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Steady-State Stresses in a Half-Space Due to Moving Wheel-Type Loads with Finite Contact Patch

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Abstract. *In this paper, the steady-state stresses in a homogeneous isotropic half-space under a moving wheel-type load with constant subsonic speed, prescribed on a finite patch on the boundary, are investigated. Navier's equations of motion in 2D case were modified via Stokes-Helmholtz resolution to a system of partial differential equations. A double Fourier-Laplace transformation procedure was employed to solve the system of partial differential equations in a new moving reference system, regarding the boundary conditions. The effects of force transmission from the contact patch to the half-space have been considered in the boundary conditions. Utilizing a property of Laplace transformation leads to transformed steady-states stresses for which inverse Fourier transformation yielded the steady-state stresses. Considering two types of uniform and parabolic force transmission mechanism and a comparison between the pertaining results demonstrated that the parabolic load transmission induce lower stresses than the uniform one. Results of the problem for various speeds of moving loads showed that the stresses increase as the moving loads' speeds increase to an extremum speed known as CIS. After the CIS speed, stresses' absolute values decrease for higher speeds. Eventually CIS values for homogeneous half-spaces with different material properties were obtained.*

Keywords: *Moving load; Half-space; Wave propagation; Contact patch.*

INTRODUCTION

The importance of the transportation engineering is intensifying due to the increase in all kinds of communications. New developments in technologies are employed to provide more facilities in transportation engineering. Hence structures which are modeled as half-spaces are subjected to larger moving loads with higher speeds. Thus understanding the exact dynamic behavior of structures subjected to moving loads, which enhance the design and construction of these structures, is of significant interest. Dynamic responses of simple structures such as beams and plates, subjected to moving loads, have been investigated by many researchers. Recent investigations for simple structures under moving loads are devoted to special conditions and case studies. Andersen et al. [1] reviewed numerical methods for analysis of structure and ground vibration from moving loads. They investigated the main theoretical aspects of analysis of moving loads in a local coordinate system, and addressed the steps in the finite-element and boundary element method formulations. They also studied, in their work, problems in describing material dissipation in the moving reference system.

In case studies of simple structures, identification of moving loads on bridges was investigated by Yu and Chan [2], who provided a review on recent progresses on identification of moving loads on bridges. Yu and Chan in their study introduced the theoretical background of four identification methods and performed numerical simulations, illustrative examples and comparative studies on the effects of different parameters.

Xia et al. [3] investigated the dynamic interaction of a long suspension bridge under running trains. They found that the dynamic responses of the long suspension bridge under the running train are relatively

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small, and the effects of bridge motion on the runability of the railway vehicles are insignificant.

Mehri et al. [4] presented an exact and direct modeling technique based on the dynamic Green function for modeling beam structures with various boundary conditions, subjected to a constant load moving at constant speed.

Influences of the inertial loads on simple structures were investigated in many papers. Moving loads with inertial effects are known as moving masses. In this field, Akin and Mofid [5] introduced an analytical-numerical method to solve the governing partial differential equation of the beam under a moving mass. They ignored the effects of the speed of the moving load. Dehestani et al. [6] investigated the stresses in thin-walled finite beams with various boundary conditions subjected to a moving mass under a pulsating force. They have used separation of variables method to solve the partial differential equations of beams wherein the effects of the speeds of moving masses were considered.

Accomplished works on the semi-infinite regions under moving loads are not numerous compared with works on the influences of moving loads on simple structures such as beams and plates. In this field, Celebi and Schmid [7] studied ground vibrations induced by moving loads. They presented two numerical frequency domain methods in order to analyze the propagation of surface vibrations in the free field adjacent to railway lines induced by specific moving loads acting on the surface of a layered and homogeneous half-space with different material properties.

Bierera and Bode [8] presented a semi-analytical model in time domain, for moving loads with subsonic speeds. They obtained the vertical displacements induced by moving loads with constant and time-varying amplitudes at a fixed observation point at the surface of the 3D half-space in time domain. The authors also introduced a semi-analytical, discrete model based on Green's functions for a suddenly applied, stationary surface point load with Heaviside time dependency to solve the non-axisymmetric, initial boundary value problem. They compared results for the transient and the steady-state ground motions to the analytical solutions.

Galvin and Dominguez [9] employed a 3D time domain boundary element formulation for elastic solids to evaluate the soil motion due to high-speed moving loads and in particular, to high-speed trains. Their results demonstrated that the boundary element approach can be used for actual analyses of high-speed train-induced vibrations.

In this paper, steady-state stresses in a homogeneous, isotropic half-space under a moving load with constant subsonic speeds are investigated. Transmission of the load to the half-space is accomplished through a finite contact patch which is more realistic and was ignored in previous studies.

MATHEMATICAL MODEL

Description of the Model

Consider a homogeneous, isotropic, elastic half-space $y \geq 0$ with no body forces, as shown in Figure 1, subjected to a wheel-type load with a finite rectangular, $2a \times 2b$, patch. Half-space is under the action of a load $P(x,t)$ which is prescribed through the patch on the surface boundary and moves at right angle to the positive direction of axis x from infinity to infinity at uniform speed V.

