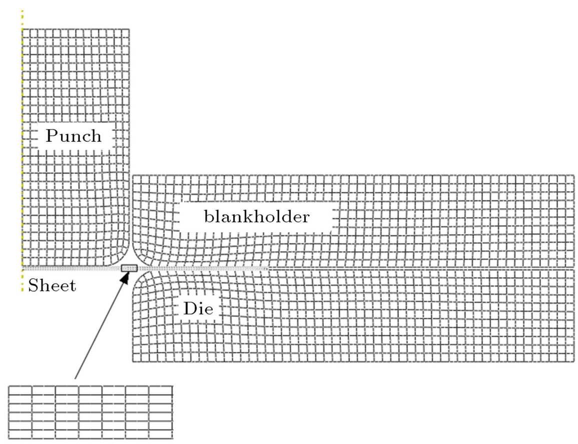

Figure 2. Finite element model of warm deep drawing process.

deep drawing process. The sheet is meshed with six elements along the thickness, based on optimized meshing.

3. Results and discussion

After starting the analysis, the punch, at room temperature, would be in contact with the preheated sheet. So, according to consideration of the interface heat transfer and because of high thermal conductivity and the low heating capacity of magnesium alloy, the temperature of the sheet decreases quickly. Figure 3 indicates the temperature variation of the sheet center with punch displacement for LDR of 2.3 at the forming temperature of 200°C during the drawing process.

Initial punch temperature is an important parameter for improving the formability of a magnesium alloy sheet. It should be noted that the temperature of a deformed cup depends on the initial temperature of the workpiece and tools, the heating of plastic deformation, the heating of friction, and the heat transfer between the workpiece and the tools with the environment. Contacting the punch with the sheet and the thermal interchange between them leads to an increase in the strength of the sheet at the bottom and corner of the punch. Thus, the formability of magnesium alloy sheet in the warm deep drawing process increases.

3.1. Punch force

Punch force is one of the most important parameters in metal forming processes that is equal to the force for deforming the workpiece plus the force for overcoming the friction between the tools and the sheet. Figure 4 shows the comparison of the punch force curve with the simulation results of the present work, the experimental results of Droder [1] and the numerical analysis of Palaniswamy et al. [5] for LDR of 2.3 at the forming temperature of 200°C.

The trend of changing the punch force during the process, obtained from FEA (present work and Palaniswamy et al. [5]), matches with experimental results in which maximum punch force occurs in a punch displacement of 20 mm to 40 mm. It is also observed that the punch force curve obtained from the present work was closer to experimental investigation than the result of Palaniswamy et al. [5]. It should be noted that after the peak point, the punch force decreases gradually, due to the decrease in the length of the sheet under the blankholder.

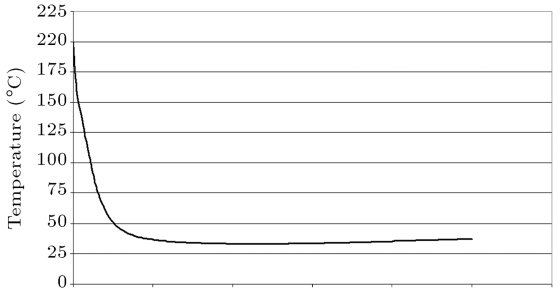

Figure 3. The temperature variation of sheet center with punch displacement for LDR of 2.3 at the forming temperature of 200°C.

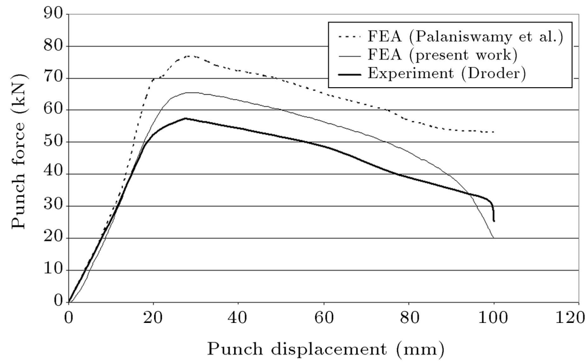

Figure 4. Punch force obtained from numerical simulations and experiment for LDR of 2.3 at the forming temperature of 200°C.

