Transaction E: Industrial Engineering
Vol. 17, No. 1, pp. 12–24

SCIENTIA
IRANICA

Phase II Monitoring of Autocorrelated Polynomial Profiles in AR(1) Processes

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Abstract. *In many practical situations, the quality of a process or product can be characterized by a function or profile. Here, we consider a polynomial profile and investigate the effect of the violation of a common independence assumption, implicitly considered in most control charting applications, on the performance of the existing monitoring techniques. We specifically consider a case when there is autocorrelation between profiles over time. An autoregressive model of order one is used to model the autocorrelation structure between error terms in successive profiles. In addition, two remedial methods, based on time series approaches, are presented for monitoring autocorrelated polynomial profiles in phase II. Their performances are compared using a numerical simulation runs in terms of an Average Run Length (ARL) criterion. The effects of assignable cause and autocorrelation coefficient on the shape of profiles are also investigated.*

Keywords: *Statistical process control; Polynomial profiles; Autocorrelation; Average run length; Assignable cause; Phase II.*

INTRODUCTION

In some statistical process control applications, the quality of a process or product is characterized by a relationship between a response variable and one or more explanatory variables, which is referred to as a profile. A number of authors, including Stover and Brill [1], Kang and Albin [2], Mahmoud and Woodall [3], Woodall et al. [4] and Wang and Tsung [5], have discussed the practical applications of profiles. Many authors, including Mestek et al. [6], Stover and Brill [1], Kang and Albin [2], Kim et al. [7], Mahmoud and Woodall [3] and Mahmoud et al. [8] have studied the phase I monitoring of simple linear profiles. The purpose of the phase I analysis is to evaluate the stability of a process and to estimate process parameters. Authors including Kang and Albin [2], Kim et al. [7], Noorossana et al. [9], Gupta et al. [10], Zou et al. [11] and Niaki et al. [12] have investigated the phase II monitoring of simple linear profiles. In the phase II analysis, we are interested in detecting shifts in the process parameters as soon as possible. Sometimes, more complicated models are needed to represent profiles. Kazemzadeh et al. [13] extended three phase I methods for monitoring polynomial profiles. Zou et al. [14] proposed a multivariate EWMA control chart for monitoring general linear profiles in phase II. Kazemzadeh et al. [15] transformed polynomial regression to an orthogonal polynomial regression model and proposed a method based on using EWMA control charts to monitor the parameters of an orthogonal polynomial regression model in phase II. In all the mentioned research, it is assumed that the error terms of models are independent and identically distributed normal random variables. However, in some cases, these assumptions can be violated. Noorossana et al. [16] studied the effect of the non-normality of error terms on the performances of the EWMA/R method by Kang and Albin [2]. Jensen et al. [17] proposed a linear mixed model to account for the autocorrelation within a linear profile. Noorossana et al. [18] considered a case in which there is a first order autoregressive autocorrelation between linear profiles over time. They proposed three methods based on a time series approach and evaluated the performance of their methods. Nonlinear profile monitoring has also been discussed by several authors, including Jin and

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Received 4 March 2008; received in revised form 14 June 2008; accepted 6 October 2008

Shi [19], Walker and Wright [20], Ding et al. [21] and Williams et al. [22]. Jensen and Birch [23,24] used nonlinear mixed models to monitor the nonlinear profiles in phase I to account for the correlation structure within nonlinear profiles. They showed that the use of mixed models could have significant advantages when nonlinear regression models are used. Woodall [25] reviewed current research on profile monitoring and encouraged further research in this area.

In this paper, we consider a kth order polynomial profile between a response variable and one explanatory variable. In some practical applications, such as paint shops in the auto industry, where different locations on a body are painted by different nozzles and the time lapse between two successive painted bodies is short, the error terms or observations in successive profiles, relating paint thickness (Y) to body location (X), can be correlated. Since the nozzles operate independently, it is reasonable to assume that the profile autocorrelation is negligible. Hence, in this paper, it is assumed that the error terms in successive profiles are autocorrelated. We consider an autoregressive model of order one to model the autocorrelation structure that exists between error terms in successive profiles and show its impact on the ARL performance of the T^2 control chart extended by Kazemzadeh et al. [14]. Two methods are provided to eliminate the effect of autocorrelation between profiles and their performances are evaluated using ARL criterion.

The problem model is given in the following section. Then the two monitoring methods are presented and the effect of autocorrelation on the T^2 control chart and comparison of the performance of the two remedial methods are investigated. Following that the effects of assignable cause and correlation coefficient on the shape of profiles are investigated, respectively, and an illustrative example is also presented. Our concluding remarks are presented in the final section.

THE PROBLEM MODEL

We assume that for the jth sample collected over time, we have observations $(x_i, x_i^2, \cdots, x_i^k, y_{ij})$ $i = 1, 2, \ldots, n$. In other words, the subscript i shows the ith observations within each profile, and subscript j shows the jth profile collected over time. Each profile includes n observations as $(x_i, x_i^2, \cdots, x_i^k, y_{ij})$. In this paper, it is assumed that a kth order polynomial regression in one variable is well fitted to the observations of each profile and relates the response variable, Y, to the explanatory variable, X. The range of x-values and their scales are case-based and different from one application to the other one. As explained in the introduction of the paper, sometimes the error terms or observations in successive profiles can be correlated. However, due to the nature of the process, there is no autocorrelation between observations within each profile. In other words, the error terms or equivalent observations at different values of x within each profile are assumed to be independent of each other. Assuming a first order autoregressive model for the error terms, the explained problem can be formulated as follows:

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k + \varepsilon_{ij},$$

$$\varepsilon_{ij} = \phi \varepsilon_{i(j-1)} + a_{ij}, \tag{1}$$

where ε_{ij}'s are the correlated error terms and a_{ij}'s are independent and identically distributed normal random variables with mean zero and variance σ^2. It is assumed that the x-values are fixed and constant from profile to profile. In this paper, we consider the phase II case in which the in-control values of parameters $A_0, A_1, \cdots, A_k$ and σ^2 are assumed to be known.

It can be easily shown that the existing autoregressive structure between errors, defined in Equation 1, leads to autocorrelation between observations at different values of x in successive profiles. In other words, the observations in successive profiles can be expressed as:

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k + \varepsilon_{ij},$$

and:

$$y_{i(j-1)} = A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k + \varepsilon_{i(j-1)},$$

leading to:

$$\begin{aligned} y_{ij} - &(A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k) \\ &= \phi[y_{i(j-1)} - (A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k)] \\ &+ a_{ij}. \end{aligned} \tag{2}$$

The data framework is depicted schematically in Figure 1 as follows:

THE PROPOSED METHODS

The forecasting observation vectors are calculated using Equation 3 and the residual vectors corresponding to each profile are determined by using Equation 4.

