

Interaction of Quantum Dot Molecules with Multi-Mode Radiation Fields

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Abstract. *In this article, the interaction of an arbitrary number of quantum dots behaving as artificial molecules with different energy levels and a multi-mode electromagnetic field is studied. We make the assumption that each quantum dot can be represented as an atom with zero kinetic energy, and that all excitonic effects except dipole-dipole interactions may be disregarded. We use the Jaynes-Cummings-Paul model with applications to quantum systems based on a time-dependent Hamiltonian and entangled states. We obtain a system of equations describing the interaction, and present a method to solve the equations analytically for a single mode field within the Rotating-Wave Approximation. As an example of the applicability of this approach, we solve the system of two, two-level quantum dots in a lossless cavity with two modes of electromagnetic field. We, furthermore, study the evolution of entanglement by defining and computing the concurrency.*

Keywords: *Quantum optics; Quantum dot molecules; Nanotechnology; Entanglement; Nano-photonics.*

INTRODUCTION

Quantum entangled states have become a popular topic for research in the last decade due to their potential application in quantum communication and information [1-3]. Generally, the trend of research is towards increasing the number of entangled bodies and to our knowledge entanglement of eight particles has been state of the art [4]. Parallel to experiments, theoretical work has also been devoted to modeling phenomena associated with entanglement [5-10]. The simplest model for description of interaction within the framework of Quantum Electrodynamics (QED) is the so-called Jaynes-Cummings-Paul Model (JCPM) [11], which provides a closed-form and explicit solution to the case of a two-level atom with zero kinetic energy in a cavity with a single mode electromagnetic field. In the real world, however, the problem is much more complex; for instance neither the field is necessarily single mode nor do the atoms possess only two energy levels. Also, quantum dots with various configurations may be exploited as artificial atoms; combinations of quantum dots give rise to the concept of quantum

dot (artificial) molecules. Hence, more sophisticated models have been proposed to include multi-atom systems [12], multi-phonon transitions [13], intensity-dependent entanglement [14], three and more energy levels [15] and electromagnetic modes [15,16].

In this paper, we provide a generalization of the JCPM to investigate a system of an arbitrary number of quantum dots (or atoms with negligible kinetic energy) with an arbitrary number of energy levels in a multi-mode electromagnetic (EM) field. First, we will find an appropriate Hamiltonian for the system. Such a Hamiltonian should account for dot-dot and dot-field interaction, while being simple enough to avoid increasing the solution complexity. We employ the Heisenberg's interaction picture [17] to obtain such a Hamiltonian, here referred to as the image Hamiltonian. We devise a rigorous and algorithmic method of substituting the image Hamiltonian into the time dependent Schrödinger's equation, to construct a system of simultaneous differential equations. This system of linear ordinary differential equations is regarded as a generalization of well-known Rabi equations in the JCPM. By solving this system of equations, we find the time evolution of the system and, therefore, its entanglement through evaluation of a proper functional, which is referred to as the concurrency. We ignore spin and excitonic effects in quantum dots to simplify the formulation. However,

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the latter effect has been thoroughly discussed in a recent paper of our group [18].

Finding a solution for the mentioned system of equations analytically seems to be a sophisticated task in the general case. Actually, further assumptions, such as the Rotating Wave Approximation (RWA), can greatly help in simplifying the problem. We show that for single-mode fields, the RWA leads to an analytically solvable problem. However, finding an analytical solution is at least much more complex for multi-mode EM fields. In such cases, numerical methods can replace analytical methods. We present examples for both cases of single- and multi-mode EM fields. In the former case, we follow the analytical approach while for the latter case, we have implemented a simple finite difference scheme to calculate the time evolution of the system. It is, furthermore, shown that if the EM field is not single mode, the system should be described through a system of differential equations with the number of equations dependent on the photon numbers of the EM modes. Unfortunately, the number of equations blows up rapidly as the number of EM modes increases, and this imposes an undesirable computational burden.

Evaluating the degree of entanglement is the final step in this work. Several types of work have been previously performed to provide an appropriate tool to measure multi-particle entanglement [3,19,20] from among which we have used the generalized definition of concurrency, as mentioned in [21], to present entanglement in the last example, i.e. two, two-level dots in a cavity with two EM modes.

BASIC DEFINITIONS

In this article, the interaction of a collection of k -dots with arbitrary energy levels, and a reservoir of w -modes of an electromagnetic field is investigated. Each dot interacts with light through specified constants according to the following Hamiltonian [17]:

$$\mathbb{H}_{r,E} = \sum_{n,i < j} (\gamma_{nij} \hat{\sigma}_{i,j}^n + \gamma_{nij}^* \hat{\sigma}_{j,i}^n) \sum_{\nu} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}), \quad (1)$$

where coefficients γ_{nij} are matrix elements of the dipole operator of the dot indexed by n and coefficient $g_{nij\nu}$ determines the coupling strength of dot n with the field of mode indexed by ν . Both γ_{nij} and $g_{nij\nu}$ are assumed to be dependent on the two energy levels, i and j between which, the electron of dot n does a transition through interaction with the field. Operator $\hat{\sigma}_{i,j}^n$ is the transition operator from level j into level i for dot n , a_{ν} and a_{ν}^{\dagger} are annihilation and creation operators of the photon corresponding to mode ν respectively. By indicating the i th eigenket of dot n as $|n\rangle_i$, we may

adopt the following definitions of atomic operators:

$$\hat{\sigma}_{i,j}^n = |n\rangle_i \langle n|_j, \quad (2)$$

$$\hat{\sigma}_i^n \triangleq \hat{\sigma}_{i,i}^n = |n\rangle_i \langle n|_i, \quad (3)$$

$$\hat{\sigma}_{i,j}^n \hat{\sigma}_{k,i'}^n = \hat{\sigma}_{i,i'}^n \delta_{j,k}, \quad (4)$$

$$\hat{\sigma}_{i,j}^n |n\rangle_k = |n\rangle_i \langle n|_j |n\rangle_k = |n\rangle_i \delta_{j,k}, \quad (5)$$

$$\hat{\sigma}_f^s \hat{\sigma}_g^s = \delta_{f,g} \hat{\sigma}_f^s, \quad (6)$$

$$\sum_f \hat{\sigma}_f^s = 1, \quad (7)$$

where $\delta_{f,g}$ represents the Kronecker delta function. Equation 6 is a result of orthogonality of the eigenstates and Equation 7 is correct, due to the completeness property of the eigenstates for each dot.

Each pair of dots is allowed to have a dipole interaction through the coefficients depending on the type of dots and the energy levels between which the electrons of these dots undergo a transition. The related Hamiltonian of dipole interaction is given by [17]:

$$\mathbb{H}_{r,r} = \sum_{n < m, i < j} (\eta_{nij} \hat{\sigma}_{i,j}^n + \eta_{nij}^* \hat{\sigma}_{j,i}^n) (\eta_{mij} \hat{\sigma}_{i,j}^m + \eta_{mij}^* \hat{\sigma}_{j,i}^m). \quad (8)$$

Coefficient η_{nij} depends on the strength of the dipole generated by transition of dot n between two energy levels, i and j , associated with the atom.

Ignoring interactions between dots and the electromagnetic (EM) field, the basic Hamiltonian is [17]:

$$\mathbb{H}_0 = \sum_{n,i} E_i^n \hat{\sigma}_i^n + \sum_{\nu} \hbar \Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}, \quad (9)$$

where the zero point energies of radiation fields are dropped. In the latter relation, the value of energy level i of dot n is indicated by E_i^n , and the frequency of mode ν is indicated by Ω_{ν} . For each a , b and n , we assume the convention that if $a < b$, then $E_a^n < E_b^n$. The total Hamiltonian can be then represented as $\mathbb{H} = \mathbb{H}_0 + \mathbb{H}_{r,E} + \mathbb{H}_{r,r}$.