In Figure 1, ρ represents the density of the half-space material and, λ and μ are Lame's elastic constants for plane stress condition in which:

$$\lambda = \frac{Ev}{1-v^2}, \tag{1}$$

$$\mu = \frac{E}{2(1+v)}, \tag{2}$$

where E and v are elastic modulus and Poisson's ratio, respectively.

Governing Equations

Navier's Equation of motion for a homogeneous, isotropic half-space with no body forces can be expressed as:

$$\rho\ddot{\mathbf{u}} = \mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla\nabla.\mathbf{u}, \tag{3}$$

where $\mathbf{u}$ stands for the displacement vector component. Applying the Stokes-Helmholtz resolution to the displacement field yields:

$$\mathbf{u} = \mathbf{grad}\phi + \mathbf{curl}\psi, \tag{4}$$

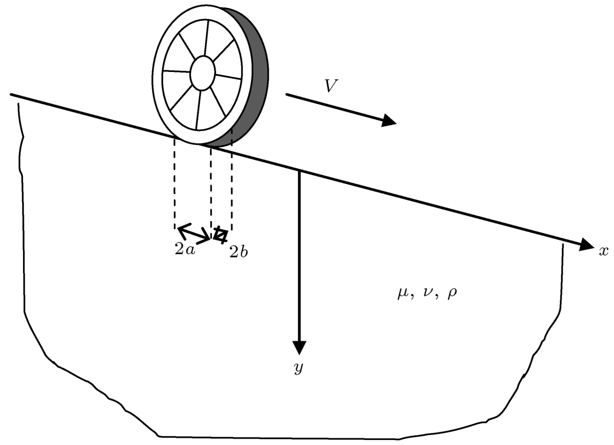

Figure 1. Moving wheel-type load on a homogeneous, elastic, isotropic half-space.

where ϕ and ψ are scalar and vector-valued functions, respectively, for which on account of the definiteness, we should have:

$$\psi_{i,i} = 0. \tag{5}$$

Substituting Equation 4 into Equation 3 leads to the fact that Navier's equation is satisfied, if the functions ϕ and ψ are solutions of the wave-type equations:

$$\nabla^2 \phi = \frac{1}{c_1^2} \frac{\partial^2 \phi}{\partial t^2}, \tag{6}$$

$$\nabla^2 \psi_k = \frac{1}{c_2^2} \frac{\partial^2 \psi_k}{\partial t^2}, \tag{7}$$

where $c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$ and $c_2 = \sqrt{\frac{\mu}{\rho}}$ are dilatational disturbance propagation velocity and shear waves propagation velocity, respectively.

Boundary Conditions

Surface of the half-space is assumed to be subjected to wheel-type loads. In order to investigate the force transmission mechanism in contact patch between load and the boundary, two types of influence mechanism are considered. For the first case, assume that the load $Q(t)$ is applied uniformly on the patch for which the traction on the surface takes the form:

$$P_1(x,t) = \frac{Q(t)}{4ab} \{\mathcal{H}(x-Vt+a) - \mathcal{H}(x-Vt-a)\}, \tag{8}$$

where $\mathcal{H}$ denotes the Heaviside function. It is much more accurate to consider the normal force (pressure) distribution on the contact patch to be parabolic [10]. Thus in the second case, the load $Q(t)$ is applied through a parabolic function to the patch. Therefore after some manipulations the traction on the surface would be obtained as:

$$P_2(x,t) = \frac{3Q(t)}{8ab} \left(1 - \left(\frac{x-Vt}{a}\right)^2\right) \{\mathcal{H}(x-Vt+a) - \mathcal{H}(x-Vt-a)\}. \tag{9}$$

Hence the problem is to obtain the distribution of the stresses in a 2D elastic half-space medium with the boundary conditions for $y = 0$ to be:

$$\sigma_{22} = -P_i(x,t), \tag{10}$$

$$\sigma_{12} = 0. \tag{11}$$

The other boundary conditions can be obtained from the fact that infinitely far into the bulk of the medium the stresses must be vanished. Thus the functions ϕ and ψ must be obtained in such a way that all stresses approach to zero as y tends to infinity.

ANALYTICAL SOLUTION

Regardingthe special type of the influence for the moving loads on structures, it is convenient to implement a moving reference system instead of a fixed reference system as:

$$\begin{cases} \eta = x - Vt \\ y = y \\ \tau = t \end{cases} \tag{12}$$

Thus the governing Equations 6 and 7 take the forms:

$$(1-k_1^2V^2)\frac{\partial^2\phi}{\partial\eta^2} - k_1^2\frac{\partial^2\phi}{\partial\tau^2} + 2k_1^2V\frac{\partial^2\phi}{\partial\eta\partial\tau} + \frac{\partial^2\phi}{\partial y^2} = 0, \tag{13}$$

$$(1-k_2^2V^2)\frac{\partial^2\psi}{\partial\eta^2} - k_2^2\frac{\partial^2\psi}{\partial\tau^2} + 2k_2^2V\frac{\partial^2\psi}{\partial\eta\partial\tau} + \frac{\partial^2\psi}{\partial y^2} = 0, \tag{14}$$

where $k_1^2 = c_1^{-2}$ and $k_2^2 = c_2^{-2}$. In most of the applicable cases, speeds of the surface moving loads are not higher than c_2 of that half-space. Hence in this study, it is assumed that the speeds are in subsonic range in which $k_2^2V^2 < 1$.