3.2. Thickness distribution

Figures 5 and 6 show the comparison of the thickness distribution obtained from FEA with the experimental result for LDR of 2.3 at the forming temperatures of 200°C and 250°C. The trend of change in thickness (%) obtained from the present work matches completely with the experimental result and has some improvements in comparison with the numerical results of Palaniswamy et al. [5].

It should be mentioned that maximum thinning occurs in the length 60 mm to 80 mm of the cup. This shows that the maximum thinning observed in the cup wall is opposite to cold warm deep drawing, where maximum thinning occurs in the punch radius region.

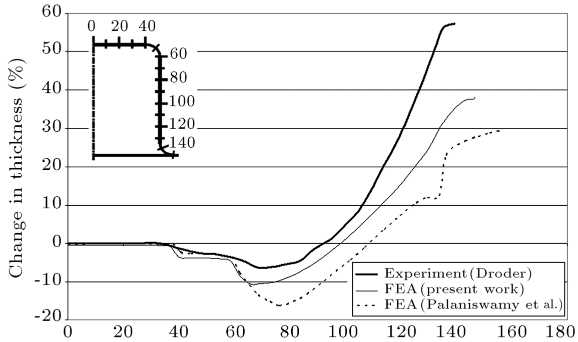

Figure 5. Thickness distribution obtained from numerical simulations and experiment for LDR of 2.3 at the forming temperature of 200°C.

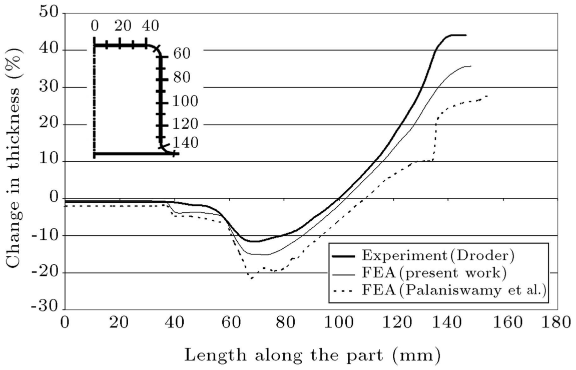

Figure 6. Thickness distribution obtained from numerical simulations and experiment for LDR of 2.3 at the forming temperature of 250°C.

3.3. *The effect of forming temperature on limit drawing ratio*

When thinning rate exceeds 25%, it can be concluded that the sheet is considered to fail [5]. In addition to this, the drawing process has been run at different LDRs and temperatures for investigating thinning and failure in the sheet. Figure 7 indicates the failed sheets that thinned in the die radius region under different conditions. Figure 8 displays the dependency of the limit drawing ratio on the forming temperature obtained from simulation, which is compared with experimental results. It shows that the limit drawing ratio increases with an increase in the forming temperature. Maximum LDR was obtained as 2.5 at the forming temperature of 200°C, which demonstrates good agreement between numerical and experimental results, whereas Palaniswamy et al. [5] obtained a maximum LDR of 2.4 at the forming temperature of 200°C. After the maximum point, the LDR decreases with an increase in the forming temperature, because work hardening decreases with an increase in forming temperature.

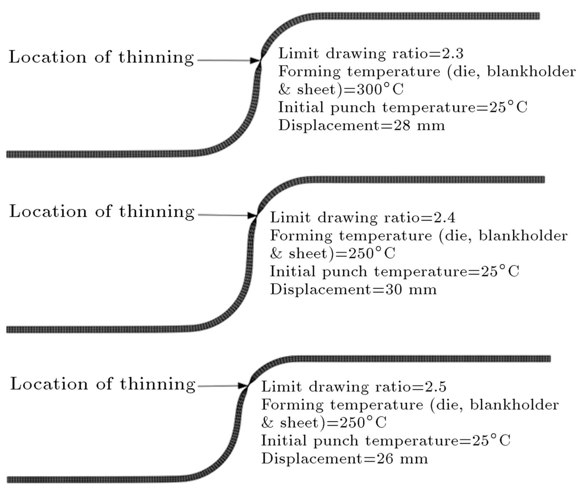

Figure 7. Location of thinning at different temperatures and LDRs obtained from numerical simulation.

