

# Stochastic Study of the Effect of Strong Ground Motion Variables on Input Energy

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**Abstract.** The input energy to a structure during an earthquake is an important measure of seismic demand. The elastic input energy in a multi-degree-of-freedom system can be computed from a Fourier Amplitude Spectrum (FAS) and the real part of the relative velocity transfer function of different modes. One of the essential characteristics of the seismological method is that it distills what is known about the various factors affecting ground motions into different functional forms and, for this reason, the modal analysis-based method in the frequency domain is very efficient in the computation and evaluation of earthquake input energy. The earthquake input energy reliability is dependent on ground motion variables. In this paper, to compare the effect of strong ground motion variables, the contribution of these sources of variability to the input energy's uncertainty is examined by using a stochastic analysis. The analytical results show that earthquake source factors and soil condition variables are the main source of uncertainty in the input energy spectra, while path variables, such as source-site distance, anelastic attenuation and upper crust attenuation, have relatively little effect.

Keywords: Input energy; Modal analysis; Frequency domain; Seismological Fourier spectrum.

## INTRODUCTION

The idea of using energetic concepts in structural design to resist earthquake has been discussed early in the development of earthquake engineering. Hudson and Housner, at the end of the 1950s, demonstrated that structures failed when the energy demand imposed by an earthquake exceeded the energy supply, determined by structural properties [1,2]. Most energy design methods are based on the premise that the energy demand can be predicted, therefore, suitable member sizes can be provided to dissipate the input energy within an acceptable limit state [3]. The input energy to a structure during an earthquake is an important measure of both the ground motion characteristics and structural properties. The earthquake input energy transmitted to a structure consists of the kinetic energy, elastic strain energy, damping energy, and hysteretic energy [4]. Zahrah and Hall [5] and Akiyama [4] believe that ductility and damping do not have a significant influence on the earthquake input energy. Therefore, in developing an energy-based design approach and assessing the damage potential of structures, one must know the earthquake input energy.

Earthquake input energy has usually been computed in the time domain. The time-domain approach has several advantages, e.g. the availability for nonlinear structures, the description of the time-history response of input energy and the possibility of expressing the input energy rate. But, the time domain approach is not necessarily appropriate for probabilistic analysis. For that purpose, the frequency domain approach is suitable because it uses the Fourier Amplitude Spectrum (FAS) of input ground accelerations and the time invariant transfer functions of the structure [6-10]. Prior research demonstrates that the input energy spectrum could be exactly made with the FAS and without information of phases [7,10].

For regions where recorded ground motion data are scarce, it becomes imperative to use physical models to represent the ground motion generation and propagation. An advantage of physical over empirical models is that meaningful parameters pertaining to source, path attenuation and site effects can be inferred from the data, thus promoting physical understanding of the underlying processes of strong ground motion generation and attenuation [11,12]. Also, most studies

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based on combined recorded ground motion data sets came from different-size earthquakes and were recorded in different regions. Thus, the isolation of individual factors from various influences is limited by the nature of the data.

This paper studies the effect of variability in ground motion variables (such as source, path and site) on the stochastic input energy of the different frames modeled. The relative contributions of these sources of variability to the overall variability in input energy are assessed. Therefore, in developing an energybased design approach, these results open the door to understanding appropriate estimating of the variables for the stochastic generation of ground motions; in some part with a lack of sufficient recorded data.

#### COMPUTATION INPUT ENERGY

Uang and Bertero [13] proposed two procedures for computing the earthquake input energy: one based on absolute motion and the other on relative motion. The difference between the two procedures is less important in damage assessment, and the damage potential of structures is independent of the approach used. Bruneau and Wang [14], and Chopra [15] believe that the input energy, in terms of relative motion, is more meaningful than the input energy in terms of absolute motion, since internal forces within structures are computed using relative displacements and velocities. Therefore, the procedure based on relative displacement is used in this study. Consider the equation of motion of a proportionally damped linear elastic Multi-Degree-Of-Freedom (MDOF) system subjected to unidirectional horizontal ground acceleration,  $\ddot{u}_{q}(t)$ :

