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A Shortest Path Problem in an Urban Transportation Network Based on Driver Perceived Travel Time

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Abstract. *This paper proposes a method to solve shortest path problems in route choice processes when each link's travel time is a fuzzy number, called the Perceived Travel Time (PTT). The PTT is a subjective travel time perceived by a driver. The algorithm solves the fuzzy shortest path problem (FSPA) for drivers in the presence of uncertainty regarding route travel time. For congested networks, the method is able to find the shortest path in terms of perceived travel time and degree of saturation (congestion) along routes at the same time. The FSPA can be used to support the fuzzification of traffic assignment algorithms. The applicability of the resulting FSPA for the traffic assignment was tested in conjunction with incremental traffic loading and was applied to a large-scale real network. The results of the traffic assignment based on the FSPA, User Equilibrium (UE) and a stochastic loading network model (Dial's assignment algorithm) were compared to the observed volume for certain links in the network. We conclude that the proposed method offers better accuracy than the UE or Dial's assignment algorithm for the network under testing.*

Keywords: *Fuzzy sets; Perceived travel time; Route choice; Shortest path; Urban network.*

INTRODUCTION

An important consideration in the route choice process for urban transportation networks is how drivers perceive link travel time and find the appropriate path to their destination based on path travel time. Traditional assignment algorithms (e.g. incremental assignment or User Equilibrium (UE)), assume that all drivers experience the same link travel time. In reality, this is not the case, because drivers may have different perceptions of travel time for a particular path. Another assumption behind traditional assignment algorithms is that each individual uses a single value for travel time. For a more realistic description of the problem, this assumption can be disregarded when one knows that the traffic demand and consequently the volume of each link are stochastic in nature or that an unexpected event may influence a link's travel time. The variation in traffic volume in conjunction with the degree of saturation and unexpected events may affect the driver's perception of travel time. Generally a driver's perception of link travel time arises from his/her experience in previous route choice scenarios between the same origin and destination.

Many studies have been carried out on travel time uncertainty in route choice. Two different methods are applicable to model travelers' choices under uncertainty. One uses a probabilistic choice model that is often based on utility functions. The other method, discussed in this paper, makes use of fuzzy logic. Fuzzy logic is a powerful technique used for considering uncertainties related to human perception in modeling contexts. Here, we establish an innovative fuzzy definition for Perceived Travel Time (PTT) using membership functions and the route choice process. Also, a Fuzzy Shortest Path Algorithm (FSPA) based on a labeling procedure is proposed for application to traffic assignments in an urban transportation network in which a PTT is assigned to each link.

A great deal of recent research has attempted to use fuzzy concepts to solve the shortest path problem. The most recent papers addressing the fuzzy shortest path problem include those by Chang and Lee (1999)

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who used an overall existence ranking index to assign different types of status (optimistic, pessimistic or neutral person) to drivers [1], and Okada and Soper (2000) who introduced an order relation between fuzzy numbers based on the concept of a fuzzy minimum [2]. In another paper, Okada (2004) considered interactivity among fuzzy numbers assigned to arc length by defining a new comparison index [3]. Blue et al. (2002) formulated several standard graph-theoretic problems (shortest path and minimum cut) for fuzzy graphs using a unified approach that is distinguished by its uniform application of guiding principles, such as the construction of membership grades via the ranking of fuzzy numbers etc. [4]. Chuang and Kang (2005) assigned a fuzzy set with a triangular membership function to each arc length and proposed a new algorithm to deal with the fuzzy shortest problem [5]. They also proposed a heuristic procedure to find the Fuzzy Shortest Path Length (FSPL), among all possible paths in a network, based on the idea that a crisp (non-fuzzy) number is a minimum if, and only if, any other number is larger than or equal to it. They then proposed a way to measure the degree of similarity between the FSPL and each fuzzy path length. The path with the highest degree of similarity was chosen as the shortest path. Moazeni (2006) defined an order relation between fuzzy quantities with finite support [6]. Ji et al. (2007) considered different decision criteria to specify the shortest path in a fuzzy environment and to solve the model using hybrid intelligent methods. The hybrid intelligent methods included a combination of simulations and a genetic algorithm [7]. Hernandes et al. (2007) proposed an iterative algorithm that uses a generic ranking index for comparing the fuzzy numbers involved in the problem, in such a way that each time the decision-maker wants to solve a concrete problem, (s)he can choose (or propose) the ranking index that best suits that problem [8].