IMAGE HAMILTONIAN

The Image Hamiltonian in the Heisenberg's interaction picture [17] can be expressed as:

$$\mathbb{H}_{int}^{(I)} = \exp \left[\frac{i}{\hbar} \mathbb{H}_0 t \right] \mathbb{H}_{int} \exp \left[\frac{-i}{\hbar} \mathbb{H}_0 t \right], \quad (10)$$

where the total interaction Hamiltonian is:

$$\mathbb{H}_{int} = \mathbb{H}_{r,E} + \mathbb{H}_{r,r}. \quad (11)$$

Our goal is now to calculate Equation 10, term by term. At first, we define $\hbar\omega_i^n \triangleq E_i^n$ and $\xi \triangleq e^{it}$. Using Equation 9, we have:

$$\begin{aligned} \exp\left[\frac{i}{\hbar}\mathbb{H}_0 t\right] &= \exp\left[i\left(\sum_{n,i}\omega_i^n\hat{\sigma}_i^n + \sum_{\nu}\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu}\right)t\right] \\ &= \prod_n \exp\left(\sum_i i\omega_i^n\hat{\sigma}_i^n t\right) \exp\left(\sum_{\nu} i\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu} t\right). \end{aligned} \quad (12)$$

In Equation 12, we have used relation $\exp(\hat{A} + \hat{B}) = \exp(\hat{A})\exp(\hat{B})$ which is in turn true, if $[\hat{A}, \hat{B}] = 0$. Definition of the exponential operator and Relation 6 results in:

$$\exp\left(\sum_i i\omega_i^n\hat{\sigma}_i^n t\right) = \sum_i \exp(i\omega_i^n t) \hat{\sigma}_i^n = \sum_i \xi^{\omega_i^n} \hat{\sigma}_i^n, \quad (13)$$

where $\xi = \exp(it)$ is taken for convenience. Also, due to the fact that $[a_{\nu}^{\dagger}a_{\nu}, a_{\nu'}^{\dagger}a_{\nu'}] = 0$, one can conclude that:

$$\exp\left(\sum_{\nu} i\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu} t\right) = \prod_{\nu} \xi^{\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu}}. \quad (14)$$

Combining Equations 12, 13 and 14 finally leads to:

$$\exp\left[\frac{i}{\hbar}\mathbb{H}_0 t\right] = \prod_n \sum_i \xi^{\omega_i^n} \hat{\sigma}_i^n \cdot \prod_{\nu} \xi^{\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu}}. \quad (15)$$

Through a similar procedure we have:

$$\exp\left[-\frac{i}{\hbar}\mathbb{H}_0 t\right] = \prod_n \sum_i \xi^{-\omega_i^n} \hat{\sigma}_i^n \prod_{\nu} \xi^{-\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu}}. \quad (16)$$

Using Equations 1, 8, 11 and 15, one can conclude that:

$$\begin{aligned} \exp\left[\frac{i}{\hbar}\mathbb{H}_0 t\right] \mathbb{H}_{int} &= \prod_s \sum_k \xi^{\omega_k^s} \hat{\sigma}_k^s \prod_{\nu} \xi^{\Omega_{\nu}a_{\nu}^{\dagger}a_{\nu}} \\ &\times \left[\sum_{n,i < j} (\gamma_{nij}\hat{\sigma}_{i,j}^n + \gamma_{nij}^* \hat{\sigma}_{j,i}^n) \sum (g_{nij\nu}a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \right. \\ &+ \sum_{n < m, i < j, p > q} (\eta_{nij}\hat{\sigma}_{i,j}^n + \eta_{nij}^* \hat{\sigma}_{j,i}^n) \\ &\left. (\eta_{mpq}\hat{\sigma}_{p,q}^m + \eta_{mpq}^* \hat{\sigma}_{q,p}^m) \right]. \end{aligned} \quad (17)$$

To further simplify the calculation of Equation 10, we first notice that for any arbitrary function f we have:

$$\begin{aligned} \prod_s \sum_k f(s, k, t) \hat{\sigma}_k^s \hat{\sigma}_{i,j}^n \\ &= \prod_{s \neq n} \sum_k f(s, k, t) \hat{\sigma}_k^s \sum_{k'} f(n, k', t) \hat{\sigma}_{k'}^n \hat{\sigma}_{i,j}^n \\ &= \prod_{s \neq n} \sum_f f(s, k, t) \hat{\sigma}_f^s f(n, i, t) \hat{\sigma}_{i,j}^n \end{aligned} \quad (18)$$

$$\begin{aligned} \hat{\sigma}_{i,j}^n \prod_s \sum_k f(s, k, t) \hat{\sigma}_k^s \\ &= \hat{\sigma}_{i,j}^n \prod_{s \neq n} \sum_k f(s, k, t) \hat{\sigma}_k^s \sum_{k'} f(n, k', t) \hat{\sigma}_{k'}^n \\ &= \prod_{s \neq n} \sum_k f(s, k, t) \hat{\sigma}_k^s \hat{\sigma}_{i,j}^n f(n, j, t). \end{aligned} \quad (19)$$

Equation 19 is a result of the fact that $[\sigma_{a,b}^s, \hat{\sigma}_{i,j}^n] = 0$ for $s \neq n$, that is, the transition operators of the two different dots commute with themselves. Now, let $f(s, k, t) = \xi^{\omega_k^s}$. We then get the following from Equations 17 through 19:

$$\begin{aligned} \prod_s \sum_k \xi^{\omega_k^s} \hat{\sigma}_k^s \hat{\sigma}_{i,j}^n &= \prod_{s \neq n} \sum_k \xi^{\omega_k^s} \hat{\sigma}_k^s \sum_k \xi^{\omega_k^n} \hat{\sigma}_k^n \hat{\sigma}_{i,j}^n \\ &= \prod_{s \neq n} \sum_f \xi^{\omega_f^s} \hat{\sigma}_f^s \xi^{\omega_i^n} \hat{\sigma}_{i,j}^n. \end{aligned}$$

Using Equations 6, 17 and 18, we arrive at:

$$\begin{aligned} \exp\left[\frac{i}{\hbar}\mathbb{H}_0 t\right] \mathbb{H}_{int} &= \sum_{n,i < j} \left\{ \left[\gamma_{nij} \prod_{s \neq n} \sum_f \xi^{\omega_f^s} \hat{\sigma}_f^s \xi^{\omega_i^n} \hat{\sigma}_{i,j}^n \right. \right. \\ &+ \gamma_{nij}^* \prod_{s \neq n} \sum_f \xi^{\omega_f^s} \hat{\sigma}_f^s \xi^{\omega_j^n} \hat{\sigma}_{j,i}^n \left. \right] \\ &\times \prod_{\nu'} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \sum_{\nu} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \left. \right\} \\ &+ \sum_{n < m, i < j, p < q} \left\{ \eta_{nij} \prod_{\nu''} \xi^{\Omega_{\nu''} a_{\nu''}^{\dagger} a_{\nu''}} \prod_{s \neq n, m} \sum_f \xi^{\omega_f^s} \hat{\sigma}_f^s \right. \\ &\left[\eta_{mpq} \xi^{\omega_i^n + \omega_p^m} \hat{\sigma}_{i,j}^n \hat{\sigma}_{p,q}^m + \eta_{mpq}^* \xi^{\omega_i^n + \omega_q^m} \hat{\sigma}_{i,j}^n \hat{\sigma}_{p,q}^m \right] \\ &+ \eta_{nij}^* \prod_{\nu''} \xi^{\Omega_{\nu''} a_{\nu''}^{\dagger} a_{\nu''}} \prod_{s \neq n, m} \sum_f \xi^{\omega_f^s} \hat{\sigma}_f^s \\ &\left. \left[\eta_{mpq} \xi^{\omega_j^n + \omega_p^m} \hat{\sigma}_{j,i}^n \hat{\sigma}_{p,q}^m + \eta_{mpq}^* \xi^{\omega_j^n + \omega_q^m} \hat{\sigma}_{j,i}^n \hat{\sigma}_{p,q}^m \right] \right\}, \end{aligned} \quad (20)$$

Using Equations 7, 16, 19 and 20, we can calculate the image Hamiltonian within the Heisenberg interaction picture as:

$$\begin{aligned}
\mathbb{H}_{int}^{(I)} &= \exp\left(\frac{i}{\hbar}\mathbb{H}_0 t\right) \mathbb{H}_{int} \exp\left(-\frac{i}{\hbar}\mathbb{H}_0 t\right) = \\
&\sum_{n,i < j} \left\{ \left(\gamma_{nij} \xi^{\omega_{ij}} \hat{\sigma}_{i,j}^n + \gamma_{nij}^* \xi^{\omega_{ji}} \hat{\sigma}_{j,i}^n \right) \right. \\
&\times \sum_{\nu} \left(g_{nij\nu} a_{\nu} \xi^{-\Omega_{\nu}} + g_{nij\nu}^* a_{\nu}^{\dagger} \xi^{\Omega_{\nu}} \right) \left. \right\} \\
&+ \sum_{n < m, i < j, p < q} \left\{ \eta_{nij} \eta_{mpq} \xi^{\omega_{ij}^n + \omega_{pq}^m} \hat{\sigma}_{i,j}^n \hat{\sigma}_{p,q}^m \right. \\
&+ \eta_{nij} \eta_{mpq}^* \xi^{\omega_{ij}^n + \omega_{pq}^m} \hat{\sigma}_{i,j}^n \hat{\sigma}_{p,q}^m \\
&+ \eta_{nij}^* \eta_{mpq} \xi^{\omega_{ji}^n + \omega_{pq}^m} \hat{\sigma}_{j,i}^n \hat{\sigma}_{p,q}^m \\
&\left. + \eta_{nij}^* \eta_{mpq}^* \xi^{\omega_{ji}^n + \omega_{pq}^m} \hat{\sigma}_{j,i}^n \hat{\sigma}_{p,q}^m \right\}, \quad (21)
\end{aligned}$$

in which $\hbar\omega_{ij}^n \triangleq E_j^n - E_i^n$ represents the transition frequency between i th and j th states of the n th dot.