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\}\ddot{u}_g, \tag{1}$$

where  $\{x\}$  is the vector relative displacement and [M], [K] and [C] are the mass, stiffness and damping matrix, respectively. An over-dot denotes differentiation with respect to time and:

$$\{1\} = \{1 \cdots 1\}^T$$

The input energy to a MDOF shear building system by ground motion from t = 0 to  $t = t_0$  (time of loading) can be defined by the work of the ground on the MDOF system [8,13], and is expressed by:

$$E_I = \int_0^{t_0} \{1\}^T [M](\{1\}\ddot{u}_g + \{\ddot{x}\})\dot{x}dt.$$
(2)

Integration by parts of Equation 2 and assumption  $\{\dot{x}\} = \{0\}$  at t = 0, and  $\dot{u}_g = 0$  at t = 0 and  $t = t_0$  provide:

$$E_I = -\int_0^{t_0} \{\dot{x}\}^T [M] \{1\} \ddot{u}_g dt.$$
(3)

In view of the definition of an Inverse Fourier trans-

form, the relative nodal velocity can be expressed as:

$$\{\dot{x}\}^T = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{\dot{X}\}^T e^{i.\omega.t} d\omega, \qquad (4)$$

where  $\{\dot{X}\}$  denotes the Fourier transformation of  $\{\dot{x}\}$ , which is the vector relative displacement. Replacing Equation 4 in Equation 3 and changing the order of integration and acceptance of  $\int_{-\infty}^{\infty} \ddot{u}_g e^{i\omega t} dt$  as the Fourier transform of  $\ddot{u}_g$ , evaluated at frequency  $-\omega$ , lead to:

$$E_I = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \{\dot{X}\}^T [M] \{1\} A(-\omega) d\omega, \qquad (5)$$

where  $A(\omega)$  is the Fourier transform of ground acceleration. Let the following coordinate transformation be introduced:

$$\{x\} = [\Phi]\{q\}, \tag{6}$$

where  $[\Phi]$  is the modal matrix and:

$$\{q\} = \{\cdots q_n \cdots \}^T. \tag{7}$$

Substitution of the coordinate transformation  $\{\dot{x}\} = [\Phi]\{\dot{q}\}$  its Fourier transform  $\{\dot{X}\} = [\Phi]\{\dot{Q}\}$ , and the velocity transfer relation  $\{\dot{Q}\} = \{H_v\}A(\omega)$  into Equation 5 provides:

$$E_{I} = -\frac{1}{\pi} \int_{0}^{\infty} \operatorname{Re}[\{H_{V}\}^{T}[\Phi]^{T}[M]\{1\}] \Big| A(\omega) \Big|^{2} d\omega, \quad (8)$$

where  $\{H_V\}$  is the velocity transfer function vector, with respect to base acceleration, and expressed by:

$$\{H_V\} = \{\cdots H_{V_n} \cdots \}^T, \tag{9a}$$

$$H_{Vn} = -i\omega/(\Omega_n^2 - \omega^2 + 2i\xi_n\Omega_n\omega), \qquad (9b)$$

where Re[.] denotes the real part of a complex number, and  $\Omega_n$  and  $\zeta_n$  are natural frequency and damping ratio for the *n*th mode.

Equation 8 indicates that the earthquake input energy to damped linear elastic MDOF system depends explicitly on the dynamic properties of the system and smoothed version of the amplitude spectrum of ground motion and does not depend on the phase information of ground motion. Equation 8 implies that the energy transfer function and FAS information of ground motion play an important role in the evaluation of earthquake input energy. It should be noted that there is a vast amount of research aimed towards predicting amplitude Fourier; coming especially from the engineering seismology field. Such models have usually been developed in the context of the stochastic modeling approach and random theory [16].

# FOURIER AMPLITUDE SPECTRUM BASED ON SEISMOLOGICAL VIEW

In the view of the seismological relation, FAS can be expressed as a product of number factors [12,16]:

$$A(M_0, R, \omega) = S(M_0, \omega) \cdot G(R) \cdot An(\omega) \cdot D(\omega) \cdot V(\omega), \quad (10)$$

where:

$S(M_0,\omega)$	is the source factor,
G(R)	is the geometric attenuation factor,
$An(\omega)$	is the anelastic whole path attenuation
	factor,
$D(\omega)$	is the upper crust attenuation factor,

 $V(\omega)$  is the upper crust amplification factor.