In order to solve the system of Equations 13 and 14 as the governing equations (regarding the boundary conditions) a concurrent Fourier-Laplace integral transformation of position and time is considered as:

$$\hat{\bar{\phi}}(\xi,y,s) = \int_0^\infty \int_{-\infty}^\infty \phi(\eta,y,\tau) e^{i\xi\eta} e^{-s\tau} d\eta d\tau \to \phi(\eta,y,\tau)$$
$$= \frac{1}{4\pi^2 i} \int_{-\infty}^\infty \int_{s_1-i\infty}^{s_1+i\infty} \hat{\bar{\phi}}(\xi,y,s) e^{s\tau} e^{-i\xi\eta} ds d\xi, \tag{15}$$

$$\hat{\bar{\psi}}(\xi,y,s) = \int_0^\infty \int_{-\infty}^\infty \psi(\eta,y,\tau) e^{i\xi\eta} e^{-s\tau} d\eta d\tau \to \psi(\eta,y,\tau)$$
$$= \frac{1}{4\pi^2 i} \int_{-\infty}^\infty \int_{s_1-i\infty}^{s_1+i\infty} \hat{\bar{\psi}}(\xi,y,s) e^{s\tau} e^{-i\xi\eta} ds d\xi. \tag{16}$$

Employing the concurrent Fourier-Laplace integral transformation to Equations 13 and 14, regarding the boundary conditions, would give rise to:

$$\hat{\bar{\phi}}(\xi,y,s) = A(\xi,s) \exp\left(-(k_1^2(s+iV\xi)^2 + \xi^2)^{\frac{1}{2}} y\right), \tag{17}$$

$$\hat{\bar{\psi}}(\xi,y,s) = B(\xi,s) \exp\left(-(k_2^2(s+iV\xi)^2 + \xi^2)^{\frac{1}{2}} y\right), \tag{18}$$

where functions $A(\xi,s)$ and $B(\xi,s)$ should be obtained using boundary conditions in Equations 10 and 11.

Equation 4 and elastic constitutive relation between strains and stresses in elastodynamic theory yield the transformed stresses in the 2D case as:

$$\hat{\bar{\sigma}}_{11}(\xi,y,s)=2\mu\left((0.5k_2^2-k_1^2)(s+iV\xi)^2-\xi^2\right)\hat{\bar{\phi}}$$

$$+2\mu i\xi\left(k_2^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}\hat{\bar{\psi}}, \qquad (19)$$

$$\hat{\bar{\sigma}}_{12}(\xi,y,s)=2\mu i\xi\left(k_1^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}\hat{\bar{\phi}}$$

$$+2\mu\left(0.5k_2^2(s+iV\xi)^2+\xi^2\right)\hat{\bar{\psi}}, \qquad (20)$$

$$\hat{\bar{\sigma}}_{22}(\xi,y,s)=2\mu\left(0.5k_2^2(s+iV\xi)^2+\xi^2\right)\hat{\bar{\phi}}$$

$$-2\mu i\xi\left(k_2^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}\hat{\bar{\psi}}. \qquad (21)$$

Transformed types of boundary conditions in Equations 10 and 11 and transformed stresses given by Equations 20 and 21 for $y=0$ establish a system of equation from which functions $A(\xi,s)$ and $B(\xi,s)$ can be obtained as:

$$A(\xi,s)=\frac{-\left(0.5k_2^2(s+iV\xi)^2+\xi^2\right)}{2\mu F(\xi,s)}\bar{P}(\xi,s), \qquad (22)$$

$$B(\xi,s)=\frac{i\xi\left(k_1^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}}{2\mu F(\xi,s)}\bar{P}(\xi,s), \qquad (23)$$

where:

$$F(\xi,s)=\left(0.5k_2^2(s+iV\xi)^2+\xi^2\right)^2$$

$$-\xi^2\left(k_2^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}\left(k_1^2(s+iV\xi)^2+\xi^2\right)^{\frac{1}{2}}. \qquad (24)$$

Transformed Boundary Stresses

Concurrent Fourier-Laplace integral transforms for the boundary stresses given by Equations 8 and 9 can be obtained by the following equation:

$$\hat{\bar{P}}(\xi,s)=\int_0^\infty\int_{-\infty}^\infty P(\eta,\tau)e^{i\xi\eta}e^{-s\tau}d\eta d\tau, \qquad (25)$$

as:

$$\hat{\bar{P}}_1(\xi,s)=\frac{\hat{Q}(s)}{2b}\frac{\sin(a\xi)}{a\xi}, \qquad (26)$$

$$\hat{\bar{P}}_2(\xi,s)=\frac{3\hat{Q}(s)}{8b}\left(\frac{\sin(a\xi)}{a^3\xi^3}-\frac{\cos(a\xi)}{a^2\xi^2}\right), \qquad (27)$$

where:

$$\hat{Q}(s)=\mathcal{L}\{Q(\tau)\}=\int_0^\infty Q(\tau)e^{-s\tau}d\tau. \qquad (28)$$