In derivation of Equation 21, we have used the following relation:

$$\begin{aligned}
&\prod_{\nu'} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \sum_{\nu} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \prod_{\nu''} \xi^{-\Omega_{\nu''} a_{\nu''}^{\dagger} a_{\nu''}} \\
&= \sum_{\nu} (g_{nij\nu} a_{\nu} \xi^{-\Omega_{\nu}} + g_{nij\nu}^* a_{\nu}^{\dagger} \xi^{\Omega_{\nu}}). \quad (22)
\end{aligned}$$

The validity of Equation 22 may be established as follows:

$$\begin{aligned}
&\prod_{\nu'} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \sum_{\nu} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \prod_{\nu''} \xi^{-\Omega_{\nu''} a_{\nu''}^{\dagger} a_{\nu''}} \\
&= \sum_{\nu} \prod_{\nu'} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \\
&\prod_{\nu''} \xi^{-\Omega_{\nu''} a_{\nu''}^{\dagger} a_{\nu''}} = \sum_{\nu} \prod_{\nu' \neq \nu} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \xi^{\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}} \\
&(g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \xi^{-\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}} \prod_{\nu' \neq \nu} \xi^{-\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \\
&= \sum_{\nu} \prod_{\nu' \neq \nu} \xi^{\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \prod_{\nu' \neq \nu} \xi^{-\Omega_{\nu'} a_{\nu'}^{\dagger} a_{\nu'}} \xi^{\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}} \\
&(g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \xi^{-\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}} \\
&= \sum_{\nu} \xi^{\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}} (g_{nij\nu} a_{\nu} + g_{nij\nu}^* a_{\nu}^{\dagger}) \xi^{-\Omega_{\nu} a_{\nu}^{\dagger} a_{\nu}}
\end{aligned}$$

$$= \sum_{\nu} (g_{nij\nu} a_{\nu} \xi^{-\Omega_{\nu}} + g_{nij\nu}^* a_{\nu}^{\dagger} \xi^{\Omega_{\nu}}).$$

RABI EQUATIONS

Assuming that l is an integer, and $\mathcal{A} = [r_i]_{1 \times k} = [r_1 \ r_2 \ \dots \ r_k]$ and $\mathcal{F} = [f_i]_{1 \times \omega} = [f_1 \ f_2 \ \dots \ f_{\omega}]$ are two vectors, we take the following notations for the rest of the calculations:

$$\mathcal{A}_{r_n \rightarrow i} = [r_1 \ r_2 \ \dots \ r_{n-1} \ i \ r_{n+1} \ \dots \ r_k],$$

$$\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow j} =$$

$$[r_1 \ r_2 \ \dots \ r_{n-1} \ i \ r_{n+1} \ \dots \ r_{m-1} \ j \ r_{m+1} \ \dots \ r_k],$$

$$|\mathcal{A}\rangle = \bigotimes_{n=1}^k |n\rangle_{r_n} = |1\rangle_{r_1} |2\rangle_{r_2} \dots |k\rangle_{r_k},$$

$$|\mathcal{A}_{r_n \rightarrow i}\rangle = \bigotimes_{n' \neq n} |n'\rangle_{r_{n'}} |n\rangle_i,$$

$$|\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow j}\rangle = \bigotimes_{\substack{n' \neq n \\ n' \neq m}} |n'\rangle_{r_{n'}} = |n\rangle_t |m\rangle_j,$$

$$\mathcal{F}_{f_{\nu} \rightarrow f_{\nu+1}} = [f_1 \ f_2 \ \dots \ f_{\nu-1} \ f_{\nu} + l \ f_{\nu+1} \ \dots \ f_{\omega}],$$

$$|\mathcal{F}\rangle = \bigotimes_{\nu=1}^{\omega} |f_{\nu}\rangle = |f_1\rangle |f_2\rangle \dots |f_{\omega}\rangle,$$

$$\sum_{\mathcal{A}, \mathcal{F}} (\cdot) = \sum_{r_1, r_2, \dots, r_k, f_1, f_2, \dots, f_{\omega}},$$

$$\sum_{\mathcal{A} = \{r_n\}, \mathcal{F}} (\cdot) = \sum_{r_1, r_2, \dots, r_{n-1}, r_{n+1}, \dots, r_k, f_1, f_2, \dots, f_{\omega}} (\cdot),$$

$$\sum_{\mathcal{A} = \{r_n, r_m\}, \mathcal{F}} (\cdot) = \sum_{r_1, r_2, \dots, r_{n-1}, r_{n+1}, \dots, r_{m-1}, r_{m+1}, \dots, r_k, f_1, f_2, \dots, f_{\omega}} (\cdot).$$

The general time-dependent state of the ensemble of dots and photons at any time may be expanded on the basis outer products of individual dots and field eigenstates as:

$$|\varphi(t)\rangle = \sum_{\mathcal{A}, \mathcal{F}} \Phi(\mathcal{A}, \mathcal{F}) |\mathcal{A}\rangle |\mathcal{F}\rangle, \quad (23)$$

in which $|\mathcal{A}\rangle = \bigotimes_{n=1}^k |n\rangle_{r_n}$ denotes the eigenstate of the quantum dot molecule with $|n\rangle_{r_n}$ denoting the r_n th

state of dot n . Also, $\left| \mathcal{F} \right\rangle = \bigotimes_{\nu=1}^{\omega} \left| f_{\nu} \right\rangle$ represents the field state when the ν th mode has photon number f_{ν} where $0 \leq f_{\nu} \leq \infty$. Note that the total number of dots in the dot molecule is denoted by k , while the number of energy levels in dot n is given by B_n . Also, the total number of modes of the EM field is denoted by ω , so that the indices are bounded by $1 \leq n \leq k, 1 \leq r_n \leq B_n$ and $1 \leq \nu \leq \omega$.

Equation 23 states that, at any time, $|\varphi(t)\rangle$ is a superposition of all possible states of the system, including atom and field states, and each state has a time-dependent coefficient equal to $\Phi(\mathcal{A}, \mathcal{F})$. According to the time-dependent Schrödinger equation we have:

$$i\hbar \frac{\partial}{\partial t} \left| \varphi(t) \right\rangle = \mathbb{H}_{int}^{(I)} \left| \varphi(t) \right\rangle. \quad (24)$$

By substituting the representation of Equation 23 in the left side of Equation 24 we obtain:

$$i\hbar \frac{\partial}{\partial t} \left| \varphi(t) \right\rangle = i\hbar \sum_{\mathcal{A}, \mathcal{F}} \dot{\Phi}(\mathcal{A}, \mathcal{F}) \left| \mathcal{A} \right\rangle \left| \mathcal{F} \right\rangle. \quad (25)$$

Also, we have the following properties for the photon number annihilation and creation operators:

$$a_{\nu} \left| f_{\nu} \right\rangle = \sqrt{f_{\nu}} \left| f_{\nu} - 1 \right\rangle, \quad (26.1)$$

$$a_{\nu}^{\dagger} \left| f_{\nu} \right\rangle = \sqrt{f_{\nu} + 1} \left| f_{\nu} + 1 \right\rangle. \quad (26.2)$$

By using Equations 21, 23 and 26, the right-hand-side of Equation 24 can be expressed as:

$$\begin{aligned} \mathbb{H}_{int}^{(I)} \left| \varphi(t) \right\rangle &= \sum_{n, i < j} \left\{ \eta_{nij} \xi^{\omega_{ij}^n} \left[\sum_{\mathcal{A} - \{r_n\}, \mathcal{F}} \right. \right. \\ &\quad \left. \sum_{\nu} \Phi(\mathcal{A}_{r_n \rightarrow j}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} + 1}) \times g_{nij\nu} \xi^{-\Omega_{\nu}} \sqrt{f_{\nu} + 1} \left| \mathcal{F} \right\rangle \right. \\ &\quad \left. \left| \mathcal{A}_{r_n \rightarrow i} \right\rangle + \sum_{\mathcal{A} - \{r_n\}, \mathcal{F}} \sum_{\nu} \Phi(\mathcal{A}_{r_n \rightarrow j}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} - 1}) \right. \\ &\quad \left. g_{nij\nu}^* \xi^{\Omega_{\nu}} \sqrt{f_{\nu}} \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow i} \right\rangle \right] + \eta_{nij}^* \xi^{\omega_{ji}^n} \left[\sum_{\mathcal{A} - \{r_n\}, \mathcal{F}} \right. \\ &\quad \left. \sum_{\nu} \Phi(\mathcal{A}_{r_n \rightarrow i}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} + 1}) g_{nij\nu} \times \xi^{-\Omega_{\nu}} \sqrt{f_{\nu} + 1} \left| \mathcal{F} \right\rangle \right. \\ &\quad \left. \left| \mathcal{A}_{r_n \rightarrow j} \right\rangle + \sum_{\mathcal{A} - \{r_n\}, \mathcal{F}} \sum_{\nu} \Phi(\mathcal{A}_{r_n \rightarrow i}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} - 1}) g_{nij\nu}^* \xi^{\Omega_{\nu}} \right. \end{aligned}$$

