

Three-Dimensional Elasticity Solution for Laminated Cross-Ply Panels Under Localized Dynamic Moment

A. Alibeigloo^{1,*} and M. Shakeri²

Abstract. *Three-dimensional elasticity solutions have been presented for a thick laminated cross-ply circular cylindrical panel. The cylindrical panel is under a dynamic localized patch moment, is simply-supported at all edges and has finite length. Ordinary differential equations with variable coefficients are formulated using an expansion of Fourier series applied to the displacement field in circumferential and axial direction. The resulting ordinary differential equations are solved by the Galerkin finite element method. Numerical results are presented for [0/90/0], [90/0/90], [0/90/0/90/0], [0/90/0/90/0/90/0] and [0/90/0/90/0/90/0/90/0] stacking sequences.*

Keywords: *Dynamic; Patch moment; Elasticity; Panel; Multi-layered; Cross-ply.*

INTRODUCTION

Modeling of composite cylindrical shells and panels is usually based on one of the following three types of theory: The Classical Laminated Theory (CLT), shear deformation theories, or the three-dimensional theory of elasticity. Because of the high ratio of in-plane Young's modulus to transverse shear modulus, ignorance of the shell transverse shear deformation in CLT can lead to serious errors, even for thin cylindrical shells/panels. These theories also fail to accurately predict the stresses and displacements in the vicinity of discontinuous loads and for thick panels. To increase the range of validity of classical shell theories, the effects of shear deformation and transverse normal stresses are incorporated in shear deformation theories. However, to establish the validity and limitations of these approximate theories, three-dimensional elasticity solutions are necessary.

Various approaches that have been used for the analysis of composite shells are reviewed in [1]. The plane strain deformation of transversely loaded,

simply-supported cross-ply cylindrical panels using a stress-function approach was considered and the results were provided in the form of maximum displacements and stresses for various stacking sequences in [2]. The more general case of finite length cross-ply cylindrical shells subjected to transverse loading, varying sinusoidally in both the axial and circumferential directions, was analyzed later by employing the Frobenius power series method [3,4]. Elasticity solutions have also been obtained for simply-supported angle-ply cylindrical shells undergoing: (1) Bending in the circumferential direction, alone, i.e. the generalized plane strain problem [5] and (2) Axisymmetric deformation [6]. An exact closed-form solution has been obtained in the first case which, as the second case, has been analyzed using Taylor series. All of the above work is restricted to smoothly varying sinusoidally distributed transverse loads. The problem of a simply-supported finite cross-ply cylindrical shell pinched by diametrically opposite localized radial forces was analyzed in [7]. It was shown that this analysis is a severe test case for judging the accuracy of the two-dimensional shell theory for the case of smooth transverse loading. An exact three-dimensional elasticity solution for an infinitely long, thick, transversely isotropic circular cylindrical shell under a static patch load was investigated in [8]. In this paper, the boundary value problem is reduced to Bessel's differential equation and the result shows

1. Department of Mechanical Engineering, Bu Ali Sina University, Hamedan, P.O. Box 65175-4161, Iran.

2. Department of Mechanical Engineering, Amir Kabir University of Technology, Tehran, Iran.

*. Corresponding author. E-mail: a_beigloo@yahoo.com

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that the transverse shear stress distribution over the shell thickness is not parabolic at or near the vicinity of a highly discontinuous load. Elasticity solutions for laminated orthotropic cylindrical shells subjected to a localized static moment were studied in [9]. It was shown that a classical approach fails to capture the localized deformations and stresses correctly, even for quite thin shells. It has also been pointed out that localized moment loading is more severe and rigorous than the cases of smooth and localized radial loading for judging the range of applicability of a two-dimensional laminate theory. Shuvalov and Soldatos [10] used successive approximation methods for the three-dimensional analysis of radially inhomogeneous tubes with arbitrary cylindrical anisotropy. Chen et al. [11] conducted a 3D coupled vibration analysis of fluid-filled orthotropic cylindrical shells of functionally graded materials. The elasticity method was developed to study the bending and free vibration of simply-supported angle-ply laminated cylindrical panels in cylindrical bending [12]. The dynamic stability analysis of thin, laminated panels under static and periodic axial forces is presented, using the mesh-free kp-Ritz method [13]. The mesh-free kernel particle estimate is employed to approximate the 2D transverse displacement field. Effects of lamination schemes such as the instability regions were examined in detail.

A review of the published literature shows that the elasticity solution of laminated cross-ply cylindrical panels under a dynamic patch moment has not yet been investigated. Recently, the author's have studied the response of multi-layered anisotropic cylindrical panels and shallow panels under dynamic radial normal loading with a constant distribution, using the theory of elasticity [14,15]. In the present investigation, a three-dimensional elasticity solution for a thick laminated cross-ply circular cylindrical panel, simply-supported along four edges and subjected to a dynamic patch moment, has been presented.

FORMULATION

A laminated cylindrical panel composed of N homogeneous orthotropic thick layers is considered (Figure 1). The material axes coincide with the geometric axes r , θ and z . The four edges of the panel are simply-supported and the constitutive equations of each layer are stated as:

$$\begin{bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \\ \tau_{zr} \\ \tau_{z\theta} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{r\theta} \\ \gamma_{zr} \\ \gamma_{z\theta} \end{bmatrix} \quad (1)$$

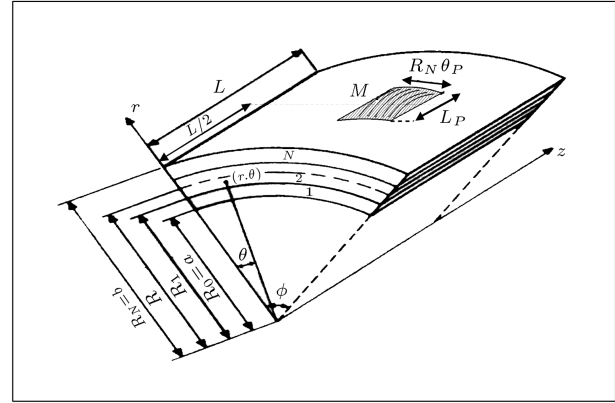


Figure 1. Geometry and the coordinate system of laminated panel.