Because of the fact that for the case of a moving object, the length of the contact patch $2a$ is very small, the evaluation of the inverse formidable integrals whose integrands are given in Equations 19, 20 and 21 may not be worth the effort. Thus it is convenient to obtain the asymptotic expansions of Equations 26 and 27 as a tends to zero:

$$\hat{\bar{P}}_1(\xi,s)=\frac{\hat{Q}(s)}{2b}p_1(\xi), \qquad (29)$$

$$\hat{\bar{P}}_2(\xi,s)=\frac{\hat{Q}(s)}{2b}p_2(\xi), \qquad (30)$$

where:

$$p_1(\xi)=1-\frac{a^2\xi^2}{6}+\frac{a^4\xi^4}{120}+O(\xi^6), \qquad (31)$$

$$p_2(\xi)=\frac{1}{4}-\frac{a^2\xi^2}{40}+\frac{a^4\xi^4}{1120}+O(\xi^6). \qquad (32)$$

Obtaining the Steady-State Stresses

The steady-state stresses are known as the stresses after sufficiently large time duration which takes a constant value with respect to the time. In order to obtain the steady-state stresses, the stresses should be evaluated for time τ tending to infinity. To this aim, consider the Laplace transformation for the first derivative of the stresses [11]:

$$\mathcal{L}\left\{\frac{\partial\sigma_{ij}(\eta,y,\tau)}{\partial\tau}\right\}=\int_0^\infty\frac{\partial\sigma_{ij}(\eta,y,\tau)}{\partial\tau}e^{-s\tau}d\tau$$

$$=s\mathcal{L}\{\sigma_{ij}(\eta,y,\tau)\}-\sigma_{ij}(\eta,y,0). \qquad (33)$$

Use of Equation 33 would lead to:

$$\sigma_{ij}^{ss}(\eta,y)=\lim_{\tau\to\infty}\sigma_{ij}(\eta,y,\tau)=\lim_{s\to 0}(s\hat{\sigma}_{ij}(\eta,y,s))$$

$$=\frac{1}{2\pi}\lim_{s\to 0}\int_{-\infty}^\infty s\hat{\bar{\sigma}}_{ij}(\xi,y,s)e^{-i\xi\eta}d\xi. \qquad (34)$$

Performing the limitations in the expanded form of the stresses yield the steady-state stresses as:

$$\sigma_{11}^{ss}(\eta,y)=\frac{q}{4\pi b}\int_{-\infty}^\infty\Big(C_{11}e^{-|\xi|\gamma_1 y}$$

$$+D_{11}e^{-|\xi|\gamma_2 y}\Big)p_i(\xi)e^{-i\eta\xi}d\xi, \qquad (35)$$

$$\sigma_{12}^{ss}(\eta,y)(\eta,y)=\frac{iq}{4\pi b}\int_{-\infty}^\infty|\xi|\xi^{-1}\Big(C_{12}e^{-|\xi|\gamma_1 y}$$

$$+D_{12}e^{-|\xi|\gamma_2 y}\Big)p_i(\xi)e^{-i\eta\xi}d\xi, \qquad (36)$$

$$\sigma_{22}^{ss}(\eta, y) = \frac{q}{4\pi b}\int_{-\infty}^{\infty}\left(C_{22}e^{-|\xi|\gamma_1 y}\right.$$

$$\left.+D_{22}e^{-|\xi|\gamma_2 y}\right)p_i(\xi)e^{-i\eta\xi}d\xi, \quad (37)$$

where $\gamma_i = \sqrt{1-\beta_i^2}$, $\beta_i = k_i V$ and $p_i(\xi)$ should be substituted from Equations 31 or 32, depending on the case. q stands for the steady-state form of the load which is equal to:

$$q = \lim_{\tau\to\infty} Q(\tau) = \lim_{s\to 0}(s\hat{Q}(s)). \quad (38)$$

Constant parameters C_{ij} and D_{ij} can be expressed as:

$$C_{11} = \frac{-(\beta_1^2 - 0.5\beta_2^2 - 1)(1-0.5\beta_2^2)}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}, \quad (39)$$

$$C_{12} = \frac{-(1-0.5\beta_2^2)\gamma_1}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}, \quad (40)$$

$$C_{22} = \frac{-(1-0.5\beta_2^2)^2}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}, \quad (41)$$

$$D_{11} = \frac{-\gamma_1\gamma_2}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}, \quad (42)$$

$$D_{12} = \frac{(1-0.5\beta_2^2)\gamma_1}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}, \quad (43)$$

$$D_{22} = \frac{\gamma_1\gamma_2}{(1-0.5\beta_2^2)^2 - \gamma_1\gamma_2}. \quad (44)$$

