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Research Note

Health Monitoring of Structures Using Few Frequency Response Measurements

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Abstract. *The development of damage detection techniques for offshore jacket structures is vital for preventing catastrophic events. This paper applies a frequency response based method for the purpose of structural health monitoring. In this approach, the concept of a minimum rank perturbation theory is used. The feasibility of using a finite number of sensors and its effect on damage detection capabilities is investigated. In addition, the performance of the proposed method is evaluated in the case of multiple damages. The aforementioned points are illustrated using the numerical study of a two dimensional jacket platform.*

Keywords: *Damage detection; Structural health monitoring; Frequency response function; Offshore jacket platform; Minimum rank perturbation theory.*

INTRODUCTION

Structures face various loadings and confront different circumstances as they are built and used. This situation causes the aging structures to deteriorate, which leads to a decrease in reliability and safety. In recent decades, the need for systems to assure the integrity of structures in terms of age, usage and level of safety, when experiencing infrequent and extreme forces such as earthquakes, tornados, hurricanes and large waves, has been seriously recognized. These are often referred to as Structural Health Monitoring (SHM) systems in the literature. Overall, the field of SHM aims to identify, localize and size any defect in the structure as it happens. The main objective of such a system is to increase the reliable operating lifetime.

Generally, structural damage detection can be classified into local damage detection and global damage detection. Local damage detection techniques refer to Non-Destructive Testing (NDT), such as X-ray methods, eddy current approaches, thermal imaging and ultrasonic methods, because it is mainly used to detect local damage in structures [1]. Local damage detection is applicable only for small and regular structures, such as pressure vessels, and for detecting only the finite suspicious components of large structures. In response to this limitation, a set of more global vibration-based approaches has been used. Therefore, global vibration-based damage detection is especially essential for large and complicated structures in order to detect the location of damage, and then with primary knowledge of the location of the defect, the inspection group can trace the damage right to the specified region, utilizing one of the local damage detection techniques. In the case of offshore structures, utilizing such global vibration based damage detection techniques is not only necessary, but also inevitable due to some of its exclusive characteristics, which can be summarized as:

1. Offshore structures being so important, expensive and huge that their failure or collapse would be a catastrophic event.
2. Poor visibility and concealment of damage by marine growth causing other techniques to be accompanied with prohibitive cost.
3. Cyclic wave loading, severe storms, sea quakes and hostile environments harshly affecting the integrity of the structure.

Most vibration-based damage detection techniques re-

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quire a significant amount of modal test data. These requirements make the damage detection procedure costly, time consuming and impractical. Some research using modal data has been recently developed. Kaouk and Zimmerman [2] used eigenvalues and eigenvectors, and adopted the concept of a minimum rank perturbation theory to locate and size the damage in a two-dimensional truss and a cantilevered beam. Li et al. [3] applied the cross-model cross-mode method for damage detection in offshore jacket structures, where spatially incomplete modal data is available.

On the other hand, some literature has concentrated on the use of a Frequency Response Function (FRF) directly, as opposed to modal data extracted from FRF measurements. There are two main advantages of using FRF data. Firstly, modal data can be contaminated by modal identification errors in addition to measurement errors, because they are derived data sets. Secondly, a complete set of modal data can only be measured in the simplest structures. FRF data can provide much more information on damage at a desired frequency range compared to modal data that is extracted from a very limited range around resonances [4]. Maia et al. [5] discussed some modal-based and FRF-based damage detection techniques, and compared the results on a simple beam. In addition, they introduced an indicator of damage as a FRF-based damage index. Hwang and Kim [6] found the location and amount of damage through computational iterations by matching experimental FRF and analytical FRF.

The objective of this paper is to present a promising methodology which applies FRF data at some frequency points to arrive at perturbations to the stiffness matrix due to some defects in the structure. The method is demonstrated numerically on a spring mass system (shear building) and then applied to an offshore jacket platform. The authors' effort was to consider a set of more probable and realistic damages in the jacket platforms relative to other similar works.