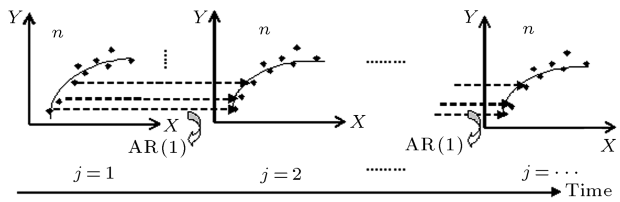

Figure 1. Data framework.

Although there is an AR(1) relationship between observations corresponding to each x in different profiles, the residuals are independent and can be monitored by traditional control charts.

The estimated values of y_{ij} are calculated using the following equation:

$$\hat{y}_{ij} = \phi y_{i(j-1)} + (1-\phi)(A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k). \quad (3)$$

And the residual values are as follows:

$$e_{ij} = y_{ij} - \hat{y}_{ij} = y_{ij} - \phi y_{i(j-1)} - (1-\phi)(A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k). \quad (4)$$

The expected value and variance of the residuals are equal to 0 and σ^2, respectively (they are proven in Appendix A).

As a first method, we use an EWMA control chart to monitor the average value of the residuals, in combination with a R-chart, to detect shifts in the process variance. The proposed control charts are the same as the control charts proposed by Kang and Albin [2] for monitoring linear profiles in which the errors in different profiles are independent. The only difference is the way we calculate the residuals. First, the residuals are calculated using Equation 4, then the average value of the residuals for the jth profile can be calculated using $\bar{e}_j = \sum_{i=1}^{n} e_{ij}/n$.

The EWMA control chart statistic, denoted by z_j, for $j = 1, 2, \cdots$, is given by:

$$z_j = \theta \bar{e}_j + (1-\theta) z_{j-1}. \quad (5)$$

$\theta (0 < \theta \leq 1)$ is a smoothing constant and $z_0 = 0$. The lower and upper control limits for the EWMA chart are:

$$\text{LCL} = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}},$$

and:

$$\text{UCL} = L\sigma\sqrt{\frac{\theta}{(2-\theta)n}}, \quad (6)$$

respectively, where $L(> 0)$ is a constant selected to give a specified in-control ARL.

The R control chart statistic denoted by R_j is calculated by $R_j = \max(e_{ij}) - \min(e_{ij})$. The lower and upper control limits for the R chart are:

$$\text{LCL} = \sigma(d_2 - Ld_3),$$

and:

$$\text{UCL} = \sigma(d_2 + Ld_3), \quad (7)$$

respectively, where $L(> 0)$ is a constant chosen to give a specified in-control ARL. The values of d_2 and d_3 are constants that depend on the sample size, n.

As a second method, we propose a modified T^2 control chart by Kang and Albin [2]. It should be noted that the proposed T^2 statistics are different from the ones used by Kang and Albin [2]. They used the estimates of the regression parameters to construct their T^2 statistics. Our simulation studies in section 4 showed that the autocorrelation structure leads to poor performance of the T^2 control charts by Kang and Albin [2]. It is shown in Appendices B and C that the AR(1) model between error terms in successive profiles transfers to vector of regression parameter estimators and the T^2 statistics, respectively. This is why the T^2 control chart by Kang and Albin [2] is not applicable here. Hence, we used the residuals in Equation 4 to construct the T^2 statistic. The modified T^2 statistic and the upper control limit are given, respectively, as follows:

$$T_j^2 = (\underline{e}_j - \underline{0}) \sum_{\underline{e}_j}^{-1} (\underline{e}_j - \underline{0})^T,$$

$$\text{UCL} = \chi^2_{n,\alpha}, \quad (8)$$

where $\underline{e}_j = (e_{1j}, e_{2j}, \cdots, e_{nj})^T$, $\sum_{\underline{e}j} = \sigma^2 I$ (I is identity matrix and $\underline{0}$ is zero vector). In the vector of $\underline{e}_j$, n defines the number of x values. $\chi^2_{n,\alpha}$ is the 100(1-α) percentile of the chi-square distribution with n degrees of freedom.

SIMULATION STUDIES

In this section, we first study the effect of autocorrelation between profiles on the performance of the T^2 control chart used by Kazemzadeh et al. [15] for monitoring polynomial profiles, based on the proposed control chart by Kang and Albin [2]. Then, we compare the performance of the proposed methods.

The following example is used to evaluate the performance of the T^2 control chart in the presence of autocorrelation between errors in the successive profiles:

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij},$$

$$\varepsilon_{ij} = \phi \varepsilon_{i(j-1)} + a_{ij},$$

where $a_{ij} \sim N(0, 1)$ and x =1, 2, 3, 4, 5, 6, 7, 8, 9, 10. The sample size used by Kang and Albin [2] in a simple linear case is equal to 4. It should be noted that a sample size greater than 4 is needed to fit a polynomial regression model to the observations more adequately. Hence, 10 observations similar to those in the paper by Kazemzadeh et al. [13] are applied here.

Table 1. The effect of autocorrelation coefficient on ARL performance for T^2 control chart under different shifts in intercept, second parameter, third parameter and error standard deviation.

		λ (Shift in the Intercept)										
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	200	165.6	103.9	57.7	31.6	17.6	10.3	6.4	4.2	2.9	2.2
ϕ	0.1	189.9	157.8	99.5	56.2	30.9	17.7	10.5	6.6	4.3	3.1	2.3
Autocorrelation	0.3	119.9	102.3	70.6	43.1	26.0	15.7	10.1	6.7	4.6	3.3	2.5
Coefficients	0.5	51.9	47.3	37.2	26.6	18.6	13.0	9.1	6.5	4.7	3.5	2.7
	0.7	18.9	18.2	16.5	14.4	11.9	9.7	7.7	6.1	4.8	3.7	2.9
	0.9	8.1	8.1	7.8	7.5	7.1	6.5	5.8	5.2	4.4	3.7	3.1
		β (Shift in the Second Parameter)										
		0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
	0	200	180.7	138.2	95.3	62.8	40.2	26.3	17.3	11.8	8.2	5.9
ϕ	0.1	189.9	170.3	131.9	92.1	60.6	39.5	26.2	17.6	11.9	8.4	6.0
Autocorrelation	0.3	119.9	111.1	89.4	65.6	46.0	32.1	22.2	15.7	11.4	8.3	6.2
Coefficients	0.5	51.9	49.3	43.1	35.4	27.7	21.6	16.7	12.8	9.9	7.8	6.1
	0.7	18.9	18.6	17.6	16.2	14.5	12.9	11.2	9.6	8.2	6.9	5.8
	0.9	8.1	8.1	8.0	7.8	7.6	7.2	6.8	6.4	6.0	5.5	5.1
		δ (Shift in the Third Parameter)										
		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
	0	200	160.4	93.4	48.7	24.9	13.4	7.6	4.7	3.1	2.2	1.7
ϕ	0.1	189.9	151.8	89.2	47.1	24.6	13.5	7.9	4.9	3.3	2.4	1.8
Autocorrelation	0.3	119.9	100.1	64.3	36.9	21.1	12.6	7.9	5.1	3.5	2.5	1.9
Coefficients	0.5	51.9	46.1	34.9	23.8	16.1	10.8	7.4	5.2	3.7	2.7	2.1
	0.7	18.9	18.1	16.1	13.5	11.0	8.6	6.8	5.2	3.9	3.0	2.3
	0.9	8.1	8.0	7.8	7.4	6.8	6.2	5.4	4.7	3.9	3.1	2.5
		γ (Shift in the Standard Deviation)										
		1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
	0	200	70.9	32.9	18.1	11.5	7.8	5.8	4.6	3.7	3.1	2.7
ϕ	0.1	189.9	67.9	31.6	17.7	11.1	7.8	5.8	4.5	3.7	3.2	2.8
Autocorrelation	0.3	119.9	48.1	24.0	14.3	9.5	6.9	5.3	4.2	3.5	3.1	2.7
Coefficients	0.5	51.9	25.3	15.1	10.0	7.2	5.6	4.5	3.8	3.2	2.8	2.5
	0.7	18.9	12.1	8.6	6.6	5.3	4.4	3.7	3.2	2.9	2.6	2.4
	0.9	8.1	6.5	5.4	4.6	4.0	3.5	3.1	2.8	2.6	2.4	2.2

