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Suppression of Condensation Shock in Wet Steam Flow by Injection of Water Droplets in Different Regions of a Laval Nozzle

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Abstract. *In this paper, a compressible, one-dimensional and two-phase flow is analytically modeled by using a homogeneous classical nucleation rate equation in a Laval nozzle. For droplet growth calculations, the heat transfer between the droplets and the surrounding steam is modeled by a free molecular flow model and a semi-empirical two-layer model which is deemed to be valid over a wide range of Knudsen numbers. In order to reduce the condensation shock strength, the water droplets are injected at the inlet, and just after the throat of the Laval nozzle. According to the results, the nucleation rate is considerably suppressed due to the small droplet injection, and, therefore, the condensation shock nearly disappears, particularly for the inlet injection of water droplets.*

Keywords: *Two-phases; Nucleation; Condensation shock; Injection; Laval nozzle.*

INTRODUCTION

In low-pressure steam turbines used for electricity generation, the presence of condensation leads to problems of blade erosion and to a loss of turbine efficiency.

The flow is initially single phase, but after reaching the Wilson point, water droplets are developed and a non-equilibrium two-phase flow is generated. The growing water droplets release their latent heat to the flow and this heat addition to the flow causes a pressure rise, called condensation shock.

Because of the irreversible heat transfer in this region, the entropy will increase tremendously and cause thermodynamic loss. The drag between large droplets and vapour generates a friction loss. Also, these droplets are not able to follow the vapor steam path and generate a braking loss, which also leads to erosion.

Reducing the wetness fraction of wet steam flow in the last stages of steam turbines significantly promotes stage efficiency and prevents erosion of rotors.

Regarding the literature review of this paper, please refer to [1-9]. In these references, the homogeneous steam two-phase flow is generally modeled through one- and two-dimensional analytic and numerical (CFD) methods, and the measurement of pressure has been stated, particularly for laboratory cases.

In this paper, an innovative analytic method for the one-dimensional homogeneous flow is proposed that uses a two-layer heat transfer through a four sentence Virial state equation along with the injection of identical droplets with different sizes in the convergent and divergent sections.

It can be assumed that the specific volume of droplets is much smaller in comparison with that of the vapour. Hence, the volume of the liquid phase can be neglected [2].

Considering this assumption, the mixture is taken as the summation of droplets of a specified size, which are uniformly distributed in the bulk vapour, and the space between them is filled by the vapour of given pressure and temperature. The bulk vapour can be condensed by the growth of existing droplets or the formation of new ones. Formation of new droplets can be calculated using various nucleation models. The model used in this paper is a classical homogeneous nucleation model with Courtney and Kantrowitz corrections [3]. This nucleation model assumes that condensation occurs in the absence of any impurities. The condensation releases the latent

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heat to the surrounding vapour and, for predicting the flow behavior, it is necessary to derive the equations of heat and mass exchange between droplets and the bulk vapour.

On the other hand, the mixture, as a whole, must satisfy the conservation laws of mass, momentum and energy. Two-phase effects enter into these equations through density and enthalpy. These are non-equilibrium mixture quantities and, therefore, depend on the mass fraction of the liquid and on the vapour subcooling. Finally, a suitable equation of state is needed for the surrounding vapour phase treatment. The flow can be described completely using this set of equations.

Using a two-phase Eulerian-Lagrangian model, conservation equations for the two-phase mixture are solved in a Eulerian frame of reference, but nucleation and droplet heat and mass exchange equations are calculated following fluid particles in steady one-dimensional flows. Therefore, four unknowns, namely temperature, pressure, velocity and density of vapour, can be calculated [4].

TWO-PHASE FLOW EQUATIONS NUCLEATION RATE MODEL

For non-equilibrium flow calculations, there must be a criterion for the onset of nucleation and the appearance of the second phase (liquid). The present model considers only homogenous nucleation in a pure substance and relies heavily on the classical nucleation theory. The relevant nucleation theory is briefly presented here.

The essence of the model is that, in order for nucleation to occur in a flowing stream undergoing expansion, molecular clusters (the minute embryo of a liquid droplet) must overcome a free-energy barrier associated with increase in the surface-free-energy of the droplets for the onset of phase change. As a substance is increasingly supercooled, the chances of a molecular cluster moving over the free energy barrier increase.

The change of Gibbs free energy for a mass, m_r, of a supercooled vapour at constant pressure and temperature, which forms a liquid droplet at the same pressure and temperature, can be calculated in the absence of magnetic and electrical fields [5]:

$$\Delta G = -m_r R T_G[\Delta G_1] + 4\pi r^2 \sigma_r,$$

$$\Delta G_1 = \ln\left(\frac{\rho}{\rho_S(T_G)}\right) + 2B_1[\rho - \rho_S(T_G)] + \frac{3}{2}B_2[\rho^2 - \rho_S^2(T_G)] + \frac{4}{3}B_3[\rho^3 - \rho_S^3(T_G)], \quad (1)$$

where ΔG is the change of Gibbs free energy, m_r is the droplet mass, R is the gas constant for vapour, T is the temperature, r is the droplet radius, σ is the surface tension, ρ is the density, and B parameters are the virial coefficients given in the Appendix. Furthermore, the subscripts r, G and s denote the radius, vapour and saturation phases, respectively.