The integrations in Equations 35, 36 and 37 can be carried out, concerning the identities:

$$\int_0^\infty \xi^{2m}\cos(\eta\xi)e^{-\gamma_i y\xi}d\xi = I_i^{(m)}(\eta, y)\cos(m\pi), \quad (45)$$

$$\int_0^\infty \xi^{2m}\sin(\eta\xi)e^{-\gamma_i y\xi}d\xi = J_i^{(m)}(\eta, y)\cos(m\pi), \quad (46)$$

where $m = 0, 1, 2, \cdots$ and:

$$I_i^{(m)}(\eta, y) = \frac{\partial^{2m}}{\partial\eta^{2m}}\left(\frac{\gamma_i y}{\gamma_i^2 y^2 + \eta^2}\right), \quad (47)$$

$$J_i^{(m)}(\eta, y) = \frac{\partial^{2m}}{\partial\eta^{2m}}\left(\frac{\eta}{\gamma_i^2 y^2 + \eta^2}\right). \quad (48)$$

Hence final expressions for the steady state stresses in the half-space for the two cases introduced in the previous sections can be obtained as:

Case 1:

$$\sigma_{11}^{ss}(\eta, y) = \frac{q}{2\pi b}\left\{C_{11}\left(I_1^{(0)} - \frac{a^2}{6}I_1^{(1)} + \frac{a^4}{120}I_1^{(2)}\right)\right.$$

$$\left.+D_{11}\left(I_2^{(0)} - \frac{a^2}{6}I_2^{(1)} + \frac{a^4}{120}I_2^{(2)}\right)\right\}, \quad (49)$$

$$\sigma_{12}^{ss}(\eta, y)(\eta, y) = \frac{q}{2\pi b}\left\{C_{12}\left(J_1^{(0)} - \frac{a^2}{6}J_1^{(1)} + \frac{a^4}{120}J_1^{(2)}\right)\right.$$

$$\left.+D_{12}\left(J_2^{(0)} - \frac{a^2}{6}J_2^{(1)} + \frac{a^4}{120}J_2^{(2)}\right)\right\}, \quad (50)$$

$$\sigma_{22}^{ss}(\eta, y) = \frac{q}{2\pi b}\left\{C_{22}\left(I_1^{(0)} - \frac{a^2}{6}I_1^{(1)} + \frac{a^4}{120}I_1^{(2)}\right)\right.$$

$$\left.+D_{22}\left(I_2^{(0)} - \frac{a^2}{6}I_2^{(1)} + \frac{a^4}{120}I_2^{(2)}\right)\right\}. \quad (51)$$

Case 2:

$$\sigma_{11}^{ss}(\eta, y) = \frac{q}{2\pi b}\left\{C_{11}\left(\frac{1}{4}I_1^{(0)} - \frac{a^2}{40}I_1^{(1)} + \frac{a^4}{1120}I_1^{(2)}\right)\right.$$

$$\left.+D_{11}\left(\frac{1}{4}I_2^{(0)} - \frac{a^2}{40}I_2^{(1)} + \frac{a^4}{1120}I_2^{(2)}\right)\right\}, \quad (52)$$

$$\sigma_{12}^{ss}(\eta, y)(\eta, y)$$

$$= \frac{q}{2\pi b}\left\{C_{12}\left(\frac{1}{4}J_1^{(0)} - \frac{a^2}{40}J_1^{(1)} + \frac{a^4}{1120}J_1^{(2)}\right)\right.$$

$$\left.+D_{12}\left(\frac{1}{4}J_2^{(0)} - \frac{a^2}{40}J_2^{(1)} + \frac{a^4}{1120}J_2^{(2)}\right)\right\}, \quad (53)$$

$$\sigma_{22}^{ss}(\eta, y) = \frac{q}{2\pi b}\left\{C_{22}\left(\frac{1}{4}I_1^{(0)} - \frac{a^2}{40}I_1^{(1)} + \frac{a^4}{1120}I_1^{(2)}\right)\right.$$

$$\left.+D_{22}\left(\frac{1}{4}I_2^{(0)} - \frac{a^2}{40}I_2^{(1)} + \frac{a^4}{1120}I_2^{(2)}\right)\right\}. \quad (54)$$

NUMERICAL EXAMPLE

Consider an isotropic, homogeneous semi-infinite region, shown in Figure 1, with $\mu = 50$ MPa, $\upsilon = 0.33$ and $\rho = 2000$ kg/m^3, subjected to a moving load of 20 KN with constant speeds and prescribed on a patch with $2a = 0.08$ m and $2b = 0.2$ m.

Steady-state stresses with respect to the position of the moving load η for constant depth of $y = 0.5$ m and constant speed of $V = 0.3\ k_2^{-1}$, in Cases 1 and 2, have been evaluated and are shown in Figures 2 and 3, respectively. A comparison between the values of the stresses in two cases demonstrates that the half space experiences higher stresses when the load