The governing differential equations of motion in the three-dimensional theory of elasticity are:

$$\begin{aligned} \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{z\theta}}{r \partial \theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} &= \rho \frac{\partial^2 U_z}{\partial t^2}, \\ \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\partial \sigma_\theta}{r \partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} &= \rho \frac{\partial^2 U_\theta}{\partial t^2}, \\ \frac{\partial \tau_{zr}}{\partial z} + \frac{1}{r} \frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} &= \rho \frac{\partial^2 U_r}{\partial t^2}. \end{aligned} \quad (2)$$

Strain-displacement relations are expressed as:

$$\begin{aligned} \varepsilon_r &= \frac{\partial U_r}{\partial r}, & \varepsilon_\theta &= \frac{U_r}{r} + \frac{\partial U_\theta}{r \partial \theta}, \\ \varepsilon_z &= \frac{\partial U_z}{\partial z}, & \gamma_{zr} &= \frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z}, \\ \gamma_{r\theta} &= \frac{-U_\theta}{r} + \frac{\partial U_\theta}{\partial r} + \frac{\partial U_r}{r \partial \theta}, & \gamma_{z\theta} &= \frac{\partial U_\theta}{\partial z} + \frac{\partial U_z}{r \partial \theta}. \end{aligned} \quad (3)$$

Combining Equations 1 to 3, the governing equations, in terms of displacements for each layer of a cylindrical panel, become:

$$\begin{aligned} C_{11}^k \frac{\partial^2 U_z}{\partial z^2} + C_{12}^k \left(\frac{\partial U_r}{r \partial z} + \frac{\partial^2 U_\theta}{r \partial z \partial \theta} \right) + C_{13}^k \frac{\partial^2 U_r}{\partial r \partial z} \\ + C_{66}^k \frac{1}{r} \left(\frac{\partial^2 U_\theta}{\partial z \partial \theta} + \frac{\partial^2 U_z}{r \partial \theta^2} \right) \\ + C_{55}^k \left[\left(\frac{\partial^2 U_z}{\partial r^2} + \frac{\partial^2 U_r}{\partial r \partial z} \right) \right. \\ \left. + \frac{1}{r} \left(\frac{\partial U_z}{\partial r} + \frac{\partial U_r}{\partial z} \right) \right] = \rho^k \frac{\partial^2 U_z}{\partial t^2}, \end{aligned}$$

$$\begin{aligned}
& C_{66}^k \left(\frac{\partial^2 U_\theta}{\partial z^2} + \frac{\partial^2 U_z}{r \partial \theta \partial z} \right) + C_{12}^k \frac{\partial^2 U_z}{r \partial r \partial \theta} \\
& + C_{22}^k \frac{1}{r^2} \left(\frac{\partial U_r}{\partial \theta} + \frac{\partial^2 U_\theta}{\partial \theta^2} \right) + C_{23}^k \frac{\partial^2 U_r}{r \partial r \partial \theta} \\
& + C_{44}^k \left[-\frac{\partial U_\theta}{r \partial r} + \frac{U_\theta}{r^2} + \frac{\partial^2 U_\theta}{\partial r^2} + \frac{\partial^2 U_r}{r \partial r \partial \theta} - \frac{\partial U_r}{r^2 \partial \theta} \right. \\
& \left. + \frac{2}{r} \left(-\frac{U_\theta}{r} + \frac{\partial U_\theta}{\partial r} + \frac{\partial U_r}{r \partial \theta} \right) \right] = \rho^k \frac{\partial^2 U_\theta}{\partial t^2}, \\
& C_{44}^k \frac{1}{r} \left(-\frac{\partial U_\theta}{r \partial \theta} + \frac{\partial^2 U_\theta}{\partial r \partial \theta} + \frac{1}{r} \frac{\partial^2 U_r}{\partial \theta^2} \right) \\
& + C_{23}^k \left(\frac{1}{r} \frac{\partial U_r}{\partial r} - \frac{U_r}{r^2} + \frac{\partial^2 U_\theta}{r \partial r \partial \theta} - \frac{\partial U_\theta}{r^2 \partial \theta} \right), \\
& + \frac{1}{r} \left[(C_{13}^k - C_{12}^k) \frac{\partial U_z}{\partial z} + (C_{23}^k \right. \\
& \left. - C_{22}^k) \left(\frac{U_r}{r} + \frac{\partial U_\theta}{r \partial \theta} \right) + (C_{33}^k - C_{23}^k) \frac{\partial U_r}{\partial r} \right] \\
& C_{55}^k \left(\frac{\partial^2 U_z}{\partial r \partial z} + \frac{\partial^2 U_r}{\partial z^2} \right) + C_{33}^k \frac{\partial^2 U_r}{\partial r^2} + C_{13}^k \frac{\partial^2 U_z}{\partial r \partial z} \\
& = \rho^k \frac{\partial^2 U_r}{\partial t^2}. \tag{4}
\end{aligned}$$

The simply-supported boundary conditions are taken as:

$$\begin{aligned}
U_r = \sigma_\theta = \tau_{\theta z} = 0 \quad \text{at} \quad \theta = 0, \phi, \\
U_r = \sigma_z = \tau_{z\theta} = 0 \quad \text{at} \quad z = 0, L. \tag{5}
\end{aligned}$$

For a laminate consisting of N laminae, the continuity conditions to be enforced at any arbitrary interior k th interface can be written as:

$$\begin{aligned}
(\sigma_r)_k = (\sigma_r)_{k+1}, \quad (\tau_{r\theta})_k = (\tau_{r\theta})_{k+1}, \\
(\tau_{rz})_k = (\tau_{rz})_{k+1}, \tag{6a}
\end{aligned}$$

$$\begin{aligned}
(U_r)_k = (U_r)_{k+1}, \quad (U_\theta)_k = (U_\theta)_{k+1}, \\
(U_z)_k = (U_z)_{k+1}, \tag{6b}
\end{aligned}$$

where suffixes k and $k+1$ represent the corresponding stresses and displacements at the k th and $(k+1)$ th laminae.

Since the panel is under the longitudinal patch moment M , applied in the form of a linear variation of radial stress (σ_r) distributed over a small rectangular patch $\frac{(L-L_p)}{2} \leq Z \leq \frac{(L+L_p)}{2}$ and $\frac{(\phi-\theta_p)}{2} \leq \theta \leq \frac{(\phi+\theta_p)}{2}$

on the outer surface, the boundary conditions on the outer and inner surfaces of the panel are taken as:

$$\begin{aligned}
\sigma_r = \frac{-6M}{R_b \theta_p L_p^3} (L - 2Z), \quad \tau_{zr} = \tau_{r\theta} = 0, \\
\text{at } r = R_b, \tag{7a}
\end{aligned}$$

$$\sigma_r = \tau_{zr} = \tau_{r\theta} = 0 \quad \text{at } r = R_a. \tag{7b}$$

SOLUTION

The unknown displacements that satisfy the boundary conditions (Equations 5) are expanded into a Fourier series as follows:

$$\begin{aligned}
u_r &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \beta_m \theta \sin P_n z \cdot U_r(r, t), \\
u_\theta &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \cos \beta_m \theta \sin P_n z \cdot U_\theta(r, t), \\
u_z &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \beta_m \theta \cos P_n z \cdot U_z(r, t), \tag{8}
\end{aligned}$$

where:

$$p_n = \frac{n\pi}{L}, \quad \beta_m = \frac{m\pi}{\phi}, \quad (m, n = 1, 2, \dots).$$