Based on Brune's source model and typical geometric, anelastic whole path and upper crust attenuation functions [11,12]:

$$A(M_0, R, \omega) = \frac{\langle R_{\theta\phi} \rangle PF}{4\pi\rho_s \beta_s^3} \times \frac{M_0 \omega^2}{1 + (\omega/\omega_c)^2} \times \frac{1}{R}$$
$$\times \exp\left(\frac{-0.5\omega R}{C_Q \cdot Q_0 \left(\frac{1}{2\pi}\omega\right)^n}\right) \times \exp\left(-\frac{\omega\kappa}{2}\right)$$
$$\times V(\omega), \tag{11}$$

where  $M_0$  is the seismic moment, R is the sourcesite distance,  $\omega$  is the circular frequency of the wave,  $\langle R_{\theta\phi} \rangle$  is the wave radiation factor (taken here as 0.55), F is free surface amplification factor (taken to be 2), and P is the factor that partitions the energy into orthogonal directions (taken to be  $\sqrt{2}/2$ ).

The parameter of  $\rho_s$  is the density of the rock within the top 10 km of the earth crust, and is typically 2.8 ton/m<sup>3</sup>.  $\beta_s$  is the shear-wave velocity in the vicinity of the source. The geometrical attenuation factor, G(R), which represents geometrical damping, is given by a piecewise continuous series of straight lines [12]. In this study, it is assumed to be  $R^{-1}$  for simplicity. The loss of energy along the wave travel path is very complex. The An(f) factor includes all losses that have not been accounted for by the geometrical attenuation factor and is defined by the exponent expression [17].  $Q_0$  and n are the regional dependent factors of the wave transmission quality factor,  $Q(\omega)$ , which is defined by the exponent expression. The parameter,  $C_Q$ , is the seismic velocity used in the determination of  $Q(\omega)$ .

The attenuation (or diminution) operator,  $D(\omega)$ , in Equation 10 accounts for the path independent loss of high-frequencies in ground motions. This loss may be due to a source effect, a site effect or a combination of these effects, where  $\kappa$  is the attenuation parameter that accounts for the high-frequency cutoff. In Equation 10,  $V(\omega)$  is the upper crust amplification factor and the quarter-wavelength method proposed by Boore and Joyner [18] is used to model the amplification factor of site soil. They proposed that the site-amplification factor,  $V(\omega)$ , is a function of the average shear wave velocities,  $(\bar{V}_S)$ , representing the soil conditions in the upper 30 m.

Following the Brune assumption, the corner frequency is given by the following equation:

$$\omega_c = (2\pi) \times 4.9 \times 10^6 \beta_s (\Delta \sigma / M_0)^{1/3}, \tag{12}$$

where, in this equation, the stress drop,  $\Delta \sigma$ , has units of bars,  $\omega_c$  has units of Hz;  $\beta_s$  in km/s, and  $M_0$  has units of dyne-cm. The seismic moment,  $M_0$ , is often expressed in terms of the moment magnitude,  $M_w$ , which is defined as follows [19]:

$$M_w = \frac{2}{3} \log M_0 - 10.7. \tag{13}$$

By substituting Equations 11-13 in Equation 8, the corresponding input energy can be computed. The presented relation in calculation input energy is based on the well-known stochastic model for generating strong motion, which is customarily used to calculate earthquake design in places where there is a lack of sufficient recorded data. The stochastic point source model, based on the static corner frequency used here, despite some theoretical deficiencies, gives results that are similar to those of the finite-fault models, at least in medium and far away distances from the fault and for the frequency ground motion of most interest to engineers [20,21].

Figure 1 shows three shear building models, denoted as case 1 to case 3, which are one story, two stories and five stories in height. For all models, the floor masses, the story stiffness and damping have been illustrated.

To examine the accuracy of the presented frequency domain method, the earthquake input energy



Figure 1. Models of frames [15].

has been computed by FAS based on seismological information, recorded ground motions, and the time domain method for different recorded ground motions [18,22,23] as indicated in Table 1. The earthquake input energy for different models is shown in Table 2. Discrepancies of several percent can be found in the frequency domain and time domain methods based on FAS and time-series of recorded ground motions. These discrepancies are caused by the difference in the integration procedure. Figure 2 shows an example of input energy spectra recorded ground motion. The acceptable mismatch between results based on seismological FAS and recorded information is shown in Table 2 and Figure 2. For regions where recorded ground motion data are scarce, it becomes imperative to use the proposed models to represent the earthquake input energy in the energy based design of structures.

