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Effective Hamiltonian and Effective Penguin Model on *b* Quark Decays

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Abstract. *In this research, we investigated b quark decays by two different approaches; firstly, according to the structure of penguin decays, and secondly, based on an effective Hamiltonian theory. Working with the standard model, the QCD penguin terms for various b and $\bar{b}$ decays are calculated. We also studied decay rates of the matter-antimatter of b quark decays. The gluonic penguin of b decays, $b \to q_k g \to q_k q_i q_{\bar{j}}$, is studied through the Wilson coefficients of the effective Hamiltonian. We obtained the decay rates of the tree and penguin and magnetic dipole terms all together to compare them with the effective Hamiltonian current-current and penguin operators. We described the effective Hamiltonian theory and applied it to the calculation of current-current ($Q_{1,2}$), QCD penguin ($Q_{3,\cdots,6}$) and magnetic dipole (Q_8) decay rates. Based on the effective penguin model, the simple coefficients, $d_1, \cdots, d_{6,8}$, are defined according to the gluon penguin structure and used in the effective Hamiltonian theory. In the other section of this research, the decay rates of processes like $b \to cd\bar{c}(\bar{b} \to \bar{c}\bar{d}c)$, $b \to cs\bar{c}(\bar{b} \to \bar{c}\bar{s}c)$, $b \to ud\bar{u}(\bar{b} \to \bar{u}\bar{d}u)$ and $b \to us\bar{u}(\bar{b} \to \bar{u}\bar{s}u)$ are obtained based on the Effective Hamiltonian (EH) and Effective Penguin Model (EPM). Decay rates and branching ratios are very similar in all models, but in the Effective Hamiltonian Magnetic Dipole, the total decay rate is about 10% larger than the simple tree or Effective Hamiltonian. On the other hand, including the penguin induces matter-antimatter asymmetries. These are largest in the rate decays $b \to ud\bar{u}$, the decay rate of which is about 7% smaller than the decay rate $\bar{b} \to \bar{u}\bar{d}u$. Also, rate $b \to su\bar{u}$ is larger than rate $\bar{b} \to \bar{s}\bar{u}u$.*

Keywords: *b quark; QCO Penguin; Effective Penguin model; Magnetic dipole.*

INTRODUCTION (PENGUIN)

In the Standard Model, flavor-changing neutral currents are forbidden, for example there is no direct coupling between the b quark and s or d quarks. Effective flavor-changing neutral currents are induced by one-loop or "penguin" diagrams where a quark emits and reabsorbs a W, thus changing flavors twice as in the $b \to t \to s$ transition. Penguin decays have become increasingly appreciated in recent years [1-3]. These loop diagrams with their interesting combination of CKM matrix elements give insight into the Standard Models [4]. In addition, they are quite sensitive to new physics. The weak couplings of the quarks are given by the CKM matrix. For the Standard Model with three generations, the CKM matrix can be described completely by three Euler-type angles and a complex phase.

Various types of the penguin process are [5]: electromagnetic, electroweak and gluonic. In electromagnetic penguin decays, such as $b \to s\gamma$, a charged particle emits an external real photon. The hard photon emitted in these decays is an excellent experimental signature. The inclusive rate is dominated by short distance (perturbative) interactions and can be reliably predicted. The QCD corrections enhance the rate and have been calculated precisely. The electromagnetic penguin decay, $b \to d\gamma$, is further suppressed by $|V_{td}|^2/|V_{ts}|^2$ and gives an alternative to $B^0-\overline{B}^0$ mixing for extraction $|V_{td}|$ [6,7]. Experimentally, inclusive $b \to d\gamma$ has large backgrounds from the dominant $b \to s\gamma$ decays, which must be rejected using good particle identification or kinematics separation. The decay $b \to s\ell^+\ell^-$ can proceed via an electroweak penguin diagram where an emitted virtual photon or Z^0 produces a pair of leptons. This decay can also proceed via a box diagram [8]. The Standard Model prediction for the $b \to s\ell^+\ell^-$ decay rate is two orders

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of magnitude smaller than the $b \to s\gamma$ rate [9,10]. The rate for $b \to s\nu\overline{\nu}$ is enhanced relative to $b \to s\ell^+\ell^-$, primarily due to summing the three neutrino flavors. These decays are expected to be dominated by the weak penguin, since neutrinos do not couple to photons. The predicted rate is only a factor of 10 lower than for $b \to s\gamma$ [2]. Unfortunately, the neutrinos escape detection making this mode difficult to observe.

Another category of penguin is the so-called vertical or annihilation penguin where the penguin loop connects the two quarks in the B meson. These rates are expected to be highly suppressed in the Standard Model, since they involve a $b \to d$ transition and are suppressed by $(f_B/m_B)^2 \approx 2 \times 10^{-3}$ [11] where f_B is the B-meson decay constant that parameterizes the probability that the two quarks in the B meson will "find each other", and m_B is the B meson mass. The $B \to \gamma\gamma$ decay is suppressed [12] relative to $b \to s\gamma$ by an additional $\alpha_{\rm QCD}$. The $B \to \ell^+\ell^-$ decays are helicity-suppressed [13,7]. Because these decays are so suppressed in the Standard Model, they provide a good opportunity to look for non SM effects.

An on- or off-shell gluon can also be emitted from the penguin loop. While the on-shell $b \to sg$ rate had been calculated to be $O(0.1\%)$ [14], the inclusive on-plus off-shell $b \to sg^*$ rate includes contribution from $b \to sq\overline{q}$ and $b \to sgg$, which increase the inclusive rate to 0.5-1% [2,15]. The $b \to dg^*$ penguin rate is smaller by $|V_{td}/V_{ts}|^2$.

Unfortunately, there are several difficulties associated with gluonic penguins. There is no good signature for the inclusive $b \to sg^*$ decay, unlike the $b \to s\gamma$ case. The branching fraction of individual exclusive gluonic penguin channels is typically quite small and hadronization effects are difficult to calculate [16,17]. In addition, many gluonic penguin final states are accessible via other diagrams, so the gluonic penguin is difficult to assess. Thus, the penguin processes, such as $B^0 \to \phi K^0$, which have contributions only from gluonic penguins, are eagerly sought.

While the gluonic penguin gives rise only to hadronic final states, several other processes can contribute to the same final states. One important contribution is from the tree-level, $b \to u$, decay. For example, the $b \to us\overline{u}$ transition and the $b \to sg^*$ penguin transition both contribute to $B^0 \to K^+\pi^-$. However, the $b \to us\overline{u}$ transition is Cabibbo-suppressed, so the penguin process is expected to dominate [18-21]. On the other hand, in $B \to \pi^+\pi^-$, for example, the small $b \to dg^*$ contribution is expected to be dominant by the non-cabibbo-suppressed tree-level $b \to ud\overline{u}$ transition. In general, most decays to the hadronic final state with ϕ mesons or non-zero net strangeness are expected to be dominated by the gluonic penguin, and hadronic final states with zero net strangeness are expected to be dominated by tree-level, $b \to u$.

The Electroweak penguin also contributes to hadronic final states. Every gluonic penguin can be converted to an electroweak penguin by replacing the gluon with a Z^0 or γ. Electroweak penguins with internal Z^0 or γ emissions are suppressed, relative to the corresponding strong gluonic penguin. In the hairpin process, gluon Z^0 or γ are emitted externally and subsequently form a meson.

The vertical electroweak penguin diagram with the lepton pair replaced by a de-quark pair is highly suppressed and is important only for decays such as $B^0 \to \phi\phi$ where no other diagrams contribute [22,23]. In the annihilation diagram, the b and $\overline{u}$ quarks in a B^- meson annihilate to form a virtual W^-. The annihilation diagram is suppressed by $|V_{ub}|$ and by f_B/m_B and is expected to be mostly negligible. In the exchange diagram, a $b \to u$ transition and a $\overline{d} \to \overline{u}$ transition occur simultaneously via the exchange of a W between the b and $\overline{d}$ quarks in a $\overline{B}^0$ meson. The exchange process is also suppressed by $|V_{ub}|$ and f_B/m_B, and is also expected to be negligible except in decays such as $B^0 \to K^+K^-$ where no flavored diagrams contribute [24-26].

Although $s \to u$ loop diagrams are important in K decays, those decays are typically dominated by large non-perturbative effects. A notable exception is $K^+ \to \pi^+\nu\overline{\nu}$. This decay is expected to be dominated by electroweak penguins and could eventually provide a measurement of $|V_{td}|$. Penguin processes are also possible in c and t decays, but these particles have the CKM-flavored decays, $c \to s$ and $t \to b$, accessible to them. Since the b quark has no kinematically-allowed CKM-flavored decay, the relative importance of the penguin decay is greater. The mass of the top quark - the main contributor to the loop - is large, and the coupling of the b quark and the t quark, $|V_{tb}|$, is very close to unity, both strengthening the effect of the penguin. The $b \to s(b \to d)$ penguin transition is sensitive to $|V_{ts}|/|V_{td}|$, which will be extraordinarily difficult to measure in top decay. Information from the penguin decay will complement information on $|V_{ts}|$ and $|V_{td}|$ from $B_s - \overline{B}_s$ and $B^0 - \overline{B}^0$ mixing [27]. Since the Standard Model loops involve the heaviest known particles (t, W, Z), rates for these processes are very sensitive to non-SM extension with heavy charged Higge or supersymmetric particles. Therefore, the measurement of loop processes constitutes the most sensitive low energy probes for such extensions to the Standard Model.