After substituting Equation 8 into Equation 4, the partial differential equations reduce to ordinary differential equations with variable coefficients. By considering linear shape functions for three field variable, U_r , U_θ and U_z , as:

$$\begin{aligned}
U_r &= [N_i \quad N_j] \begin{bmatrix} U_{ri} \\ U_{rj} \end{bmatrix}, \quad U_\theta = [N_i \quad N_j] \begin{bmatrix} U_{\theta i} \\ U_{\theta j} \end{bmatrix}, \\
U_z &= [N_i \quad N_j] \begin{bmatrix} U_{zi} \\ U_{zj} \end{bmatrix}, \tag{9}
\end{aligned}$$

and then by applying the formal Galerkin finite element method to the first governing ordinary differential equation as:

$$\begin{aligned}
& \int_i^j \left[-C_{11}^k P_n^2 U_x + C_{12}^k \frac{P_n}{r} (U_r - \beta_m U_\theta) \right. \\
& \quad + C_{13}^k P_n \frac{\partial U_r}{\partial r} \\
& \quad + C_{66}^k \frac{1}{r} \left(-\beta_m P_n U_\theta - \frac{\beta_m^2}{r} U_x \right) \\
& \quad + C_{55}^k \left[\left(\frac{\partial^2 U_x}{\partial r^2} + P_n \frac{\partial U_r}{\partial r} \right) \right. \\
& \quad \left. \left. + \frac{1}{r} \left(\frac{\partial U_x}{\partial r} + P_n U_r \right) \right] \rho^k \frac{\partial^2 U_x}{\partial t^2} \right] \\
& \times N_i dr = 0, \tag{10}
\end{aligned}$$

the following result is obtained:

$$A_4 u_{ri} + B_4 u_{rj} + C_4 u_{\theta i} + D_4 u_{\theta j} + E_4 u_{zi} + F_4 u_{zj} + G_4 \ddot{u}_{zi} + \frac{G_4}{2} \ddot{u}_{zj} = C_{55}^k \frac{\partial u_z}{\partial r} \Big|_i, \quad (11)$$

where $A_4, B_4, \dots$ and G_4 are constants (Appendix A) and the indices i and j depend on the number of nodes in a radial direction.

By integrating the second and third ordinary differential equations, two equations similar to Equation 11 are obtained. By changing N_i to N_j and then repeating the above procedure, three other equations are obtained. The result is written in the following dynamic finite element equilibrium equation for each non-boundary element:

$$[M]_k \{\ddot{X}\}_k + [K]_k \{X\}_k = \{F(t)\}_k, \quad (12)$$

where:

$$\{X\}_k = \{U_{ri} \ U_{\theta i} \ U_{zi} \ U_{rj} \ U_{\theta j} \ U_{zj}\}^{-1}.$$

By referring to the first equation of Equations 9 and using Equations 1 and 3 and by expressing the pertinent derivatives in backward and forward finite differences for k th and $(k+1)$ th layers, U_{rkI} , $U_{\theta kI}$ and U_{zkI} can be obtained in term of the displacement values of neighboring nodes as follows:

$$\begin{aligned} U_{rkI}^k &= U_{rkI+1}^{k+1} = A \cdot U_{rkI-1}^k + B \cdot U_{rkI+2}^{k+1} \\ &+ C \cdot U_{\theta kI-1}^k + D \cdot U_{\theta kI+2}^{k+1} + E \cdot U_{zkI-1}^k \\ &+ F \cdot U_{zkI+2}^{k+1}, \\ U_{\theta kI}^k &= U_{\theta kI+1}^{k+1} = A' \cdot U_{rkI-1}^k + B' \cdot U_{rkI+2}^{k+1} \\ &+ C' \cdot U_{\theta kI-1}^k + D' \cdot U_{\theta kI+2}^{k+1} \\ &+ E' \cdot U_{zkI-1}^k + F' \cdot U_{zkI+2}^{k+1}, \\ U_{zkI}^k &= U_{zkI+1}^{k+1} = A'' \cdot U_{rkI-1}^k + B'' \cdot U_{rkI+2}^{k+1} \\ &+ C'' \cdot U_{\theta kI-1}^k + D'' \cdot U_{\theta kI+2}^{k+1} \\ &+ E'' \cdot U_{zkI-1}^k + F'' \cdot U_{zkI+2}^{k+1}, \end{aligned} \quad (13)$$

where $A, B, \dots, F''$ are constant coefficients (Appendix B).

Substituting Equation 13 into Equation 12, the dynamic finite element equilibrium equations for the two neighboring elements at the interface of k th and $(k+1)$ th layers, become:

$$[M]_K \{\ddot{X}\} + [K]_k \{X\}_k = \{0\}, \quad (14a)$$

$$[M]_{k+1} \{\ddot{X}\}_{k+1} + [K]_{k+1} \{X\}_{k+1} = \{0\}. \quad (14b)$$

Applying the boundary conditions (Equations 7a and 7b) for the first and last nodes on the inner and outer surfaces, the displacement values for these nodes will be as follows:

$$\begin{aligned} U_{r1} &= C_{10} U_{r2} + D_{10} U_{\theta 2} + E_{10} U_{z2}, \\ U_{\theta 1} &= C'_{10} U_{r2} + D'_{10} U_{\theta 2} + E'_{10} U_{z2}, \\ U_{z1} &= C''_{10} U_{r2} + D''_{10} U_{\theta 2} + E''_{10} U_{z2}, \end{aligned} \quad (15)$$

$$\begin{aligned} U_{rMI} &= G_{10} U_{rMI-1} + H_{10} U_{\theta MI-1} + I_{10} U_{zMI-1} \\ &+ F_{10} \cdot M(t), \end{aligned}$$

$$\begin{aligned} U_{\theta MI} &= G'_{10} U_{rMI-1} + H'_{10} U_{\theta MI-1} + I'_{10} U_{zMI-1} \\ &+ F'_{10} \cdot M(t), \end{aligned}$$

$$\begin{aligned} U_{zMI} &= G''_{10} U_{rMI-1} + H''_{10} U_{\theta MI-1} + I''_{10} U_{zMI-1} \\ &+ F''_{10} \cdot M(t), \end{aligned} \quad (16)$$

where $C_{10}, D_{10}, \dots$ and F''_{10} are constants (Appendix C).

From Equations 12, 15 and 16 the dynamic equations for the first and last elements become:

$$[M]_1 \{\ddot{X}\}_1 + [K]_1 \{X\}_1 = \{F\}_1, \quad (17a)$$

$$[M]_{MI} \{\ddot{X}\}_{MI} + [K]_{MI} \{X\}_{MI} = \{F\}_{MI}. \quad (17b)$$

Assembling Equations 12, 14a, 14b, 17a and 17b the general dynamic finite element equilibrium equations are obtained as:

$$[M] \{\ddot{X}\} + [K] \{X\} = \{F\}. \quad (18)$$

The Newmark direct integration method with a suitable time step is used, and then Equation 18 is solved.