Based on the thermodynamic equilibrium, there is a minimum radius, r^* (the critical radius), that must be attained in order to form a stable nucleus from supersaturated vapour. For this condition, the Gibb's free energy of formation of a critical cluster is denoted by ΔG^*. By differentiating Equation 1 with respect to r, ΔG^* is obtained [6]:

$$\Delta G^* = \frac{16\pi\sigma_r^3}{3\rho_L^2(\Delta G_1)^2}, \quad (2)$$

where subscript L denotes the liquid phase.

The critical radius is equal to [6]:

$$r^* = \frac{2\sigma_r}{3\rho_L^2(\Delta G_1)^2}. \quad (3)$$

r^* becomes increasingly smaller as the level of supercooling increases, since $P/P_s(T_G)$ increases with higher levels of supercooling. In Equation 3, liquid surface tension, σ_r for a flat surface is used.

In the vapour phase, there exists a statistically steady state population of molecular clusters of various sizes. As supercooling increases and r^* becomes smaller, there will eventually be enough clusters at the size of r^* (or greater) to start the condensation process and the reversion back to equilibrium. Using a Boltzmann distribution function to represent the cluster population, combined with the change in free energy at r^*, and subjecting the result to refinements by Courtney and Kantrowitz leads to the following equation as given in [3]:

$$J_{Ka} = \left\{1 + \frac{q_c \rho_G}{\alpha_r}\left(\frac{RT_G}{2\pi}\right)^{0.5}\left(\frac{l^2}{RT_G^2} - \frac{l}{2T_G}\right)\right\}^{-1} \times \sqrt{\frac{2N^3}{\pi}\frac{qP^2\sqrt{\sigma_r}}{\rho_L(RT_G)^2}} \exp\left[\frac{-16\pi N\sigma_r^3}{3\rho_L^2(RT_G)^3\left\{\ln\left(\frac{P}{P_s(T_G)}\right)\right\}^2}\right]. \quad (4)$$

J_{Ka} gives the steady-state number of droplets nucleated per unit volume per unit time. In Equation 4, q_c is the condensation coefficient, K is Boltzmann's constant, l is the equilibrium latent heat, P is the pressure and N is the number of molecules per unit mass.

It should be noted that no universally applicable nucleation model is yet available for condensing flows, however, the general form described here has been successfully used for steam at low pressures by a number of researchers [7,8].

DROPLETS GROWTH EQUATIONS IN SUPERSATURATED VAPOUR

The droplets grow by capturing molecules from bulk vapour. But, this mass transfer is affected also by the transfer rate of latent heat of phase change from droplets to the surrounding vapour. Bakhtar and Zidi presented a semi-empirical relation for droplet growth, as explained in [9]:

$$\begin{aligned}\frac{dm_r}{dt} &= 4\pi r^2 \rho \frac{dr}{dt} \\ &= \frac{q_c\sqrt{8\pi R}}{1+a_r}\left[\rho_G\sqrt{T_G} - \rho_s(T_L, r)\sqrt{T_L}\right]. \qquad (5)\end{aligned}$$

Denoting the energy released per unit mass of vapour, the rate of heat loss from the droplet to the vapour may be written as:

$$l\frac{dm_r}{dt} = 4\pi r^2 \alpha_r (T_L - T_G) + \frac{d(C_L m_r)}{dt}. \qquad (6)$$

In this expression, α_r is the coefficient of heat transfer from a droplet with radius r to the surrounding vapour, and l is latent heat. In the right hand side of the above expression, the second term can be neglected, with respect to the first term, and therefore, the heat transfer equation can be written as [10]:

$$l\frac{dm_r}{dt} = 4\pi r^2 \alpha_r (T_L - T_G). \qquad (7)$$

The Knudsen number, Kn, plays a key role in the coefficient of heat transfer due to the vast range of droplet radii. The Knudsen number is the ratio of the mean free path, $\bar{l}$, of vapour molecules to droplet diameter.

$$\text{Kn} = \frac{\bar{l}}{2r}. \qquad (8)$$

If the Knudsen number is much greater than unity, intermolecular collisions are not important. This is known as the free molecular flow and an expression for the coefficient of heat transfer can be obtained from the kinetic theory [9]:

$$\alpha_r = \frac{P_G}{\sqrt{2\pi R T_G}} \frac{\gamma_G + 1}{\gamma_G} C_{PG}, \qquad (9)$$

where γ is the ratio of specific heat and C_{PG} is the vapour specific heat at constant pressure.

At the other extreme, for a Knudsen number much less than unity, the vapour phase behaves as a continuum [11].

$$\alpha_r = \frac{\lambda_G}{r}, \qquad (10)$$

where λ is the coefficient of thermal conductivity. Figure 1 illustrates Knudsen numbers versus droplet diameter with temperature as a parameter. The critical radius of flow in a steam turbine is of the order between 10^{-3} and 10^{-4} μm, whilst the mean diameter of fine droplets after the last stage is of the order between 0.1 and 1 μm. Therefore, the Knudsen number has a wide range from free molecular flow to so-called slip flow. Unfortunately, it is difficult to model the coefficient of heat transfer in the intermediate Knudsen number range.

By hypothesizing that the surrounding space of a droplet consists of two separate layers, it is assumed that the exchange regime in the internal layer is free molecular and, in the external layer, a continuum regime is dominant. A two-layer model, as shown in Figure 2, provides an expression for the heat transfer coefficient applicable to a wide range of Knudsen numbers.

The droplet is assumed enclosed in a volume governed by free molecular flow. Surrounding this, the outer region is governed by continuum flow. The division between the two regions occurs at β times the mean free path.