DAMAGE DETECTION FORMULATION

Basic Theory

The basic theory of this type of damage detection initiates with the second order structural equation of motion:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f\}, \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are undamaged mass, damping and stiffness matrices, respectively, $\{x\}$ is the vector of positions, $\{f\}$ is the vector of applied forces, and the over dots represent differentiation with respect to time. If the structure is excited by a set of forces all at the same frequency, ω, but with individual amplitudes and phases, then:

$$\{f(t)\}=\{F(\omega)\}.e^{i\omega t}, \tag{2}$$

by neglecting the transient response and concerning the steady state:

$$\{x(t)\}=\{X(\omega)\}.e^{i\omega t}, \tag{3}$$

where $\{F\}$ and $\{X\}$ are vectors of time-independent amplitudes. The equation of motion then becomes:

$$\{X(\omega)\}=([K]+i\omega[C]-\omega^2[M])^{-1}\{F(\omega)\},$$

$$i=\sqrt{-1},$$

$$\{F(\omega)\}=([K]+i\omega[C]-\omega^2[M])\{X(\omega)\},$$

$$[H(\omega)]=([K]+i\omega[C]-\omega^2[M])^{-1},$$

$$[Z(\omega)]=([K]+i\omega[C]-\omega^2[M]), \tag{4}$$

where $[H(\omega)]$ is standard FRF and $[Z(\omega)]$ is inverse FRF. In the undamaged condition, it can be written as:

$$\{F(\omega)\}=[Z(\omega)]\{X(\omega)\}. \tag{5}$$

But due to damage interference, Equation 5 changes to the following form:

$$\{F(\omega)\}=[Z(\omega)+\Delta Z(\omega)]\{X(\omega)\}, \tag{6}$$

where $[\Delta Z(\omega)]$ represents the effect of damage on the inverse FRF.

The force damage vector can be defined by a slight manipulation of Equation 6:

$$\{d(\omega)\}=\{F(\omega)\}-[Z(\omega)]\{X(\omega)\}=[\Delta Z]\{X(\omega)\}. \tag{7}$$

Assuming that the inverse FRF has been measured at p discrete frequencies, and the introduced defect has only affected one of the structural property matrices (either $[M]$, $[C]$ or $[K]$), Equation 7 can be rewritten as:

$$[\Delta Z][X]=[D], \tag{8}$$

where the frequency and space information of $\{X(\omega)\}$ and $\{d(\omega)\}$ were arranged as the rectangular matrices. $[X]$ and $[D]$, respectively:

$$[X]=[X(\omega_1)\cdots X(\omega_p)], \tag{9}$$

$$[D]=[d(\omega_1)\cdots d(\omega_p)]. \tag{10}$$

Minimum Rank Perturbation Theory

Equation 8 can be solved by using the same approach as used in the minimum rank perturbation theory [2]. In [7], the symmetric minimum rank solution of Equation 8 was derived and mathematical characteristics of the solution were investigated.

The minimum rank perturbation theory provides the unique minimum rank solution for Equation 8 as:

$$[\Delta Z] = [D]([D]^T[X])^{-1}[D]^T. \tag{11}$$

This solution is motivated by the application of damage detection where the perturbations could be assumed to be limited to a few isolated locations. The minimum rank stiffness matrix perturbation can be thought of as the stiffness matrix perturbation with the smallest number of nonzero values.

It should be decided, by engineering judgment, which property matrix (either $[M]$, $[C]$ or $[K]$) has caused the perturbation $[\Delta Z]$. For example, if damage has affected the stiffness of the structure, then the displacement should be measured by sensors, and $[\Delta Z]$ would be equal to $[\Delta K]$. All required information, such as damage location and the extent of stiffness reduction, is contained in the matrix $[\Delta K]$.

Computational Improvement

The key issue in the damage detection scheme is the ability to identify the matrix $[D]$.