We used 50,000 simulation runs and showed the effect of different autocorrelation coefficients under different shifts in the intercept, the second parameter, the third parameter and error standard deviation, using an average run length criterion. The results are summarized in Table 1. In this table, λ, β, δ and γ are shifts in the intercept, the second parameter, the third parameter and standard deviation, respectively, and are measured in units of σ. The results show that in-control ARLs of T^2 control chart decrease in the presence of autocorrelation between profiles and lead to its poor performance. When the autocorrelation coefficient gets larger, this effect is more considerable. In addition, decreasing the in-control ARL values for the T^2 control chart leads to lower out-of-control ARL values.

Now we compare the ARL performance of our proposed methods using the same example we introduced in our initial simulation studies. In our study, we considered two autocorrelation coefficients $\phi = 0.1$ and $\phi = 0.9$ (both weak and strong autocorrelations), and designed the proposed methods to have the overall in-control ARL of 200. The smoothing constant θ in the EWMA chart is set equal to 0.2. In the EWMA and R control charts under both $\phi = 0.1$ and $\phi = 0.9$ autocorrelation coefficients, the value of L is set equal to 3.08, in order to obtain an overall in-control ARL of 200. In the T^2 control chart, UCL is set equal

Table 2. ARL (top) and SDRL (bottom) comparisons under intercept shifts from A_0 to $A_0 + \lambda\sigma$ with $\phi = 0.1$ and $\phi = 0.9$.

$\boldsymbol{\phi = 0.1}$	$\boldsymbol{\lambda}$										
Proposed Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\boldsymbol{T^2}$	198.5 (200.9)	183.8 (184.7)	152.3 (152.3)	111.5 (111.8)	77.4 (76.5)	50.5 (49.5)	32.3 (31.6)	20.7 (20.2)	13.7 (13.1)	9.1 (8.6)	6.3 (5.7)
EWMA/R	197.3 193.8	103.6 (97.4)	33.7 (28.6)	14.8 (10.3)	9.0 (5.1)	6.2 (2.9)	4.8 (1.9)	3.9 (1.4)	3.3 (1.1)	2.9 (0.9)	2.6 (0.7)
$\boldsymbol{\phi = 0.9}$	$\boldsymbol{\lambda}$										
Proposed Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$\boldsymbol{T^2}$	199.2 (199.2)	201 (202)	200 (204)	195 (193)	196 (193)	195 (195)	193 (189)	191 (189)	188 (189)	188 (188)	184 (188)
EWMA/R	200.8 (201.9)	198 (191)	190 (186)	182 (181)	170 (165)	158 (155)	141 (138)	129 (127)	114 (109)	100 (94)	90 (85)

Table 3. ARL (top) and SDRL (bottom) comparisons under shifts form A_1 to $A_1 + \beta\sigma$ with $\phi = 0.1$ and $\phi = 0.9$.

$\boldsymbol{\phi = 0.1}$	$\boldsymbol{\beta}$										
Proposed Methods	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$\boldsymbol{T^2}$	198.5 (200.9)	170.8 (172.9)	108.9 (109.2)	57.6 (57.1)	29.1 (28.3)	14.9 (14.2)	8.1 (7.5)	4.7 (4.2)	2.9 (2.4)	2.1 (1.5)	1.6 (0.9)
EWMA/R	197.3 (193.8)	66.7 (60.8)	17.5 (12.5)	8.3 (4.5)	5.4 (2.4)	4.0 (1.5)	3.2 (1.0)	2.7 (0.8)	2.3 (0.6)	2.1 (0.5)	1.9 (0.4)
$\boldsymbol{\phi = 0.9}$	$\boldsymbol{\beta}$										
Proposed Methods	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$\boldsymbol{T^2}$	199.2 (199.2)	199 (199)	202 (203)	192 (193)	195 (195)	192 (189)	187 (189)	182 (184)	172 (172)	168 (167)	165 (166)
EWMA/R	200.8 (201.9)	200 (197)	184 (185)	168 (165)	151 (151)	132 (129)	110 (109)	93 (90)	77 (74)	64 (59)	55 (50)

to 25.1882, to give an in-control ARL of 200. We used 10,000 simulation runs to achieve out-of-control ARL under different shifts in the intercept, the second parameter, the third parameter and the error standard deviation. The standard deviations of run length values are also reported in parentheses to estimate the standard error of simulations, as well as confirming the validity of our simulation studies. Since run length values follow a geometric distribution in cases where the parameters are known, the standard deviation of RL will be close to the mean whenever type one error is small [26]. The results are summarized in Tables 2 through 5.

In both weak and strong autocorrelation situations ($\phi = 0.1$ and $\phi = 0.9$) under the intercept shift from A_0 to $A_0 + \lambda\sigma$, the EWMA/R method uniformly performs better than the T^2 control chart in the entire range of the intercept shift. In the case of strong autocorrelation, the out-of-control ARLs are larger than the weak autocorrelation situation because the shifts in the expected value of the residuals for $\phi = 0.9$ are highly small. When a shift of $\lambda\sigma$ in the intercept of a reference profile occurs, a shift of $\lambda\sigma$ in that period and a shift of $\lambda\sigma(1-\phi)$ at following periods in the expected value of the residuals will also occur (see Appendix D).