GLUON PENGUIN

Conservation of the gluonic current requires the $b \to q_k g$ vertex to have the structure [28,29]:

$$\Gamma^a_\mu(q^2) = (ig_s/4\pi^2)\overline{u}_k(p_k)T^a V_\mu(q^2)u_b(p_b), \tag{1}$$

where:

$$V_\mu(q^2) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma^\nu [F_1^L(q^2)P_L + F_1^R(q^2)P_R] + i\sigma_{\mu\nu}q^\nu [F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (2)$$

Here, F_1 and F_2 are the electric (monopole) and magnetic (dipole) form factors, $q = q_g = p_b - p_k$ is the gluon four momentum, $P_{L(R)} \equiv (1 \mp \gamma_5)/2$ are the chirality projection operators and $T^a(a = 1, \cdots, 8)$ are the $SU(3)_c$ generators normalized to $Tr(T^aT^b) = \delta^{ab}/2$. The $\overline{b} \to \overline{q}_k g$ vertex is:

$$\overline{\Gamma}^a_\mu(q^2) = -(ig_s/4\pi^2)\overline{v}_b(p_b)T^a\overline{V}_\mu(q^2)v_k(p_k). \quad (3)$$

Here, $\overline{V}_\mu$ has the form of Equation 2 with the form factors $F^{L,R}_{1,2}(q^2)$ replaced by $\overline{F}^{L,R}_{1,2}(q^2)$. To the lowest order in α_s the penguin amplitude for the decay process, $b \to q_k g \to q_k q'\overline{q}'(q_k q_i \overline{q}_j)_{i=j}$, is:

$$M^{\text{Peng}} = -i(\alpha_s/\pi)[\overline{u}_k(p_k)T^a\Lambda_\mu u_b(p_b)] [\overline{u}'_q(p'_q)\gamma^\mu T^a v_{\overline{q}'}(p_{\overline{q}'})], \quad (4)$$

where $\alpha_s = g_s^2/4\pi$ and:

$$\Lambda_\mu \equiv \gamma_\mu[F_1^L(q^2)P_L + F_1^R(q^2)P_R] + (i\sigma_{\mu\nu}q^\nu/q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \quad (5)$$

Similarly, for $\overline{b} \to \overline{q}_k q'\overline{q}'$, the amplitude is:

$$\overline{M}^{\text{Peng}} = i(\alpha_s/\pi)[\overline{v}_k(p_k)T^a\overline{\Lambda}_\mu v_b(p_b)] [\overline{u}'_q(p'_q)\gamma_\mu T^a v_{\overline{q}'}(p_{\overline{q}'})], \quad (6)$$

where $\overline{\Lambda}_\mu$ is obtained from Relation 5 by the replacement of all $F(q^2)$ form factors by $\overline{F}(q^2)$ form factors. The top quark dominates in the sum for F_2. Hence, at a value of q^2 (a good approximation), we have $F_2^L(q^2) \approx F_2^L(0)$ and $F_2^R(q^2) \approx F_2^R(0)$ [30], so:

$$F_1^L(q^2) = \left(G_F/\sqrt{2}\right)\sum_{i=u,c,t} V^*_{ik}V_{ib}f_1(x_i, q^2),$$

$$F_1^R(0) = 0, \quad (7)$$

$$F_2^L(0)/m_q = F_2^R(0)/m_b = \left(G_F/\sqrt{2}\right)\sum_{i=u,c,t} V^*_{iq}V_{ib}f_2(x_i), \quad (8)$$

where $x_i \equiv m_i^2/M_W^2 (i = u, c, t)$ and:

$$f_2(x) = -(x/4(1-x)^4)[2+3x-6x^2+x^3+6x\ln x], \quad (9)$$

$$f_1(x) = (1/12(1-x)^4)[18x - 29x^2 + 10x^3 + x^4 - (8 - 32x + 18x^2)\ln x], \quad (10)$$

$$f_1(x_i, q^2) = (10/9) - (2/3)\ln x_i + (2/3z_i) - (2(2z_i + 1)/3z_i)g(z_i). \quad (11)$$

Here, $z_i \equiv q^2/4m_i^2$ and [31,8]:

$$g(z) = \begin{cases} \sqrt{\frac{1-z}{z}}\arctan\left(\sqrt{\frac{z}{1-z}}\right), & z < 1 \\ \frac{1}{2}\sqrt{\frac{z-1}{z}}\left[\ln\left(\frac{\sqrt{z}+\sqrt{z-1}}{\sqrt{z}-\sqrt{z-1}}\right) - i\pi\right], & z > 1 \end{cases} \quad (12)$$

For the u quark, z_i is large and we use the asymptotic form of Equation 11:

$$f_1(x_u, q^2) = (10/9) - (2/3)[\ln(q^2/M_W^2) - i\pi]. \quad (13)$$

We find $F_1^L >> F_1^R$ and $F_2^R >> F_2^L$. For the $b \to dq'\overline{q}'$ amplitude, we find that F_1^L is dominant. Processes like $b \to ds\overline{s}$ and $\overline{b} \to \overline{d}s\overline{s}$ are expected to be penguin dominated [6] and F_1^L dominates all the other form factors. In the $b \to sq'\overline{q}'$ transition, we again find that $F_1^L >> F_1^R$, $F_2^R >> F_2^L$ and that the F_1^L amplitude is dominant.

Now, a very important issue is the generation of QCD corrections to penguin operators. Consider, for example, the local operator $(\overline{u}_\alpha b_a)_{V-A}(\overline{d}_\beta u_\beta)_{V-A}$, which is directly induced by W-boson exchange. In this case, additional QCD correction diagrams with a gluon contribute and, as a consequence, four operators are involved in the mixing under renormalization instead of two. These are [32,33]:

$$Q_3 = (\overline{d}_\alpha b_\alpha)_{V-A}\sum_q(\overline{q}_\beta q_\beta)_{V-A},$$

$$Q_4 = (\overline{d}_\beta b_\alpha)_{V-A}\sum_q(\overline{q}_\alpha q_\beta)_{V-A},$$

$$Q_5 = (\overline{d}_\alpha b_\alpha)_{V-A}\sum_q(\overline{q}_\beta q_\beta)_{V+A},$$

$$Q_6 = (\overline{d}_\beta b_\alpha)_{V-A}\sum_q(\overline{q}_\alpha q_\beta)_{V+A}. \quad (14)$$

α and β are color indices. The sum over q runs over all quark flavors that exist in the effective theory in question. Since the gluon coupling is, of course, flavor conserving, it is clear that penguins cannot be generated from the operator current due to the gluon coupling in the lower part. For convenience, this vector structure is decomposed into a $(V - A)$ and a $(V + A)$ part according to chiral representation;

$$(\overline{q}_{i\alpha}b_\alpha)_{V\mp A}(\overline{q}_{k\beta}q_{j\beta})_{V\mp A} = (\overline{q}_{i\alpha}\gamma^\mu((1 \mp \gamma_5)/2)b_\alpha) (\overline{q}_{k\beta}\gamma_\mu((1 \mp \gamma_5)/2)q_{j\beta}). \quad (15)$$

And two component of spinors are given by:

$$\overline{q}_{i\alpha}\gamma^\mu((1 - \gamma_5)/2)b_\alpha = q^\dagger_{i\alpha L}\tilde{\sigma}^\mu b_{\alpha L},$$

$$\overline{q}_{k\beta}\gamma_\mu((1 - \gamma_5)/2)q_{j\beta} = q^\dagger_{k\beta L}\tilde{\sigma}_\mu q_{j\beta L},$$

$$\overline{q}_{i\alpha}\gamma^{\mu}((1+\gamma_5)/2)b_{\alpha} = q^{\dagger}_{i\alpha R}\sigma^{\mu}b_{\alpha R},$$

$$\overline{q}_{k\beta}\gamma_{\mu}((1+\gamma_5)/2)q_{j\beta} = q^{\dagger}_{k\beta R}\sigma_{\mu}q_{j\beta R}. \quad (16)$$

For each of these, two different color forms arise due to the color structure of the exchanged gluon. The amplitude (Equation 4) can be written as [34]:

$$M^{\text{Peng}} = -i(G_F/\sqrt{2})\{(\alpha_s(M_W/8\pi)\left[\sum_{i=u,c} V_{ib}V^*_{iq}f_1(x_i,q^2) + V_{tb}V^*_{tq}f_1(x_t)\right]Q_P + (1/2)\sum_{i=u,c,t} V_{ib}V^*_{iq}f_2(x_i)Q_8\right\}. \quad (17)$$

Q_8 is the chromomagnetic dipole operator:

$$Q_8 = 4\alpha_s^2 m_b[\overline{q}_{i\alpha}\sigma^{\mu\nu}(1+\gamma_5)T^a_{\alpha\beta}b_\beta](q_\nu/q^2)[\overline{q}_{\overline{j}}\gamma_\mu T^a q_k]. \quad (18)$$

Here:

$$Q_P = Q_4 + Q_6 - (1/3)(Q_3 + Q_5). \quad (19)$$

EFFECTIVE HAMILTONIAN

As a weak decay under the presence of the strong interaction, B meson decays require special techniques [35]. The main tool to calculate such B meson decays is the effective Hamiltonian theory [36,8]. It is a two-step program starting with an Operator Product Expansion (OPE) and performing a Rrenormalization Group Equation (RGE) analysis afterwards [37,8]. The necessary machinery has been developed over the last few years.

The derivation starts as follows: If the kinematics of the decay are of the kind where the masses of the internal particle, M_i, are much larger than the external momenta, P, $M_i^2 >> p^2$, then the heavy particle can be integrated out. This concept takes concrete form with functional integral formalism. It means that the heavy particles are removed as dynamical degrees of freedom from the theory, and hence their fields no longer appear in the (effective) Lagrangian. Their residual effect lies in the generated effective vertices [38]. In this way, an effective low energy theory can be constructed from a full theory like the Standard Model [39]. A well known example is the four-Fermi interaction where the W-boson propagator is made local for $M_W^2 >> q^2$ (q denotes the momentum transfer through the W):

$$-i(g_{\mu\nu})/(q^2 - M_W^2) \rightarrow ig_{\mu\nu}[(1/M_W^2) + (q^2/M_W^4) + \cdots], \quad (20)$$

where the ellipses denote terms of higher order in $1/M_W$.

Apart from the t quark, the basic framework for weak decay quarks is the effective field theory, relevant for scales $M_W, M_Z, M_t >> \mu$ [40,31]. This framework, as we have seen above, brings in local operators which govern "effectively" the transition in question. From the point of the decaying quark, it represents the generalization of the Fermi theory as formulated by Sudershan and Marshak and Feynman and Gell-Mann forty years ago.

It is well known that the decay amplitude is the product of two different parts whose phases are made of a weak (Cabbibo-Kobayashi-Maskawa) and a strong (final state interaction) contribution. The weak contributions to the phases change sign when going to the CP-conjugate process while the strong ones do not. Indeed, the simplest effective Hamiltonian without QCD effects ($b \rightarrow ud\overline{u}$) is:

$$H^0_{\text{eff}} = 2\sqrt{2}G_F V_{ub}V^*_{ud}Q_1, \quad (21)$$

where G_F is the Fermi constant, V_{ij} are the relevant CKM factors and:

$$Q_1 = (\overline{u}_\alpha b_\alpha)_{V-A}(\overline{d}_\beta u_\beta)_{V-A}, \quad (22)$$

where $(V-A)$ is a current-current local operator. This simple tree amplitude introduces a new operator, Q_2, and is modified by the QCD effect to:

$$H_{\text{eff}} = 2\sqrt{2}G_F V_{ub}V^*_{ud}(C_1Q_1 + C_2Q_2). \quad (23)$$

Here:

$$Q_2 = (\overline{u}_\beta b_\alpha)_{V-A}(\overline{d}_\alpha u_\beta)_{V-A}, \quad (24)$$

where C_1 and C_2 are Wilson coefficients. The situation in the Standard Model is, however, more complicated because of the presence of additional interactions and in particular penguins which effectively generate new operators. These are in particular the gluon, photon and Z^0-boson exchanges and penguin b quark contributions as seen before.