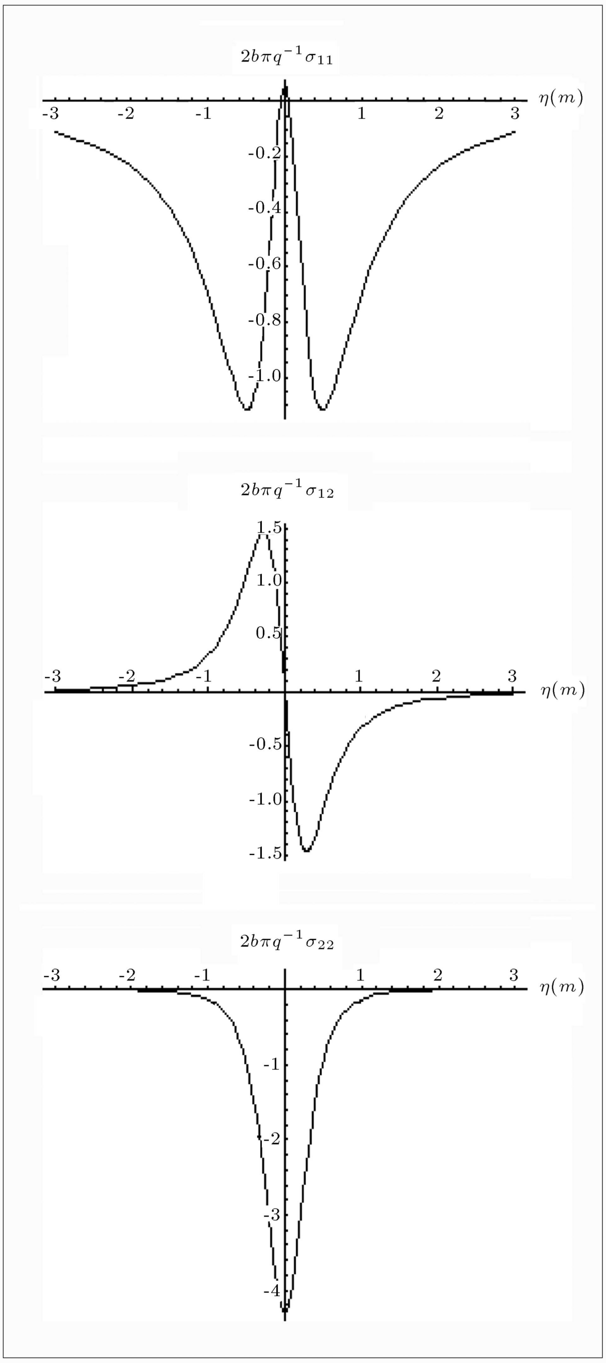

Figure 2. Variations of stresses at $y = 0.5$ m with respect to the position of a moving load with $V = 0.3k_2^{-1}$ in Case 1.

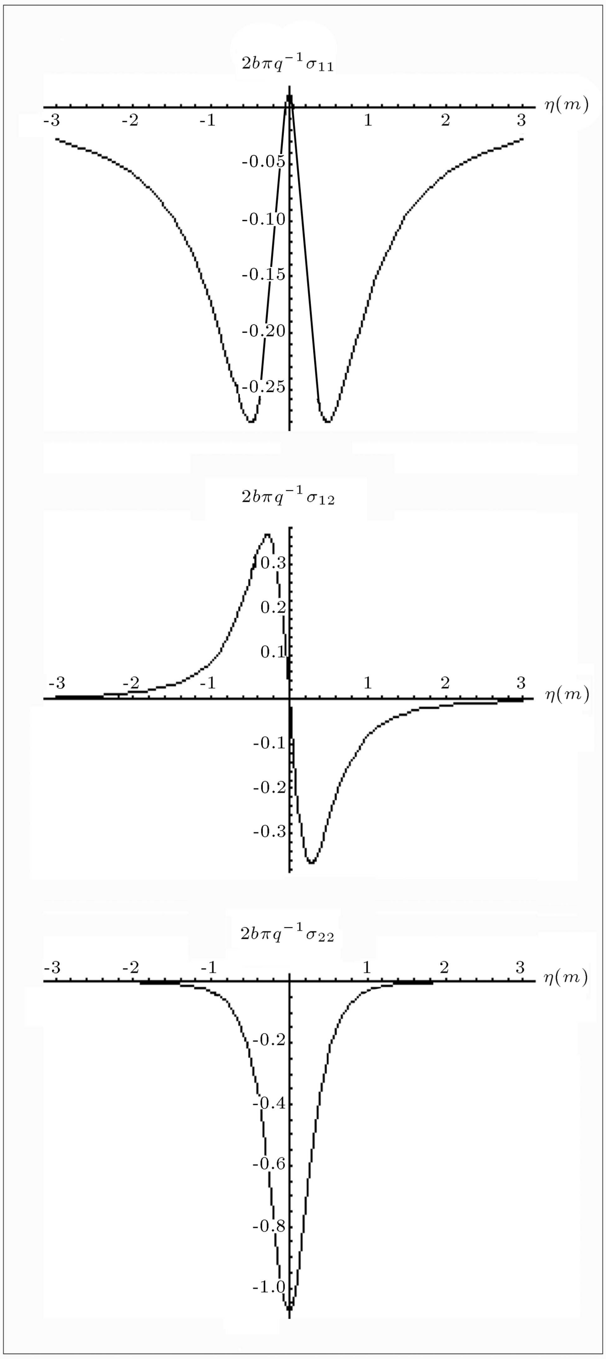

Figure 3. Variations of stresses at $y = 0.5$ m with respect to the position of a moving load with $V = 0.3k_2^{-1}$ in Case 2.

is prescribed uniformly on the finite patch, compared with the case in which the load is prescribed in the parabolic form. This shows the importance of the circumstance in which the load is prescribing on the half-space boundary.