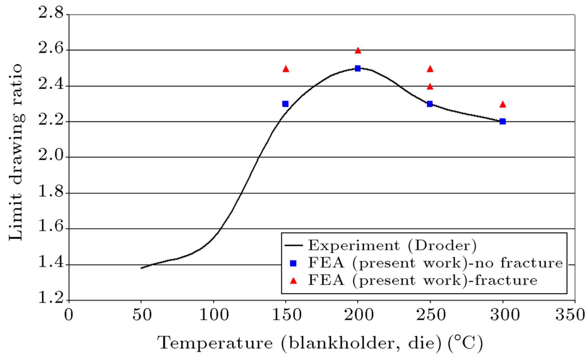

Figure 8. The effect of forming temperature on limit drawing ratio.

3.4. *The effect of heat transfer on improving formability of magnesium alloy sheet*

The effect of heat transfer and temperature distribution on increased formability has been investigated, as uniform temperatures are considered for the tools and the sheet. Figure 9 shows the comparison between isothermal and non-isothermal deep drawing for LDR of 2.5 at the forming temperature of 200°C. According to Figure 9(a), it is concluded that heat transfer between the tools and the sheet leads to successful drawing. However, Figure 9(b) shows that neglecting heat transfer between the tools and the sheet leads to thinning occurring in the punch radius region. Figure 10 also displays the comparison between isothermal and non-isothermal deep drawing for LDR of 2.5 at the forming temperature of 250°C. It is clearly observed that the punch displacement of non-isothermal drawing is more than in the isothermal one. This is because of the fact that heat transfer between the punch and the sheet leads to increase the strength of the sheet in the punch radius region. So, the thinning happens in the die radius region. But, in isothermal deep drawing, similar to conventional deep drawing, due to uniform temperature in the sheet, thinning and failure occur in the punch radius region.

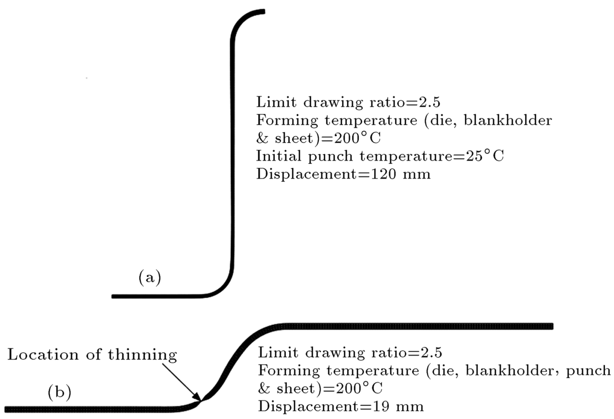

Figure 9. The effect of heat transfer on improving formability: (a) Non-isothermal deep drawing; and (b) isothermal deep drawing.

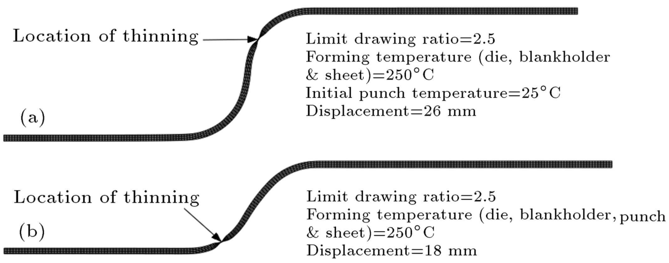

Figure 10. The effect of heat transfer on improving formability: (a) Non-isothermal deep drawing; and (b) isothermal deep drawing.

3.5. The effect of friction coefficient on limit drawing ratio at different temperatures

Lubrication between tools and the sheet is one of the most significant parameters in the deep drawing process and has great influence on the forming force. The high friction coefficient causes an increase in the friction force and the punch force, which leads to thinning in the sheet. Figure 11 shows the effect of the friction coefficient on the limit drawing ratio at the forming temperatures of 150°C and 200°C. It shows that the maximum friction coefficient for successful drawing decreases with an increase in the limit drawing ratio. In other words, if LDR increases, the lubrication rate for producing acceptable parts should be increased. It is also observed that with an increase in temperature from 150°C to 200°C, the maximum friction coefficient for successful drawing increases. So, it can be concluded that an increase in the forming temperature leads to a decrease in lubrication rate between the tools and the sheet.