Under shifts in the second parameter from A_1 to $A_1 + \beta\sigma$, under the strong autocorrelation coefficient, the EWMA/R method is uniformly better than the T^2 control chart. Except in very large shifts, the same result under the weak autocorrelation coefficient is obtained. From Table 3, we can conclude that the out-of-control ARLs for the strong autocorrelation case are larger than the weak autocorrelation situation. Similar justification as that discussed for the intercept case is applicable here (see Appendix D).

Table 4. ARL (top) and SDRL (bottom) comparisons under shifts from A_2 to $A_2 + \delta\sigma$ with $\phi = 0.1$ and $\phi = 0.9$.

$\phi = 0.1$	δ										
Proposed Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
T^2	198.5 (200.9)	49.7 (49.8)	6.2 (5.6)	1.7 (1.0)	1.1 (0.3)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)
EWMA/R	197.3 (193.8)	9.3 (5.4)	3.4 (1.2)	2.1 (0.6)	1.6 (0.5)	1.2 (0.4)	1.0 (0.1)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)	1.0 (0.0)
$\phi = 0.9$	δ										
Proposed Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
T^2	199.2 (199.2)	196.0 (193.4)	182.1 (182.2)	164.8 (164.2)	140.5 (140.0)	120.5 (120.1)	100.1 (99.2)	78.3 (77.1)	64.3 (63.9)	48.8 (48.0)	38.5 (38.1)
EWMA/R	200.8 (201.9)	173.8 (170.1)	117.3 (113.4)	70.7 (67.8)	44.4 (39.3)	28.7 (23.8)	19.6 (14.8)	14.8 (10.3)	11.5 (7.3)	9.3 (5.5)	7.8 (4.2)

Table 5. ARL (top) and SDRL (bottom) comparisons under standard deviation shifts from σ to $\gamma\sigma$ with $\phi = 0.1$ and $\phi = 0.9$.

$\phi = 0.1$	γ										
Proposed Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
T^2	198.5 (200.9)	45.3 (44.2)	15.5 (14.8)	7.2 (6.6)	4.3 (3.7)	2.9 (2.4)	2.2 (1.6)	1.8 (1.2)	1.5 (0.9)	1.4 (0.7)	1.3 (0.6)
EWMA/R	197.3 (193.8)	59.3 (57.5)	23.4 (22.3)	11.8 (11.0)	6.8 (6.0)	4.5 (3.9)	3.2 (2.6)	2.5 (1.9)	2.1 (1.5)	1.8 (1.1)	1.6 (0.9)
$\phi = 0.9$	γ										
Proposed Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
T^2	199.2 (199.2)	45.4 (45.1)	15.7 (15.2)	7.4 (7.0)	4.2 (3.6)	3.0 (2.4)	2.2 (1.6)	1.8 (1.2)	1.5 (0.9)	1.4 (0.7)	1.3 (0.6)
EWMA/R	200.8 (201.9)	58.8 (57.7)	23.8 (23.2)	11.5 (10.9)	6.8 (6.0)	4.4 (3.8)	3.2 (2.6)	2.5 (1.9)	2.1 (1.4)	1.8 (1.2)	1.6 (0.9)

For the third parameter, under the strong autocorrelation coefficient, the results are similar to the intercept and the second parameter, and are summarized in Table 4. Under the weak autocorrelation coefficient, the EWMA/R method performs better than the T^2 method in a very small shift. However, the T^2 method is better than the EWMA/R method in medium shifts. As the magnitude of shift increases, the performance of the two methods would be the same. It should be mentioned that the out-of-control ARLs under shifts in A_2 are smaller than the out-of-control ARLs under shifts in A_0 and A_1 despite the fact that the shifts considered in the third parameter of the reference profile are smaller than shifts in the intercept and second parameter. The reason is that when a shift of size $\lambda\sigma$ in the Kth ($K = 0, 1, \cdots, k$) regression parameter of the reference profile occurs, then a shift of size $\lambda\sigma x_i^K$ in that period and a shift of $(1-\phi)\lambda\sigma x_i^K$ in the following periods, in the expected value of residuals, will occur (see Appendix D). This implies that the size of shifts in the expected values of residuals is affected by the values of x_i and the location of the parameters in the polynomial model, in addition to the autocorrelation coefficient.

In both weak and strong autocorrelation situations, under the standard deviation shift from σ to $\gamma\sigma$, the T^2 control chart performs uniformly better than EWMA/R. In addition, the results are the same for both weak and strong autocorrelations. This means that the autocorrelation coefficient does not affect the out-of-control ARL under the standard deviation shift. Also, under a strong autocorrelation situation, the out-of-control ARLs are not as large as the results obtained in the regression parameters. The reason is that when a shift of $\gamma\sigma$ in the standard deviation of the reference profile occurs, a shift of $\sigma\sqrt{(\gamma^2-\phi^2)/(1-\phi^2)}$ in that period and a shift of $\gamma\sigma$ at following periods in the standard deviation of residuals will occur. Hence, shifts in the standard deviation of the residuals are the same as shifts in the standard deviation of the reference profile. The proof is the same as in a linear profile case which is given by Noorossana et al. [18].

In real world applications, there are some factors which affect the parameters of polynomial profiles. Any changes in these factors will cause changes in the parameters of polynomial profiles. Sometimes a factor will affect both linear and quadratic coefficients and sometimes one of them. In the latter case, it would be possible to expect a shift in the linear coefficient with no change in the quadratic coefficient or vice versa. This case is investigated in Tables 3 and 4. However, simultaneous shifts in the linear and quadratic parameters can occur in the former case. Hence, this case is also investigated and the results are illustrated in Figures 2 and 3.

As shown in Figure 2, the EWMA/R is uniformly better than the T^2 method under the strong autocorrelation coefficient. However, under the weak autocorrelation coefficient in Figure 3, the performance of the two methods is roughly the same.

To investigate the effect of a quadratic model on the values of ARL, different quadratic models are tested under the same x-values used in the previous section and the same results are obtained. Hence, it can be concluded that the results do not depend on the specific quadratic model used in the simulation studies. However, changing the number of set points, n, and the x-values affect the ARl_1 values of both methods, as Kang and Albin [2] discuss for simple linear profiles. The T^2 statistic follows a non-central chi-square distribution under shifts in process parameters

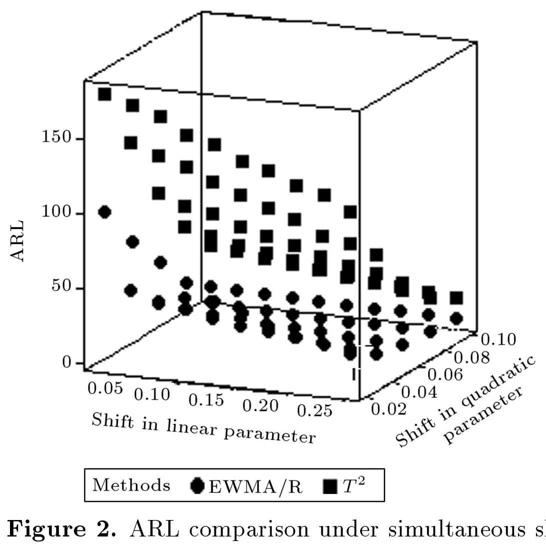

Figure 2. ARL comparison under simultaneous shift in the second and third parameters with $\phi = 0.9$.