RESULTS AND DISCUSSION

Stacking sequences that were mentioned in the abstract for cross-ply laminated cylindrical panels are considered. The external dynamic moment in the form of a Fourier series is for example:

$$\begin{aligned} M &= M_0(1 - e^{-\alpha t}) \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn}(1 - e^{-\alpha t}) \sin \beta_m \theta \sin p_n z, \end{aligned} \quad (19)$$

where:

$$\begin{aligned} q_{mn} &= \frac{192 M_0 L}{m \pi^3 n^2 \theta_p R_b L_p^3} \cos \left(\frac{n \pi}{2} \right) \sin \left(\frac{m \pi \theta_p}{2 \phi} \right) \sin \left(\frac{m \pi}{2} \right) \\ &\left[\sin \left(\frac{n \pi L_p}{2 L} \right) - \left(\frac{n \pi L_p}{2 L} \right) \cos \left(\frac{n \pi L_p}{2 L} \right) \right]. \end{aligned}$$

($m, n = 1, 2, \dots$) and α is the time constant, which in this paper is chosen as 13100. The material properties are considered as follows:

$$\frac{E_L}{E_T} = 40, \quad \frac{G_{LT}}{E_T} = 0.5, \quad \frac{G_{TT}}{E_T} = 0.2,$$

$$\nu_{LT} = \nu_{TT} = 0.25, \quad \rho = 1600(\text{kg/m}^3).$$

Numerical results are obtained in the form of maximum non-dimensional displacements and stresses which are as:

$$\bar{U}_r = \frac{E_L R_0^2 U_r}{M_0},$$

$$(\bar{U}_z, \bar{U}_\theta) = \frac{10 E_L R_0^2}{M_0} (U_z, U_\theta),$$

$$\bar{r} = \frac{r - R}{H}, \quad S = \frac{R_0}{H},$$

$$(\bar{\sigma}_i, \bar{\tau}_{ij}) = (\sigma_i, \tau_{ij}) \frac{R_0^3}{M_0} \quad \text{for} \quad i, j = r, \theta, z.$$

The external load at time $T = 0.3$ msec reaches its peak value of M_0 and then remains constant at this value.

The time history of the radial displacement of three-, five-, seven- and nine-layered panels are shown in Figures 2a to 2d. As expected, the panels vibrate in a radial direction due to initial conditions, $(U_r, \dot{U}_r, \ddot{U}_r)$. By increasing the number of layers, the amplitude of vibration decreases and the natural frequencies increase. These results are acceptable, as the stiffness of the panel is increased by increasing the number of layers. The variation of radial, circumferential and axial displacements, $(\bar{U}_r, \bar{U}_\theta, \bar{U}_z)$, with time at $r = R_0$ and $\theta = \frac{\phi}{2}$, are shown in Figures 3a to 3c for a half length of the panel. As expected, by increasing the loading duration up to 0.35 msec, the displacements are first increased and then decreased due to the vibration characteristic of the problem.

The main advantage of an elasticity solution, compared with other approximate solutions, is that all in the elasticity solution (boundary conditions as well as inter-laminar continuity conditions) can be satisfied, whereas this is not the case for other methods.

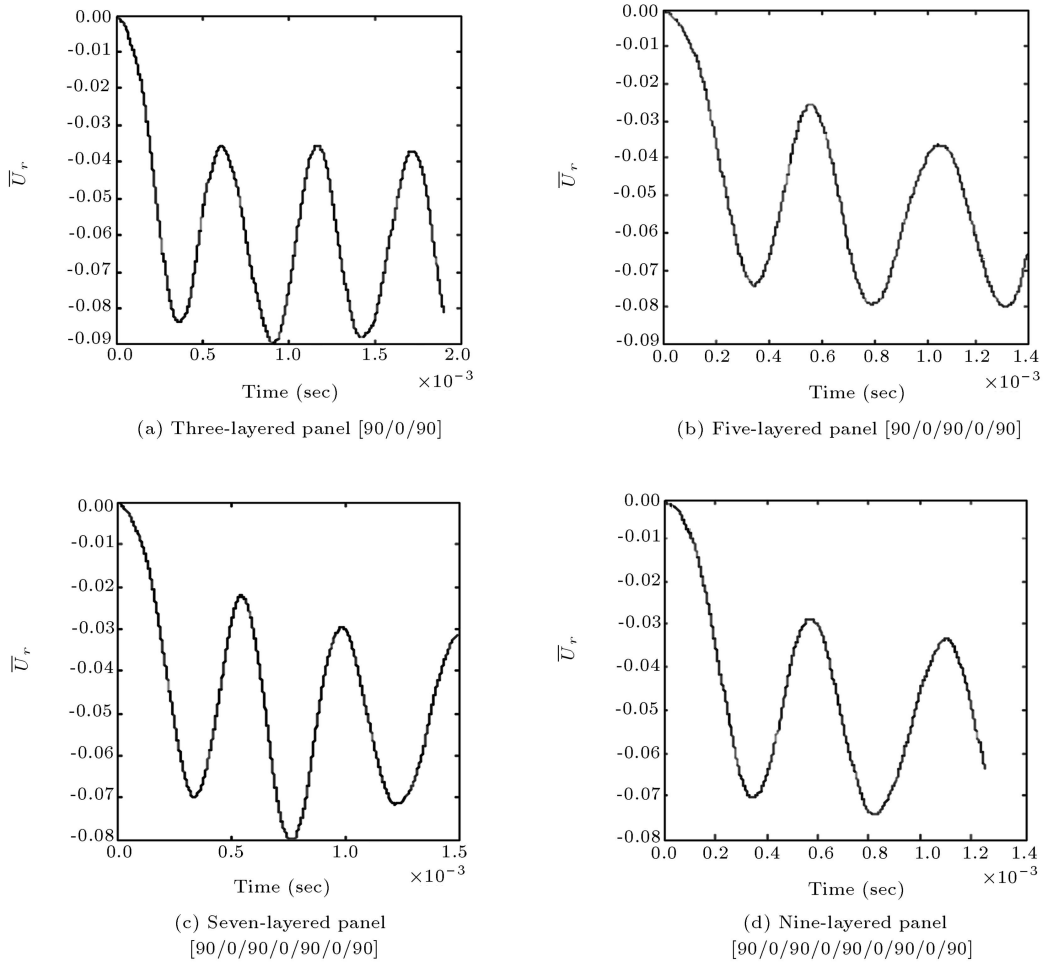


Figure 2. Time history of displacements at $\frac{z}{L} = 0.48$, $r = R_0$ and $S = 5$.