3.6. The effect of punch speed on limit drawing ratio

Punch speed is an important factor, which affects heat transfer between the tools and the sheet. The maximum cup height with low punch speed is higher than maximum cup height with high punch speed. The reason is that the heat transfer is more effective at low punch speed than at high punch speed [7]. In this study, the punch speed was considered to be 5 mm/s at all temperatures and LDRs, based on the experimental work of Dorder [1]. For analyzing the effect of punch speed on fracture and thinning, the punch speed was increased gradually until thinning occurred in the sheet.

Figure 12 shows the deformed sheet at different punch speeds and LDRs at the forming temperature of 200°C. The sheet is thinned in the die radius region. Figure 13 displays the curve of the limit drawing ratio-punch speed for LDR of 2.3, 2.4 and 2.5 at the forming temperature of 200°C.

It shows that the maximum punch speed for acceptable drawing decreases with an increase in the limit drawing ratio. This is because the necessary time for heat transfer between the tools and the sheet should be increased, with an increase in the limit drawing ratio, which leads to a decrease in punch speed.

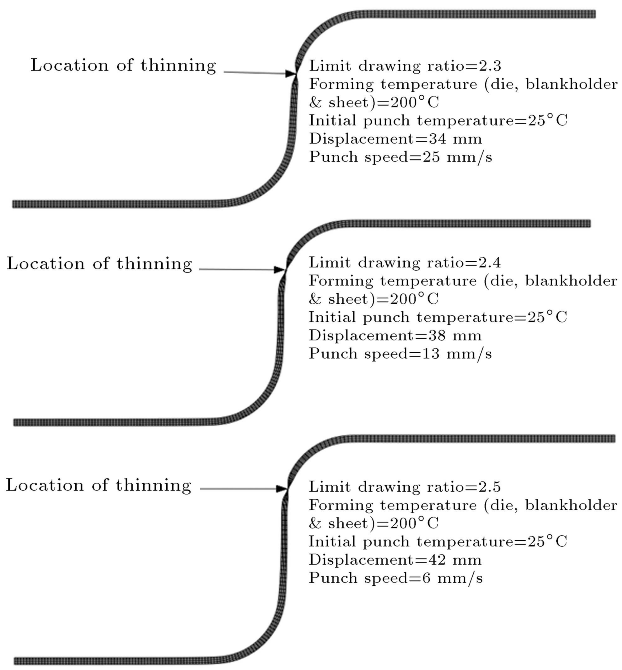

Figure 12. Location of thinning at different punch speeds and LDRs obtained from numerical simulation.

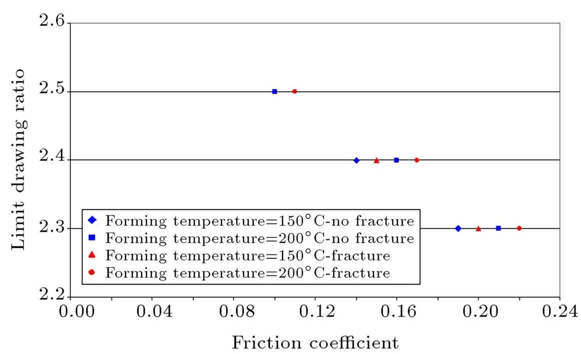

Figure 11. The effect of friction coefficient on limit drawing ratio at different temperatures.

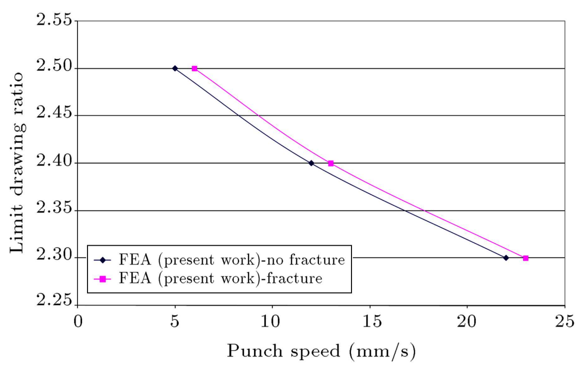

Figure 13. The effect of punch speed on limit drawing ratio.