Consequently, the relevant effective Hamiltonian for B-meson decays generally involves several operators, Q_i, with various colors and Dirac structures that are different from Q_1. The operators can be grouped into two categories [41]: $i = 1, 2$-current-current operators and $i = 3, \cdots, 6, 8$-gluonic penguin operators. Moreover, each operator is multiplied by a calculable Wilson coefficient, $C_i(\mu)$:

$$H_{\text{eff}} = 2\sqrt{2}G_F \sum_{i=1}^{6,8} d_i(\mu)Q_i(\mu), \quad (25)$$

where scale μ is discussed below. $d_i(\mu) = V_{\text{CKM}} C_i(\mu)$ and V_{CKM} denote the relevant CKM factors which are:

$$d_{1,2} = V_{ib} V_{jk}^* C_{1,2},$$

$$d_{3,\cdots,6} = -V_{tb} V_{tk}^* C_{3,\cdots,6},$$

$$d_8 = -(\alpha_s/4\pi) V_{tb} V_{tk}^* C_8. \tag{26}$$

For tree-level, the $d_{1,2}$ coefficients are:

$$d_1 = V_{ib} V_{kj}^*,$$

$$d_2 = 0. \tag{27}$$

And for an Effective Penguin Model, the $d_{3,\cdots,6,8}$ coefficients are:

$$d_3 = d_5 = (1/6)(\alpha_s/4\pi) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_1(x_i),$$

$$d_4 = d_6 = -3d_3,$$

$$d_8 = -(m_b/2)(\alpha_s/4\pi) \sum_{i=u,c,t} V_{iq}^* V_{ib} f_2(x_i). \tag{28}$$

At this stage, it should be mentioned that the usual Feynman diagram containing full W-propagators, Z^0-propagators and top-quark-propagators really represent the happening at scales $O(M_W)$, whereas the true picture of a decaying hadron is more correctly described by the local operators in question. Thus, whereas at scale $O(M_W)$ we have to deal with the full six-quark theory containing the photon, weak gauge bosons and gluon, at scale $O(1 \text{ GeV})$, the relevant effective theory contains only three light quarks, u, d, and s, gluons and the photon. At intermediate energy scales, $\mu = O(m_b)$ and $\mu = O(m_c)$, relevant for beauty and charm decays, effective five-quark and effective four-quark theories have to be considered, respectively [42].

The usual procedure then is to start at a high energy scale, $O(M_W)$, and consecutively integrate out the heavy degrees of freedom (heavy with respect to the relevant scale, μ) from explicitly appearing in the theory. The word explicitly is very essential here. The heavy field did not disappear. Their effects are merely hidden in the effective gauge coupling constants, running masses and, most importantly, the coefficients describing the effective strength of the operators at scale μ; the Wilson coefficient functions, $C_i(\mu)$ [43]. It is also straightforward to apply H_{eff} to B- and D-meson decays by changing the quark flavors appropriately. μ is some low-energy scale of $O(1 \text{ GeV})$, $O(m_c)$ and $O(m_b)$ for K, D, and B meson decays, respectively. The argument, μ, of the operators, $Q_i(\mu)$, means that their matrix elements are to be normalized at scale μ.

MAGNETIC DIPLOE AMPLITUDE OF $b \to q_k q_i \overline{q}$

A charge particle in orbital motion generates a magnetic dipole moment of a magnitude proportional to its orbital angular momentum. Furthermore, a particle with intrinsic angular momentum or spin has an intrinsic magnetic moment. The magnetic dipole term in the penguin amplitude, according to Relation 5, is:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu} q^\nu/q^2)[F_2^L(q^2)P_L + F_2^R(q^2)P_R]. \tag{29}$$

Also, according to Equation 8, the magnetic (dipole) form factor at $q^2 = 0(q^2/M_W^2 << 1)$ is:

$$\frac{F_2^L(0)}{m_k} = \frac{F_2^R(0)}{m_b} = \frac{g_2^2}{8M_W^2} \sum_i V_{ik}^* V_{ib} f_2(x_i). \tag{30}$$

The top quark is dominant for $F_2^R(0)$, so we can write:

$$F_2^R(0) = m_b(G_F/\sqrt{2})(V_{tk}^* V_{tb}) f_2(x_t). \tag{31}$$

Here, $f_2(x_t)$ is defined by Equation 9 and $x_t = m_t^2/M_W^2$. Also, we saw that $F_2^L(0) << F_2^R(0)$ because $m_k << m_b$, so the magnetic dipole term becomes:

$$\Lambda_\mu \equiv (i\sigma_{\mu\nu} q^\nu/q^2) F_2^R(0) P_R. \tag{32}$$

Putting in the penguin amplitude according to Equation 4, we have:

$$M^{\text{dip}} = \frac{g_s^2}{4\pi^2}[\overline{u}_k(p_k) T^a (i\sigma_{\mu\nu} q^\nu/q^2) F_2^R(0) P_R u_b(p_b)] [\overline{u}_i(p_i)\gamma^\mu T^a v_{\overline{j}}(p_{\overline{j}})]. \tag{33}$$

The magnetic dipole of penguin amplitude is given by (see Appendix A):

$$M^{\text{dip}} = A_8 d_8 [\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \tag{34}$$

Here:

$$d_8 = -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^* V_{ib} f_2(x_i),$$

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2). \tag{35}$$

Now, we must calculate each term of the above equation for b spins project -1/2 and 1/2, then square these terms, add all of them and at least averaging. The penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by (see Appendix A):

$$M^{\rm dip}_{(+1/2)} = A_8 d_8\{[-\sin((\theta_b+\theta_i-\theta_k-\theta_j)/2) + \sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)] + [-\sin((\theta_b-\theta_i-\theta_k+\theta_j)/2) + \sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)]\}, \quad (36)$$

$$M^{\rm dip}_{(-1/2)} = A_8 d_8\{[-\cos((\theta_b-\theta_i-\theta_k+\theta_j)/2) - \cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)] + [-\cos((\theta_b+\theta_i-\theta_k-\theta_j)/2) - \cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)]\}. \quad (37)$$

EFFECTIVE HAMILTONIAN DECAY RATES OF $b \to q_k q_i \bar{q}_j$

We cannot add the amplitude of the magnetic dipole for b spin project -1/2 (or b spin project 1/2) because there must be added amplitudes of terms of operators $Q_1, \cdots, Q_6$. Then we can add all of the terms b spin -1/2 and +1/2. So, for a Left-Left handed operator as well as operators Q_1, Q_2, Q_3, Q_4, Q_8, according to Relations A22, A25 and A26, the total amplitude is given by [44]:

$$A_{1L-L(-1/2)} = 2(d_1+d_2+d_3+d_4) [-\sin((\theta_b-\theta_j)/2)\sin((\theta_i-\theta_k)/2)] - d'_8[\cos((\theta_b-\theta_i)/2)\cos((\theta_k-\theta_j)/2)], \quad (38)$$

$$A_{2L-L(+1/2)} = 2(d_1+d_2+d_3+d_4) [\cos((\theta_b-\theta_j)/2)\sin((\theta_i-\theta_k)/2] - d'_8[\sin((\theta_b-\theta_i)/2)\cos((\theta_k-\theta_j)/2)]. \quad (39)$$

And, for Right-Right handed operators (Q_5, Q_6, Q_8), according to Equations A23, A27 and A28, the total amplitude is given by:

$$A_{3L-R(-1/2)} = 2(d_5+d_6) [\sin((\theta_b-\theta_i)/2)\sin((\theta_k-\theta_j)/2)] - d'_8[\cos((\theta_b-\theta_j)/2)\cos((\theta_i-\theta_k)/2)], \quad (40)$$

$$A_{4L-R(+1/2)} = 2(d_5+d_6) [-\cos((\theta_b-\theta_i)/2)\sin((\theta_k-\theta_j)/2)] - d'_8[\sin((\theta_b-\theta_j)/2)\cos((\theta_i-\theta_k)/2)], \quad (41)$$

where according to the effective Hamiltonian theory, the coefficients of $d_1, \cdots, d_6$ defined by Equations 26 and d_8 and d'_8 are:

$$d'_8 = 2A_8 d_8,$$

$$d_8 = -(\alpha_s/4\pi)V_{tb}V^*_{tk}C_8, \quad (42)$$

where A_8 is defined by Equation A21. In addition, according to Relations 27 and 28, for the Effective Penguin Model, we can write this coefficient as:

$$d_1 = V_{ib}V^*_{kj},$$

$$d_2 = 0,$$

$$d_3 = (1/6)(\alpha_s/4\pi)\sum_{i=u,c,t} V^*_{iq}V_{ib}f_1(x_i),$$

$$d_4 = -(1/2)(\alpha_s/4\pi)\sum_{i=u,c,t} V^*_{iq}V_{ib}f_1(x_i),$$

$$d_5 = d_3,$$

$$d_6 = d_4,$$

$$d_8 = -(m_b/2)(\alpha_s/4\pi)\sum_{i=u,c,t} V^*_{iq}V_{ib}f_2(x_i). \quad (43)$$

Now, we want to consider only terms of d_8 and interference with d_8 and note the first used (Equation 42) for coefficients. Averaging over b spin, the square of amplitude of terms of d_8 and interference with d_8 are given by (see Appendix B):

$$|M_{d8}|^2 = (32/3)|d_8|^2(1/2E_iE_kE_j)(1/q^2) \times [p_b.p_ip_i.p_k + p_b.p_jp_k.p_j],$$

$$|M_{\rm int8}|^2 = 4\sqrt{2}\sqrt{32/3}(1/E_iE_kE_j) \times [{\rm Re}(d^*_8(d_1+d_4))p_i.p_k + {\rm Re}(d^*_8d_6)p_j.p_k]. \quad (44)$$

The total decay rate of the effective Hamiltonian magnetic dipole is given by:

$$\Gamma^{\rm TOTAL}_{d8+{\rm int}8} = (\Gamma_{0b}/m_b)\{4[I^1_{d8}+I^2_{d8}] - 16\sqrt{2}\sqrt{32/3}\,[I^1_{\rm int8}+I^2_{\rm int8}]\}. \quad (45)$$

The phase spaces are:

$$I^1_{d8} = \int_0^1 dx \int_0^1 dy\ \xi_8 h_{xy} 6xy[1-h_{abc}],$$

$$I_{d8}^2=\int_0^1 dx\int_0^1 dy\ \xi_8 h_{yx}6xy[1-h_{bca}],$$

$$I_{\text{int}8}^1=\int_0^1 dx\int_0^1 dy\ \xi_{\text{int}}^1 6xy.[1-h_{abc}],$$

$$I_{\text{int}8}^2=\int_0^1 dx\int_0^1 dy\ \xi_{\text{int}}^2 6xy.[1-h_{bca}]. \qquad (46)$$

Here, Γ_{0b}, ξ_8, ξ_{int}^1, ξ_{int}^2, h_{abc}, h_{bca}, h_{xy} and h_{yx} are defined in Appendix B. Putting $d_8=0$, we can obtain the decay rates of the Effective Hamiltonian of $Q_1,\cdots,Q_6$ and putting $Q_2,\cdots,Q_6=0$, we can obtain the Tree-Level decay rates.