Considering the conservation of energy at the middle plate:

$$l\frac{dm_r}{dt} = 4\pi r^2 \frac{P_i}{\sqrt{2\pi R T_i}} \left(\frac{\gamma+1}{2\gamma}\right) C_p (T_L - T_i), \qquad (11)$$

$$l\frac{dm_r}{dt} = 4\pi (r + \beta\bar{l})^2 \frac{\lambda}{r + \beta\bar{l}} (T_i - T_G). \qquad (12)$$

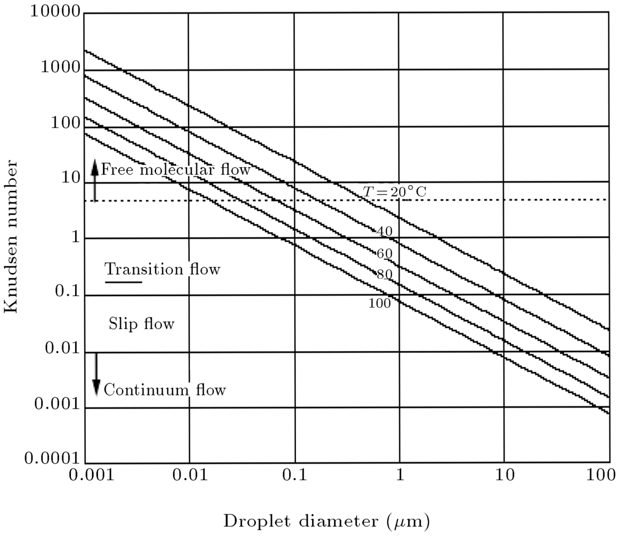

Figure 1. Knudsen number versus droplet diameter [12].

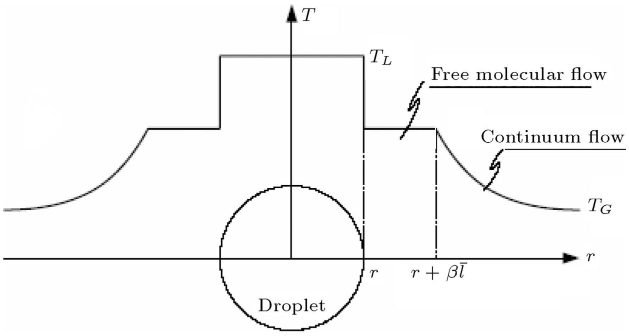

Figure 2. Temperature distribution around a droplet in the Langmuir model [12].

Eliminating T_i between the two above expressions and rearranging the result, it is concluded that [12]:

$$l\frac{dm_r}{dt} = 4\pi r^2 \frac{\lambda}{r\left[\frac{1}{(1+2\beta Kn)} + 3.78\frac{Kn}{\mathrm{Pr}}\right]}(T_L - T_G). \tag{13}$$

Hence, the heat transfer coefficient in this model can be calculated as:

$$\alpha_r = \frac{\lambda}{r\left[\frac{1}{(1+2\beta \mathrm{Kn})} + 3.78\frac{\mathrm{Kn}}{\mathrm{Pr}}\right]},$$

$$\beta = 0.75, \qquad \mathrm{Pr} = \frac{C_P \mu_G}{\lambda}, \tag{14}$$

where μ_G is the kinematic viscosity of vapour.

TWO-PHASE GAS DYNAMICS EQUATIONS

By considering the steady one-dimensional flow over an incremental distance dx along the channel, the fundamental equation of flow can be written as:

$$\frac{d\rho_G}{\rho_G} + \frac{dA}{A} + \frac{du}{u} + \frac{dW_L}{W - W_L} = 0, \tag{15}$$

where, A is the area, W is the overall mass flow rate and u is the velocity.

State Equation [10]:

$$\frac{P}{\rho_G R T_G} = 1 + B_1\rho_G + B_2\rho_G^2 + B_3\rho_G^3, \tag{16}$$

where B parameters are virial coefficients; these are functions of temperature as given in the Appendix.

$$\frac{dP}{P} - X\frac{d\rho_G}{\rho_G} - Y\frac{dT_G}{T_G} = 0,$$

where:

$$X = \frac{\rho_G}{P}\left(\frac{\partial P}{\partial \rho_G}\right)_{T_G}$$

$$= \frac{1 + 2B_1\rho_G^2 + 3B_2\rho_G^2 + 4B_3\rho_G^3}{1 + B_1\rho_G^1 + B_2\rho_G^2 + B_3\rho_G^3},$$

$$Y = \frac{T_G}{P}\left(\frac{\partial P}{\partial \rho_G}\right)_{\rho_G}$$

$$= 1 + \frac{\rho_G T_G\left(\frac{dB_1}{dT_G} + \rho_G\frac{dB_2}{dT_G} + \rho_G^2\frac{dB_3}{dT_G}\right)}{1 + B_1\rho_G^1 + B_2\rho_G^2 + B_3\rho_G^3}. \tag{17}$$

MACH NUMBER

The speed of sound, a, in a single phase gas is:

$$a = \sqrt{\frac{\gamma P}{\rho_G}}. \tag{18}$$

Thus:

$$Z = \mathrm{Ma}^2 = \frac{u^2}{\left(\frac{\gamma P}{\rho_G}\right)}. \tag{19}$$