Figures 4 and 5 show the variations of stresses with respect to various depths in the half-space for points under the moving load, $\eta = 0$, and constant speed of $V = 0.3\ k_2^{-1}$. Results demonstrated that the stresses decay quickly with increase in the depth. It should be noted that for $\eta = 0$, the shear stresses in two cases should be vanished due to the symmetry of the problem definition.

Figures 6 and 7 show the variations of the stresses for the point with coordinates $(\eta, y) = (0, 0.5)$ subjected to moving loads with various subsonic speeds in two cases. Results demonstrate that the stresses increase as the moving loads' speeds increase to an

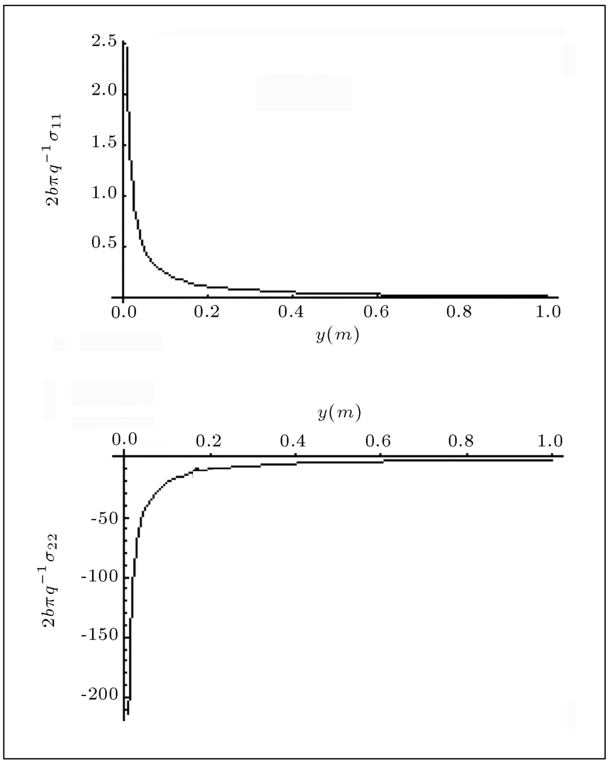

Figure 4. Variations of the stresses for $\eta = 0$ at various depths in the half space subjected to a moving load with $V = 0.3k_2^{-1}$ in Case 1.

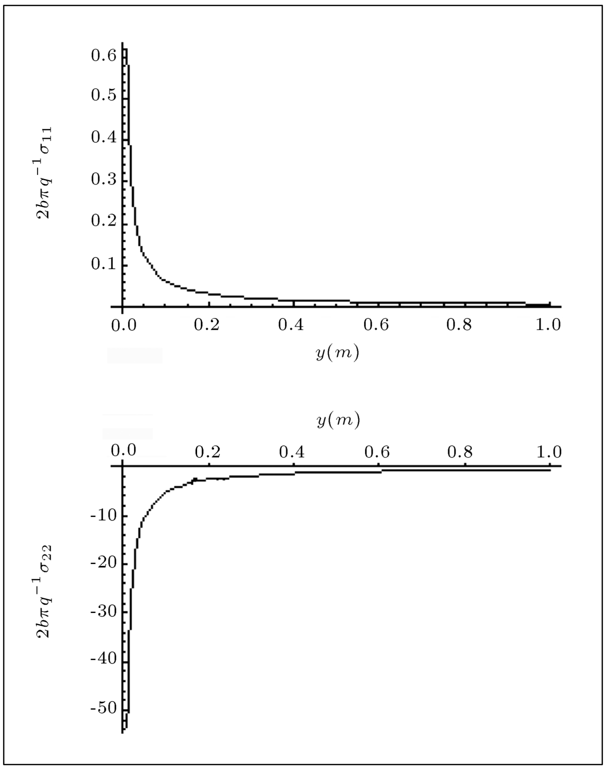

Figure 5. Variations of the stresses for $\eta = 0$ at various depths in the half space subjected to a moving load with $V = 0.3k_2^{-1}$ in Case 2.