Some studies have used fuzzy theory in route choice contexts to address behavioral characteristics. The most important studies are by the following authors: Teodorovic and Kikuchi who proposed [9] an approximate reasoning model used for traffic assignment between two alternative routes on a highway network; Murat and Uludag [10] who compared a fuzzy logic model with a linear regression model to show how the application of fuzzy logic to defining link travel times increases route choice accuracy; and Binetti and De Mitri [11] who used fuzzy numbers to represent the imprecision in path costs for a road network. Additionally, Binetti and De Mitri utilized the method of successive averages to generate user equilibrium flows in a simple network. The model yielded realistic results. Finally, Arslan and Khisty [12] presented a heuristic way of handling fuzzy perceptions and used the Analytical Hierarchy Process (AHP) to explain route choice behavior in transportation systems. This method provided intuitive and promising results.

In comparison to those mentioned above, the present study demonstrates a fuzzy definition of PTT by developing the route choice process using a shortest path algorithm. The algorithm utilizes a labeling procedure in an urban transportation network with perceived fuzzy travel time. The proposed FSPA is defined in such a way that it can easily be used in fuzzy assignment algorithms. The criterion used in this paper to compare link travel times as fuzzy numbers is more consistent with the actual route choice process.

In the next section, the method of formulating a PTT using fuzzy sets is described. After that, a route choice decision-making process is presented followed by the FSPA. The results for a real network are shown in the subsequent section and the conclusions are presented in the last section.

PTT DEFINITION AND CONCEPTS

Generally, there are two kinds of uncertainty. The first type of uncertainty is derived from the imprecision that is inherent in taking measurements or surveys, which can be affected by unpredicted events (e.g. an accidents or weather conditions) or demand fluctuations. The second type is linked with human perception. According to the work of Zadeh [13,14], probability theory is very useful for dealing with the uncertainty inherent in measurements; however, it is not very useful for dealing with the uncertainty associated with human perceptions. The former involves crisp sets, while the latter involves fuzzy sets [15]. Because PTT uncertainty is derived from human perceptions, it is more useful to use fuzzy concepts when considering the uncertainty in drivers' deductions for PTT.

Although many researchers have attempted to determine travel time uncertainty using fuzzy concepts, no distinct process has been applied to fuzzy traffic assignment models to build and define Membership Functions (MF) [16]. In fact, the definition of a MF in existing fuzzy models is not well-constructed. Generally, a triangular MF is used for the PTT to simplify the computation process. The triangular MF has three specification parameters: the MF center, the right limit, and the left limit. The PTT MF center denotes the most common travel time of a link. The right limit is associated with the highest possible travel time. The left limit is associated with the minimum possible travel time. Typically, the computed travel time of a link based on its assigned volume is assumed to be the center of the PTT MF for that link.

To define the left and right limits of the MFs in this work, a method is proposed that reflects the traffic state of each link. Basically, this method assumes that the volume of each link varies from day to day. This variation causes differences in travel time and consequently users have a set of travel times in mind. The PTT is modeled using fuzzy numbers. The assumption of variable volumes for each link is not unique for this study, as it is also used in probabilistic traffic assignment, where link volume is a random variable [17]. The method assumes that the volume of each link varies between a lower and an upper bound that are $(1-\alpha_l)$ and $(1+\alpha_r)$ times the most observed volume (i.e. the volume assigned to the link within the assignment steps), respectively (α_l and α_r are two numbers specify a lower and upper bound for link volume, respectively). As shown in Figure 1, the PTT is assumed to have a triangular membership function. The left and right limits of the PTT MF correspond to the travel times of the lower and upper bounds of the volume, respectively. As mentioned earlier, the travel time for the most observed volume is the center of the PTT MF. Hence, knowing (α_l), (α_r) and the volume assigned to the link, the PTT MF can be constructed. The PTT triangular membership function parameters, $\tilde{t}$, are the left, center and right limits of the PTT membership function and are denoted as t^l, t^c, t^r, respectively. Function $t[x]$ calculates the link travel time as a function of the link traffic volume shown by x.

The specification parameters of the triangular MF (i.e. the left, center and right limits shown in Figure 1) are highly dependent on the problem at hand. These parameters can be calculated by comparing the results of the traffic assignment used in the PTT MF, shown in Figure 1, to the volume of the observed links. A computational procedure for the estimation of α is suggested in the PTT application section using a real network assignment algorithm. However, a PTT MF can also be determined through use of questionnaires that ask drivers for their PTT for a specific link, but in this paper, no PTT data are available.