4. Conclusions

In this study, the warm deep drawing of a magnesium alloy AZ31B sheet in a circular die was simulated in the implicit finite element software, ABAQUS/Standard, using elastic-plastic fully coupled thermo-mechanical analysis. The obtained results were compared with the experimental work of Droder [1] and the numerical investigation of Palaniswamy et al. [5] in order to validate the FE analysis, and then further parameters of forming were studied. The conclusions are as follows:

1. The forming force at the forming temperature of 200°C and different LDRs has been investigated and it is concluded that punch force increases with an increase in the limit drawing ratio at constant temperature. The trend of the punch force-displacement curve obtained from the numerical simulation for LDR of 2.3 at the forming temperature of 200°C matches well with the experimental work of Droder [1], and had some improvements compared to the numerical study of Palaniswamy et al. [5]. It must be noted that maximum punch force occurs in a punch displacement of 20 mm to 40 mm in all investigations.
2. The trend of the thickness distribution obtained from the present work for LDR of 2.3 at the forming temperatures of 200°C and 250°C has good agreement with the experimental work of Droder [1] and had some improvements compared to the numerical investigation of Palaniswamy et al. [5]. It should be also noticed that maximum thinning occurs in the length 60 mm to 80 mm of the cup. This could be due to the fact that temperature distribution from the sheet center to the flange region increased and thus the strength of the sheet decreased. So, it can be concluded that maximum thinning occurs in the cup wall.
3. The obtained results show that the limit drawing ratio increases with an increase in forming temperature. Maximum LDR was obtained as 2.5 at the forming temperature of 200°C, which demonstrates good agreement between the numerical analysis of the present work and the experimental result, whereas Palaniswamy et al. [5] obtained a maximum LDR of 2.4 at the forming temperature of 200°C. After the maximum point, the LDR decreased with an increase in the forming temperature.
4. The reasons behind the improvements of the simulation results of the present work being non-comparable to the numerical investigation of Palaniswamy et al. [5] are:
   - The sheet was meshed with six elements along the thickness in the present work whereas the sheet was meshed with four elements in the numerical study of Palaniswamy et al. [5].
   - Exact implementation of Newton's method (fully coupled analysis) was used in the present work, while Palaniswamy et al. [5] probably used an approximate implementation of Newton's method (weak coupled analysis) in their investigation.
5. The heat transfer effect and location of sheet failure have been studied by comparing isothermal and non-isothermal warm deep drawing. In isothermal deep drawing, as in conventional deep drawing, thinning occurs in the punch radius region due to uniform temperature in the sheet. But, in non-isothermal deep drawing, heat transfer between the punch and the sheet leads to an increase in the strength of the sheet in the punch radius region and delays the thinning to that of the die radius. Therefore, the punch displacement of non-isothermal drawing is more than that of isothermal drawing.
6. The friction coefficient, as an important parameter in the deep drawing process, was investigated. Results show that an increase in the friction coefficient leads to an increase in the punch force and a decrease in the limit drawing ratio. In addition, results show that the maximum friction coefficient to produce parts without defect increases with an increase in forming temperature.
7. Punch speed, as an effective factor in the deep drawing process, was also investigated. Results show that the maximum punch speed required to produce acceptable parts decreased with an increase in the limit drawing ratio.

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Behzad Soltani was born in Abadan, Khouzestan, Iran, in 1960. He received his BS and MS degrees in Mechanical Engineering from Isfahan University of Technology, Iran, in 1988 and 1991, respectively, and his PhD degree in Mechanical Engineering from Chalmers University of Technology, Sweden, in 1996. He is currently Assistant Professor in the Department of Mechanical Engineering at the University of Kashan. His research interests include continuum mechanics, nonlinear finite element and mesh free methods.