It is seen that at the simply-supported edge where $z = 0$, u_r and u_θ become zero, while the panel is free to move in an axial direction, ($\bar{U}_z \neq 0$). At mid-length of the panel ($z = \frac{L}{2}$), the radial and circumferential displacements are zero because of symmetry. The inter-laminar continuity conditions are also satisfied for all three components of displacement.

To show the satisfaction of boundary and inter laminar continuity conditions for stresses, the distribution of stress components across the thickness of the panel at $T = 0.3$ msec is shown in Figures 4a to 4d. It is seen that the boundary conditions at the inner and outer surfaces and the continuity conditions between the layers are satisfied. Figure 4d also shows the

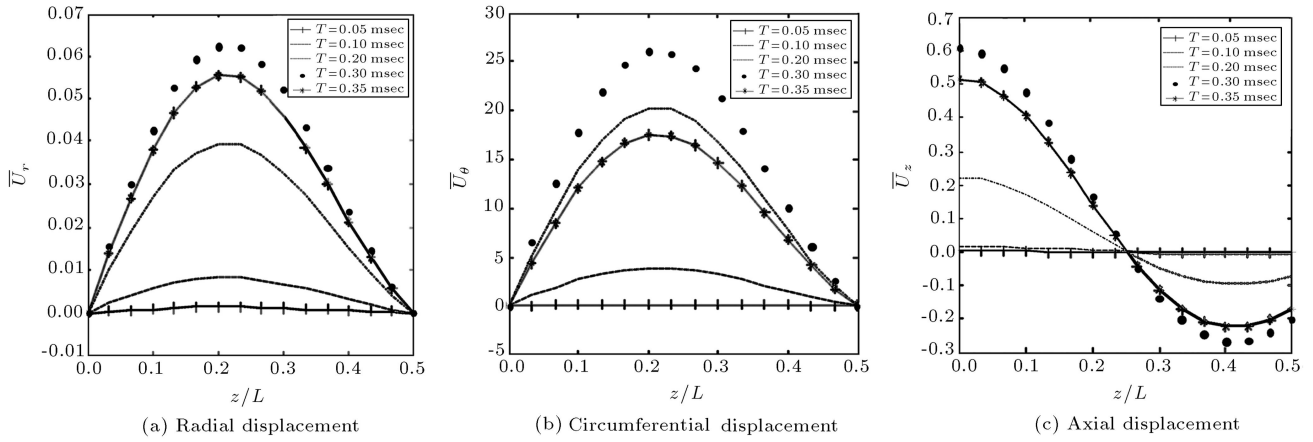


Figure 3. Variation of displacements with time versus z at $\theta = \frac{\pi}{6}$, $r = R_0$, $T = 0.3$ msec $[90/0/90]$.

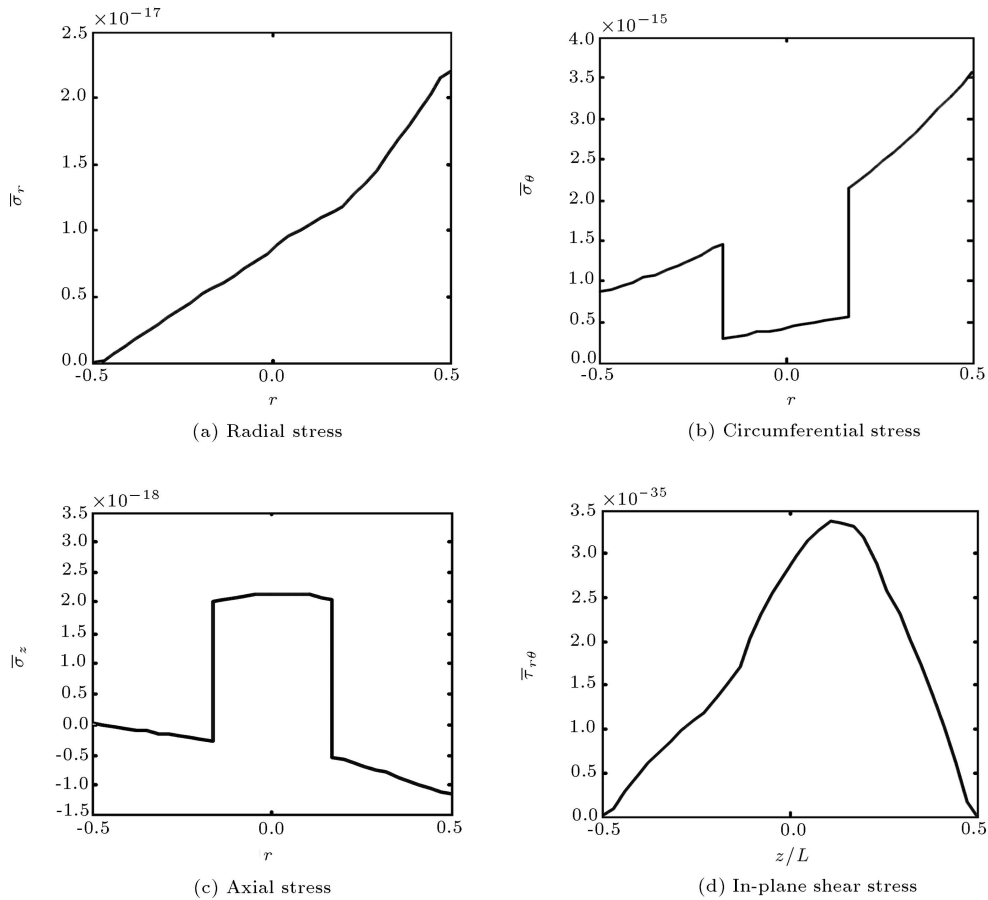


Figure 4. Distribution of stresses across the thickness at $T = 0.3$ msec, $S = 20$, $\theta = \frac{\pi}{6}$, $\frac{z}{L} = 0.48$ $[90/0/90]$.