The PTT MF for each path is computed using the fuzzy summation of the PTT MFs for all links. One of the advantages of triangular MFs over other types of (non-linear) fuzzy numbers is that there is a closed form for the summation of these numbers. For example, the

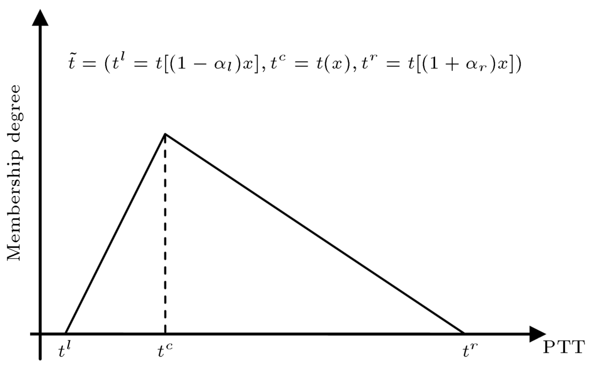

Figure 1. PTT MF shape and parameters.

PTT MF for path K is calculated as follows:

$$\tilde{t}_K = \left(\sum_{a\in K} t_a[(1-\alpha_l)x_a], \sum_{a\in K} t_a(x_a), \sum_{a\in K} t_a[(1+\alpha_r)x_a] \right), \tag{1}$$

where:

$\tilde{t}$:	PTT for route k,
$t_a[x_a]$:	Link a travel time as a function of link a traffic volume
x_a:	Link a traffic volume,
α_l, α_r:	Coefficients of the observed traffic volume specifying the traffic volume lower and upper bounds, respectively.

The advantage of constructing the MF in this manner is that it takes the degree of saturation of a link into consideration as an important factor in path assignment. Two links with the same fuzzy travel time center will not have equal right and left limits when their degrees of saturation differ. Because the fuzzy travel time center is the same as the travel time for the UE, it means that this method of MF construction incorporates the effect of congestion in route choice, unlike the UE algorithm. Then, one may ask the question: "What is the relationship between congestion and the value of the right and left limit?" In order to answer this question, two links with the same travel time and different degrees of saturation were analyzed. It is assumed that link travel time follows the Bureau Public Roads (BPR) function. The general form of the BPR function is:

$$t(x) = t^0 \left[1 + \alpha \left(\frac{x}{C} \right)^\beta \right], \tag{2}$$

where x = link volume, C = link capacity, t^0 = free-flow link travel time, and $t(x)$ = final travel time.

Suppose links "a" and "b" have equal travel times ($t_a = t_b$) and that $X_b < X_a$ (where X_i is the degree of saturation for link "i"), then, according to Equation 2 the following equations can be constructed:

$$t_a^0 \left[1 + \alpha \left(\frac{x_a}{C_a} \right)^\beta \right] = t_b^0 \left[1 + \alpha \left(\frac{x_b}{C_b} \right)^\beta \right]. \tag{3}$$

Substituting $X_a = \frac{x_a}{C_a}$ and $X_b = \frac{x_b}{C_b}$ into Equation 3 gives:

$$t_b^0 - t_a^0 = t_a^0.\alpha.(X_a^\beta) - t_b^0.\alpha.(X_b^\beta). \tag{4}$$

If $t_a^0 < t_b^0$, then according to Equation 4:

$$t_b^0 - t_a^0 = t_a^0.\alpha.(X_a^\beta) - t_b^0.\alpha.(X_b^\beta) > 0. \tag{5}$$

If the left side of Equation 5 is multiplied by a positive number $(1+\alpha_r)^\beta(>0)$,

$$t_b^0 - t_a^0 < (1+\alpha_r)^\beta \times \lfloor t_a^0.\alpha.(X_a^\beta) - t_b^0.\alpha.(X_b^\beta)\rfloor, \tag{6}$$

$$\Rightarrow t_b^0 + (1+\alpha_r)^\beta \times \lfloor t_b^0.\alpha.(X_b^\beta)\rfloor < t_a^0 + (1+\alpha_r)^\beta \times \lfloor t_a^0.\alpha.(X_a^\beta)\rfloor, \tag{7}$$

$$\Rightarrow t_b^0 \lfloor 1 + (1+\alpha_r)^\beta \times \alpha \times (X_b^\beta)\rfloor < t_a^0 \lfloor 1 + (1+\alpha_r)^\beta \times \alpha \times (X_a^\beta)\rfloor. \tag{8}$$

According to Equations 2 and 4, Equation 8 will change to the following:

$$\Rightarrow t_b\lfloor(1+\alpha_r)(X_b)\rfloor < t_a\lfloor(1+\alpha_r)(X_a)\rfloor. \tag{9}$$

The above computation proves that the right limit of the PTT MF of link "a" is bigger than the right limit of link "b", because the degree of saturation of link "a" is bigger than that of link b; however, the center of the PTT MF for both links is equal. Because other travel time functions use similar concepts, this proof is approximately true for other cases. Following this reasoning, it can be proven that the left limit of the PTT MF for link "a" is smaller than that of link "b".