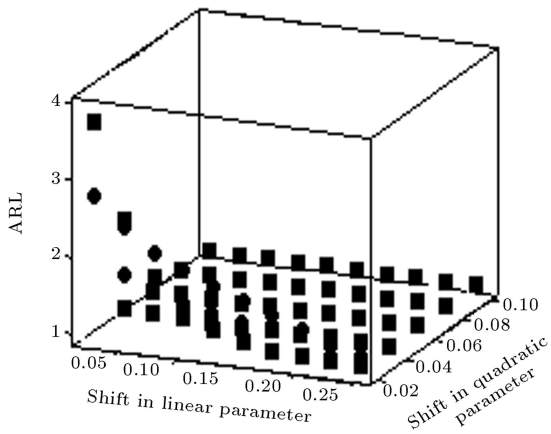

Figure 3. ARL comparison under simultaneous shift in the second and third parameters with $\phi = 0.1$.

(see [27]). Kang and Albin [2] state that the non-central parameter for the distribution is $\tau = \zeta^T \sum^{-1} \zeta$ where ζ is equal to $(\lambda\sigma, \beta\sigma)^T$ for a simple linear profile, $\sum$ is the covariance matrix of parameter estimators and $\lambda\sigma$ and $\beta\sigma$ are shifts in the intercept and the slope, respectively. For a quadratic model, substituting ζ by $(\lambda\sigma, \beta\sigma, \delta\sigma)$ and $\sum$ by the following equation:

$$\sum^{-1} = [\sigma^2(X^TX)^{-1}]^{-1}$$

$$= \frac{1}{\sigma^2}\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^4 \end{bmatrix}, \tag{9}$$

it can be easily shown that:

$$\tau = n\lambda^2 + 2\lambda\beta\sum_{i=1}^{n} x_i + (2\lambda\delta+\beta^2)\sum_{i=1}^{n} x_i^2$$

$$+2\beta\delta\sum_{i=1}^{n} x_i^3 + \delta^2\sum_{i=1}^{n} x_i^4, \tag{10}$$

where $\lambda\sigma$ and $\beta\sigma$ and $\delta\sigma$ are shifts in the intercept and the second and third parameters, respectively. Equation 10 implies that n, $\sum_{i=1}^{n} x_i^2$ and $\sum_{i=1}^{n} x_i^4$ affect the non-central parameter under a separate shift in the intercept, second parameter and third parameter, respectively. Also, Wierda [28] shows that the power of the T^2 control chart increases as the non-central parameter τ increases. Therefore, if one increases x-values without changing the number of set points, the ARL_1 values do not change under a shift in the intercept. However, due to an increase in x-values, the ARL_1 values decrease under a shift in the second and third parameters. If one increases the number of set points, the ARL_1 under a shift in the intercept, second and third parameters decrease. Our simulation studies (not reported here) confirm these results for the EWMA/R method as well. In addition, our simulation studies show that increasing x-values has no effect on the ARL_1 of both methods under a shift in standard deviation. However, increasing n leads to a decrease in out-of-control ARL in both methods.

EFFECT OF ASSIGNABLE CAUSE ON THE SHAPE OF PROFILES

To investigate the effect of assignable cause on the shape of profiles, the example in the fourth section is used and the effect of parameter changes, including the intercept, linear and quadratic coefficients and standard deviation, are taken into consideration. Two models for each parameter, one in-control and the other out-of-control, are used and the figures are plotted in Figures 4 to 7. To illustrate the effect of shifts in the intercept and the second parameter more clearly, changing the scale of Figures 4 and 5, a part of the

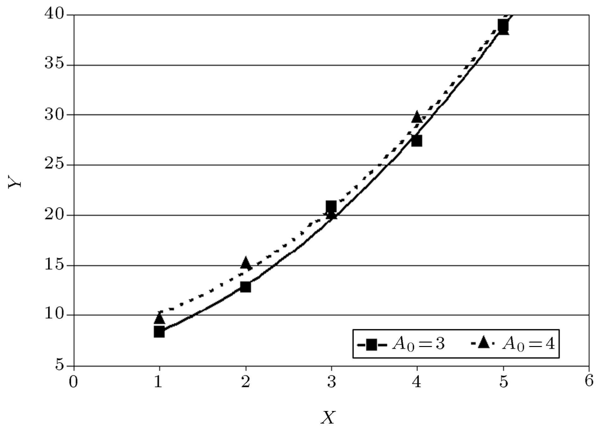

Figure 4. Step shift in intercept from 3 to 4.

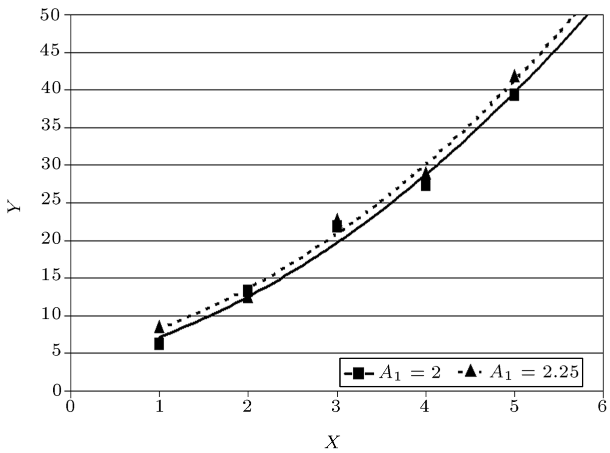

Figure 5. Step shift in second parameter from 2 to 2.25.

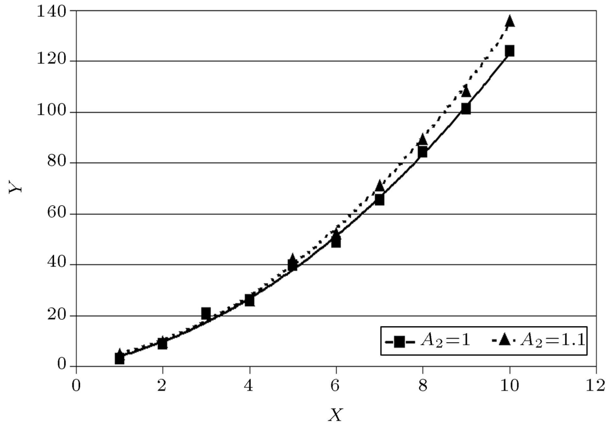

Figure 6. Step shift in third parameter from 1 to 1.1.