The components of this matrix are just associated with the measured degrees of freedom. Therefore, the size of the equation takes effect from the measured degrees of freedom (number of sensors). Note that vector $\{d\}$, which appeared in Equation 7, is a kind of residual force vector. Indeed, one interpretation of $\{d\}$ is as a collection of externally applied loads acting on the undamaged structure to give a response similar to that of the damaged structure [8]. In numerical examples without the presence of noise, matrix $[D]$ is rank deficient. In other words, it has several singular values that are very close to zero. But in practical examples due to the presence of noise, matrix $[D]$ is close to full rank. This fact causes Equation 8 to be an ill-conditioned numerical problem. As mentioned above, the source of this numerical problem is mainly because of matrix $[D]$, so operations to improve the numerical condition concern matrix $[D]$.

The subspace selection algorithm, proposed by Zimmerman [9], consists of determining a matrix $[Q]$ such that Equation 12 is numerically well-conditioned:

$$[\Delta Z][X][Q] = [D][Q]. \tag{12}$$

Consider the singular value decomposition of $[D]$ as:

$$[D] = [U][S][V]^T, \tag{13}$$

where $[S]$ is the diagonal matrix of non-negative singular values in decreasing order, and U and V are the left and right singular vectors, respectively. Here, we need a criterion for partitioning S, U and V in the following order:

$$[D] = [U_1][U_2]\begin{bmatrix}\Sigma & 0\\ 0 & \varepsilon\end{bmatrix}[V_1][V_2]^T, \tag{14}$$

where Σ is the matrix of top 'm' singular values (m is user-defined).

With the selected m columns of $[U]$, matrix $[U_1]$ can represent $[D][Q]$, the right hand side of Equation 12, which will improve computational efficiency by excluding damage vectors of smaller singular values:

$$[D][Q] = [U_1]. \tag{15}$$

Thus matrix $[Q]$ can be computed by using the pseudo-inverse of $[D]$ as:

$$[Q] = [D]^+[U_1]. \tag{16}$$

Then, with the substitution of $[X][Q]$ and $[D][Q]$ for $[X]$ and $[D]$ in Equation 11, the minimum rank solution can be easily derived as:

$$[\Delta Z] = [D][Q]([Q]^T[D]^T[X][Q])^{-1}[Q]^T[D]^T. \tag{17}$$

NUMERICAL STUDIES

Two examples are presented here to illustrate the characteristics of some of the developed theories. The first example is an idealized 3 degrees of freedom shear building system. In this example, a perturbation to the stiffness property matrix is used to illustrate the numerical procedure of damage detection using FRF data. The second example involves the identification and localization of some small damages, which are artificially introduced to the two-dimensional model of an offshore jacket platform.

Idealized Shear Building System

This example consists of 3 degrees of freedom, as shown in Figure 1. Consider the undamaged model of the system to have the following parameters;

$$\{k_1, k_2, k_3\} = \{10, 10, 10\}, \tag{18}$$

$$\{m_1, m_2, m_3\} = \{0.1, 0.1, 0.1\}, \tag{19}$$

which have the undamaged mass and stiffness matrices:

$$[M_u] = \begin{bmatrix}0.1 & 0 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix}, \tag{20}$$

$$[K_u] = \begin{bmatrix}20 & -10 & 0\\ -10 & 20 & -10\\ 0 & -10 & 10\end{bmatrix}. \tag{21}$$

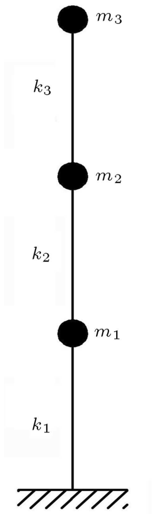

Figure 1. Idealized shear building system.