#### ANALYSIS METHOD

Variations in source, path and site variables of ground motions affect the uncertainty in the input energy spectra. To study the stochastic input energy spectra, the earthquake's magnitude,  $M_w$ , source-site distance, R, static stress drop,  $\Delta \sigma$ , quality factor, Q, highfrequency attenuation parameter,  $\kappa$ , and amplification factors,  $V(\omega)$ , were modeled as random variables. Each



Figure 2. Input energy spectra for  $\xi = 5\%$  damping of 1985, Nahanni, Canada ground motion. Continuous line: computation in the frequency domain based on FAS of recorded ground motion; dashed line: computation in the time domain, and bold dashed line: computation in the frequency domain based on seismological FAS.

random variable is modeled as follows [24]:

$$Y = \mu_Y (1 + \alpha_Y), \tag{14}$$

where  $\mu_Y$  is the mean value and  $\alpha_Y$  is a random variable with a zero mean. The stochastic input energy of frames resulting from the variability of ground motion variables is evaluated by using the perturbation

Random Variables	1990 Upland, California	1979 Imperial Valley, California	1985, Nahanni, Canada
$M_w$	5.6, 7.5	6.5	6.8
Source-site distance, $R$ (km)	74	26	23
Density, $\rho_s ~({ m gr/cm^3})$	2.7	2.8	2.8
Shear-wave Vel., $\beta_s$ (km/s)	3.7	3.7	3.7
Stress drop, $\Delta \sigma$ (bar)	70	120	134
Quality factor, $Q(\omega)$	$72.9\omega^{0.56}$	$301\omega^{0.5}$	$351\omega^{0.36}$
Kappa parameter, $\kappa$ (s)	0.058	0.04	0.005
Amplification factor, $V(\omega)$	Boore [22]	Boore and Joyner [18]	Boore et al. [23]

Table 1. Set of variables of recorded strong ground motion.

Table 2. Earthquake input energy by frequency and time domain analysis for cases 2 and 3 (unit is Joule).

		Upland, $M_w$ 5.6	Upland, $M_w$ 7.5	Imperial Valley	Nahanni
	T	18.7	1.42 E+3	8.20 E+3	7.0 E+4
Model 2	F1	18.9	1.43 E+3	8.22 E+3	6.97 E+4
	F2	20.0	1.64 E+3	8.91 E+3	6.60 E+4
	T	179	1.41 E+3	4.15 E+4	1.98 E+5
Model 3	F1	178	1.42 E+4	4.18 E+4	1.97 E+5
	F2	167	1.66 E+4	4.88 E+4	1.83 E+5

T: Time domain;

F1: Frequency domain based on Fourier amplitude spectra of recorded earthquake;

F2: Frequency domain based on seismological Fourier amplitude spectra.

method. The perturbation method which is based on a Taylor series expansion of the function of random variables [25,26] that describes the input of the system, is used to evaluate the mean and variance of the input energy. The relative contributions of the variances in the random variables, and the covariance between these random variables to the variance in input energy of the structural frame can be calculated by the following equation:

$$Cov[EI, EI] = \sum_{i=1}^{n} \sum_{j=1}^{n} EI_{i}^{1} (EI_{j}^{1})^{T} Cov[\alpha_{i}, \alpha_{j}], \quad (15)$$

where n is the number of random variables, and  $EI_i^1$ and  $EI_j^1$  are the coefficient vectors of the first-order rates of change. These coefficients are evaluated at the mean value of all random variables. The covariance matrix of random variables is defined by the following equation:

$$Cov[\alpha_i, \alpha_j] = V_i V_j \rho_{ij}, \tag{16}$$

where  $V_i$  and  $V_j$  are the Coefficients Of Variation (COV) of the random variables  $\alpha_i$  and  $\alpha_j$ , and  $\rho_{ij}$  is the correlation coefficient of  $\alpha_i$  and  $\alpha_j$ . For simplicity,  $\rho_{ij}$  is taken as 1.0 for each random variable to itself, as 0.5 for the earthquake magnitude to stress drop, earthquake magnitude to kappa, kappa to quality factor, kappa to amplification factor, kappa to stress drop, and as 0.0 for the other variables [18,27,28]. The overall variance in input energy of the structural frame is affected by the variances in each of the source, path and site variables. The COV of these variables plays an important role in the variation of the response. The previous studies revealed that the variation of moment magnitude and distance is less than other variables [18, 29-33]. In this study, the COV is assumed to be equal to 0.02, 0.05, and 0.2 for earthquake moment magnitude, source-site distance and all other variables, respectively.

## DISCUSSION

The formulation in the frequency domain is essential for deriving arbitrary sensitivities of the input energy and with respect to uncertain earthquake variables. The total input energy of the frame is evaluated based on the values of all structural and ground motion variables. At first, we deal with case 1, the Single Degree Of Freedom (SDOF) system, as an example. The effect of the source-site distance on the earthquake input energy spectra can be examined by comparing response spectra from the same source, path, and site variables at different distances. The earthquake magnitude influences the input energy shape and values of the input energy spectrum. The mean values of



Figure 3. The mean values of input energy for different earthquake magnitudes. These energy spectra were normalized by the peak amplitude in the case of  $M_w7$ .

the input energy at different periods are illustrated in Figure 3 for different earthquake magnitudes. These energy spectra were normalized by the peak amplitude in the case of  $M_w7$  to compare the spectral shape. Note, in Figure 3, how the input energy spectrum's value changes as the earthquake's magnitude grows, which shows a steady increase in medium and high rise structures. So far, in structures with periods smaller than 0.1 s, the magnitude of earthquake has a minor influence on input energy amplification. Figure 4 compares the mean values of input energy at three different distances. The normalized spectral amplitude initially gradually increases with increasing period, reaches a peak value, and slowly decreases at high periods. The spectral amplitude of each distance group steadily decreases with increasing distance.

Figure 5 shows the effect of soil conditions on the input energy spectra for the SDOF system. The normalized mean values for generic rock ( $V_{30} = 620 \text{ m/s}$ ) and very hard rock ( $V_{30} = 2880 \text{ m/s}$ ) are shown. The amplification for very hard rock is substantially below



Figure 4. Comparison of input energy spectra in three different distances. These energy spectra were normalized by the peak amplitude in the case of R = 40 km.



Figure 5. The effect of soil conditions on input energy spectra for SDOF system. These energy spectra were normalized by the peak amplitude in the case of generic rock.

that for a generic rock site. The spectral values are very large for a wide period range, especially for generic rock. As seen from the figure, for generic rock, the spectrum has a maximum at a period of nearly 1.0 s and the curve has slopes on both sides of this point.

The overall variance in the input energy is affected by the variances in each of the random variables. Figure 6 shows the relative contributions of the variances to the variables and the covariance between them to the variance in the input energy, in cases 1 and 2, when the distance takes the values of 20, 40, and 80 km at periods of 0.3 and 1.0 s. Figure 6 indicates that the source and soil condition variables are the main sources of uncertainty affecting the probabilistic input energy of the structures. The input energy spectral values decrease with an increase in distance but the relative contribution of variables is not more sensitive to a variation of distance. Figure 6 shows that the relative contribution of earthquake magnitude to total variance in input energy at near distance is more important than at far distances.

The study of attenuation of seismic waves is useful in predicting ground motion in seismic hazard analysis. The attenuation of material is often modeled by multiplying by the anelastic path,  $An(\omega)$ , and by using a high-cut filter,  $D(\omega)$ . The high-cut filter process is described by "kappa" and has often been used to refer more specifically to the distance-independent attenuation operator. The anelastic attenuation's effect is described through the Q factor, which is the distancedependent operator. Figure 6 shows that the relative contribution of kappa is not a function of distance, but the relative contribution of the Q factor at near distance is slightly larger than for far distances.