Effective Penguin Model

Using Equation 43 for coefficients, we can obtain the decay rates of the Effective Penguin Model:

$$\Gamma_{d8+\text{int}8}^{T+P}=(\Gamma_{0b}/m_b)\{4[I_{d8}^1+I_{d8}^2] - 16\sqrt{2}\sqrt{32/3}\ [I_{\text{int}8}^1+I_{\text{int}8}^2]\}. \qquad (47)$$

NUMERICAL RESULTS

As an example of the use of the above formalism, we use the standard Particle Data Group [45] parameterization of the CKM matrix with the central values, $\theta_{12}=0.221$, $\theta_{13}=0.0035$, $\theta_{23}=0.041$ and choose the CKM phase δ_{13} to be $\pi/2$. Following Ali and Greub [41], we treat internal quark masses in tree-level loops with the values (GeV) $m_b=4.88$, $m_s=0.2$, $m_d=0.01$, $m_u=0.005$ and $m_c=1.5$. According to Table 1 [34], that is, effective Wilson coefficients C_i^{eff} at renormalization scale, $\mu=2.5$ GeV, for the various $b\to q(\bar{b}\to\bar{q})$ transitions, we can obtain the various decay rates.

Putting the constants of Equations 26 and 42 at Equations 45, and Equation 43 at Equation 47, we obtained the decay rates for Effective Hamiltonian and Effective Penguin Models, respectively. The total decay rates of Tree-Level (TL), Effective Hamiltonian $Q_1,\cdots,Q_6$ (EH), Effective Hamiltonian plus Magnetic Dipole $Q_1,\cdots,Q_6,Q_8$ (EHMD) and Effective Penguin Model (EPM) for particle and antiparticle are given by:

$$\Gamma_{(Q_1)\text{P}}^{\text{TL}}(b\to\text{anything})=3.103\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1)\text{AP}}^{\text{TL}}(b\to\text{anything})=3.103\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6)\text{P}}^{\text{EH}}(b\to\text{anything})=3.153\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6)\text{AP}}^{\text{EH}}(b\to\text{anything})=3.152\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6,Q_8)\text{P}}^{\text{EHMD}}(b\to\text{anything})=3.267\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6,Q_8)\text{AP}}^{\text{EHMD}}(b\to\text{anything})=3.268\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6,Q_8)\text{P}}^{\text{EPM}}(b\to\text{anything})=3.129\times10^{-13}\ \text{GeV},$$

$$\Gamma_{(Q_1,\cdots,Q_6,Q_8)\text{AP}}^{\text{EPM}}(b\to\text{anything})=3.131\times10^{-13}\ \text{GeV}.$$

The decay rates and Branching Ratios of Tree-Level (TL) (see Figure 1), Effective Hamiltonian $Q_1,\cdots,Q_6$ (EH) (see Figure 2), Effective Hamiltonian plus Magnetic Dipole $Q_1,\cdots,Q_6,Q_8$ (EHMD) and Effective Penguin Model (EPM) (see Figure 3) ($\alpha_s=0.2$) for both particles and antiparticles $b\to ud\bar{u}, us\bar{u}, cd\bar{c}, cs\bar{c}$ and $b\to cd\bar{u}$ are shown in Tables 2 and 3, respectively. We see that the decay rate for antiparticle $\bar{b}\to\bar{u}\bar{d}u$ is greater than the decay rate particle, $b\to ud\bar{u}$, and the decay rate antiparticle, $\bar{b}\to\bar{c}\bar{d}c$, is less than decay rate particle $b\to cd\bar{c}$, and so on. We consider that modes $b\to c\bar{u}d$ and $b\to c\bar{c}s$ are dominant. Also, we see that the magnetic term is small and we can negligible this term in the total decay rate.

CONCLUSIONS

We showed the decay rates of the b quark at the tree-level, the penguin and tree plus penguin and for

Table 1. Effective Wilson coefficients C_i^{eff} at the renormalization scale $\mu=2.5$ GeV for the various $b\to q(\bar{b}\to\bar{q})$ transitions.

	$b\to d$	$\bar{b}\to\bar{d}$	$b\to s$	$\bar{b}\to\bar{s}$
C_1^{eff}	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i	1.1679+0.0000i
C_2^{eff}	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i	-0.3525+0.0000i
C_3^{eff}	0.0217+0.0018i	0.0234+0.0047i	0.0232+0.0030i	0.0231+0.0029i
C_4^{eff}	-0.0498-0.0054i	-0.0543-0.0142i	-0.0535-0.0091i	-0.0531-0.0086i
C_5^{eff}	0.0156+0.0018i	0.0173+0.0047i	0.0171+0.0030i	0.0170+0.0029i
C_6^{eff}	-0.0625-0.0054i	-0.0678-0.0142i	-0.0670-0.0091i	-0.0667-0.0086i

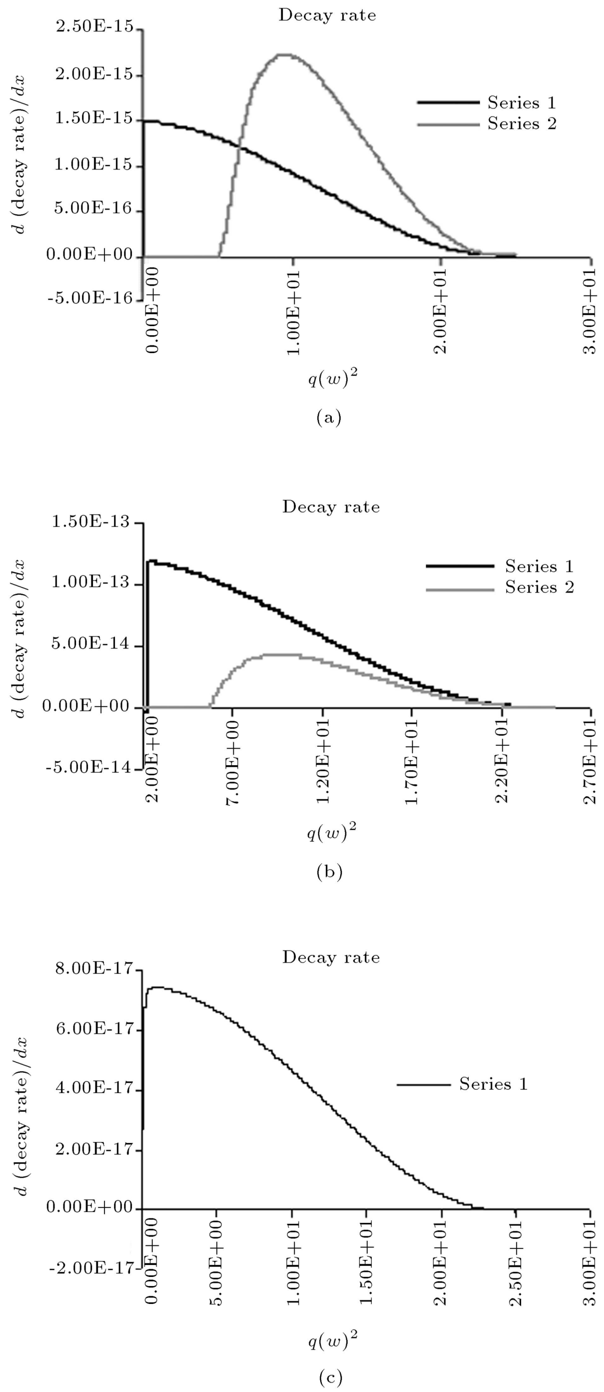

Figure 1. Differential decay rates of Tree-Level (TL) for $b \to ud\overline{u}$ (a-series 1), $b \to us\overline{u}$ (c-series 1), $b \to cd\overline{c}$ (a-series 2), $b \to cs\overline{c}$ (b-series 2) and $b \to cd\overline{u}$ (b-series 1).

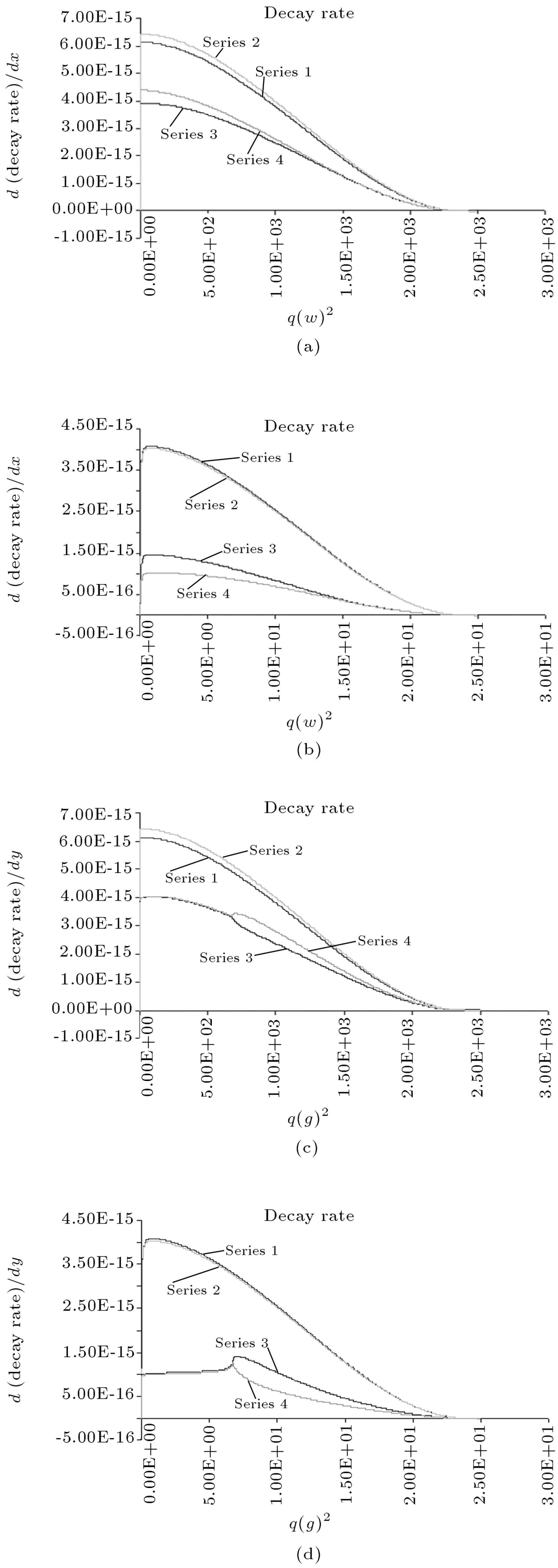

Figure 2. Differential decay rates of Effective Penguin Model (EPM) for $b \to ud\overline{u}$ (series 1), $\overline{b} \to \overline{u}\overline{d}u$ (series 2), $b \to us\overline{u}$ (series 3) and $\overline{b} \to \overline{u}\overline{s}u$ (series 4).

the first time, the Effective Penguin Model and the Effective Hamiltonian model, including the magnetic dipole terms. According to Table 2, the dominant mode in b quark in the hadronic decay is, $b \to cd\overline{u}$, because the decay rates of $b \to c$ channel are much bigger than $b \to u$, since $V_{cb} >> V_{ub}$. In addition, the dominant mode in the pure penguin decays is $b \to s$.