Differentiating Equation 19 leads to:

$$\frac{dZ}{Z} = \frac{2d\mathrm{Ma}}{\mathrm{Ma}} = 2\frac{du}{u} + \frac{d\rho_G}{\rho_G} - \frac{dP}{P}. \tag{20}$$

MOMENTUM EQUATION

Considering the flow as a whole, the change in momentum of the system with the friction factor of f across dx can be written as:

$$AdP + A\frac{f\rho_G u_G^2}{2D_e}dx = -d[W_G u_G + W_L u_L]. \tag{21}$$

Expanding this equation, denoting $dW_G = -dW_L$ and dividing it by $A \times P$, regarding $\frac{(u_G - u_L)}{AP}dW_L$ as being small in comparison with other terms, and re-arranging yields [4]:

$$\frac{dP}{P} = -\frac{f\rho_G u_G^2}{2P}\frac{dx}{D_e} - \frac{(W - W_L)u_G}{AP}\frac{du_G}{u_G}$$

$$- \frac{W_L u_L}{AP}\frac{du_L}{u_L}. \tag{22}$$

ENERGY EQUATION

Considering the flow of energy into and out of the control volume and assuming the heat loss through the sides of the duct to be small, the energy equation can be written as:

$$d\left[(W-W_L)\left(h_G+\frac{u^2}{2}\right)+W_L\left(h_L+\frac{u^2}{2}\right)\right]=0. \tag{23}$$

Since the phase change occurs within the control volume, the term $dW_L(h_G - h_L)$ will appear while expanding this equation. It represents the heat released to, or absorbed from, the flow by condensation onto, or evaporating from, the droplets. Having considered the released energy associated with the phase change, the bulk of the vapour may then be regarded as a perfect gas for which the term dh_G can be replaced by $C_p dT_G$. Substituting this into Equation 23, denoting $(h_G - h_L)$ by l, regarding the term $\frac{u_G^2-u_L^2}{2}$ as being small compared to l, neglecting $W_L dl$, dividing throughout by WC_PT_G, and rearranging results in [13], yields:

$$\frac{dT_G}{T_G}=-\left(1-\frac{W_L}{W}\right)\frac{u_G^2}{C_pT_G}\frac{du_G}{u_G}+\frac{l}{C_pT_G}\frac{dW_L}{W}-\frac{W_L}{W}\frac{u_L^2}{C_pT_G}\frac{du_L}{u_L}. \tag{24}$$

Equations 17, 20, 22 and 24 can be solved for the four unknowns $\frac{dT_G}{T_G}$, $\frac{dP}{P}$, $\frac{du}{u}$ and $\frac{d\rho_G}{\rho_G}$, providing that $\frac{du_L}{u_L}$, $\frac{dA}{A}$ and $\frac{dW_L}{W_L}$ are known as explained in [2] or [6].

CALCULATING THE INTERACTION BETWEEN THE INJECTED AND NEW DROPLETS

There are two groups of droplets inside each element of Δx. Group (a) includes the droplets formed and grown within the element in the case of a considerable rate of nucleation. Group (b) includes the droplets formed in the upstream of the element entrance, which grow within the element of Δx. In the element of Δx, the time of Δt is divided by n to give a suitable time scale (δt):

$$\Delta t=\frac{\Delta x}{u},$$

and:

$$\delta t=\frac{\Delta t}{n}. \tag{25}$$

The new droplets grow with a radius of r^* and the mass of the liquid phase related to the droplets of group (a) at the time of nucleation is calculated as:

$$W_{L,a}=\left(J_{st}\delta t(AU)\frac{4}{3}\pi\right)\times\left(\rho_L\frac{r^{*3}}{2}+\rho_{L1}r_1^3+\rho_{L2}r_2^3+\cdots+\rho_{Ln}\frac{r_n^3}{2}\right). \tag{26}$$

Taking the number of droplets in group (b) as N_i, if r_i is the mean input radius and r_e is the mean output radius of the element, the net mass of the grown droplets within the element of Δx is equal to:

$$W_{L,b}=N_i\frac{4}{3}\pi(\rho_{l_e}r_e^3-\rho_{l_i}r_i^3). \tag{27}$$

It should be noted that in this research, the droplets of group (b) are initially injected and, then, new droplets are produced in the nucleation zone.

The total mass of the injected and formed droplets in the element of Δx is:

$$W_L=\frac{4}{3}\pi N_i r_i^3\rho_{Li}+W_L(a)+W_L(b),$$

$$N=N_i+J_{st}\Delta t(A\Delta x). \tag{28}$$

In order to decrease the calculation complexity, the mean diameters and temperatures of the droplets at the end of element Δx are used as inputs for the next element.

Using the method of mean square root, the droplet radius at the end of the element is determined as:

$$\bar{r}=\left(\frac{SA}{4\pi N}\right)^{\frac{1}{2}},$$

where:

$$SA=4\pi N_i r_e^2+J\delta t(A\Delta x)4\pi\left[\frac{r^{*2}}{2}+r_1^2+\cdots+\frac{r_n^2}{2}\right]. \tag{29}$$

Calculations are continued for other elements using this method.