Now, consider a damage case in which parameter k_1 decreases one unit:

$$[K_d] = \begin{bmatrix} 19 & -10 & 0 \\ -10 & 20 & -10 \\ 0 & -10 & 10 \end{bmatrix}, \tag{22}$$

where the subscripts $(.)_u$ and $(.)_d$ denote undamaged and damaged conditions, respectively. We want to detect damage by considering the frequency points of 1, 2 and 3 Hertz. Assuming that only the stiffness matrix is to be perturbed, displacement should be measured by sensors under arbitrary excitation. Therefore, $[X]$ and $[D]$ are computed by Equations 7, 9 and 10.

$$\{F\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix},$$

$$[X] = \begin{bmatrix} -0.154 & 0.555 & -0.043 \\ -0.232 & 0.178 & 0.07 \\ -0.218 & -0.48 & -0.067 \end{bmatrix},$$

$$[D] = \begin{bmatrix} 0.154 & -0.555 & 0.043 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{23}$$

Then, perturbation to the stiffness property matrix from Equation 17 yields:

$$[\Delta Z] = [\Delta K] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{24}$$

which is the exact anticipated perturbation to the stiffness matrix:

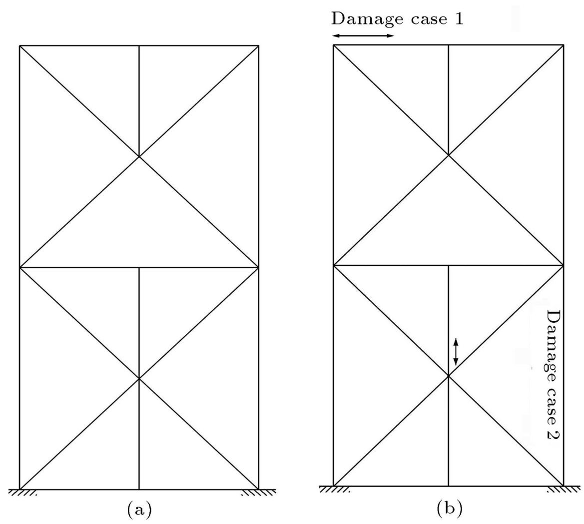

Figure 2. (a) The sketch of the structure. (b) The locations of damage cases.

$$[K_u] + [\Delta K] = [K_d]. \tag{25}$$

Two-Dimensional Jacket Platform

A two-dimensional jacket platform used in this example is shown in Figure 2a. The structure consists of two stories and is fixed to the ground. The height of the two stories and the length of the beams are 18.3 meters. The material properties of the steel tabular members are: elastic modulus $E = 200$ (Gpa), Poisson's ratio $\nu = 0.3$ and mass density $\rho = 7800$ (kg/m^3).

For obtaining a clearer insight into the effect of the number of measured degrees of freedom (number of sensors), we have tried to implement the damage detection procedure with two classes of sensor placement. The first class involves only 4 sensors, whereas the second class utilizes 18 sensors. The position of sensors in each class is illustrated in Figure 3.

In this example, the damage is simulated by reducing the thickness of members. Three damage cases have been investigated. Locations of the damage cases are illustrated in Figure 2b. Damage case number one involves the upper-left beam of the second story with a length of 9.1 centimeters. Damage case number two engages the upper zipper column of the first story with a length of 9.1 centimeters. The third damage case involves both the other two damage cases. Notably, matrix $[\Delta K]$ has some negative values that could not be displayed easily on a three-dimensional plot. Thus the authors made them positive artificially. Stiffness matrix perturbations due to damage cases one, two and three are shown in Figures 4, 5 and 6, respectively. Matrix $[\Delta K]$ has localized damage by displaying greater values at corresponding DOFs, as shown in Figures 4

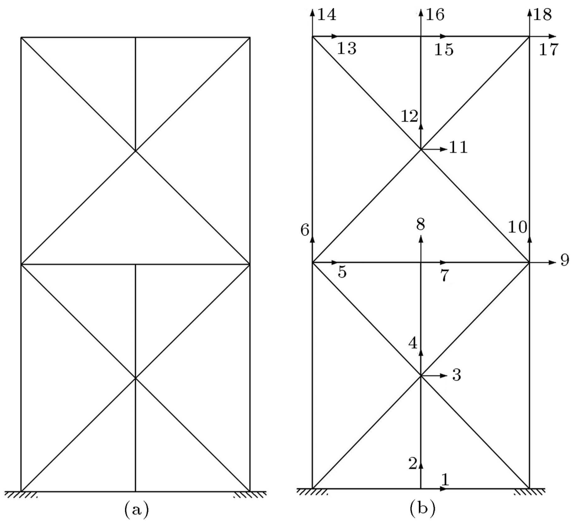

Figure 3. Measured DOFs (sensors' placement). (a) First class; (b) second class.