The magnetic dipole terms are small for b quark

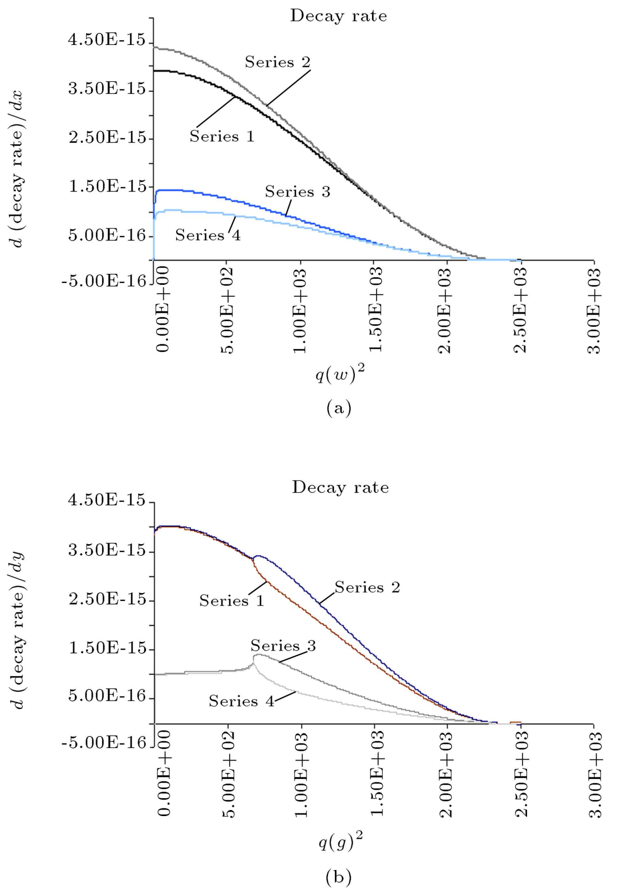

Figure 3. Differential decay rates of Effective Hamiltonian (EH) for $b \to ud\bar{u}$ (a, c series 1), $\bar{b} \to \bar{u}\bar{d}u$ (a, c series 2), $b \to us\bar{u}$ (b, d series 1), $\bar{b} \to \bar{u}\bar{s}u$ (b, d series 2), and for tree plus penguin for $b \to ud\bar{u}$ (a, c series 3), $\bar{b} \to \bar{u}\bar{d}u$ (a, c series 4), $b \to us\bar{u}$ (b, d series 3) and $\bar{b} \to \bar{u}\bar{s}u$ (b, d series 4).

decay rates (the magnetic dipole contributions are small) and the decay rate of the tree-level, Effective Hamiltonian, Effective Hamiltonian Magnetic Dipole and Effective Penguin Model are also not very different (see Table 1).

The decay rates of b quark and $\bar{b}$ antiquark at the tree-level are exactly the same, but in the Effective Hamiltonian, Effective Hamiltonian Magnetic Dipole and Effective Penguin Model, they are different. For example, $\Gamma_{b\to ud\bar{u}} < \Gamma_{\bar{b}\to\bar{u}\bar{d}u}$, $\Gamma_{b\to cd\bar{u}} > \Gamma_{\bar{b}\to\bar{c}\bar{d}u}$ and $\Gamma_{b\to cs\bar{c}} \approx \Gamma_{\bar{b}\to\bar{c}\bar{s}c}$, because the total decay rates of b quark and $\bar{b}$ antiquark must be equal, $\Gamma_b^{\text{total}} = \Gamma_{\bar{b}}^{\text{total}}$.

Also, the decay rates and branching ratios are very similar in all the models, but the Effective Hamiltonian Magnetic Dipole total decay rate is about 10% larger than the simple tree or Effective Hamiltonian. On the other hand, including the penguin induces matter-antimatter asymmetries. These are largest in the rate decays $b \to ud\bar{u}$, the decay rate of which is about 7% smaller than the decay rate $\bar{b} \to \bar{u}\bar{d}u$. Also, the rate $b \to su\bar{u}$ is larger than the rate $\bar{b} \to \bar{s}\bar{u}u$.

Table 2. Decay rates of Tree-Level (TL), Effective Hamiltonian $Q_1, \cdots, Q_6$ (EH), Effective Hamiltonian plus Magnetic Dipole $Q_1, \cdots, Q_6, Q_8$ (EHMD) and the Effective Penguin Model (EPM) ($\alpha_s = 0.2$) for both particles and antiparticles ($\times 10^{-15}$ GeV).

Process	TL	EH	EHMD	EPM
$b \to ud\bar{u}$	2.216	2.499	2.594	2.364
$b \to ud\bar{c}$	1.130	1.130	1.130	1.130
$b \to us\bar{u}$	1.057	2.162	13.15	1.781
$b \to us\bar{c}$	2.025	2.025	2.025	2.025
$\bar{b} \to \bar{u}\bar{d}u$	2.216	2.391	2.485	2.286
$\bar{b} \to \bar{u}\bar{d}c$	1.130	1.130	1.130	1.130
$\bar{b} \to \bar{u}\bar{s}u$	1.057	2.131	1.312	1.706
$\bar{b} \to \bar{u}\bar{s}c$	2.025	2.025	2.025	2.025
$b \to cd\bar{c}$	7.324	8.123	8.115	7.891
$b \to cd\bar{u}$	138.8	138.8	138.8	138.8
$b \to cs\bar{c}$	149.9	151.8	152.1	150.2
$b \to cs\bar{u}$	8.754	8.754	8.754	8.754
$\bar{b} \to \bar{c}\bar{d}c$	7.324	8.094	8.085	7.729
$\bar{b} \to \bar{c}\bar{d}u$	138.8	138.8	138.8	138.8
$\bar{b} \to \bar{c}\bar{s}c$	149.9	151.9	152.3	150.6
$\bar{b} \to \bar{c}\bar{s}u$	8.754	8.754	8.754	8.754

Table 3. Branching ratios of Tree-Level (TL), Effective Hamiltonian $Q_1, \cdots, Q_6$ (EH), Effective Hamiltonian plus Magnetic Dipole $Q_1, \cdots, Q_6, Q_8$ (EHMD) and the Effective Penguin Model (EPM) ($\alpha_s = 0.2$) for both particles and antiparticles (BR $\times 10^{-2}$).

Process	TL	EH	EHMD	EPM
$b \to ud\bar{u}$	0.714	0.792	0.794	0.755
$b \to ud\bar{c}$	0.360	0.358	0.346	0.361
$b \to us\bar{u}$	0.341	0.368	0.402	0.569
$b \to us\bar{c}$	0.652	0.642	0.619	0.647
$\bar{b} \to \bar{u}\bar{d}u$	0.714	0.758	0.761	0.730
$\bar{b} \to \bar{u}\bar{d}c$	0.360	0.358	0.498	0.361
$\bar{b} \to \bar{u}\bar{s}u$	0.341	0.359	0.401	0.544
$\bar{b} \to \bar{u}\bar{s}c$	0.652	0.642	0.619	0.647
$b \to cd\bar{c}$	2.360	2.576	2.484	2.521
$b \to cd\bar{u}$	44.72	44.01	42.53	44.34
$b \to cs\bar{c}$	48.33	48.15	46.53	48.04
$b \to cs\bar{u}$	2.821	2.776	2.679	2.797
$\bar{b} \to \bar{c}\bar{d}c$	2.360	2.568	2.473	2.468
$\bar{b} \to \bar{c}\bar{d}u$	44.72	44.04	42.51	44.34
$\bar{b} \to \bar{c}\bar{s}c$	48.33	48.23	46.62	48.13
$\bar{b} \to \bar{c}\bar{s}u$	2.821	2.778	2.678	2.796

REFERENCES

1. Datta, A. and London, D. "Obtaining CKM phase information from B penguin decays", *JHEP 0404*, p. 072 (2004).
2. Silvestrini, L. "Searching for new physics in b to s hadronic penguin decays", arXiv:0705.16 24.
3. Eigen, G. "Radiative penguin decays from BABAR", eConf C0304052, WG204 (2003).
4. Pich, A. "The standard model of electroweak interactions", arXiv:0705.4264.
5. Fleischer, R. and Recksiegel, S. "General lower bounds for $b- > d$ penguin processes", *Phys. Rev.*, **D71**, p. 051501 (2005).
6. Buras, A.J., Fleischer, R., Recksiegel, S. and Schwab, F. "Electroweak penguin hunting through $B \rightarrow \pi\pi$, π K and Rare K and B decays", hep-ph/0512059.
7. Kim, C.S., Oh, S. and Yu, C. "Strong phase shifts and color-suppressed tree amplitudes in $B \rightarrow DK^{(*)}$ and $B- > D\pi$, $D\rho$ decays", *Phys. Lett.*, **B621**, pp. 259-268 (2005).
8. Buras, A.J., Gambino, P. and Haisch, U. "Electroweak penguin contributions to non-leptonic $\delta F = 1$ decays at NNLO", *Nucl. Phys.*, **B570**, pp. 117-154 (2000).
9. Koppenburg, P. "Electroweak penguin B decays at belle", *Eur. Phys. J.*, **C33**, pp. S274-S276 (2004).
10. Besmer, T. and Steffen, A. "R-parity violation and the decay $b \rightarrow s\gamma$", *Phys. Rev.*, **D63**, p. 055007 (2001).
11. Yang, K.C. "Annihilation in factorization-suppressed B decays involving a PP, PV meson and search for new-physics signals", *Phys. Rev.*, **D72**, p. 034009 (2005); *Erratum-ibid*, **D72**, p. 059901 (2005).
12. Villa, S. "Search for the decay $B^0 \rightarrow \gamma\gamma$", *Phys. Rev.*, **D73**, p. 051107 (2006).
13. Aubert, B. "Search for the rare decay $B \rightarrow \pi^+\pi^-$", *Phys. Rev. Lett.*, **99**, p. 051801 (2007).
14. Fleischer, R. and Recksiegel, S. "$b \rightarrow d$ penguins: CP violation, general lower bounds on the branching ratios and standard model tests", hep-ph/0511325.
15. Beneke, M., Feldmann, T. and Seidel, D. "Exclusive radiative and electroweak $b \rightarrow d$ and $b \rightarrow s$ penguin decays at NLO", *Eur. Phys. J.*, **C41**, pp. 173-188 (2005).
16. Fleischer, R. and Recksiegel, S. "General lower bounds for $b \rightarrow d$ penguin processes", *Phys. Rev.*, **D71**, p. 051501 (2005).
17. Pham, T.N. "Charming penguin in nonleptonic B decays", *AIP Conf. Proc.*, **602**, pp. 206-211 (2001).
18. Fleischer, R. "Exploring CP violation and penguin effects through $B_d^0 \rightarrow D^+D^-$ and $B_s^0 \rightarrow D_s^+D_s^-$", arXiv:0705.4421.
19. Athanas, M. "First observation of the cabibbo suppressed decay $B^+ \rightarrow \overline{D}^0 K^+$", *Phys. Rev. Lett.*, **80**, pp. 5493-5497 (1998).
20. Deshpande, A., He, X.G. and Hou, W.S. "Electroweak penguins, final state interaction phases and CP violatioin in $B \rightarrow K\pi$ decays", *Phys. Rev. Lett.*, **82**, pp. 2240-2243 (1999).
21. Mishima, S. and Yoshikawa, T. "Large electroweak penguin contribution in $B \rightarrow k\pi, \pi\pi$ decay modes", *Phys. Rev.*, **D70**, p. 094024 (2004).
22. Fleischer, R. "Flavour physics and CP violation", hep-ph/0608010.
23. Du, D.S. and Yang, M.Z. "Electroweak penguin effects in some B_s^0 two-body decays", *Phys. Lett.*, **B358**, pp. 123-128 (1995).
24. Lesniak, L., Furman, A., Kaminski, R. and Loiseau, B. "Final state interactions and long-distance effects in B to pion pion K and B to K anti KK decays", *PoS HEP2005*, p. 249 (2005).
25. Furman, A., Kaminski, R., Lesniak, L. and Loiseau, B. "Long-distance effects and final state interactions in B to pion pion K and $B \rightarrow K$ anti-KK decays", *Phys. Lett.*, **B622**, pp. 207-217 (2005).
26. Datta, A. and London, D. "Measuring new-physics parameters in B penguin decays", *Phys. Lett.*, **B595**, pp. 453-460 (2004).
27. Chua, C.K. "Extracting the unitarity angle gamma in Bs to D0 h0, D0bar h0 Decays", *Phys. Lett.*, **B633**, pp. 70-74 (2006).
28. Xiao, Z., Li, C.S. and Chao, K.T. "Gluon penguin enhancements to inclusive charmless decays of b quark in the 2HDM with flavor changing couplings", *Phys. Lett.*, **B473**, pp. 148-156 (2000).
29. Beneke, M., Rohrer, J. and Yang, D. "Enhanced electroweak penguin amplitude in $B \rightarrow VV$ decays", *Phys. Rev. Lett.*, **96**, p. 1418 (2006).
30. Abel, S.A., Cottingham, W.N. and Wittingham, I.B. "Gluon and gluino penguins and the charmless decays of the b quark", *Phys. Rev.*, **D58**, p. 0730 (1998).
31. Xiao, Z., Li, W., Guo, L. and Lu, G. "Charmless decays $B- >$ PP, PV, and effects of new strong and electroweak penguins in topcolor-assisted technicolor model", *Eur. Phys. J.*, **C18**, pp. 681-709 (2001).
32. Lu, C.D., Shen, Y.L. and Wang, W. "Role of electromagnetic dipole operator in the electroweak penguin dominated vector meson decays of B meson", *Chin. Phys. Lett.*, **23**, pp. 2684-2687 (2006).
33. Beneke, M. and Jager, S. "Spectator scattering at NLO in non-leptonic B decays: Leading penguin amplitudes", *Nucl. Phys.*, **B768**, pp. 51-84 (2007).
34. Cottingham, W.N., Mehrban, H. and Wittingham, I.B. "Hadronic B decays: Supersymmetric enhancement and a simple spectator model", *Phys. Rev.*, **D60**, p. 114029 (1999).
35. Li, H.N. "QCD aspects of exclusive B meson decays", *Czech. J. Phys.*, **53**, pp. 657-666 (2003).
36. Xiao, Z., Li, W., Guo, L. and Lu, G. "Charmless decays $B- >$ PP, PV, and effects of new strong and electroweak penguins in topcolor-assisted technicolor model", *Eur. Phys. J.*, **C18**, pp. 681-709 (2001).