ENTROPY CALCULATION

The explored flow entropy at each point is a combination of three parameters, namely; vapour entropy, liquid entropy and the entropy related to the formation of the droplet surface, each of which can be calculated from the following equations:

$$S=S_G+S_L+S_s. \tag{30}$$

Using the equation of state, along with thermodynamic equations, the following equation is derived for the special entropy of gas or vapour [7]:

$$S_G=R\left[(-B_1\rho)+\left(-\rho\frac{dB_1}{dT}\right)\right]+S_0, \tag{31}$$

where S_0 is calculated from:

$$S_0=0.30773-0.46153\ln(\rho)+1.1095\ln(T_G)+7.11756^*10^{-4}T_G-\frac{3495}{T_G^2}. \tag{32}$$

According to classic thermodynamics, the special entropy of water droplets at temperature T_L can be calculated with acceptable accuracy using:

$$S_L = C_L \ln\left(\frac{T_L}{T_D}\right). \tag{33}$$

In this equation, C_L is the special temperature of water and T_D is the datum temperature, which is equal to 273.15 Kelvin.

According to the thermodynamics of droplet surface formation, the entropy per unit surface is calculated from the following equation [14]:

$$S_s = -A_D\left(\frac{\partial\sigma}{\partial T}\right)_P. \tag{34}$$

Considering all the above points, the total entropy would be:

$$S = W_G S_G + W_L S_L + S_s. \tag{35}$$

Dividing the above equation by the total discharge, the special entropy of the total flow is calculated from:

$$s = (1-w)S_G + wS_L + \frac{S_s}{W}. \tag{36}$$

Considering the fact that the flow is adiabatic and the wetness at the inlet is constant, the outlet entropy can be a good basis for estimating the generated entropy.

When the radius of the droplet at the inlet is small (e.g. $R = 0.2$ micron), after a rising trend, the supersaturation ratio starts to decrease and, at the channel end ($X/L = 2.0$), reaches a low value. Considering the significant decrease in the outlet entropy in this state, presuming more condensation for the outlet flow will not have much effect on the increase of outlet entropy.

INVESTIGATION OF THE HEAT TRANSFER COEFFICIENT

The two-phase compressible flow field in a one dimensional Laval nozzle is solved, using both models for the heat transfer coefficient, which was described previously. Figure 3 shows the pressure ratio distribution along the nozzle computed by both models. By comparing the results to experimental results reported by Young [15], it is clearly observed that both non-equilibrium condensation flow calculations predict the pressure increase due to sudden heat release from growing droplets, but agreement with the experiment is better for the two-layer model.

Hence, in this research, the two layer heat transfer model is used to solve the droplet growth along the nozzle.

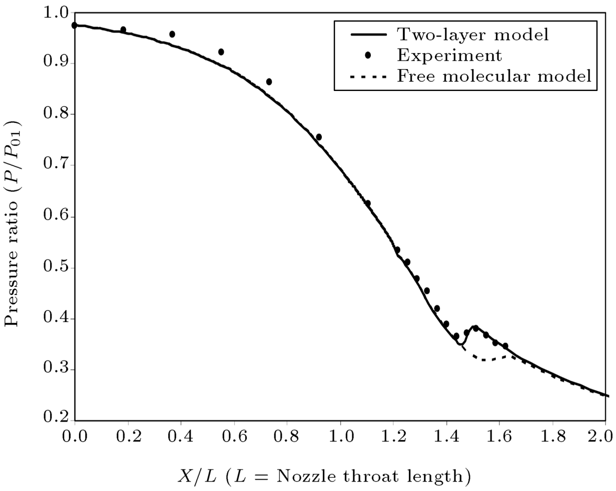

Figure 3. Comparison of computed pressure ratio along the nozzle with the experiment ($P_0 = 150$ kPa , $T_0 = 37$ 7K).

INJECTING WATER DROPLETS AT THE INLET OF THE LAVAL NOZZLE

In order to calculate the flow variables in the convergent-divergent nozzle, they are studied in the two-phase mixture, and the saturation line is crossed by the supersonic wet steam non-equilibrium flow in the convergent part.

The rate of injecting the liquid droplets at the inlet of the Laval nozzle is calculated by giving the wetness fraction and droplet diameter. It is assumed that the mass of vapour phase is decreased by the same amount as the mass of injected droplets.

The number of droplets can be varied by changing the wetness fraction for a given droplet diameter or by changing the diameter for a given wetness fraction.

RESULTS AND DISCUSSION

Droplets are injected at the inlet and just after the throat of the nozzle with a geometry that is shown in Figure 4.

Stagnation pressure and temperature at the inlet of the nozzle are considered as 70 kPa and 366 K.

Injected droplets are performed with 0.5% of wetness fraction and radiuses between 2 and 0.2 microns. Injected droplets provide surfaces for the condensation of vapour.

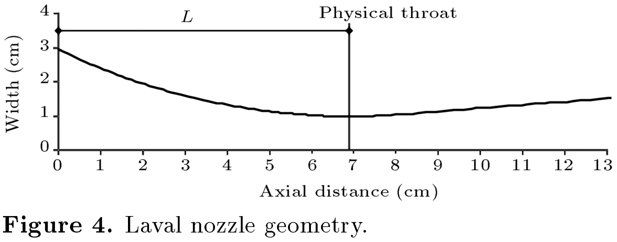

Figure 4. Laval nozzle geometry.

Since Gibbs free energy of condensation on the surface is less than that of the nucleation, it is expected that the nucleation rate will be decreased.

On the other hand, in a certain wetness fraction, the smaller injecting droplet diameter gives a higher number of droplets and, therefore, greater surfaces are provided for the condensation of vapour. So, it is confirmed that droplets with a smaller radius cause a greater reduction in nucleation rate.

Figure 5 represents the variation of nucleation rate along the nozzle length and proves this fact.