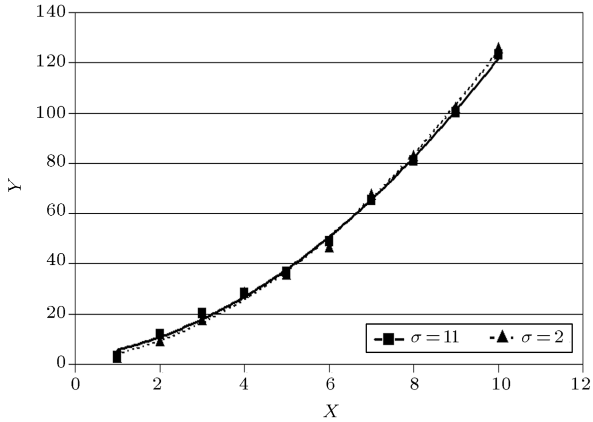

Figure 7. Step shift in standard deviation from 1 to 2.

profiles is plotted. For all parameters, the in-control model is as follows:

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij},$$

$$\varepsilon_{ij} = \phi\varepsilon_{i(j-1)} + a_{ij},$$

where $a_{ij} \sim N(0,1)$ and x =1, 2, 3, 4, 5, 6, 7, 8, 9, 10. In this section, the strong autocorrelation coefficient equal to 0.9 is used.

EFFECT OF AUTOCORRELATION COEFFICIENT ON THE SHAPE OF PROFILES

To illustrate the effect of the autocorrelation coefficient on the shape of profiles over time, first we generated 100 in-control profiles under both weak ($\phi = 0.1$) and strong $\phi = 0.9$ autocorrelation coefficients. Since the shape of profiles depends on the estimated parameters over time, in Figure 8 time series plots for the estimated intercept are depicted for both weak and strong autocorrelation coefficients.

Autocorrelation between profiles intuitively means that profiles close in time are more similar in

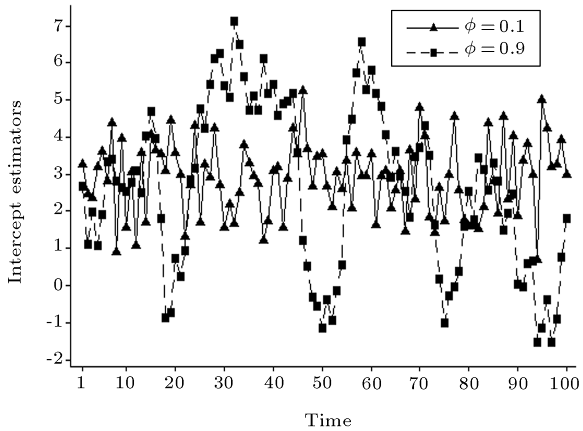

Figure 8. Intercept estimators under $\phi = 0.1$ and $\phi = 0.9$.

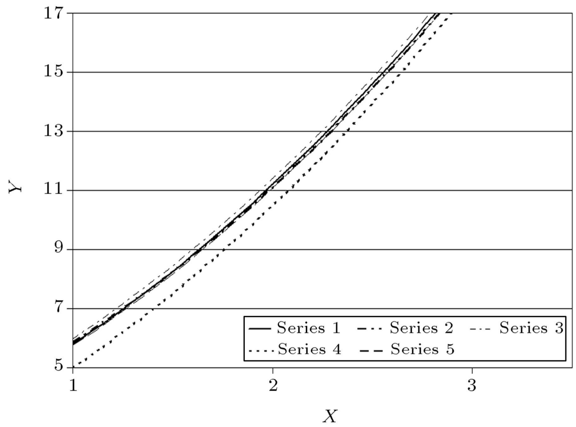

Figure 9. Five generated profiles under $\phi = 0.1$.

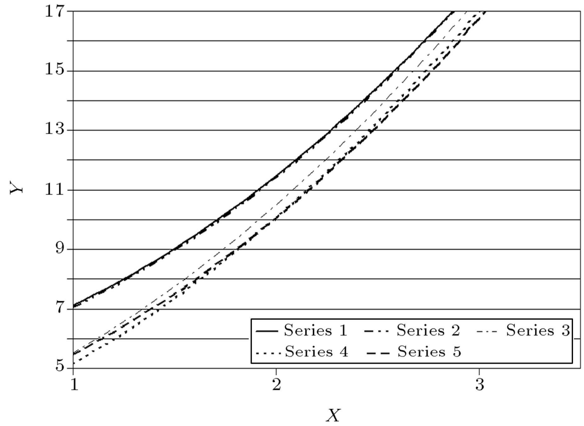

Figure 10. Five generated profiles under $\phi = 0.9$.

shape than those further apart. As the autocorrelation coefficient gets larger, the similarity between profiles close in time increases. For example, the profiles close in time under $\phi = 0.9$ are more similar than the profiles close in time under $\phi = 0.1$.

On the other hand, as the autocorrelation coefficient gets larger, the variation also increases. For instance, as depicted in Figure 8, variation of the intercept estimators under a strong autocorrelation coefficient is larger than their variation under a weak autocorrelation coefficient. This can be concluded from the following equation as well:

$$\begin{aligned}\mathrm{Var}(\hat{A}_j) &= \sigma_\varepsilon^2 (X^T X)^{-1} \\ &= \frac{\sigma^2}{1-\phi^2}(X^T X)^{-1}.\end{aligned} \tag{11}$$

Due to this variation, the intercept estimators under $\phi = 0.9$ differ significantly at some times, as illustrated in Figure 8 by circles. Similar results are obtained for linear and curvature coefficients (not reported here). Hence, the profiles under $\phi = 0.1$ generally, over time, are more similar to each other than the profiles under $\phi = 0.9$.

The above conclusions are also shown by Figures 9 and 10. In these figures, five profiles under $\phi = 0.1$ and $\phi = 0.9$ are generated, respectively. We changed the scale of the plots and showed part of the profiles to be able to show the differences clearer. If one compares the five profiles, generally, the profiles under $\phi = 0.1$ are more similar than the profiles under $\phi = 0.9$. This result is due to variations we explained formerly. On the other hand, with comparing the profiles in series 4 and 5, which are the fourth and fifth profiles out of five generated profiles, under both weak and strong autocorrelation coefficients, it is specified that these two profiles under $\phi = 0.9$ are more similar than $\phi = 0.1$. This is the issue we mentioned for profiles close in time.

ILLUSTRATIVE EXAMPLE

The example in the previous section is considered again to help clarify the approach. Assume the following in-control model:

$$\begin{aligned} y_{ij} &= 3 + 2x_i + x_i^2 + \varepsilon_{ij}, \\ \varepsilon_{ij} &= 0.3\varepsilon_{i(j-1)} + a_{ij}, \end{aligned}$$

where a_{ij}s are normally distributed with mean zero and variance 1. The fixed x-values of 1, 2, $\cdots$, 10 are used. It is clear that the above model is a numerical example we used to explain our methods step by step. However, in practice, this model is unknown in most situations and one should use historical profiles and estimate the model using phase I studies. The type of autocorrelation and the coefficient of autocorrelation should be determined by using time series analyses. Use of an autocorrelation and partial autocorrelation function for determining the type of autocorrelation is recommended.