$$\begin{aligned} &\left. \sqrt{f_{\nu}} \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow j} \right\rangle \right] + \sum_{n < m, i < j, p < q} \left\{ \eta_{nij} \eta_{mpq} \xi^{\omega_{ij}^n + \omega_{pq}^m} \right. \\ &\quad \sum_{\mathcal{A} - \{r_n, r_m\}, \mathcal{F}} \Phi(\mathcal{A}_{r_n \rightarrow j, r_m \rightarrow q}, \mathcal{F}) \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow i, r_m \rightarrow p} \right\rangle \\ &\quad + \eta_{nij}^* \eta_{mpq}^* \xi^{\omega_{ji}^n + \omega_{qp}^m} \sum_{\mathcal{A} - \{r_n, r_m\}, \mathcal{F}} \Phi(\mathcal{A}_{r_n \rightarrow j, r_m \rightarrow p}, \mathcal{F}) \\ &\quad \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow i, r_m \rightarrow q} \right\rangle \\ &\quad + \eta_{nij}^* \eta_{mpq} \xi^{\omega_{ji}^n + \omega_{pq}^m} \sum_{\mathcal{A} - \{r_n, r_m\}, \mathcal{F}} \Phi(\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow q}, \mathcal{F}) \\ &\quad \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow j, r_m \rightarrow p} \right\rangle \\ &\quad + \eta_{nij}^* \eta_{mpq}^* \xi^{\omega_{ji}^n + \omega_{qp}^m} \sum_{\mathcal{A} - \{r_n, r_m\}, \mathcal{F}} \Phi(\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow p}, \mathcal{F}) \\ &\quad \left. \left| \mathcal{F} \right\rangle \left| \mathcal{A}_{r_n \rightarrow j, r_m \rightarrow q} \right\rangle \right\}. \quad (27) \end{aligned}$$

Substituting Equations 25 and 27 in Equation 24 leads to:

$$\begin{aligned} i\hbar \dot{\Phi}(\mathcal{A}, \mathcal{F}) &= \Sigma_1(\mathcal{A}, \mathcal{F}) + \Sigma_2(\mathcal{A}, \mathcal{F}) + \Sigma_3(\mathcal{A}, \mathcal{F}) \\ &\quad + \Sigma_4(\mathcal{A}, \mathcal{F}) + \Sigma_5(\mathcal{A}, \mathcal{F}) + \Sigma_6(\mathcal{A}, \mathcal{F}), \quad (28) \end{aligned}$$

in which we have used the following notations for the sake of convenience:

$$\begin{aligned} \Sigma_1(\mathcal{A}, \mathcal{F}) &\triangleq \sum_{n, r_n < j} \gamma_{nr_n j} \xi^{\omega_{r_n j}^n} \\ &\quad \left[\sum_{\nu} g_{nr_n j \nu} \xi^{-\Omega_{\nu}} \sqrt{f_{\nu} + 1} \Phi(\mathcal{A}_{r_n \rightarrow j}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} + 1}) \right. \\ &\quad \left. + \sum_{\nu} g_{nr_n j \nu}^* \xi^{\Omega_{\nu}} \sqrt{f_{\nu}} \Phi(\mathcal{A}_{r_n \rightarrow j}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} - 1}) \right], \quad (28.1) \\ \Sigma_2(\mathcal{A}, \mathcal{F}) &\triangleq \sum_{n, i < r_n} \gamma_{nir_n}^* \xi^{\omega_{r_n i}^n} \\ &\quad \left[\sum_{\nu} g_{nir_n \nu} \xi^{-\Omega_{\nu}} \sqrt{f_{\nu} + 1} \Phi(\mathcal{A}_{r_n \rightarrow i}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} + 1}) \right. \\ &\quad \left. + \sum_{\nu} g_{nir_n \nu}^* \xi^{\Omega_{\nu}} \sqrt{f_{\nu}} \Phi(\mathcal{A}_{r_n \rightarrow i}, \mathcal{F}_{f_{\nu} \rightarrow f_{\nu} - 1}) \right], \quad (28.2) \\ \Sigma_3(\mathcal{A}, \mathcal{F}) &\triangleq \sum_{n < m, r_n < j, r_m < q} \eta_{nr_n j} \eta_{mr_m q} \xi^{\omega_{r_n j}^n + \omega_{r_m q}^m} \\ &\quad \Phi(\mathcal{A}_{r_n \rightarrow j, r_m \rightarrow q}, \mathcal{F}), \quad (28.3) \end{aligned}$$

$$\Sigma_4(\mathcal{A}, \mathcal{F}) \triangleq \sum_{n < m, r_n < j, p < r_m} \eta_{nr_n j} \eta_{mpr_m}^* \xi^{\omega_{r_n j}^n + \omega_{r_m p}^m} \Phi(\mathcal{A}_{r_n \rightarrow j, r_m \rightarrow p}, \mathcal{F}), \quad (28.4)$$

$$\Sigma_5(\mathcal{A}, \mathcal{F}) \triangleq \sum_{n < m, i < r_n, r_m < q} \eta_{nir_n}^* \eta_{mr_m q} \xi^{\omega_{r_n i}^n + \omega_{r_m q}^m} \Phi(\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow q}, \mathcal{F}), \quad (28.5)$$

$$\Sigma_6(\mathcal{A}, \mathcal{F}) \triangleq \sum_{n < m, i < r_n, p < r_m} \eta_{nir_n}^* \eta_{mpr_m}^* \xi^{\omega_{r_n i}^n + \omega_{r_m p}^m} \Phi(\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow p}, \mathcal{F}). \quad (28.6)$$

Within RWA, $\Sigma_3(\mathcal{A}, \mathcal{F})$ and $\Sigma_6(\mathcal{A}, \mathcal{F})$ may be ignored.

ANALYTICAL SOLUTION FOR THE SINGLE MODE FIELD

In this section, we assume that we have a single-mode EM field with photon number f , and the RWA is valid. These assumptions lead to the following relations:

$$i\hbar\dot{\Phi}(\mathcal{A}, f) = \Sigma_1(\mathcal{A}, f) + \Sigma_2(\mathcal{A}, f) + \Sigma_4(\mathcal{A}, f) + \Sigma_5(\mathcal{A}, f), \quad (29.1)$$

$$\Sigma_1(\mathcal{A}, f) = \sum_{n, r_n < j} \gamma_{nr_n j} g_{nr_n j} \xi^{-\Delta_{r_n j}^n} \sqrt{f+1} \Phi(\mathcal{A}_{r_n \rightarrow j}, f+1), \quad (29.2)$$

$$\Sigma_2(\mathcal{A}, f) = \sum_{n, i < r_n} \gamma_{nir_n}^* g_{nir_n}^* \xi^{\Delta_{ir_n}^n} \sqrt{f} \Phi(\mathcal{A}_{r_n \rightarrow i}, f-1), \quad (29.3)$$

$$\Sigma_3(\mathcal{A}, f) = 0, \quad (29.4)$$

$$\Sigma_4(\mathcal{A}, f) = \sum_{n < m, r_n < j, p < r_m} \eta_{nr_n j} \eta_{mpr_m}^* \xi^{\Delta_{pr_m}^m - \Delta_{r_n j}^n} \Phi(\mathcal{A}_{r_n \rightarrow j, r_m \rightarrow p}, f), \quad (29.5)$$

$$\Sigma_5(\mathcal{A}, f) = \sum_{n < m, i < r_n, r_m < q} \eta_{nir_n}^* \eta_{mr_m q} \xi^{\Delta_{r_m q}^m - \Delta_{ir_n}^n} \Phi(\mathcal{A}_{r_n \rightarrow i, r_m \rightarrow q}, f), \quad (29.6)$$

$$\Sigma_6(\mathcal{A}, \mathcal{F}) = 0, \quad (29.7)$$

in which $\Delta_{ab}^n \triangleq \Omega - \omega_{ab}^n$. Clearly, the above relations provide a distinct differential equation for each \mathcal{A} . Since the whole number of all possible vectors $\mathcal{A} = [r_1 \ r_2 \ \dots \ r_k]$, such that $1 \leq r_n \leq B_n$ is $N = \prod_{i=1}^k B_i$,