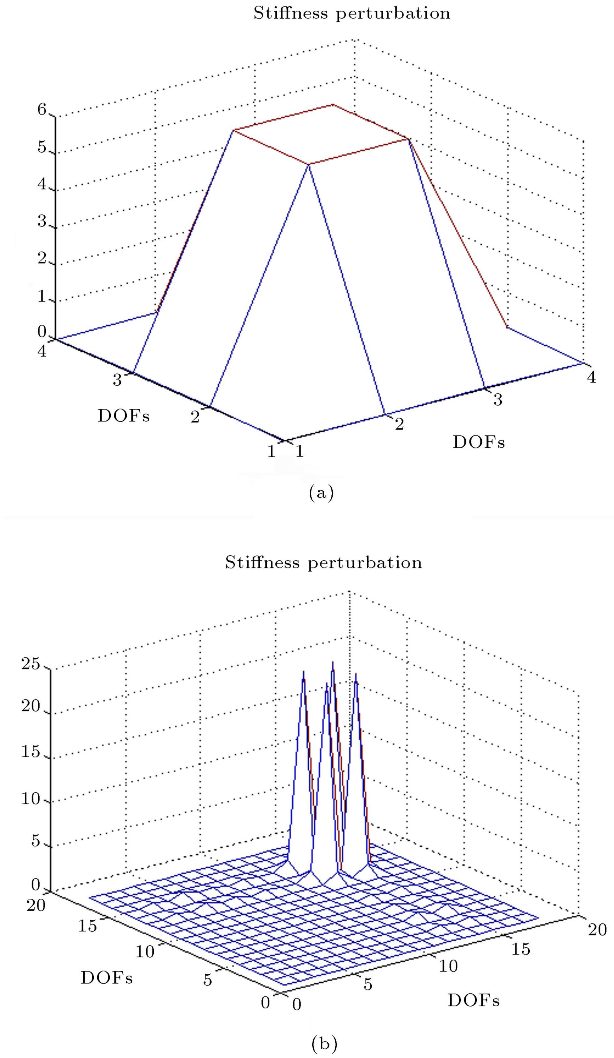

Figure 4. Stiffness matrix perturbation due to damage case one; thickness reduction 89.5%. (a) First class; (b) second class.