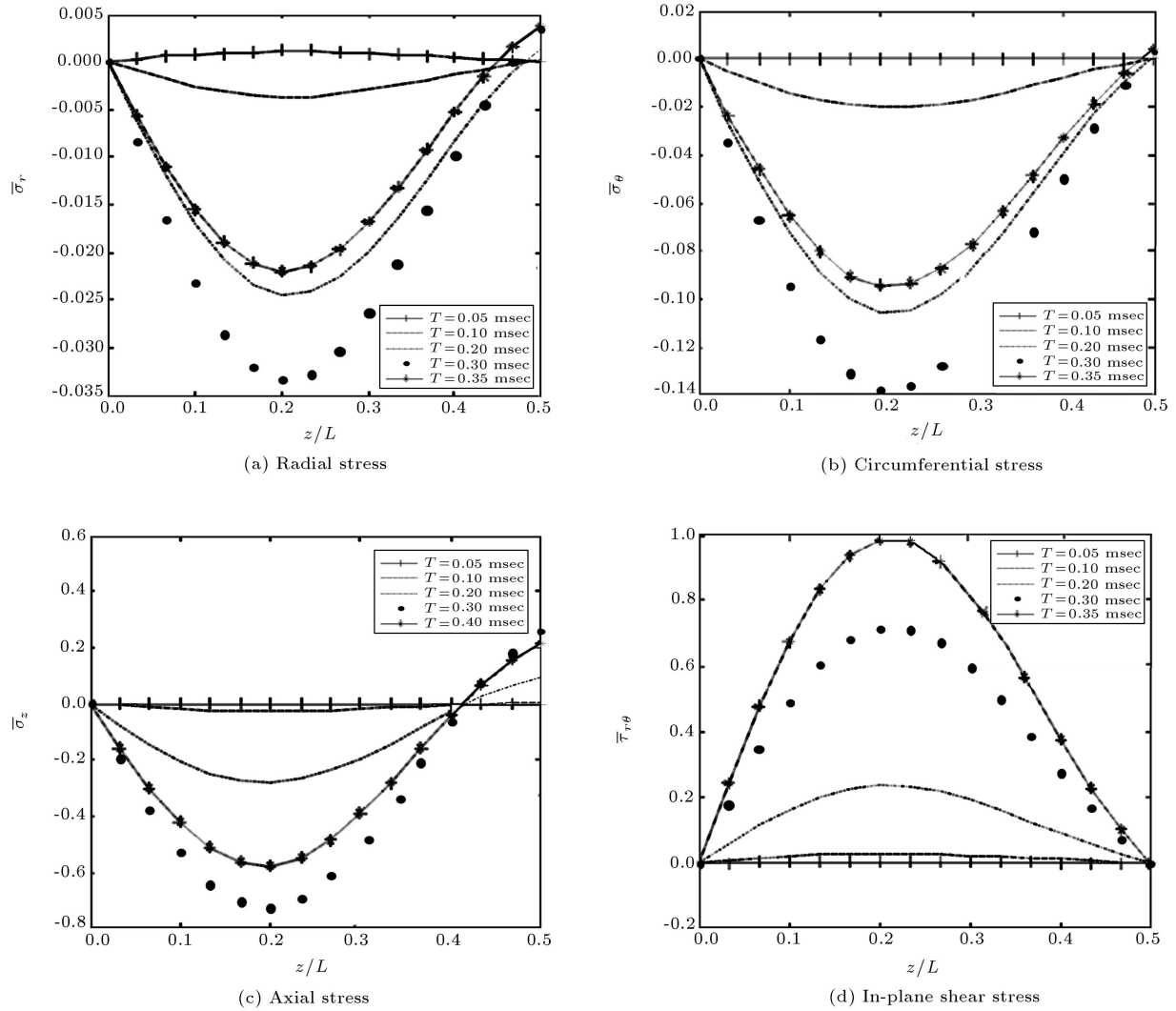


Figure 5. Variation of stresses with time versus z at $\theta = \frac{\pi}{6}$, $r = R_0$ and $S = 5$ [90/0/90].

parabolic distribution of the in-plane shear stress for each layer.

The presented elasticity solution is applicable for both thin and thick panels. Stress distributions along the z -direction are varied with time (see Figures 5a to 5d). These distributions satisfy the boundary conditions at $z = 0, L$. The stresses increase with increasing the time up to $T = 0.35$ msec, and then begin to decrease, due to the nature of external loading. In Figures 6a to 6d, the influence of a stacking sequence in a three-layered panel on stress distribution along the axial direction is shown. The axial stress distribution (Figure 6a) in the stacking sequence of [0/90/0] in mid- surface is smaller than the one for the [90/0/90] sequence lay-up. This result is evident, because where $r = R_0$, the fibers are along the circumferential direction for the [0/90/0] layered panel, but they are along the axial direction for the [90/0/90] one. The existence of fibers in a circumferential direction in the middle layer for the [0/90/0] stacking sequence

causes the circumferential stress at $r = R_0$ to be higher than that of the [90/0/90] panel (see Figure 6b). Using similar reasoning and noting that $E_2 = E_3$, the difference in radial stresses for a two stacking sequence is small (see Figure 6a). Finally, the distribution of a transverse shear stress ($\bar{\tau}_{r\theta}$) for two stacking sequences is shown in Figure 6d.

CONCLUSION

The main objective of this paper was to introduce a procedure to obtain the stress and displacement field for a panel in three dimensions while the panel is subjected to a dynamic localized patched moment. For this purpose, the three-dimensional theory of elasticity was used to derive the governing differential equations of motion in terms of displacements, and then the Galerkin finite element method was used to obtain displacements and stresses versus time. Numerical results obtained for various stacking arrangements show that