Equation 29 leads to a set of N equations. As an example, consider a single-mode EM field, $\omega = 1$, and two dots, $k = 2$, each having two energy levels. Thus, $B_1 = 2$, $B_2 = 2$ and $N = B_1 B_2 = 4$. If we assume that $\mathcal{A}_1 = [1 \ 1]$, $\mathcal{A}_2 = [1 \ 2]$, $\mathcal{A}_3 = [2 \ 1]$, $\mathcal{A}_4 = [2 \ 2]$, the total set of equations generated by Equation 29 takes the form:

$$i\hbar\dot{\Phi}(\mathcal{A}_1, f) = \wp_1 \xi^{-\Delta_2} \sqrt{f+1} \Phi(\mathcal{A}_2, f+1) + \wp_1 \xi^{-\Delta_1} \sqrt{f+1} \Phi(\mathcal{A}_3, f+1), \quad (30.1)$$

$$i\hbar\dot{\Phi}(\mathcal{A}_2, f) = \wp_1 \xi^{\Delta_2} \sqrt{f} \Phi(\mathcal{A}_1, f-1) + \wp_1 \xi^{-\Delta_1} \sqrt{f+1} \Phi(\mathcal{A}_4, f+1) + \wp_2 \xi^{\Upsilon} \Phi(\mathcal{A}_3, f), \quad (30.2)$$

$$i\hbar\dot{\Phi}(\mathcal{A}_3, f) = \wp_1 \xi^{\Delta_1} \sqrt{f} \Phi(\mathcal{A}_1, f-1) + \wp_1 \xi^{-\Delta_2} \sqrt{f+1} \Phi(\mathcal{A}_4, f+1) + \wp_2 \xi^{-\Upsilon} \Phi(\mathcal{A}_2, f), \quad (30.3)$$

$$i\hbar\dot{\Phi}(\mathcal{A}_4, f) = \wp_1 \xi^{\Delta_1} \sqrt{f} \Phi(\mathcal{A}_2, f-1) + \wp_1 \xi^{\Delta_2} \sqrt{f} \Phi(\mathcal{A}_3, f-1), \quad (30.4)$$

in which $\wp_1 = \gamma_{abc} g_{def} = \gamma_{abc}^* g_{def}^*$, $\wp_2 = \eta_{abc} \eta_{def}^* = \eta_{abc}^* \eta_{def}$, $\Delta_1 \triangleq \Omega_1 - \omega_{12}^1$, $\Delta_2 \triangleq \Omega_1 - \omega_{12}^2$ and $\Upsilon \triangleq \omega_{12}^1 - \omega_{12}^2 = \Delta_2 - \Delta_1$. One may rewrite Equation 30 as:

$$i\hbar\dot{\Phi}(\mathcal{A}_1, f) = \wp_1 \xi^{-\Delta_2} \sqrt{f+1} \Phi(\mathcal{A}_2, f+1) + \wp_1 \xi^{-\Delta_1} \sqrt{f+1} \Phi(\mathcal{A}_3, f+1),$$

$$i\hbar\dot{\Phi}(\mathcal{A}_2, f+1) = \wp_1 \xi^{\Delta_2} \sqrt{f+1} \Phi(\mathcal{A}_1, f) + \wp_1 \xi^{-\Delta_1} \sqrt{f+2} \Phi(\mathcal{A}_4, f+2) + \wp_2 \xi^{\Upsilon} \Phi(\mathcal{A}_3, f+1),$$

$$i\hbar\dot{\Phi}(\mathcal{A}_3, f+1) = \wp_1 \xi^{\Delta_1} \sqrt{f+1} \Phi(\mathcal{A}_1, f) + \wp_1 \xi^{-\Delta_2} \sqrt{f+2} \Phi(\mathcal{A}_4, f+2) + \wp_2 \xi^{-\Upsilon} \Phi(\mathcal{A}_2, f+1),$$

$$i\hbar\dot{\Phi}(\mathcal{A}_4, f+2) = \wp_1 \xi^{\Delta_1} \sqrt{f+2} \Phi(\mathcal{A}_2, f+1) + \wp_1 \xi^{\Delta_2} \sqrt{f+2} \Phi(\mathcal{A}_3, f+1).$$

Using notations $\Theta_{1,f} \triangleq \Phi(\mathcal{A}_1, f)$, $\Theta_{2,f} \triangleq \Phi(\mathcal{A}_2, f+1)$, $\Theta_{3,f} \triangleq \Phi(\mathcal{A}_3, f+1)$ and $\Theta_{4,f} \triangleq \Phi(\mathcal{A}_4, f+2)$, and multiplying the resultant equations by $1, \xi^{-\Delta_{12}^2}, \xi^{-\Delta_{12}^1}$ and $\xi^{-\Delta_{12}^1 - \Delta_{12}^2}$, respectively, results in:

$$\begin{aligned} i\hbar\dot{\Theta}_{1,f} &= \wp_1 \sqrt{f+1} \xi^{-\Delta_{12}^2} \Theta_{2,f} \\ &\quad + \wp_1 \sqrt{f+1} \xi^{-\Delta_{12}^1} \Theta_{3,f}, \\ i\hbar \xi^{-\Delta_{12}^2} \dot{\Theta}_{2,f} &= \wp_1 \sqrt{f+1} \Theta_{1,f} \\ &\quad + \wp_1 \sqrt{f+2} \xi^{-\Delta_{12}^1 - \Delta_{12}^2} \Theta_{4,f} + \wp_2 \xi^{-\Delta_{12}^1} \Theta_{3,f}, \\ i\hbar \xi^{-\Delta_{12}^1} \dot{\Theta}_{3,f} &= \wp_1 \sqrt{f+1} \Theta_{1,f} \\ &\quad + \wp_1 \sqrt{f+2} \xi^{-\Delta_{12}^2 - \Delta_{12}^1} \Theta_{4,f} + \wp_2 \xi^{-\Delta_{12}^2} \Theta_{2,f}, \\ i\hbar \xi^{-\Delta_{12}^2 - \Delta_{12}^1} \dot{\Theta}_{4,f} &= \wp_1 \sqrt{f+2} \xi^{-\Delta_{12}^2} \Theta_{2,f} \\ &\quad + \wp_1 \sqrt{f+2} \xi^{-\Delta_{12}^1} \Theta_{3,f} \end{aligned}$$

Now, we define $A \triangleq \Theta_{1,f}$, $B \triangleq \xi^{-\Delta_{12}^2} \Theta_{2,f}$, $C \triangleq \xi^{-\Delta_{12}^1} \Theta_{3,f}$, $D \triangleq \xi^{-\Delta_{12}^2 - \Delta_{12}^1} \Theta_{4,f}$, and use the property that if $X(t) \triangleq \xi^\Delta \Theta(t)$, then $\xi^\Delta \dot{\Theta} = X' - i\Delta X$, and rewrite the above set of equations into the following matrix form:

$$\begin{aligned} i\hbar\Psi' &= \Xi\Psi, \\ \Psi &\triangleq [A \quad B \quad C \quad D]^T, \\ \Xi &\triangleq \begin{bmatrix} 0 & \wp_1 f_1 & \wp_1 f_1 & 0 \\ \wp_1 f_1 & \hbar\Delta_{12}^2 & \wp_2 & \wp_1 f_2 \\ \wp_1 f_1 & \wp_2 & \hbar\Delta_{12}^1 & \wp_1 f_2 \\ 0 & \wp_1 f_2 & \wp_1 f_2 & \hbar(\Delta_{12}^1 + \Delta_{12}^2) \end{bmatrix}, \\ f_1 &= \sqrt{f+1}, \\ f_2 &= \sqrt{f+2}. \end{aligned} \quad (30.5)$$

Equation 30.5 consists of a set of linear differential equations that can be solved using standard Laplace Transform techniques either by hand or, more conveniently, using symbolic computation software such as Mathematica or Maple.

Now, we present proof that an analytical solution can be always found for Equation 29, that is for single mode quantized radiation fields; an analytical solution always exists, regardless of the number of dots. At first, we need to present a definition for the precedence:

Assume that $\mathcal{A} = \mathcal{A}_x = [r_1 \ r_2 \ \dots \ r_k]$ and $\mathcal{A}' = \mathcal{A}_y = [s_1 \ s_2 \ \dots \ s_k]$, then if for one n and for any $i \neq n$, relations $r_n < s_n$ and $r_i = s_i$ are satisfied, we say that \mathcal{A} precedes \mathcal{A}' , here denoted by either $\mathcal{A} \prec \mathcal{A}'$ or $\mathcal{A}' \succ \mathcal{A}$.