37. Actis, S. and Passarino, G. "Two-loop renormalization in the standard model Part III: Renormalization equations and their solutions", hep-ph/0612124.
38. Mendy, J.B. and Govaerts, J. "A general four-fermion effective lagrangian for Dirac and Majorana neutrino-charged matter interactions", hep-ph/0207361.
39. Deshpande, N.G., He, X.G. and Pakvasa, S. "Gluon dipole penguin contributions to ε'/ε and CP violation in Hyperon decays in the Standard Model", *Phys. Lett.*, **B326**, pp. 307-311 (1994).
40. Buras, A.J. "Theoretical review of B-physics", *Nucl. Instrum. Meth.*, **A368**, pp. 1-20 (1995).
41. Ali, A. and Greub, C. "An analysis of two-body non-leptonic B decays involving light mesons in the standard model", *Phys. Rev.*, **D57**, pp. 2996-3016 (1998).
42. Burdman, G. and Shipsey, I. "$D^0-\overline{D^0}$ mixing and rare charm decays", *Ann. Rev. Nucl. Part. Sci.*, **53**, pp. 431-499 (2003).
43. Datta, A. and London, D. "Obtaining CKM phase information from B penguin decays", *JHEP 0404*, **072** (2004).
44. Mehrban, H. "Spectator model in D Meson Decays", *Scientia Iranica*, **16**(2), pp. 140-148 (2009).
45. Particle Data Group, *European Physics J.*, C3, 1 (2006).

APPENDIX A

Penguin Amplitude of Magnetic Dipole

According to Equation 4, the penguin amplitude is given by:

$$M^{\text{dip}} = \frac{g_s^2}{4\pi^2}[\overline{u}_k(p_k)T^a(i\sigma_{\mu\nu}q^\nu/q^2)F_2^R(0)P_R u_b(p_b)] [\overline{u}_i(p_i)\gamma^\mu T^a v_{\bar{j}}(p_{\bar{j}})], \tag{A1}$$

where:

$$\sigma^{\mu\nu} = (i/2)[\gamma^\mu,\gamma^\nu] = (i/2)(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \tag{A2}$$

and:

$$\gamma^\mu\gamma^\nu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0\end{pmatrix}\begin{pmatrix} 0 & \sigma^\nu \\ \tilde{\sigma}^\nu & 0\end{pmatrix} = \begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu\end{pmatrix}. \tag{A3}$$

So:

$$\sigma^{\mu\nu} = \frac{i}{2}\begin{pmatrix} \sigma^\mu\tilde{\sigma}^\nu - \sigma^\nu\tilde{\sigma}^\mu & 0 \\ 0 & \tilde{\sigma}^\mu\sigma^\nu - \tilde{\sigma}^\nu\sigma^\mu\end{pmatrix}. \tag{A4}$$

The wave functions of b and q_k are given by:

$$\overline{u}_k\sigma_{\mu\nu}[(1+\gamma_5)/2]u_b = \frac{i}{2}\begin{pmatrix}\psi_{kL}\\ \psi_{kR}\end{pmatrix}^\dagger \begin{pmatrix} \tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu & 0 \\ 0 & \sigma_\mu\tilde{\sigma}_\nu - \sigma_\nu\tilde{\sigma}_\mu\end{pmatrix}\begin{pmatrix}0\\ \psi_{bR}\end{pmatrix} = (i/2)\psi_{kL}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)\psi_{bR}. \tag{A5}$$

Putting in the penguin amplitude, we have:

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a) [\overline{u}_{kL}\frac{q^\nu}{q^2}(\tilde{\sigma}_\mu\sigma_\nu - \tilde{\sigma}_\nu\sigma_\mu)u_{bR}][\overline{u}_i(\tilde{\sigma}^\mu + \sigma^\mu)v_{\bar{j}}]. \tag{A6}$$

Putting $q^\nu = (p_b - p_k)^\nu$ in the above equation:

$$M^{\text{dip}} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[(\overline{u}_{kL}\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)u_{bR}) - (\overline{u}_{kL}(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu u_{bR})] \times [\overline{u}_i\tilde{\sigma}^\mu v_{\bar{j}} + \overline{u}_i\sigma^\mu v_{\bar{j}}], \tag{A7}$$

or:

$$M^{dip} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[\langle k_L|\tilde{\sigma}_\mu(\sigma_\nu p_b^\nu - \sigma_\nu p_k^\nu)|b_R\rangle - \langle k_L|(\tilde{\sigma}_\nu p_b^\nu - \tilde{\sigma}_\nu p_k^\nu)\sigma_\mu|b_R\rangle] \times [\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle], \tag{A8}$$

is obtained. We know that:

$$\tilde{\sigma}_\mu p_b^\mu|b_L\rangle = m_b|b_R\rangle, \qquad \tilde{\sigma}_\mu p_k^\mu|k_L\rangle = m_k|k_R\rangle,$$
$$\sigma_\nu p_b^\nu|b_R\rangle = m_b|b_L\rangle, \qquad \sigma_\nu p_k^\nu|k_R\rangle = m_k|k_L\rangle,$$
$$\tilde{\sigma}_\mu(\sigma_\nu p_k^\nu) = p_{k\mu}, \qquad (\tilde{\sigma}_\mu p_b^\nu)\sigma_\mu = p_{b\mu}, \tag{A9}$$

and:

$$(p_b + p_k)_\mu = (2p_b - p_i - p_j)_\mu. \tag{A10}$$

Also, according to conservation of the current:

$$(p_i + p_j)_\mu[\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle] = m_i[\langle i_R|j_L\rangle + \langle i_L|j_R\rangle] - m_j[\langle i_L|j_R\rangle + \langle i_R|j_L\rangle] = 0. \tag{A11}$$

Since in the penguin decays, $m_i = m_j$ and $|j\rangle$ is the antiparticle, so:

$$\tilde{\sigma}_\mu p_{j\mu}|j_L\rangle = -m_j|j_R\rangle. \tag{A12}$$

Consequently, the magnetic dipole term of penguin amplitude becomes:

$$M^{\text{dip}} = -\frac{g_s^2}{8\pi^2}F_2^R(0)(T^aT^a)\frac{1}{q^2}[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle + m_k\langle k_R|\sigma_\mu|b_R\rangle - (p_b + p_k)_\mu\langle k_L|b_R\rangle] \times [\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle i_R|\sigma^\mu|j_R\rangle]. \tag{A13}$$

We neglected the term $m_k\langle k_R|\sigma_\mu|b_R\rangle$ because $m_k << m_b$, so:

$$M^{\rm dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - (p_b+p_k)_\mu\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - (p_b+p_k)_\mu\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A14)}$$

Using Equation A11 for the second part of the above equation, the following is obtained:

$$M^{\rm dip} = (4/3)d_8(1/q^2)[m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + m_b\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - 2p_{b_\mu}\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2p_{b_\mu}\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A15)}$$

Here, b meson is at rest ($p_{b\mu} = (m_b, \vec{0})$ and:

$$(T^aT^a) = 4/3,$$

$$d_8 = -\frac{g_s^2}{8\pi^2}F_2^R(0) = -\frac{g_s^2}{8\pi^2}m_b\frac{g_2^2}{8M_W^2}\sum_i V_{ik}^*V_{ib}f_2(x_i) = -(2\sqrt{2}G_F)(m_b/2)(\alpha_s/4\pi)\sum_i V_{ik}^*V_{ib}f_2(x_i). \quad \text{(A16)}$$

So, the magnetic dipole of penguin amplitude is given by:

$$M^{\rm dip} = (4/3)d_8(m_b/q^2)[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle - 2\langle k_L|b_R\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle - 2\langle k_L|b_R\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A17)}$$

The b quark is at rest and has spin projection -1/2 along angle θ_b. Thus, the spin projection of the b quark of +1/2 is along $\theta_b - \pi(\theta_b \to \theta_b - \pi)$:

$$b \text{ spin } (-1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2})\begin{pmatrix}-\sin(\theta_b/2)\\ \cos(\theta_b/2)\end{pmatrix},$$

$$b \text{ spin } (+1/2) \text{ and angle } \theta_b \propto (1/\sqrt{2})\begin{pmatrix}\cos(\theta_b/2)\\ \sin(\theta_b/2)\end{pmatrix}. \quad \text{(A18)}$$