The radiated far field spectrum of shear wave could be interpreted in terms of a simple point source model with just two source parameters; seismic moment and stress drop. In Brune's model [11], stress



Figure 6. Relative contributions of variances in variables and the covariance between them to the variance in the input energy in the SDOF and 2DOF systems when the source-site distance takes the values of 20, 40 and 80 km at main periods of 0.30 and 1.0 s. The COV of moment magnitude, distance and other variables are assumed to equal 0.02, 0.05 and 0.2, respectively.

drop replaces the fault dimension in the source description. Seismic moment is proportional of stress drop using  $M_0 \approx \Delta \sigma r^3$ , where r is fault dimension. Figure 6 demonstrated that the relative contribution of stress drop, in respect to distance is proportional to the relative contribution of earthquake magnitude, but, variation of earthquake magnitude is more sensitive to the variation of stress drop.

Figure 7 shows the relative contribution of variances in case 2, 2DOF, computed for three different magnitudes. This figure indicates how the relative contribution of earthquake magnitude changes as magnitude grows. This figure shows that the relative contribution of other variables is not more sensitive to a variation in earthquake magnitude.

Figure 8 shows the relative contributions of input energy in cases 1 and 3, at different periods of 0.3, 1.0, 2.0 and 4.0 s, for two groups of soil condition. This figure shows that the relative contribution of frequency dependent variables is dependent on the shape of the input energy spectrum. In the medium period, which is the almost maximum and descending branch of the spectrum, the relative contribution of these variables steadily changes with increasing periods.

As a simple way of capturing the variance of







Figure 8. The relative contributions of the input energy in the SDOF system at different periods. The COV of moment magnitude, distance and other variables are assumed to equal 0.02, 0.05 and 0.2, respectively.

Q, the attenuation operator is made up of three piecewise-continuous line segments [12]. The outer lines are specified by slopes and intercepts at specified reference frequencies, and the middle line joins the outer lines between frequencies of approximately 0.2 and 5 Hz. Figure 8 demonstrates that the relative contributions of the Q factor do not vary with the structural period in the range of 0.3 to 4 s. As mentioned above, this figure illustrates that the relative contribution of earthquake magnitude at high periods is more pronounced than at short periods.

## CONCLUSIONS

In an energy-based design approach, once the energy demand for a structure is estimated from the strong ground motion, the damage potential can be quantified by a combination of response and energy parameters. A sufficient strength and energy dissipation capacity should be provided in the structure for an acceptable damage threshold, i.e. a desired performance objective. Input energy is a measure of the energy that the earthquake introduces to the structure. This energy has to be dissipated and absorbed mainly through damping and hysteretic cycles and, therefore, it is an indication of potential structural damage.

In developing an energy-based design approach and assessing the damage potential of structures, formulation of the earthquake input energy in the frequency domain is appropriate for computing the input energy spectrum when the FAS information is available. This formulation requires only the FAS of input ground motion and the real part of the relative velocity transfer function. Some uncertainties in the design of safer structures result from lack of information due to the low occurrence rate of large earthquakes, and this problem cannot be resolved in a practical time span. It is, therefore, strongly desirable to develop a robust method, taking into account these uncertainties with limited information and enabling the design of safer structures. One of the essential characteristics of the seismological method is that it distills what is known about the various factors affecting ground motions into different functional forms. The presented expression in this study provides an important basis for a wider use of seismological theory in the understanding of the relation between seismological and structural variables.

The results reveal that source and soil condition variables are the main sources of uncertainty affecting the probabilistic input energy of the structures, while path effect variables, such as source-site distance, anelastic and upper crust attenuation properties have relatively little effect. The relative contribution of the earthquake magnitude changes as the magnitude grows, showing a steady increase, and the relative contributions of other variables are not more sensitive to variation of earthquake magnitude. The amplification for very hard rock is substantially below that for generic rock site. The relative contributions of the frequency dependent variables are dependent on the shape of the input energy spectrum.

The stochastic point source model that is based on a static corner frequency, which was used here, despite some theoretical deficiencies, gives similar results to the dynamic corner frequency version for medium and far away distances from the fault, and for ground motion frequencies of most interest to engineers (f >0.6 Hz). The dynamic behavior of a building under soil-structure interaction is very complicated due to frequency-dependent characteristics, and the frequency domain method can be developed for evaluating the earthquake input energy in these systems.

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