\mathcal{A} . We, furthermore, define $\Lambda_{x,y} \triangleq -\Delta_{r_n, s_n}^n$, in which $\Delta_{ab}^n \triangleq \Omega - \omega_{ab}^n$. For example, if $\mathcal{A} = \mathcal{A}_1 = [1 \ 3 \ 4 \ 5 \ 2]$ and $\mathcal{A}' = \mathcal{A}_2 = [1 \ 3 \ 4 \ 2 \ 2]$ then $\mathcal{A}' \prec \mathcal{A}$ and, therefore $\Lambda_{2,1} \triangleq -\Delta_{2,5}^4$. We can now rewrite Equation 29.1 as:

$$i\hbar\dot{\Phi}(\mathcal{A}_i, f) = \sum_{j=1}^N M_{i,j,f} \xi^{\chi_{i,j}} \Phi(\mathcal{A}_j, f + z_{i,j}) \quad (31)$$

in which $M_{i,j,f}$ are constants pertaining to interaction coefficients and the field photon number. Parameter $z_{i,j}$ takes on either of the values 0, -1 or +1 depending on the relation between \mathcal{A}_i and \mathcal{A}_j . Specifically, as observed in Equations 29.1 and 29.2:

$$z_{i,j} = -1, \text{ if } \mathcal{A}_i \succ \mathcal{A}_j, \quad (32.1)$$

$$z_{i,j} = +1, \text{ if } \mathcal{A}_i \prec \mathcal{A}_j. \quad (32.2)$$

Furthermore, in the two latter cases, $\chi_{i,j}$ can be obtained as:

$$\chi_{i,j} = \Lambda_{i,j}, \text{ if } \mathcal{A}_i \prec \mathcal{A}_j, \quad (32.3)$$

$$\chi_{i,j} = \Lambda_{j,i}, \text{ if } \mathcal{A}_i \succ \mathcal{A}_j. \quad (32.4)$$

For instance, in the case of the previous example, using Equations 30.1 to 30.4, we have:

$$\begin{aligned} M_{2,1,f} &= \wp_1 \sqrt{f}, \\ M_{2,2,f} &= 0, \\ M_{2,3,f} &= \wp_2, \\ M_{2,4,f} &= \wp_1 \sqrt{f+1}, \\ \chi_{2,1} &= \Delta_2, \quad \chi_{2,2} = 0, \quad \chi_{2,3} = \Upsilon, \quad \chi_{2,4} = -\Delta_1, \\ \mathcal{A}_1 &\prec \mathcal{A}_2, \quad z_{2,1} = -1, \quad \Lambda_{1,2} = -\Delta_{1,2}^2. \end{aligned}$$

Now, we present the following two lemmas, which allow the conversion of Equation 29 into an analytically-solvable form.

Lemma 1

Equation 31 is equivalent to:

$$i\hbar\dot{\Phi}(\mathcal{A}_i, f + \varsigma_i) = \sum_{k=1}^N M_{i,k,f+\varsigma_k} \xi^{\chi_{i,k}} \Phi(\mathcal{A}_k, f + \varsigma_k), \quad (33)$$

in which we have $\varsigma_1 = 0$ and $\varsigma_i = \varsigma_j + 1$, if $\mathcal{A}_i \succ \mathcal{A}_j$, $1 \leq i, j \leq N$.

Proof

We take an induction approach to prove Lemma 1. Assume that the Lemma is true for $1 \leq i$ and $j \leq p$, that is:

$$i\hbar\dot{\Phi}(\mathcal{A}_i, f + \varsigma_i) = \sum_{k=1}^N M_{i,k,f+\varsigma_i} \xi^{\chi_{i,k}} \Phi(\mathcal{A}_k, f + \varsigma_k), \quad (34.1)$$

in which $\varsigma_i = \varsigma_j + 1$, if $\mathcal{A}_i \succ \mathcal{A}_j$ and $1 \leq i, j \leq p$. Now, we must prove that the Lemma also holds for $1 \leq i, j \leq p+1$. This requires us to prove Equation 34.1 for the two following cases:

- (i) $\varsigma_i = \varsigma_{p+1} + 1$, if $\mathcal{A}_i \succ \mathcal{A}_{p+1}$, ($1 \leq i \leq p$),
- (ii) $\varsigma_{p+1} = \varsigma_j + 1$, if $\mathcal{A}_{p+1} \succ \mathcal{A}_j$, ($1 \leq j \leq p+1$).

Here, we just prove case (i); the proof of case (ii) is similar. If $\mathcal{A}_i \succ \mathcal{A}_{p+1}$ due to Equation 32 we have:

$$z_{i,p+1} = -1. \quad (34.2)$$

Replacing f with $f + \varsigma_i$ in Equation 31 leads to:

$$i\hbar\dot{\Phi}(\mathcal{A}_i, f + \varsigma_i) = \sum_{k=1}^N M_{i,k,f+\varsigma_i} \xi^{\chi_{i,k}} \Phi(\mathcal{A}_k, f + z_{i,k} + \varsigma_i). \quad (34.3)$$

A comparison between Equations 34.1 and 34.3 shows that for the summation index, $k = p+1$, we have:

$$\varsigma_{p+1} = z_{i,p+1} + \varsigma_i. \quad (34.4)$$

Using Equations 34.2 and 34.4 leads to case (i). \square

For instance, in the case of the previous example, we had $\mathcal{A}_1 = [1 \ 1]$, $\mathcal{A}_2 = [1 \ 2]$, $\mathcal{A}_3 = [2 \ 1]$ and $\mathcal{A}_4 = [2 \ 2]$. So, $\varsigma_1 = 0$, $\varsigma_2 = \varsigma_1 + 1 = 1$, $\varsigma_3 = \varsigma_1 + 1 = 1$ and $\varsigma_4 = \varsigma_2 + 1 = 2$. Equations 30.1 to 30.4 describe the system in agreement with Lemma 1.

If we now define $\Theta_{i,f} \triangleq \Phi(\mathcal{A}_i, f + \varsigma_i)$, Equation 33 can be expressed as:

$$i\hbar\dot{\Theta}_{i,f} = \sum_{j=1}^N \wp_{ijf} \xi^{\chi_{i,j}} \Theta_{j,f}, \quad (35)$$

where $\wp_{ijf} \triangleq M_{i,j,f+\varsigma_i}$.

Lemma 2

Equation 35 is equivalent to:

$$i\hbar\Gamma_i\dot{\Theta}_{i,f} = \sum_j \wp_{ijf} \Gamma_j \Theta_{j,f}, \quad (36)$$

in which we have $\Gamma_1 = 1$ and $\Gamma_j = \Gamma_i \xi^{\Lambda_{i,j}}$ if $\mathcal{A}_i \prec \mathcal{A}_j$, $1 \leq i, j \leq N$.

Proof

Again, we use induction to prove Lemma 2. Assume that the assumption holds for $1 \leq i, j \leq p$, that is:

$$i\hbar\Gamma_i\dot{\Theta}_{i,f} = \sum_k \wp_{ikf} \Gamma_k \Theta_{k,f}, \quad (37.1)$$

in which $\Gamma_j = \Gamma_i \xi^{\Lambda_{i,j}}$, if $\mathcal{A}_i \prec \mathcal{A}_j$, $1 \leq i, j \leq p$. Now, we must prove that it is also true for $1 \leq i, j \leq p+1$. That is, we must prove Equation 36 for the two following cases:

- (iii) $\Gamma_{p+1} = \Gamma_i \xi^{\Lambda_{i,p+1}}$, if $\mathcal{A}_{p+1} \succ \mathcal{A}_i$, $1 \leq i \leq p$,
- (iv) $\Gamma_j = \Gamma_{p+1} \xi^{\Lambda_{p+1,j}}$, if $\mathcal{A}_j \succ \mathcal{A}_{p+1}$, $1 \leq j \leq p+1$.

Here, we only prove case (iii); the proof of case (iv) is similar.

Assume that $\mathcal{A}_{p+1} \succ \mathcal{A}_i$. Using Equation 32.3, we have:

$$\chi_{i,p+1} = \Lambda_{i,p+1}. \quad (37.2)$$

If we multiply both sides of Equation 35 by Γ_i , we obtain:

$$i\hbar\Gamma_i\dot{\Theta}_{i,f} = \sum_{k=1}^N \wp_{ikf} \Gamma_i \xi^{\chi_{i,k}} \Theta_{k,f}, \quad (37.3)$$

A comparison between Equations 37.1 and 37.3 shows that for the summation index $k = p+1$ we have:

$$\Gamma_{p+1} = \xi^{\chi_{i,p+1}} \Gamma_i. \quad (37.4)$$

Through Equations 37.2 and 37.4, case (iii) is proved. \square

For instance, in the case of the previous example, we had $\mathcal{A}_1 = [1 \ 1]$, $\mathcal{A}_2 = [1 \ 2]$, $\mathcal{A}_3 = [2 \ 1]$ and $\mathcal{A}_4 = [2 \ 2]$. So, $\Gamma_1 = 1$, $\Lambda_{1,2} = -\Delta_{1,2}^2$, $\Gamma_2 = \Gamma_1 \xi^{\Lambda_{1,2}} = \xi^{-\Delta_{1,2}^2}$, $\Lambda_{1,3} = -\Delta_{1,2}^1$, $\Gamma_3 = \Gamma_1 \xi^{\Lambda_{1,3}} = \xi^{-\Delta_{1,2}^1}$, $\Lambda_{2,4} = -\Delta_{1,2}^1$ and $\Gamma_4 = \Gamma_2 \xi^{\Lambda_{2,4}} = \xi^{-\Delta_{1,2}^1 - \Delta_{1,2}^2}$.