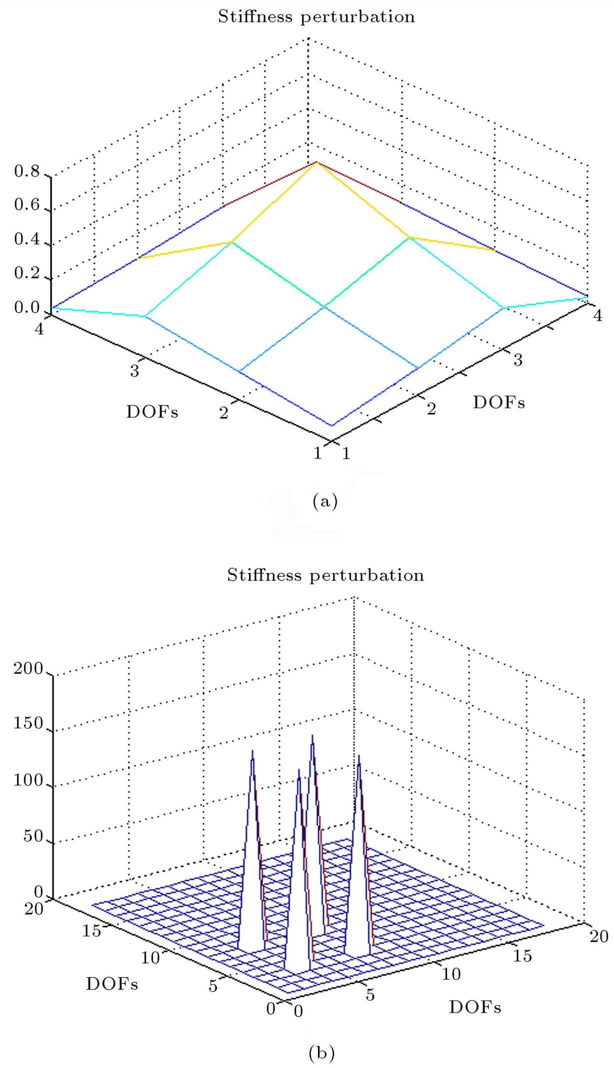

Figure 5. Stiffness matrix perturbation because of damage case two; thickness reduction 47.5%. (a) First class; (b) second class.

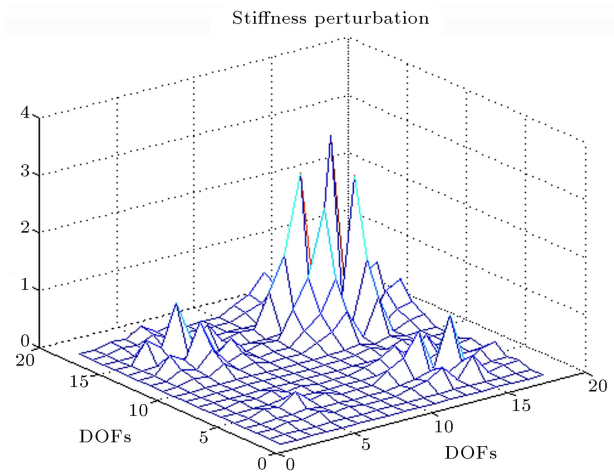

Figure 6. Stiffness matrix perturbation because of damage case three; thickness reduction 47.5% in both damage cases.

and 5b. Figure 5a shows the bad performance of the first class sensor placement for detection of zipper columns, as opposed to the second class (shown in Figure 5b).

In damage case number three, which is a combination of the first and second damage cases, the localization of damage has not occurred properly. Matrix $[\Delta K]$ mainly gets an impact from the damaged beam. The damaged zipper column displayed itself in, somehow, 'seepage' to other DOFs.

As demonstrated in the damage detection formulation, matrix $[D]$ contains damage vectors of p discrete frequency points. Matrix $[D]$, the second class sensor placement for first and third damage cases, is presented in Figure 7. The frequency of 4 (Hz) was the most sensitive frequency which is near to a mode that has a very large mode shape value in the specified damaged beam.