Figure 6 represents the variation of pressure ratio along the nozzle length. It is observed that the injected droplets reduce the condensation shock strength. Since the nucleation is reduced by injection of droplets, less latent heat is given to the vapour flow and, therefore, condensation shock strength is reduced.

As the nucleation rate reduction is greater for the smaller injected droplets, it is forecast that the strength of the condensation shock has a greater decrease for the smaller injected droplets. The thermodynamic losses in two-phase flow may be estimated by the entropy generation between the inlet and outlet of the nozzle.

Table 1 shows the outlet entropy for various diameters of injected droplets at the inlet of the Laval nozzle.

It is clearly observed that the outlet entropy is reduced significantly as the result of droplet injection. On the other hand, it is seen that with a constant wetness fraction, smaller injected droplets cause much reduction in the outlet entropy.

As expected, the more droplets injected, the less heat is given to the flow. Therefore, the increased temperature is smaller for vapour, as shown in Figure 7.

In steam turbines, the purpose is to transform the flow enthalpy to kinetic energy. Fortunately, the injection droplets into wet steam flow prevent a decrease in fluid Mach number as shown in Figure 8.

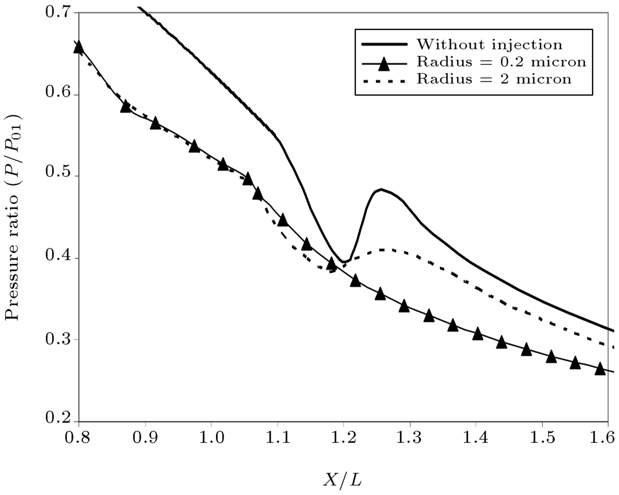

Figure 6. Effect of droplets injection on pressure changes (0.5% inlet wetness).

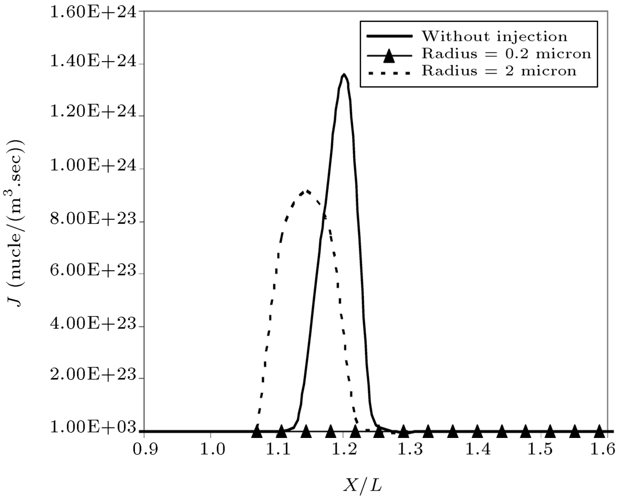

Figure 5. Effect of droplets injection on nucleation (0.5% inlet wetness).

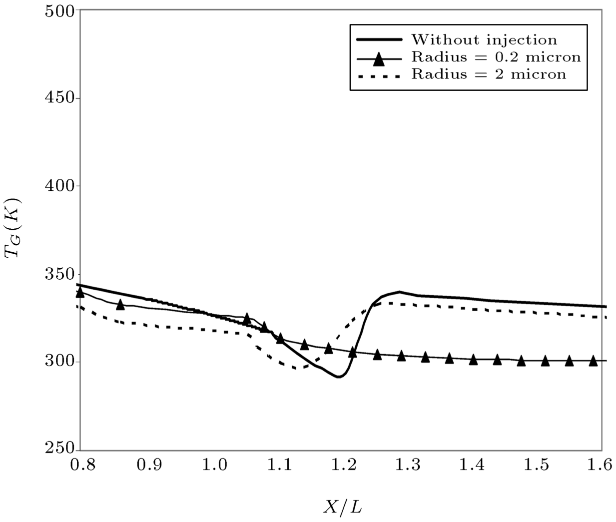

Figure 7. Effect of droplets injection on vapour temperature changes (0.5% inlet wetness).

Table 1. Effect of injected droplets on outlet entropy.

Wetness at Inlet = 0.5%						**Wetness at Inlet = 0%**
Radius (μm)	0.2	0.3	0.6	1	2	0
Outlet Entropy $\left(\frac{\text{kJ}}{\text{kg.K}}\right)$	5.65	6.62	6.81	6.85	6.91	7.51

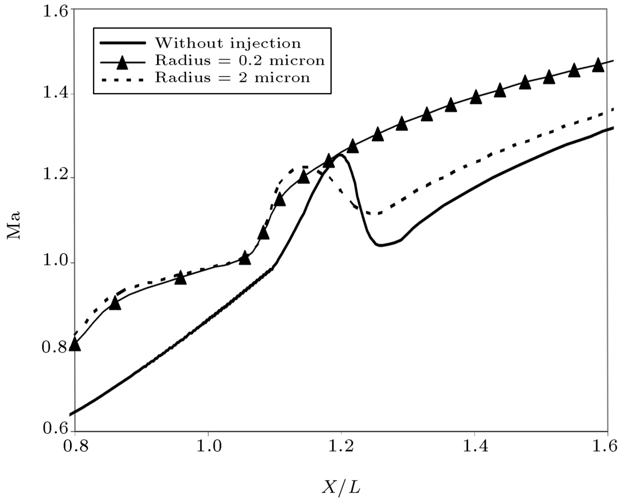

Figure 8. Effect of droplets injection on Mach number (0.5% inlet wetness).