If we define $\Gamma_i = \xi^{\theta_i}$, Equation 36 can be written as:

$$i\hbar X'_{i,f} = \sum_j \wp_{ijf} X_{j,f} - \hbar \theta_i X_{i,f},$$

which is equivalent to the following matrix form:

$$i\hbar \Psi' = \Xi \Psi, \quad (38)$$

where $\Psi \triangleq [\Psi_i]_{N \times 1}$, $\Psi_i = X_{i,f}$, $\Xi \triangleq [\Xi_{i,j}]_{N \times N}$ and $\Xi_{i,j} = \wp_{ijf} - \hbar \theta_i \delta_{i,j}$.

Equation 38 consists of a set of linear differential equations that can be solved using the Laplace Transform.

NUMERICAL EXAMPLE

Consider two two-level dots with energy levels a and b for the first dot, and c and d for the second, as shown in Figure 1, in a cavity with two modes of radiation

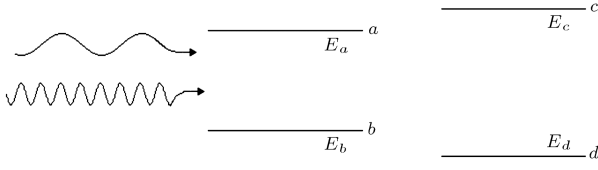


Figure 1. The investigated system. Two two-level dots are considered with different energy levels and consequently different energy gaps. Two modes of electromagnetic field are also accounted for to be entangled with the two dots.

field. We can write the time-dependent Schrödinger's equation for the system as:

$$i\hbar \frac{\partial |\varphi(t)\rangle}{\partial t} = H_{int}^{(I)} |\varphi(t)\rangle,$$

$$|\varphi(t)\rangle = \sum \left[\varphi_{a,c,n_1,n_2} |a, c, n_1, n_2\rangle + \varphi_{a,d,n_1,n_2} |a, d, n_1, n_2\rangle + \varphi_{b,c,n_1,n_2} |b, c, n_1, n_2\rangle + \varphi_{b,d,n_1,n_2} |b, d, n_1, n_2\rangle \right].$$

where n_1 and n_2 correspond to the photon numbers of the two dots. Assume, furthermore, that $\hbar\Delta_{1m} = \hbar\Omega_m - (E_a - E_b)$, $\hbar\Delta_{2m} = \hbar\Omega_m - (E_c - E_d)$, and $\hbar q = (E_a - E_b) - (E_c - E_d)$ where Ω_i is the i th mode frequency, E_k is the energy of the level k ($k = a, b, c, d$) and \hbar is Plank's constant. The following system of equations can be obtained, due to RWA, by Equation 28:

$$i\dot{A}(n_1, n_2) = g_{11}e^{-it\Delta_{11}}C(n_1 + 1, n_2)\sqrt{n_1 + 1} + g_{21}e^{-it\Delta_{21}}B(n_1 + 1, n_2)\sqrt{n_1 + 1} + g_{12}e^{-it\Delta_{12}}C(n_1, n_2 + 1)\sqrt{n_2 + 1} + g_{22}e^{-it\Delta_{22}}B(n_1, n_2 + 1)\sqrt{n_2 + 1}, \quad (39.1)$$

$$i\dot{B}(n_1, n_2) = g_{11}e^{-it\Delta_{11}}D(n_1 + 1, n_2)\sqrt{n_1 + 1} + g_{21}e^{it\Delta_{21}}A(n_1 - 1, n_2)\sqrt{n_1} + g_{12}e^{-it\Delta_{12}}D(n_1, n_2 + 1)\sqrt{n_2 + 1} + g_{22}e^{it\Delta_{22}}A(n_1, n_2 - 1)\sqrt{n_2} + \wp e^{itq}C(n_1, n_2), \quad (39.2)$$

$$i\dot{C}(n_1, n_2) = g_{21}e^{-it\Delta_{21}}D(n_1 + 1, n_2)\sqrt{n_1 + 1} + g_{11}e^{it\Delta_{11}}B(n_1 - 1, n_2)\sqrt{n_1} + g_{22}e^{-it\Delta_{22}}D(n_1, n_2 + 1)\sqrt{n_2 + 1} + g_{12}e^{it\Delta_{12}}B(n_1, n_2 - 1)\sqrt{n_2 + 1} + \wp e^{-itq}B(n_1, n_2), \quad (39.3)$$

$$i\dot{D}(n_1, n_2) = g_{21}e^{it\Delta_{21}}C(n_1 - 1, n_2)\sqrt{n_1} + g_{11}e^{it\Delta_{11}}B(n_1 - 1, n_2)\sqrt{n_1} + g_{22}e^{it\Delta_{22}}C(n_1, n_2 - 1)\sqrt{n_2} + g_{12}e^{it\Delta_{12}}B(n_1, n_2 - 1)\sqrt{n_2}, \quad (39.4)$$

where g_{mn} is the coupling coefficient associated with the m th dot and the n th mode of the field in the cavity, and \wp is the dipole-dipole coupling constant. Notice that we have used a slightly different notation for simplicity, given by:

$$A(n_1, n_2) = \varphi_{a,c,n_1,n_2},$$

$$B(n_1, n_2) = \varphi_{a,d,n_1,n_2},$$

$$C(n_1, n_2) = \varphi_{b,c,n_1,n_2},$$

$$D(n_1, n_2) = \varphi_{b,d,n_1,n_2}.$$

If the above system contained one cavity mode, we could use analytical methods, such as the Laplace transform, to solve it as discussed before. In the case of two or more cavity modes, the system is more complex and finding an analytical solution seems unlikely. Hence, we turn to the numerical solution.

We can solve the above system with a simple finite difference scheme. A forward difference scheme is used to approximate the first order derivatives, $df(t)/dt \equiv g(t) \approx (f^{n+1} - f^n)/\Delta t$, where n is the index of the time step. Therefore, discretization is as $f^{n+1} = g^n\Delta t + f^n$. As n_1 and n_2 increase, the algorithm becomes more and more time-consuming. If $D(n_1, n_2)$ is desired in Equation 39, $D(i, j)$ will be required if i and j satisfy $i + j = n_1 + n_2 + 1$. Also, $B(i, j)$ and $C(i, j)$ with $i + j = n_1 + n_2$ and $A(i, j)$ with $i + j = n_1 + n_2 - 1$ would be needed. The total number of equations to be solved is, therefore $4(n_1 + n_2)$. Tables 1 and 2 show the required variables for $n_1 + n_2 = 2$ and $n_1 + n_2 = 3$.

We solve the system of equations for the simplest case, where $n_1 + n_2 = 2$ as in Table 1. For the first example, we take $\wp = g_{i1} = \frac{1}{2\pi}\Omega_1 = \frac{1}{2}\omega_{ab} = \frac{1}{2}\omega_{cd} = 2 \times 10^{13}$ Hz and $g_{i2} = \frac{1}{2\pi}\Omega_2 = 3 \times 10^{13}$ Hz ($i = 1, 2$).

Table 1. The desired and required variables in Equation 39 for $n_1 + n_2 = 2$.

Desired	Required
$D(1,1)$	$C(1,0), C(0,1), B(1,0), B(0,1)$
$C(1,0)$	$A(0,0), D(2,0), D(1,1), B(1,0)$
$C(0,1)$	$A(0,0), D(0,2), D(1,1), B(0,1)$
$B(1,0)$	$A(0,0), D(2,0), D(1,1), C(1,0)$
$B(0,1)$	$A(0,0), D(0,2), D(1,1), C(0,1)$
$A(0,0)$	$C(1,0), C(0,1), B(1,0), C(0,1)$
$D(2,0)$	$B(1,0), C(1,0)$
$D(0,2)$	$B(0,1), C(0,1)$