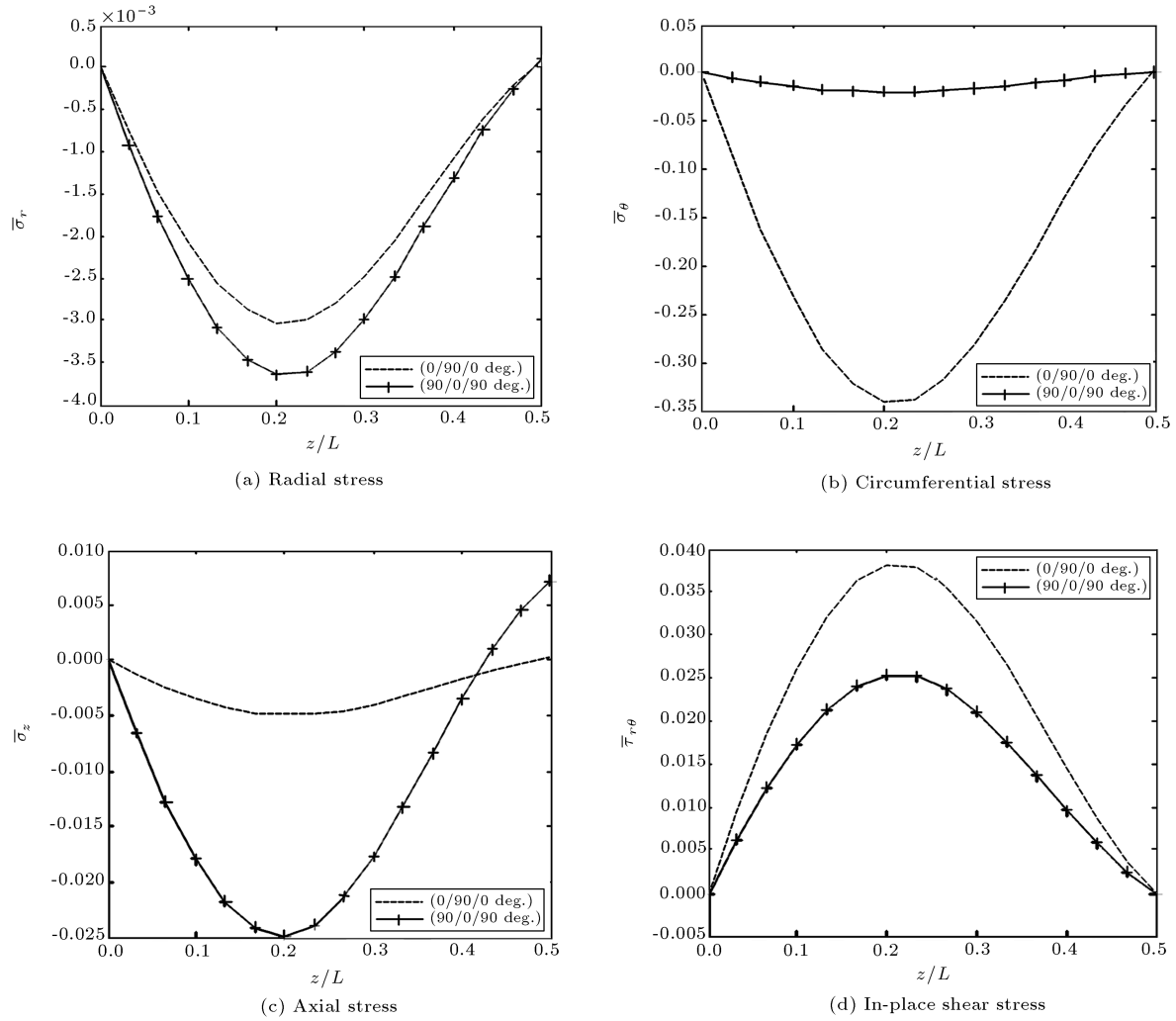


Figure 6. Influence of stacking sequence in stresses distribution versus z at $T = 0.3$ msec., $\theta = \frac{\pi}{6}$, $r = R_0$ and $S = 5$.

the magnitude of stresses and displacements depends on the stacking sequence and the number of layers. When the external moment reaches peak value (Equation 19), the panel will be under static loading. Since the damping coefficient is not considered, the panel vibrates with constant amplitude. Meanwhile, the radial stress behaviour of the panel depends strongly on the $S = \frac{R_0}{H}$. Although this procedure is not an exact solution, the authors believe that it is the only solution (elasticity solution) which satisfies all boundary and inter-laminar conditions; an almost accurate method for showing the correct patterns of displacement and stress components versus time and thickness of panel.

NOMENCLATURE

$C_{ij}(i, j = 1, 2, \dots, 6)$ constants related to elastic stiffness

E_1, E_2, G_{12}, G_{23}	elastic material constants
ν_{12}, ν_{13}	
$\{F(t)\}$	force matrix of $(3MI - 6 - 3N) \times 1$ size
H	thickness of panel
h_i	thickness of i th layer
Δh	thickness increment
$[K]$	global stiffness matrix of $(3MI - 6 - 3N) \times (3MI - 6 - 3N)$ size
L	axial dimension of panel
L_p	axial length of patch
$[M]$	global mass matrix of $(3MI - 6 - 3N) \times (3MI - 6 - 3N)$ size
MI	number of elements
N	number of layers
M_0	peak of mechanical load

R_0	mid radius
r_i, r_j	radius of i and j nodes in one element
R_a	inner surface radius
R_b	outer surface radius
t	time variable
U_r, U_θ, U_z	displacements in r, θ and z directions, respectively
$U_{ri}, U_{\theta i}, U_{zi}$	displacements of i node
$U_{rj}, U_{\theta j}, U_{zj}$	displacements of j node
$\ddot{U}_{ri}, \ddot{U}_{\theta i}, \ddot{U}_{zi}$	acceleration of i node
$\ddot{U}_{rj}, \ddot{U}_{\theta j}, \ddot{U}_{zj}$	acceleration of j node
$U_{rkI}^k, U_{\theta kI}^k, U_{zkI}^k$	displacements at kI th node of k th layer
$U_{rkI+1}^{k+1}, U_{\theta kI+1}^{k+1}, U_{zkI+1}^{k+1}$	displacements at $kI+1$ th node of $(k+1)$ th layer
α	constant related to time
$\gamma_{r\theta}, \gamma_{zr}, \gamma_{z\theta}$	shear strains
$\sigma_i (i = r, \theta, z)$	normal stresses
$\varepsilon_i (i = r, \theta, z)$	normal strains
$\tau_{r\theta}, \tau_{zr}, \tau_{z\theta}$	shear stresses
θ_p	circumferential length of patch of applied moment
ϕ	circumferential dimension

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APPENDIX A

$$A_4 = P_n \left[\frac{C_{12}^k + C_{55}^k}{(r_j - r_i)^2} \left[r_j^2 \ln \frac{r_j}{r_i} + \frac{1}{2}(r_j - r_i)(r_i - 3r_j) \right] - \frac{1}{2}(C_{13}^k + C_{55}^k) \right],$$

$$B_4 = P_n \left[\frac{C_{12}^k + C_{55}^k}{(r_j - r_i)^2} \left[\frac{1}{2}(r_j^2 - r_i^2) - r_i r_j \ln \frac{r_j}{r_i} \right] + \frac{1}{2}(C_{13}^k + C_{55}^k) \right],$$

$$C_4 = -P_n \beta_m \frac{C_{12}^k + C_{66}^k}{(r_j - r_i)^2} \left[r_j^2 \ln \frac{r_j}{r_i} + \frac{1}{2}(r_j - r_i)(r_i - 3r_j) \right],$$

$$\begin{aligned}
D_4 &= -P_n \beta_m \frac{C_{12}^k + C_{66}^k}{(r_j - r_i)^2} \left[\frac{1}{2} (r_j^2 - r_i^2) - r_i r_j \ln \frac{r_j}{r_i} \right], \\
G_4 &= \frac{1}{6} \rho (r_j - r_i), \\
E_4 &= -\frac{1}{3} C_{11}^k P_n^2 (r_j - r_i) - \frac{\beta_m^2 C_{66}^k}{(r_j - r_i)^2} \\
&\quad \left[-2r_j \ln \frac{r_j}{r_i} + (r_j - r_i) \left(1 + \frac{r_j}{r_i} \right) \right] \\
&\quad - \frac{C_{55}^k}{(r_j - r_i)^2} r_i \ln \frac{r_j}{r_i}, \\
F_4 &= -\frac{1}{6} C_{11}^k P_n^2 (r_j - r_i) - \frac{\beta_m^2 C_{66}^k}{(r_j - r_i)^2} \\
&\quad \left[(r_j + r_i) \ln \frac{r_j}{r_i} - 2(r_j - r_i) \right] + \frac{C_{55}^k}{(r_j - r_i)^2} r_j \ln \frac{r_j}{r_i}.
\end{aligned}$$