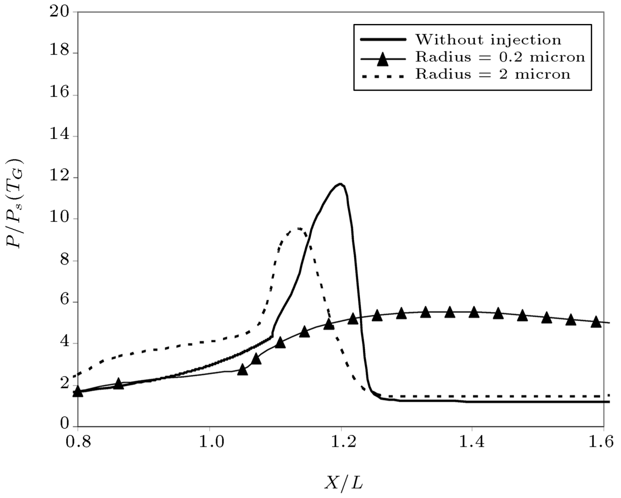

Figure 9. Effect of droplets injection on supersaturating ratio (0.5% inlet wetness).

It is observed that the injection of droplets reduces the degree of supercooling or the supersaturating ratio to a level much lower than the case without injection as shown in Figure 9.

Variation of the droplets mean radius is represented in Figure 10. Because of initial vaporization, mean radius is decreased at the inlet. On account of the nucleation process after the throat, there are many very small droplets in that region and these very small droplets reduce the mean diameter of the droplets.

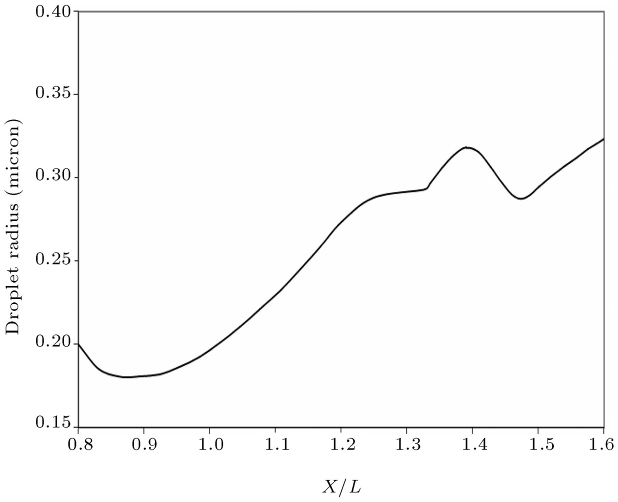

Figure 10. Variation of mean droplet radius along nozzle (0.5% inlet wetness, injected droplet radius = 0.2 micron).

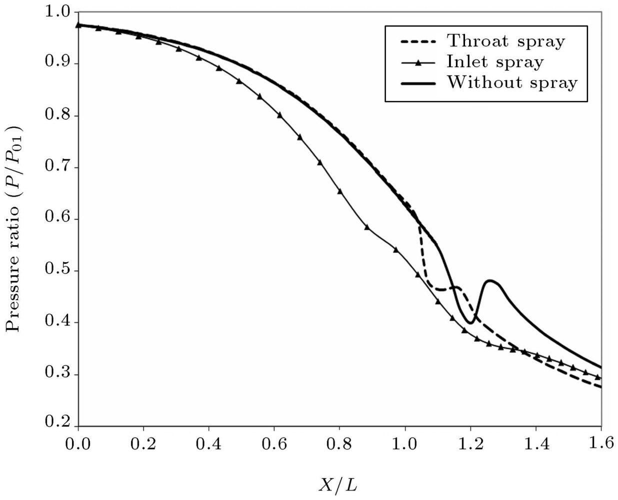

Figure 11. Variation of pressure ratio along nozzle length (0.5% inlet wetness, injected droplet radius = 0.2 micron).

LOCATION EFFECT OF INJECTION DROPLETS

The injection of droplets is performed with different inlet radius and wetness fractions at the inlet and at the throat of the Laval nozzle. In Figure 11, variation of the pressure ratio along the nozzle length for these cases, are shown. It is observed that the injection of droplets causes a tremendous reduction in the condensation shock strength. It is also observed that the injection of droplets at the inlet has better effects on the flow characteristics. Variations of wetness fractions along the nozzle length, for these cases, are shown in Figure 12. It is observed that the inlet droplet injection leads to a lower amount of wetness along the nozzle length.