Since the scope of the paper is in phase II, we assume that the parameters of the model are known.

Avoiding elaboration about phase I studies, we go forward through the phase II monitoring of autocorrelated profiles.

Our proposed approach consists of the following steps:

Step 1: Choose the desired in-control ARL value and smoothing constant, θ. Determine parameters for the control limits, i.e. L in EWMA and R control charts. The upper control limit in the T^2 control chart is $\chi^2_{n,\alpha}$. The values of ARL_0=200 and $\theta = 0.2$ leads to $L = 3.08$. The values of L can be determined by using simulation studies. For $n = 10$, d_2 and d_3 are 3.078 and 0.797, respectively. The upper control limit in the T^2 control chart, considering 10 observations and a type I error of 0.005, is 25.1882. The control limits are summarized in Table 6.

Step 2: Start monitoring the process and obtain observations, y_{ij} at fixed values of x_i for i=1, 2, $\cdots$, 10 by getting one individual sample from the process. Then, compute the statistics and compare them with the corresponding control limits. Follow this step until an out-of-control signal is detected. At this time, a corrective action is required. To illustrate the second step of the approach more clearly, we applied a shift of 0.02σ in the curvature coefficient of the model. Hence, the parameter changed from 1 to 1.02. To calculate the statistics, first, we should calculate the residuals using the following equation:

$$e_{ij} = y_{ij} - 0.3\,y_{i(j-1)} - 0.7(3 + 2x_i + x_i^2),$$

$$i = 1, 2, \cdots, 10.$$

It is clear that the observations correspondent to two successive profiles are used to calculate the residuals that are needed to construct the statistics in each time. The statistics of the EWMA and T^2 control charts are computed using Equations 5 and 8, respectively. The statistics of R control chart is computed using the equation $R_j = \max(e_{ij}) - \min(e_{ij})$. Control charts for the explained example are plotted in Figure 11 up to the time the first control chart has signaled.

Table 6. Control limits for the proposed control charts.

Control Chart	LCL	UCL
T^2	-	25.1882
EWMA	-0.3247	0.3247
R	0.6232	5.5328

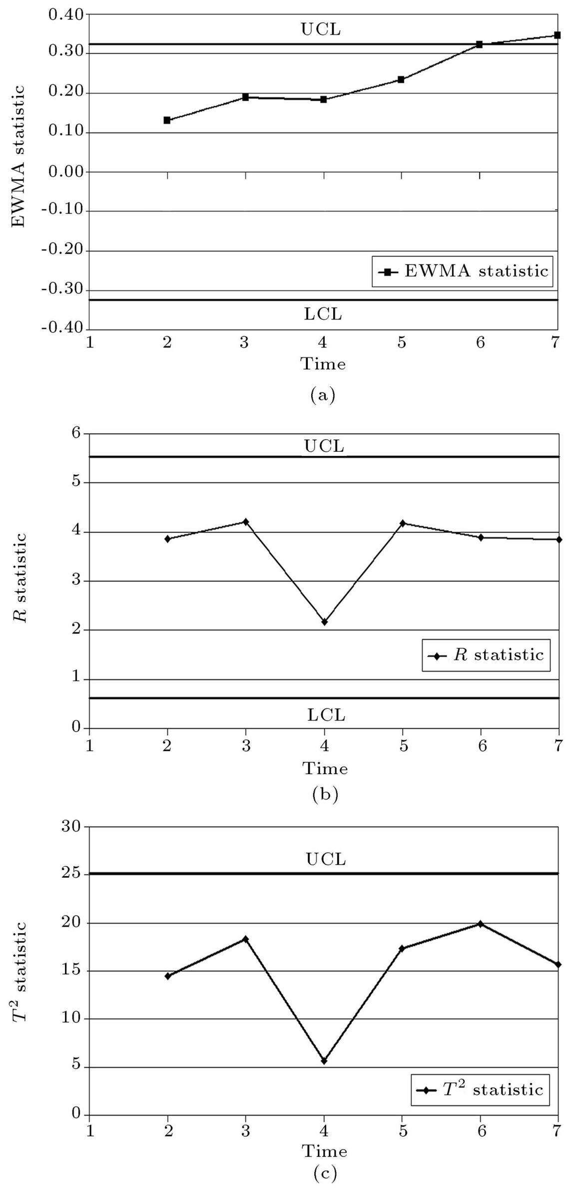

Figure 11. EWMA control chart (a), R control chart (b), and T^2 control chart (c).

CONCLUSIONS

In this paper, the effect of autocorrelation between profiles on the performance of the T^2 control chart proposed by Kang and Albin [2] was investigated and showed that autocorrelation leads to poor performance of this method. Two remedial methods, namely the T^2 control chart and EWMA/R, were used to eliminate the effect of autocorrelation. The performances of the methods were compared in terms of the average run length criterion. Simulation results showed that EWMA/R almost performs better than the T^2 method under the step shifts in the regression parameters. However, the T^2 method has better performance in comparison with the EWMA/R method under a shift in the standard deviation. This means that the R chart is not a competitive control chart for monitoring standard deviation. Hence, one can use the T^2 control chart, along with the EWMA control chart, for monitoring autocorrelated polynomial profiles. Also, it was shown that x-values and the number of set points affect the performance of both methods. In addition, the effects of assignable cause and autocorrelation coefficient on the shape of profiles were also investigated.

ACKNOWLEDGMENT

The authors gratefully acknowledge the insightful and valuable comments of the anonymous reviewers which led to the improvement of this paper.