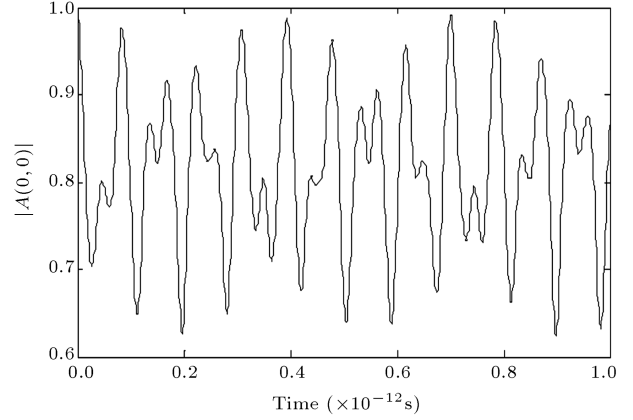
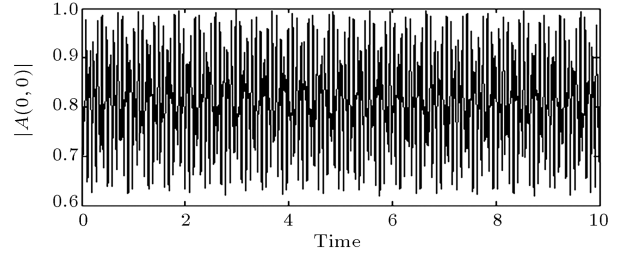
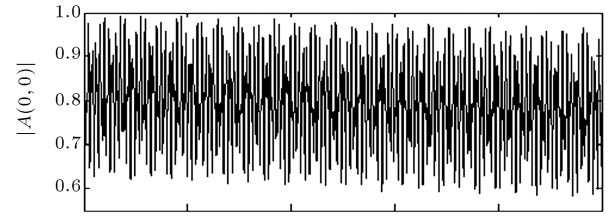
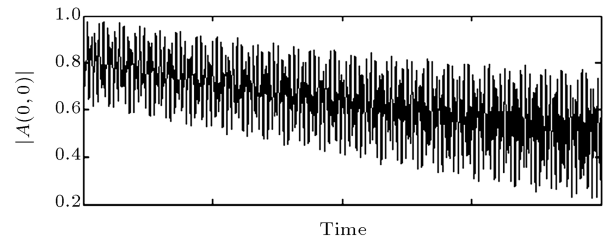
Table 2. The desired and required variables in Equation 39 for $n_1 + n_2 = 3$.

Desired	Required
$D(2,1)$	$C(2,0), C(1,1), B(2,0), B(1,1)$
$C(2,0)$	$A(1,0), D(3,0), D(2,1), B(2,0)$
$C(1,1)$	$A(1,0), A(0,1), D(1,2), D(2,1), B(1,1)$
$B(2,0)$	$A(1,0), D(3,0), D(2,1), C(2,0)$
$B(1,1)$	$A(1,0), A(0,1), D(1,2), D(2,1), C(1,1)$
$A(1,0)$	$C(2,0), C(1,1), B(2,0), B(1,1)$
$D(3,0)$	$B(2,0), C(2,0)$
$A(0,1)$	$B(0,2), B(1,1), C(0,2), C(1,1)$
$D(1,2)$	$C(0,2), C(1,1), B(0,2), B(1,1)$
$B(0,2)$	$A(0,1), D(0,3), D(1,2), C(0,2)$
$C(0,2)$	$A(0,1), D(0,3), D(1,2), B(0,2)$
$D(0,3)$	$B(0,2), C(0,2)$

These parameters represent a system that consists of two similar two-level dots with the same value of transition energy. The coupling coefficients of each dot to different cavity modes will be however taken to be different so that g_{ij} and g_{ik} are not the same. The associated simulation time step is $\Delta t = 5 \times 10^{-19}$ sec, and the initial conditions were $A(0,0) = 1, D(1,1) = D(2,0) = D(0,2) = C(1,0) = C(0,1) = B(1,0) = B(0,1) = 0$. The wave form for $|A(0,0)|$ the absolute value of $|A(0,0)|$ is plotted in Figure 2.

The convergence of the finite difference scheme is an important issue, which needs particular attention. The test for the validity of the method can be the energy conservation law. As the system is lossless, the overall energy in the system must be a fixed value; hence, if the time step is not chosen small enough, the obtained wave forms will gradually decay. In Figure 3, we show the $|A(0,0)|$ for three different Δt , where only the last one with $\Delta t = 5 \times 10^{-18}$ renders acceptable.

In the previous example, we considered two similar dots and two different cavity modes. Had we considered two different dots with $g_{ij} \neq g_{kj}$, the results

**Figure 2.** Time evolution of the absolute value of $|A(0,0)|$.(a) $\Delta t = 5 \times 10^{-19}$ (b) $\Delta t = 5 \times 10^{-18}$ (c) $\Delta t = 5 \times 10^{-17}$ **Figure 3.** Effect of size of time step on accuracy of result of finite difference method. In all above figures, the horizontal axis is the time axis, the vertical axis shows the values of $|A(0,0)|$ and the simulation is run for the overall time of $T = 10^{-12}$ s. Due to the energy conservation law, the wave form average should neither increase nor decrease as time goes on. The decay in larger time steps is a consequence of innate error in the finite difference method.

would be slightly altered, however. The common point is that the conservation of energy must be satisfied through the temporal evolution of the states. Δ_{mn} in Equation 39 appear in the exponents so that even a slight change in their values can lead to frequency shifts

in the Fourier transform of the results. On the other hand, g_{ij} exhibit less effect on the frequency shifts and mainly contribute to the amplitudes of the frequency components.

CONCURRENCE AND DEGREE OF ENTANGLEMENT

To measure the degree of entanglement quantitatively, several methods might be used [21,22]; however, we use the concurrence, as mentioned in [21]. According to [21], concurrency for the previously mentioned system of two two-level dots in a cavity with two modes of EM field can be expressed as:

$$\lambda = \left[\sum_{n_2, m_2=1}^{N_2} \sum_{n_1, m_1=1}^{N_1} |A(n_1, n_2)D(m_1, m_2) - B(n_1, n_2)C(m_1, m_2) - B(m_1, m_2)C(n_1, n_2) + A(m_1, m_2)D(n_1, n_2)|^2 \right]^{\frac{1}{2}}.$$

We have considered that $\wp = g_{11} = g_{21} = \Omega_1 = \omega_{ab} = 0.2 \times 10^{14}$ Hz and $g_{12} = g_{22} = \Omega_2 = \omega_{cd} = 0.3 \times 10^{14}$ Hz where $\hbar\omega_{ab} = E_a - E_b$ and $\hbar\omega_{cd} = E_c - E_d$. The total number of equations to be solved in Equation 39 to obtain the time evolution of $D(n_1, n_2)$, would be equal to $4(n_1 + n_2)$. Hence, each $N_{\text{sum}} = n_1 + n_2$ is associated with a set of equations, such as Equation 39. Note that we should have $N_{\text{sum}} \geq 2$ for the existence of both EM field modes. To determine N_1 and N_2 in the definition of concurrency, one can change N_{sum} such that $2 \leq N_{\text{sum}} \leq N_{\text{max}}$. For each N_{sum} , we have assumed all variables equal to zero at the start of the simulation, except $A(1, N_{\text{sum}} - 1)$, which was set to unity. Figure 4 shows the time evolution of concurrency λ evaluated for $N_{\text{max}} = 3$. As expected, the system rapidly undergoes entanglement in the first 20 fs, and the degree of entanglement remains within roughly 10% of the average value.

CONCLUSIONS

We followed the JCPM model to obtain a set of linearly coupled equations describing the interaction of an electromagnetic field of an arbitrary number of modes with a collection of dots each having arbitrary energy levels acting as a quantum dot molecule. We simplified the equations for a single mode field and applied the RWA. Then, we investigated the obtained equations and presented a method to solve them analytically after analyzing a particular example. The increased number of cavity modes causes the solution to become more complex, and analytical methods, such as the Laplace

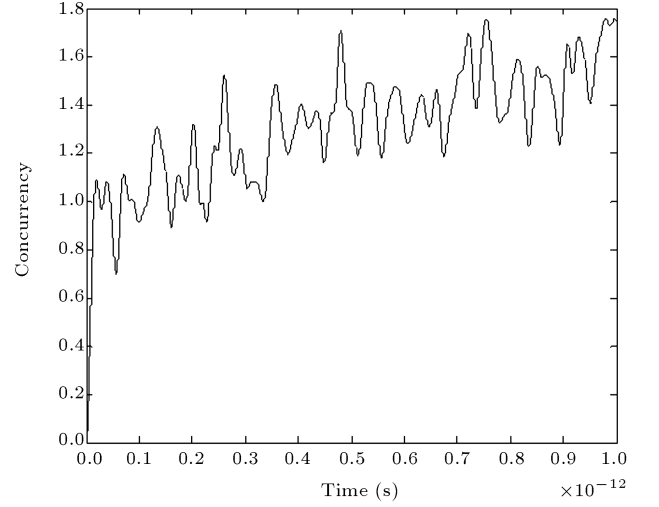


Figure 4. Time evolution of concurrency of the system of two two-level quantum dots in a cavity with two EM modes.

transform method, cannot be found as easily as before. The system governing equations are achieved and it is found that the sum of the photon number of the EM modes determines the total number of equations to be solved. The finite difference method is used as the easiest way to solve the system. However, the latter method is time consuming and its accuracy is influenced by the time step size. Finally, we demonstrated the entanglement of the system of two, two-level dots and two modes of electromagnetic field in a lossless cavity, and used concurrency as the measure to evaluate multi-particle entanglement.

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