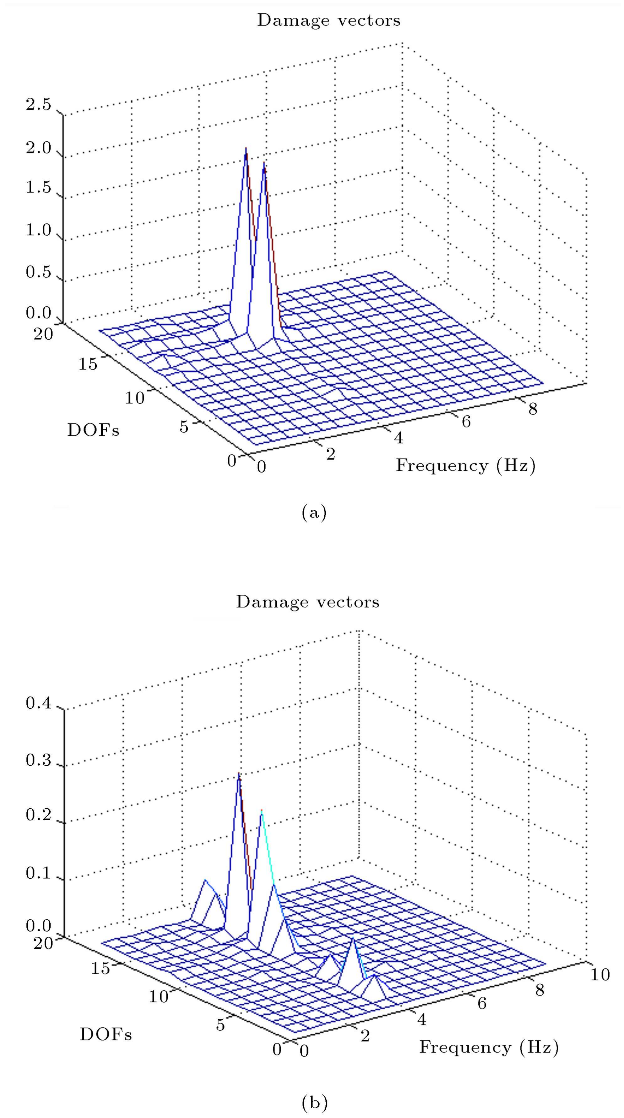

Figure 7. Damage vectors in each frequency point or matrix $[D]$ in second class. (a) Damage case one; (b) damage case three.

PRACTICAL REMARKS

As demonstrated in the damage detection formulation, the proposed method needs some information about the healthy or undamaged structure, as well as data extracted from the damaged structure. The complete matrix of inverse FRF in the undamaged situation is the only necessary data for damage detection before the occurrence of any damage. This set of data should be measured experimentally or computed analytically within a suitable frequency domain, which will probably be selected for damage detection. Now, two questions arise here. The first is how to specify a suitable frequency domain for this purpose. The second is how to select the p discrete frequency points that were used in Equations 9 and 10. For the first question, it should be noted that the 'regions' around the first several modes exhibited the highest level of FRF discrepancy between the damaged and healthy structure. But those regions of the FRF which show low coherence, due to either noise or nonlinearities, must be eliminated [10]. About the second question, Zimmerman et al. [11] investigated the effect of selecting a subset of measured frequency points by five different subset selection techniques. These selection techniques could be characterized as:

1. Evenly spaced throughout the frequency range,
2. Clustered about the resonances,
3. Clustered about the anti-resonances,
4. Placed away from the resonances and anti-resonances,
5. Placed at points of maximum percentage difference between the healthy and damaged FRF.

It was observed in this study that selection technique number 1 (evenly spaced throughout the frequency range) performed best, and provided nearly the same assessment of damage as when the full FRF data set was used.

After damage occurrence, one column of the FRF matrix within the prescribed frequency domain is the only requirement for damage detection purposes. For this purpose, the researcher imparts some dynamic input on the structure in question and obtains the resulting FRF. One column of FRF can be extracted by excitation of just one DOF. It does not matter which DOF is selected as the excited DOF, because it has no influence on the detection capabilities.

It is easier to use FRF in each frequency point directly as $\{X(\omega)\}$ and to consider $\{F(\omega)\}$ unitary at the corresponding degree of freedom. One of the excitation techniques that has been discussed in [12,13] could be applied to offshore jacket platforms.

If the researcher guesses that the existing perturbation is due to stiffness reduction, then displacement ($[Z]$ = dynamic stiffness) should be measured by sensors, otherwise, if $[\Delta Z]$ is an outcome of mass perturbation, then acceleration ($[Z]$ = apparent mass) should be measured.

CONCLUSIONS

This paper investigated the damage detection of offshore jacket platforms utilizing an FRF-based method, which applies only one column of FRF after damage occurrence. This means that, in a damaged condition, only a single arbitrary excitation point is necessary. It was demonstrated that the detection procedure could be done by using a few sensors, but for more accurate detection, it is more suitable to use as many sensors as possible. Application of more sensors will help in two ways:

1. An increment in detection resolution.
2. The global stiffness matrix takes up greater values in each component, therefore, tinier damage will cause larger perturbation and detection ability enhances as the number of measured DOFs (number of sensors) increases.

Notably, in the case of multiple damages, this method sometimes gives an imprecise output. As illustrated in damage case number three, the damage at the beam was dominant and the effect of the damaged zipper column was diminished. Therefore, if the detection outputs show damage in a specified location, we cannot strictly say that there are no other damages in the structure.

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BIOGRAPHIES

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