APPENDIX B

$$\begin{aligned}
A &= \frac{H_1 B_2 C_3}{\det A'_1}, & B &= \frac{I_1 B_2 C_3}{\det A'_1}, \\
C &= \frac{-D_2 B_1 C_3}{\det A'_1}, & D &= \frac{-F_2 B_1 C_3}{\det A'_1}, \\
E &= \frac{-E_3 B_2 C_1}{\det A'_1}, & F &= \frac{-G_3 B_2 C_1}{\det A'_1}, \\
A' &= \frac{-H_1 A_2 C_3}{\det A'_1}, & B' &= \frac{-I_1 A_2 C_3}{\det A'_1}, \\
C' &= \frac{D_2 (A_1 C_3 - A_3 C_1)}{\det A'_1}, \\
D' &= \frac{F_2 (A_1 C_3 - A_3 C_1)}{\det A'_1}, \\
E' &= \frac{E_3 A_2 C_1}{\det A'_1}, & F' &= \frac{G_3 A_2 C_1}{\det A'_1}, \\
A'' &= \frac{-H_1 B_2 A_3}{\det A'_1}, & B'' &= \frac{-I_1 A_3 B_2}{\det A'_1}, \\
C'' &= \frac{D_2 B_1 A_3}{\det A'_1}, & D'' &= \frac{F_2 B_1 A_3}{\det A'_1}, \\
E'' &= \frac{E_3 (A_1 B_2 - A_2 B_1)}{\det A'_1}, \\
F'' &= \frac{G_3 (A_1 B_2 - A_2 B_1)}{\det A'_1}.
\end{aligned}$$

where:

$$\begin{aligned}
A_1 &= \frac{C_{23}^k - C_{23}^{k+1}}{\sum_{i=1}^k h_i + R_a} + \frac{1}{\Delta h} (C_{33}^k + C_{33}^{k+1}), \\
B_1 &= \beta_m \frac{C_{23}^{k+1} - C_{23}^k}{\sum_{i=1}^k h_i + R_a}, & C_1 &= P_n (-C_{13}^k + C_{13}^{k+1}), \\
H_1 &= \frac{C_{33}^k}{\Delta h}, & I_1 &= \frac{C_{33}^{k+1}}{\Delta h}, & A_2 &= \beta_m \frac{C_{44}^k - C_{44}^{k+1}}{\sum_{i=1}^k h_i + R_a}, \\
B_2 &= \frac{C_{44}^{k+1} - C_{44}^k}{\sum_{i=1}^k h_i + R_a} + \frac{1}{\Delta h} (C_{44}^k + C_{44}^{k+1}), \\
D_2 &= \frac{C_{44}^k}{\Delta h}, & F_2 &= \frac{C_{44}^{k+1}}{\Delta h}, \\
A_3 &= P_n (C_{55}^k - C_{55}^{k+1}), \\
E_3 &= \frac{C_{55}^k}{\Delta h}, & C_3 &= \frac{1}{\Delta h} (C_{55}^k + C_{55}^{k+1}), \\
\det A' &= A_1 (B_2 C_3 - B_3 C_2) - A_2 (B_1 C_3 - B_3 C_1) \\
&\quad + A_3 (B_1 C_2 - B_2 C_1), & G_3 &= \frac{C_{55}^{k+1}}{\Delta h}.
\end{aligned}$$

APPENDIX C

$$\begin{aligned}
C_{10} &= \frac{-S_2 T_2 C_{33}^1}{\Delta h \cdot \det BB}, & D_{10} &= \frac{S_1 T_2}{\Delta h \cdot \det BB}, \\
E_{10} &= \frac{S_2 T_1}{\Delta h \cdot \det BB}, & C'_{10} &= \frac{R_2 T_2 C_{33}^1}{\Delta h \cdot \det BB}, \\
D'_{10} &= \frac{R_3 T_1 - R_1 T_2}{\Delta h \cdot \det BB}, & E'_{10} &= \frac{-R_2 T_1}{\Delta h \cdot \det BB}, \\
C''_{10} &= \frac{S_2 R_3 C_{33}^1}{\Delta h \cdot \det BB}, & D''_{10} &= \frac{-S_1 R_3}{\Delta h \cdot \det BB}, \\
E''_{10} &= \frac{S_1 R_2 - S_2 R_1}{\Delta h \cdot \det BB}, & G_{10} &= \frac{L_2 N_3}{\det AA} \frac{C_{33}^M}{\Delta h}, \\
H_{10} &= \frac{-L_1 N_3}{\Delta h \det AA}, & I_{10} &= \frac{-L_1 N_2}{\Delta h \det AA}, \\
F_{10} &= \frac{L_2 N_3}{\det AA}, & G'_{10} &= \frac{-R_2 N_3 C_{33}^M}{\Delta h \cdot \det AA},
\end{aligned}$$

$$H'_{10} = \frac{N_3 N_5 - N_2 R_3}{\Delta h \det AA}, \quad I'_{10} = \frac{R_2 N_2}{\Delta h \det AA},$$

$$F'_{10} = \frac{-R_2 N_3}{\det AA}, \quad G''_{10} = \frac{-L_2 R_3 C_{33}^M}{\Delta h \det AA},$$

$$H''_{10} = \frac{L_1 R_3}{\Delta h \det AA}, \quad I''_{10} = \frac{L_2 N_5 - L_1 R_2}{\Delta h \det AA},$$

$$F''_{10} = \frac{-L_2 R_3}{\det AA},$$

where:

$$N_2 = -P_n C_{13}^M, \quad N_3 = \frac{1}{\Delta h}, \quad N_5 = \frac{C_{23}^M}{R} + \frac{C_{33}^M}{\Delta h},$$

$$L_1 = -\beta_m \frac{C_{23}^M}{R_b}, \quad L_2 = \frac{-1}{R} + \frac{1}{\Delta h},$$

$$R_1 = \frac{C_{23}^1}{R} - \frac{C_{33}^1}{\Delta h}, \quad R_2 = \frac{\beta m}{R_a}, \quad R_3 = P_n,$$

$$S_1 = -\frac{\beta_m C_{23}^1}{R_a}, \quad S_2 = \frac{-1}{R_a} - \frac{1}{\Delta h},$$

$$T_1 = -P_n C_{13}^1, \quad T_2 = \frac{1}{\Delta h},$$

$$\det AA = N_5 L_2 N_3 - R_2 L_1 N_3 - R_3 L_2 N_2,$$

$$\det BB = R_1 S_2 T_2 - R_2 S_1 T_2 - R_3 S_2 T_1.$$