Putting the factor of $(1/\sqrt{2})$ in $M^{\rm dip}$ and negligible terms $\langle k_L|b_R\rangle$, the amplitude of magnetic dipole becomes:

$$M^{\rm dip} = A_8d_8[\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle + \langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_R|\sigma^\mu|j_R\rangle]. \quad \text{(A19)}$$

Here:

$$A_8 = (1/\sqrt{2})(4/3)(m_b/q^2). \quad \text{(A20)}$$

Terms $(\tilde{\sigma}_\mu)(\tilde{\sigma}^\mu)_{\rm LL}$ and $(\tilde{\sigma}_\mu)(\sigma^\mu)_{\rm LR}$ for spin +1/2 and -1/2 are obtained by the matrix elements of $L-L$ handed and $L-R$ handed for the b quark:

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\langle -_k|_L\tilde{\sigma}_\mu|-_j\rangle_L = \sin((\theta_k-\theta_j-\theta_i)/2) + \sin((\theta_k+\theta_j-\theta_i)/2),$$

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\langle -_k|_L\tilde{\sigma}_\mu|-_j\rangle_L = \cos((\theta_k-\theta_j-\theta_i)/2) - \cos((\theta_k+\theta_j-\theta_i)/2). \quad \text{(A21)}$$

When dealing with penguin amplitudes, we will also need the matrix elements:

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(1/2)}\rangle_L\langle -_k|_R\sigma_\mu|-_j\rangle_R = \sin((\theta_i-\theta_k-\theta_j)/2) - \sin((\theta_i+\theta_k-\theta_j)/2),$$

$$\langle -_i|_L\tilde{\sigma}^\mu|b_{(-1/2)}\rangle_L\langle -_k|_R\sigma_\mu|-_j\rangle_R = \cos((\theta_i-\theta_k-\theta_j)/2) + \cos((\theta_i+\theta_k-\theta_j)/2). \quad \text{(A22)}$$

The first term of Equation A19 for b spin project -1/2, according to the Fierz transformation,

$$\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle = -\langle k_L|\tilde{\sigma}_\mu|b_L\rangle\langle i_L|\tilde{\sigma}^\mu|j_L\rangle, \quad \text{(A23)}$$

is given by:

$$M^{\rm dip}_{1(-1/2)} \equiv -A_8d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle = A_8d_8[-\cos((\theta_b-\theta_i-\theta_k+\theta_j)/2) - \cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)]. \quad \text{(A24)}$$

And the first term for b spin project +1/2 is given by:

$$M^{\rm dip}_{1(+1/2)} \equiv -A_8d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_L|\tilde{\sigma}_\mu|j_L\rangle, = A_8d_8[-\sin((\theta_b+\theta_i-\theta_k-\theta_j)/2) + \sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)]. \quad \text{(A25)}$$

Also, the second term of Equation A19 for b spin project -1/2 is given by:

$$M^{\rm dip}_{2(-1/2)} \equiv -A_8d_8\langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_R|\sigma_\mu|j_R\rangle = A_8d_8[-\cos((\theta_b+\theta_i-\theta_k-\theta_j)/2) - \cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)]. \quad \text{(A26)}$$

In addition, the second term for b spin project +1/2 is given by:

$$M^{\rm dip}_{2(+1/2)} \equiv -A_8 d_8 \langle k_L|\tilde{\sigma}^\mu|b_L\rangle\langle i_R|\sigma_\mu|j_R\rangle$$
$$= A_8 d_8[-\sin((\theta_b-\theta_i-\theta_k+\theta_j)/2)$$
$$+\sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)]. \quad \text{(A27)}$$

So, the penguin amplitudes of magnetic dipole for b spins project -1/2 and 1/2 are given by:

$$M^{\rm dip}_{(+1/2)} = A_8 d_8\{[-\sin((\theta_b+\theta_i-\theta_k-\theta_j)/2)$$
$$+\sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)]$$
$$+[-\sin((\theta_b-\theta_i-\theta_k+\theta_j)/2)$$
$$+\sin((\theta_b-\theta_i+\theta_k-\theta_j)/2)]\}, \quad \text{(A28)}$$

$$M^{\rm dip}_{(-1/2)} = A_8 d_8\{[-\cos((\theta_b-\theta_i-\theta_k+\theta_j)/2)$$
$$-\cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)]$$
$$+[-\cos((\theta_b+\theta_i-\theta_k-\theta_j)/2)$$
$$-\cos((\theta_b-\theta_i+\theta_k-\theta_j)/2)]\}. \quad \text{(A29)}$$

APPENDIX B

Decay Rate of the Effective Hamiltonian Magnetic Dipole

According to Equations A21 and A22 for $L-L$ and $L-R$ handed operators for the d_8 term, squaring these terms for b spin project -1/2 and +1/2 is given by:

$$|M_{1L-L(-1/2)}|^2+|M_{2L-L(+1/2)}|^2$$
$$=4|A_8d_8|^2\cos^2((\theta_k-\theta_j)/2)$$
$$=2|A_8d_8|^2[2-p_k.p_j/E_kE_j]$$
$$=2|A_8d_8|^2(1/E_kE_j)(1/m_b^2)[2p_b.p_kp_b.p_j-p_b^2p_k.p_j]$$
$$=4|A_8d_8|^2(1/E_kE_j)(1/m_b^2)[(p_i+p_j).p_k(p_i+p_k).p_j$$
$$-p_k.p_j(p_i.p_k+p_i.p_j+p_k.p_j)]$$
$$=2|A_8d_8|^2(1/E_kE_j)(1/m_b^2)[q^2(p_i.p_k)], \quad \text{(B1)}$$

and:

$$|M_{3L-R(-1/2)}|^2+|M_{4L-R(+1/2)}|^2$$
$$=4|A_8d_8|^2\cos^2((\theta_k-\theta_i)/2)$$
$$=2|A_8d_8|^2[2-p_k.p_i/E_kE_i]$$
$$=2|A_8d_8|^2(1/E_kE_i)(1/m_b^2)[q^2(p_k.p_j)]. \quad \text{(B2)}$$

Averaging over b spins, the square of amplitude of term of d_8 is given by:

$$|M_{d8}|^2=\frac{1}{2}[|M_{1L-L(-1/2)}|^2+|M_{2L-L(+1/2)}|^2$$
$$+|M_{3L-R(-1/2)}|^2+|M_{4L-R(+1/2)}|^2]$$
$$=|A_8d_8|^2(1/E_iE_kE_j)(q^2/m_b^2)[p_b.p_ip_i.p_k$$
$$+p_b.p_jp_k.p_j]. \quad \text{(B3)}$$

Also for interference term, we have:

$$|M'_{1L-L(-1/2)}|^2+|M'_{2L-L(+1/2)}|^2$$
$$=d_{\rm int8}[\sin((\theta_b-\theta_j)/2)\sin((\theta_i-\theta_k)/2)$$
$$\times\cos((\theta_b-\theta_i)/2)\cos((\theta_k-\theta_j)/2)$$
$$-\cos((\theta_b-\theta_j)/2)\sin((\theta_i-\theta_k)/2)$$
$$\times\sin((\theta_b-\theta_i)/2)\cos((\theta_k-\theta_j)/2)]$$
$$=d_{\rm int8}[\sin((\theta_i-\theta_k)/2)\cos((\theta_k-\theta_j)/2)\sin((\theta_i-\theta_j)/2)]$$
$$=d_{\rm int8}(1/4)[1-\cos(\theta_i-\theta_j)+\cos(\theta_j-\theta_k)$$
$$+\cos(\theta_i-\theta_k)]$$
$$=d_{\rm int8}(1/4)[(p_i.p_j/E_iE_j)+(p_i.p_k/E_iE_k)$$
$$+(p_k.p_j/E_kE_j)]$$
$$=d_{\rm int8}(1/4E_iE_kE_j)(1/m_b)[p_b.p_kp_i.p_j$$
$$+p_b.p_jp_k.(p_i+p_j)]$$
$$=d_{\rm int8}(q^2/8E_iE_kE_j)(1/m_b)[p_b.p_k+p_k.(p_i-p_j)]$$
$$=d_{\rm int8}(q^2/4E_iE_kE_j)(1/m_b)p_i.p_k, \quad \text{(B4)}$$

where:

$$d_{\rm int8}=8{\rm Real}[(A_8d_8)^*(d_1+d_2+d_3+d_4)], \quad \text{(B5)}$$

and:

$$|M'_{3L-R(-1/2)}|^2+|M'_{4L-R(+1/2)}|^2$$

$$= d'_{\text{int8}}[-\sin((\theta_b-\theta_i)/2)\sin((\theta_k-\theta_j)/2)$$
$$\times\cos((\theta_b-\theta_j)/2)\cos((\theta_i-\theta_k)/2)$$
$$+\cos((\theta_b-\theta_i)/2)\sin((\theta_k-\theta_j)/2)$$
$$\times\sin((\theta_b-\theta_j)/2)\cos((\theta_i-\theta_k)/2)]$$
$$= d'_{\text{int8}}(q^2/4E_iE_kE_j)(1/m_b)p_j.p_k. \quad \text{(B6)}$$

Here:

$$d'_{\text{int8}} = 8\text{Real}[(A_8d_8)^*(d_5+d_6)]. \quad \text{(B7)}$$

Averaging over b spins, the square of amplitude of interference term is given by:

$$|M_{\text{int8}}|^2 = \frac{1}{2}[|M'_{1L-L(-1/2)}|^2 + |M'_{2L-L(+1/2)}|^2$$
$$+|M'_{3L-R(-1/2)}|^2 + |M'_{4L-R(+1/2)}|^2]$$
$$= 2(1/8E_iE_kE_j)(q^2/m_b)[d_{\text{int8}}p_i.p_k$$
$$+d'_{\text{int8}}p_j.p_k]. \quad \text{(B8)}$$

Adding all color factor possibilities for d_8 and the interference term, which are:

$$|(4/3)d_8|^2 \rightarrow [(4/3)^2+2(2)^2+2(2/3)^2]|d_8|^2$$
$$= (32/3)|d_8|^2,$$
$$(A_8d_8)^*(d_1+d_2+d_3+d_4) \rightarrow (A_8d_8)^*4(d_1+d_4),$$
$$(A_8d_8)^*(d_5+d_6) \rightarrow (A_8d_8)^*4d_6. \quad \text{(B9)}$$

So the factor A_8, d_{int8} and d'_{int8} according to Equations A21, B5 and B7, exchanged to:

$$A_8 = (1/\sqrt{2})(\sqrt{32/3})(m_b/q^2),$$
$$d_{\text{int8}} = 8\text{Real}[(A_8d_8)^*4(d_1+d_4)],$$
$$d'_{\text{int8}} = 8\text{Real}[(A_8d_8)^*4(d_6)]. \quad \text{(B10)}$$