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APPENDIX A

Expected Value and Variance of Residuals in Equation 4

$$E(e_{ij}) = E[y_{ij} - \phi\, y_{i(j-1)} - (1-\phi)$$
$$(A_0 + A_1x_i + A_2x_i^2 + \cdots + A_kx_i^k)]$$
$$= E(y_{ij}) - \phi\, E(y_{i(j-1)}) - (1-\phi)$$
$$(A_0 + A_1x_i + A_2x_i^2 + \cdots + A_kx_i^k) = 0,$$

$$\mathrm{Var}(e_{ij}) = \mathrm{Var}[y_{ij} - \phi\, y_{i(j-1)} - (1-\phi)$$
$$(A_0 + A_1x_i + A_2x_i^2 + \cdots + A_kx_i^k)]$$
$$= \mathrm{Var}(y_{ij}) + \phi^2\,\mathrm{Var}(y_{i(j-1)})$$
$$- 2\phi[\mathrm{Cov}(y_{ij}, y_{i(j-1)})]$$
$$= \frac{\sigma^2}{1-\phi^2} + \phi^2\frac{\sigma^2}{1-\phi^2} - 2\phi[\mathrm{Cov}(\phi y_{i(j-1)}$$
$$+ (1-\phi)(A_0 + A_1x_i + A_2x_i^2 + \cdots + A_kx_i^k),$$
$$y_{i(j-1)})] = \frac{\sigma^2}{1-\phi^2} + \phi^2\frac{\sigma^2}{1-\phi^2} - 2\phi^2\frac{\sigma^2}{1-\phi^2} = \sigma^2.$$

APPENDIX B

Relationship Between Regression Parameters Estimators in Successive Profiles

Consider the regression parameters estimators in successive profiles, j and $j-1$:

$$\hat{A}_j = (X^TX)^{-1}X^TY_j,$$

and:

$$\hat{A}_{j-1} = (X^TX)^{-1}X^TY_{j-1}.$$

Then, by multiplying the vector of parameters estimators in the $j-1$th profile by ϕ and subtracting the result from the vector of parameters estimators in the jth profile, we get:

$$\hat{A}_j - \phi\hat{A}_{j-1} = (X^TX)^{-1}X^T(Y_j - \phi Y_{j-1}). \tag{B1}$$

By substituting Equation 2 into Equation B1, we obtain:

$$\hat{A}_j - \phi\hat{A}_{j-1} = (X^TX)^{-1}X^T[(1-\phi)AX + a_j]. \tag{B2}$$

Hence:

$$\hat{A}_j - A = \phi(\hat{A}_{j-1} - A) + a'_j, \tag{B3}$$

where $a'_j = (X^TX)^{-1}X^Ta_j$ which follows a multivariate normal distribution with a mean vector of $\underline{0}$ and covariance matrix of $\sigma^2(X^TX)^{-1}$.

APPENDIX C

Relationship Between Successive T^2 Statistics by Kang and Albin [2]

Our simulation studies show that the AR(1) structure between the error terms also transfers to the T^2 statistics proposed by Kang and Albin [2]. To illustrate this result, we generated 100 profiles under an autocorrelation coefficient equal to 0.9, based on the following model:

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij},$$

$$\varepsilon_{ij} = \phi\varepsilon_{i(j-1)} + a_{ij},$$

where a_{ij}'s are normally distributed with mean zero and variance 1. The fixed x-values of 1, 2, $\cdots$, 10 are used. ACF, PACF and time series plots for the T^2 statistics are illustrated in the following figures. As shown in Figure C1, the ACF plot dies down exponentially and the PACF plot cuts off in lag 1. Hence, it is justified that there is an AR(1) model between successive T^2 statistics. The same conclusion is obtained by a time series plot in Figure C2.

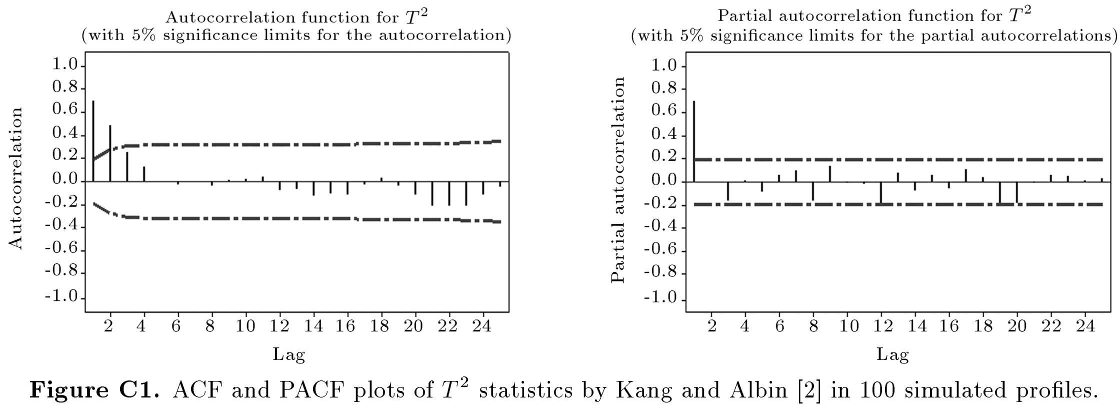

Figure C1. ACF and PACF plots of T^2 statistics by Kang and Albin [2] in 100 simulated profiles.

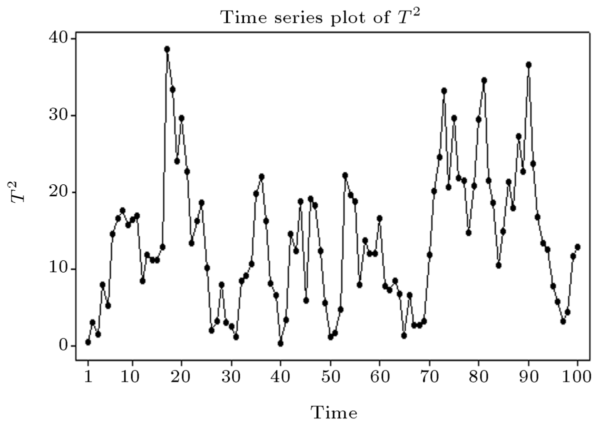

Figure C2. Time series plot of T^2 statistics by Kang and Albin [2] in 100 simulated profiles.

APPENDIX D

Relationship Between Shift in Parameters of Reference Profile and Shift in Expected Value of Residuals

As shown in Equation 4, the residuals can be calculated using the following equation:

$$\begin{aligned} e_{ij} =& y_{ij} - \hat{y}_{ij} = y_{ij} - \phi y_{i(j-1)} \\ & -(1-\phi)(A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k). \end{aligned}$$

The expected value for the above equation is equal to:

$$\begin{aligned} E(e_{ij}) =& E(y_{ij}) - \phi E(y_{i(j-1)}) \\ & -(1-\phi)(A_0 + A_1 x_i + A_2 x_i^2 + \cdots + A_k x_i^k). \end{aligned}$$

When a shift with magnitude of $\lambda\sigma$ in the Kth regression parameter ($K = 0, 1, \cdots, k$) of the reference profile occurs in period m, then:

$$\begin{aligned} & E(e_{ij}) = 0, && j < m, \\ & E(e_{ij}) = \lambda\sigma x_i^K, && j = m, \\ & E(e_{ij}) = (1-\phi)\lambda\sigma x_i^K, && j > m. \end{aligned} \tag{D1}$$

Equation D1 implies that when a shift of size $\lambda\sigma$ in the Kth regression parameter of reference profile occurs, then, a shift of size $\lambda\sigma x_i^K$ in that period and a shift of $(1-\phi)\lambda\sigma x_i^K$ in the following periods, in the expected value of residuals, will occur.

BIOGRAPHIES

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