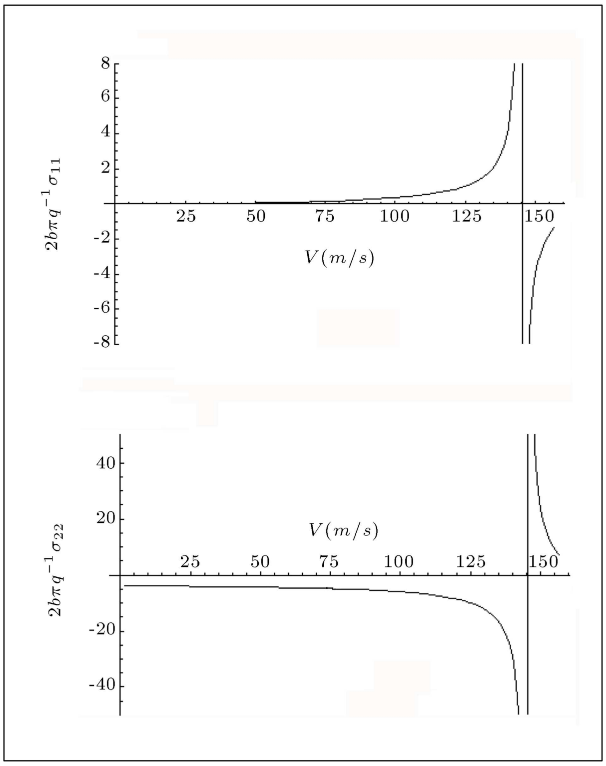

Figure 6. Variations of the stresses for the point with coordinates $(\eta, y) = (0, 0.5)$ subjected to moving loads with various subsonic speeds in Case 1.

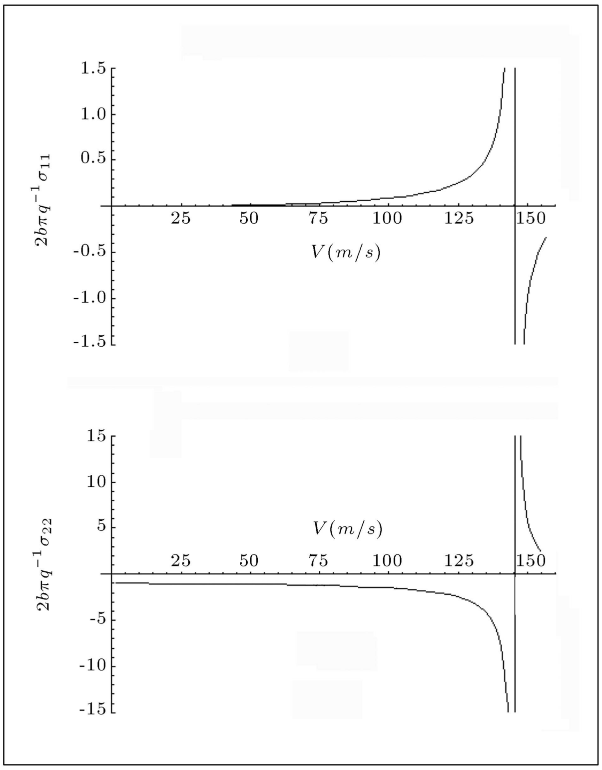

Figure 7. Variations of the stresses for the point with coordinates $(\eta, y) = (0, 0.5)$ subjected to moving loads with various subsonic speeds in Case 2.

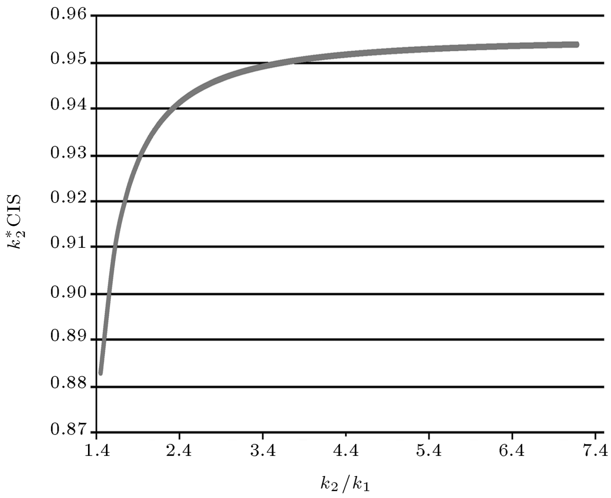

Figure 8. CIS values for various k_2/k_1.

extremum speed. As shown in Figures 6 and 7, after the extremum speed, stresses' absolute values decrease for higher speeds. The extremum speed is equivalent to the Critical Influential Speed (CIS) which is introduced in [12]. CIS which should be less than k_2^{-1} can be obtained from the solutions of the following equation:

$$\left(1 - 0.5k_2^2(\mathrm{CIS})^2\right)^2 - \sqrt{(1 - k_1^2(\mathrm{CIS})^2)\,(1 - k_2^2(\mathrm{CIS})^2)} = 0. \qquad (55)$$

Solution of Equation 55 for various half-space materials with different k_1 and k_2 values are shown in Figure 8. This figure demonstrates that with increase in the Poisson's ratio, the critical influential speeds approach to a finite limit.

CONCLUDING REMARKS

An analytical approach has been used to obtain the steady-state stresses in a homogeneous, isotropic half-space subjected to a moving load with a finite contact patch. To this aim, a concurrent double integral transformation was employed to solve the system of wave-type equations pertaining to the Navier's equation of motion. A comparison between two types of the load transmission in the contact patch demonstrated that a parabolic load transmission induce lower stresses compared with its corresponding uniform load transmission. Half-space experiences higher stresses for higher speeds of the moving loads until reaching to an extremum speed known as CIS. After the critical influential speed, the absolute values of the stresses decrease for higher speeds. Obtaining the CIS values for different half-space materials showed that the CIS values increase as the ratio between the dilatational disturbance propagation velocity and propagation velocity of shear waves, c_1/c_2, increases.

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