Putting the color factor at the amplitude for d_8 and interference terms, thus Equations B3 and B8 are summarized:

$$|M_{d8}|^2 = (32/3)|d_8|^2(1/2E_iE_kE_j)(1/q^2)$$
$$\times[p_b.p_ip_i.p_k + p_b.p_jp_k.p_j],$$
$$|M_{\text{int8}}|^2 = 4\sqrt{2}\sqrt{32/3}(1/E_iE_kE_j)$$
$$\times[\text{Re}(d_8^*(d_1+d_4))p_i.p_k + \text{Re}(d_8^*d_6)p_j.p_k]. \quad \text{(B11)}$$

We want to calculate the decay rates for d_8 and interference terms. The first step, we calculate the decay rate of the d_8 term. We know that:

$$p_b.p_i = E_bE_i - p_bp_i\cos\theta_{ib} = m_bE_i, \qquad (p_b=(m_b,\vec{0})),$$
$$p_i.p_k = E_iE_k - p_ip_k\cos\theta_{ki}, \qquad p_b.p_j = m_bE_j,$$
$$p_k.p_j = E_kE_j - p_kp_j\cos\theta_{jk}. \quad \text{(B12)}$$

The amplitude of d_8 is given by:

$$|M_{d8}|^2 = (32/3)|d_8|^2(m_b/q^2)[(E_i/E_j)(1/2)$$
$$(1-v_iv_k\cos\theta_{ki})$$
$$+(E_j/E_i)(1/2)(1-v_kv_j\cos\theta_{ik})]. \quad \text{(B13)}$$

We separated the above equation by two parts:

$$|M^1_{d8}|^2 = \xi_8(m_b/q^2)(E_i/E_j)(1/2)(1-v_iv_k\cos\theta_{ki}),$$
$$|M^2_{d8}|^2 = \xi_8(m_b/q^2)(E_j/E_i)(1/2)(1-v_kv_j\cos\theta_{ik}). \quad \text{(B14)}$$

Here:

$$\xi_8 = (32/3)|d_8|^2, \qquad v_i = p_i/E_i,$$
$$v_k = p_k/E_k, \qquad v_j = p_j/E_j. \quad \text{(B15)}$$

After integration in the phase space and change variable x and y:

$$x = 2p_i/m_b, \qquad y = 2p_k/m_b. \quad \text{(B16)}$$

We can obtain the decay rates for the d_8 term:

$$\Gamma^1_{d8} = \Gamma_{0b}I^1_{d8}, \quad \text{(B17)}$$

where:

$$I^1_{d8} = \int_0^1 dx\int_0^1 dy\,\xi_8(m_b/q^2)(E_i/E_j)6xy.f_{ab}.[1-h_{abc}]. \quad \text{(B18)}$$

Here, Γ_{0b}, f_{ab} and h_{abc} are:

$$\Gamma_{0b} = G_F^2M_b^5/192\pi^3,$$
$$f_{ab} = 2-\sqrt{x^2+a^2}-\sqrt{y^2+b^2},$$
$$h_{abc} = \frac{(f_{ab})^2-(c^2+x^2+y^2)}{2\sqrt{x^2+a^2}\sqrt{y^2+b^2}}. \quad \text{(B19)}$$

Also we know that:

$$E_i = \sqrt{m_i^2+p_i^2} = (m_b/2)\sqrt{x^2+a^2},$$
$$E_k = \sqrt{m_k^2+p_k^2} = (m_b/2)\sqrt{y^2+b^2},$$
$$E_j = m_b - E_i - E_k$$
$$= (m_b/2)(2-\sqrt{x^2+a^2}-\sqrt{y^2+b^2}). \quad \text{(B20)}$$

And the momentum of gluon is:

$$\vec{q}_g = \vec{p}_i + \vec{p}_j = \vec{p}_b - \vec{p}_k \rightarrow q_g^2 = m_b^2 - 2m_b p_k + m_k^2$$
$$\rightarrow q_g^2 = (m_b/2)^2[4(1-y) + b^2]. \quad \text{(B21)}$$

Here:

$$a = 2m_i/m_b, \quad b = 2m_k/m_b, \quad c = 2m_j/m_b. \quad \text{(B22)}$$

The decay rate becomes:

$$\Gamma_{d8}^1 = (4/m_b)\Gamma_{0b} I_{d8}^1, \quad \text{(B23)}$$

where:

$$I_{d8}^1 = \xi_8 \int_0^1 dx \int_0^1 dy h_{xy} 6xy[1 - h_{abc}]. \quad \text{(B24)}$$

Here:

$$h_{xy} = \sqrt{x^2 + a^2}/(4(1-y) + b^2). \quad \text{(B25)}$$

In the same way, the decay rate for Equation B14 is given by:

$$\Gamma_{d8}^2 = (4/m_b)\Gamma_{0b} I_{d8}^2, \quad \text{(B26)}$$

where:

$$I_{d8}^2 = \xi_8 \int_0^1 dx \int_0^1 dy h_{yx} 6xy[1 - h_{bca}]. \quad \text{(B27)}$$

Here:

$$h_{yx} = \sqrt{y^2 + c^2}/(4(1-x) + b^2),$$
$$h_{bca} = \frac{(f_{bc})^2 - (a^2 + x^2 + y^2)}{2\sqrt{x^2 + b^2}\sqrt{y^2 + c^2}}, \quad \text{(B28)}$$

and used the forms:

$$E_k = \sqrt{m_k^2 + p_k^2} = (m_b/2)\sqrt{x^2 + b^2},$$
$$E_j = \sqrt{m_j^2 + p_j^2} = (m_b/2)\sqrt{y^2 + c^2},$$
$$E_i = m_b - E_k - E_j$$
$$= (m_b/2)(2 - \sqrt{x^2 + b^2} - \sqrt{y^2 + c^2}. \quad \text{(B29)}$$

And the momentum of gluon is:

$$q_g^2 = m_b^2 - 2m_b p_k + m_k^2 \rightarrow q_g^2 = (m_b/2)^2[4(1-x) + b^2]. \quad \text{(B30)}$$

Here:

$$x = 2p_k/m_b, \qquad y = 2p_j/m_b. \quad \text{(B31)}$$

Finally, the total decay rate of the d_8 term is given by:

$$\Gamma_{d8} = \Gamma_{d8}^1 + \Gamma_{d8}^2 = (4/m_b)\Gamma_{0b}[I_{d8}^1 + I_{d8}^2]. \quad \text{(B32)}$$

The amplitude of interference term according to Equation B11, is given by:

$$|M_{\text{int8}}|^2 = 4\sqrt{2}\sqrt{32/3}\ [\text{Re}(d_8^*(d_1 + d_4)(1/E_j)$$
$$(1 - v_i v_k \cos\theta_{ik})$$
$$+ \text{Re}(d_8^* d_6)(1/E_i)(1 - v_k v_j \cos\theta_{kj})]. \quad \text{(B33)}$$

The above equation is separated by two parts:

$$|M_{\text{int8}}^1|^2 = 8\sqrt{2}\sqrt{32/3}\ \xi_{\text{int}}^1 (1/E_j)(1/2)$$
$$(1 - v_i v_k \cos\theta_{ik}),$$
$$|M_{\text{int8}}^2|^2 = 8\sqrt{2}\sqrt{32/3}\ \xi_{\text{int}}^2 (1/E_i)(1/2)$$
$$(1 - v_k v_j \cos\theta_{kj}). \quad \text{(B34)}$$

Here:

$$\xi_{\text{int}}^1 = \text{Re}(d_8^*(d_1 + d_4)),$$
$$\xi_{\text{int}}^2 = \text{Re}(d_8^* d_6). \quad \text{(B35)}$$

We can obtain the decay rates for the interference term:

$$\Gamma_{\text{int8}}^1 = \Gamma_{ob} I_{\text{int8}}^1, \quad \text{(B36)}$$

where:

$$I_{\text{int8}}^1 = \int_0^1 dx \int_0^1 dy 8\sqrt{2}\sqrt{32/3}\ \xi_{\text{int}}^1 (1/E_j) 6xy$$
$$.f_{ab}.[1 - h_{abc}]. \quad \text{(B37)}$$

Putting E_j from Equations B29 in the above equation, the decay rate becomes:

$$\Gamma_{\text{int8}}^1 = (16\sqrt{2}\sqrt{32/3}/m_b)\Gamma_{0b} I_{\text{int8}}^1, \quad \text{(B38)}$$

where:

$$I_{\text{int8}}^1 = \xi_{\text{int}}^1 \int_0^1 dx \int_0^1 dy 6xy.[1 - h_{abc}]. \quad \text{(B39)}$$

The decay rate of the second part of Equations B34 is given by:

$$\Gamma_{\text{int8}}^2 = \Gamma_{0b} I_{\text{int8}}^2. \quad \text{(B40)}$$

Here:

$$I_{\text{int8}}^2 = \int_0^1 dx \int_0^1 dy 8\sqrt{2}\sqrt{32/3}\ \xi_{\text{int}}^2 (1/E_i) 6xy$$
$$.f_{bc}.[1 - h_{bca}]. \quad \text{(B41)}$$

In the same way, putting E_i from Equations B29 in Equation B40, the decay rate is given by:

$$\Gamma^2_{\text{int8}} = (16\sqrt{2}\sqrt{32/3}/m_b)\Gamma_{0b} I^2_{\text{int8}}, \tag{B42}$$

where:

$$I^2_{\text{int8}} = \xi^2_{\text{int}} \int_0^1 dx \int_0^1 dy 6xy.[1 - h_{bca}]. \tag{B43}$$

Thus, the total decay rate of interference term is given by:

$$\Gamma_{\text{int8}} = \Gamma^1_{\text{int8}} + \Gamma^2_{\text{int8}}$$
$$= (16\sqrt{2}\sqrt{32/3}/m_b)\Gamma_{0b}[I^1_{\text{int8}} + I^2_{\text{int8}}]. \tag{B44}$$

At last we can obtain the total decay rate of the magnetic dipole (d_8 plus interference terms):

$$\Gamma^{\text{TOTAL}}_{d8+\text{int8}} = \Gamma_{d8} + \Gamma_{\text{int8}},$$

$$\Gamma^{\text{TOTAL}}_{d8+\text{int8}} = (\Gamma_{0b}/m_b)\{4[I^1_{d8} + I^2_{d8}]$$
$$- 16\sqrt{2}\sqrt{32/3}\ [I^1_{\text{int8}} + I^2_{\text{int8}}]. \tag{B45}$$

The phase spaces are:

$$I^1_{d8} = \int_0^1 dx \int_0^1 dy \xi_8 h_{xy} 6xy[1 - h_{abc}],$$

$$I^2_{d8} = \int_0^1 dx \int_0^1 dy \xi_8 h_{yx} 6xy[1 - h_{bca}],$$

$$I^1_{\text{int8}} = \int_0^1 dx \int_0^1 dy \xi^1_{\text{int}} 6xy.[1 - h_{abc}],$$

$$I^2_{\text{int8}} = \int_0^1 dx \int_0^1 dy \xi^2_{\text{int}} 6xy.[1 - h_{bca}]. \tag{B46}$$