The reason for the injection of droplets at the inlet of the converging channel being more effective than injection at the throat is as follows:

i) Pressure distribution along the channel length is steadier.

ii) When injecting small droplets at the inlet, nucle-

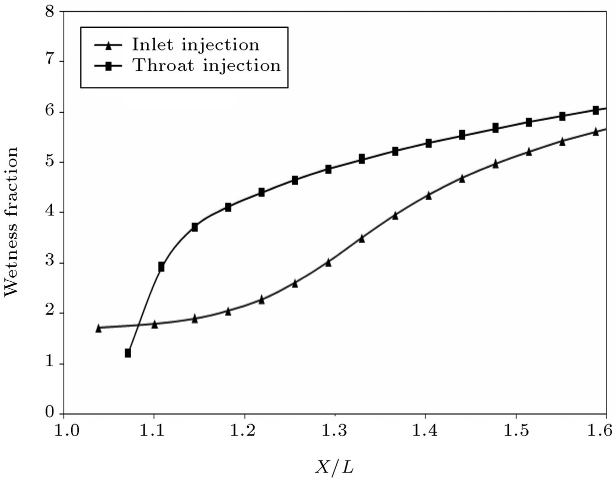

Figure 12. Variation of wetness fraction along nozzle.

ation does not occur while the injection at the throat is accompanied by nucleation.

iii) The amount of wetness in the diverging channel section when the droplets are injected at the inlet is less than that of the state where the injection is at the throat as shown in Figure 10.

iv) Due to the formation of less wetness when injecting droplets at the channel inlet, the generated entropy is less than when the droplets are injected at the throat.

In fact, the injected droplets at the inlet accelerate the flow in the convergent section and reduce the rate of nucleation in the divergent part and, therefore, the condensation shock strength is considerably decreased.

SUMMARY OF CONCLUSION

1. Using a two-layer heat transfer model, we can successfully predict the flow behavior.
2. Injecting the droplets at the inlet of the nozzle accelerates the flow in the convergent section, causes severe reduction of the nucleation rate in the diverging part and results in the disappearance of condensation shock.
3. Injection at the inlet of the nozzle is better than injection droplets after the nozzle throat in the supersonic region.

Hence, it is concluded that the injection of droplets with suitable droplet radius or wetness fraction at the inlet of actual steam turbines, causes a reduction in the nucleation rate and results in the disappearance of condensation shock, consequently, improving turbine efficiency.

NOMENCLATURE

A	area
a	speed of sound
$B1, B2, B3$	viral coefficients
C_p	specific heat at constant pressure
C_L	special temperature
f	friction factor
ΔG	change in Gibbs free energy
J	rate of formation of critical droplets per unit volume and time
Kn	Knudsen number
l	latent heat
$\bar{l}$	mean free path of vapour molecules
Ma	mach number
m_r	mass of droplet
P	vapour pressure
$P_s(T_G)$	saturation pressure at T_G
q_c	condensation coefficient
R	gas constant for vapour phase
r	radius of droplet
r^*	critical radius
T	temperature
$T_s(P)$	saturation temperature at P
ΔT	degree of supercooling $[T_s(P) - T_G]$
U	velocity
v	specific volume
W	total mass flow rate
x	distance along duct axis
X, Y	functions of temperature and density in equation of state
α_r	coefficient of heat transfer
γ	ratio of specific heats
μ_G	kinematic viscosity of vapour
ζ	dryness fraction
ρ	density of mixture
λ	coefficient of thermal conductivity
σ	surface tension
$\rho_s(T_L, r)$	density corresponding to saturation pressure at temperature T_L over a surface of curvature r

Subscripts

G	vapour phase
L	liquid phase
0	stagnation condition
s	saturation

Superscripts

$*$	critical droplet

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APPENDIX

The virial coefficients of the Vukalovich state equation:

$$B_1 = -\frac{e}{GT_G} - \phi_1 + b,$$

$$B_2 = -b\phi_1 + 4\phi_1^2\phi_2,$$

$$B_3 = 32b\phi_1^2\phi_2,$$

where:

$$\phi_1 = \frac{CG}{T_G^{W_1}}, \qquad \phi_2 = 1 - \frac{K}{T_G^{W_2}},$$

and:

$$e = 63.2, \qquad b = 0.00085$$

$$C = 0.3900 \times 10^6, \qquad G = 47.053,$$

$$K = 22.7, \qquad n = 0.355 \times 10^{-7},$$

$$m_1 = 1.968, \qquad m_2 = 2.957,$$

$$W_1 = (3 + 2m_1)/2, \quad W_2 = (3m_2 - 4m_1)/2.$$

The thermodynamic properties:

$$h = pv - RT_G^2\left(\frac{1}{v}\frac{dB_1}{dT_G} + \frac{1}{2v^2}\frac{dB_2}{dT_G} + \frac{1}{3v^3}\frac{dB_3}{dT_G}\right) + 1.111177T_G + 3.55878 \times 10^{-4}T_G^{-2} - \frac{6991.96}{T_G} + 2070.54,$$

$$s = R + 0.30773 + 1.111177\ln T_G + 7.11756 \times 10^{-4}T_G - \frac{3495.98}{T_G^2},$$

$$C_v = \frac{\partial}{\partial T_G}(h - pv)_v,$$

$$C_p = C_v - \frac{T_G\left(\frac{\partial p}{\partial T_G}\right)_v^2}{\left(\frac{\partial p}{\partial v}\right)_T},$$

$R = 0.46151 \quad \text{kJ/kg.K.}$

BIOGRAPHIES

Mohammad Reza Mahpeykar received his B.S. degree in Mechanical Engineering from Ferdowsi University of Mashhad in 1981, his M.S. degree in Thermodynamics and Fluid Mechanics from Birmingham University, UK, in 1987, and a Ph.D. degree in Energy Conversion from the same university in 1992.

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