ROUTE CHOICE DECISION-MAKING PROCESS

To choose the best path, the PTT values of all possible choices should be compared. The next issue is choosing an appropriate operator to rank fuzzy numbers. In this paper, Dubois and Prade's method for ranking fuzzy numbers using the possibility theory [18] is utilized. Their method is used to compare PTTs between a distinct origin and destination for different paths and to select the best path. For comparison, Dubois and Prade used both the possibility and the necessity theory. Supposing that "M" and "N" are two fuzzy numbers defined by two triangular MFs, Dubois and Prade defined four indices:

$$\begin{aligned} I_1(M) &= \text{Poss } (M \leq N), \\ I_2(M) &= \text{Poss } (M < N), \\ I_3(M) &= \text{Nec } (M \leq N), \\ I_4(M) &= \text{Nec } (M < N). \end{aligned} \tag{10}$$

Poss and Nec stand for possibility and necessity, respectively. Schematic definitions of these indices are shown in Figure 2.

As Figure 2 illustrates, $I_1(M)$ is the intersection point of $[N, +\infty)$ and the left side of "M"'s MF. More details about these indices are described in Henn [19].

The question may then arise as to "which of these indices is appropriate for the purpose of the shortest path when PTT is assigned to each link?" First of all, this index should discriminate between a fuzzy number (uncertainty) and a crisp number (certainty), otherwise the use of fuzzy numbers instead of deterministic travel time is meaningless. To see which of these indices has the above trait, some investigation is needed. For example, two fuzzy numbers "M" and "N" that represent the travel times on two parallel paths are compared. As shown in Figure 3, "N" is a singleton fuzzy number (displayed as line "bd"), and "M" is a fuzzy number with a triangular MF (displayed as triangle "acd").

The four indices for these two sample fuzzy numbers are as follows:

$$\begin{aligned} I_1(M) &= I_1(N) = 1, \\ I_2(M) &= I_3(N) = 1, \\ I_3(M) &= I_2(N) = 0, \\ I_4(M) &= I_4(N) = 0. \end{aligned} \tag{11}$$

Based on the indices I_1 and I_4, there is no difference between M and N; however, indices I_2 and I_3 indicate that these two fuzzy numbers are different. This means that I_1 and I_4 do not take any uncertainty into account.

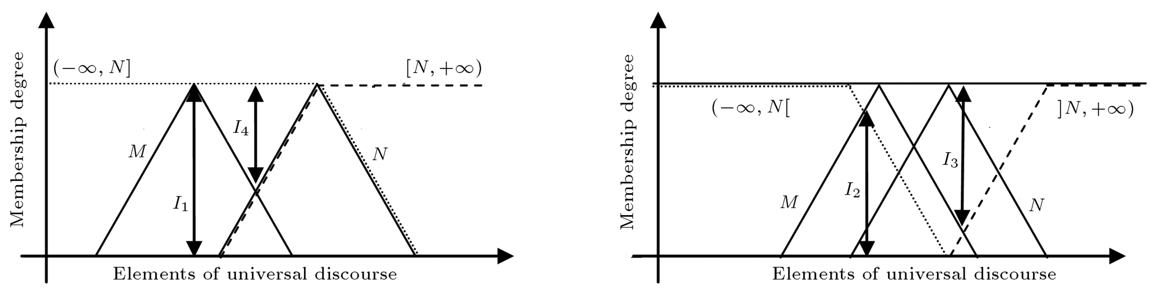

Figure 2. Schematic definitions of dubois and prade indices [18].

Yet, the aim of the fuzzy definition of PTT is to consider uncertainties, thus, these two indices do not meet our purpose.

The path with a PTT of "N" has a specific travel time, but the travel time of the other path with a PTT of "M" is uncertain. Path N is the best choice for a risk-averse driver; however, the path with uncertainty in travel time, M, is the best choice for a risk-seeking driver. The risk-seeking driver thinks that it is advantageous to reduce travel time by selecting the uncertain path, because by taking route M, there is the possibility that travel time will be less than route N. The risk-averse driver does not take the risk and selects path N to avoid potential excessive travel time in route M. Regarding this kind of behavior for risk-seeking and risk-averse users, I_2 and I_3 are appropriate for these drivers, respectively, because in the system of I_2, M is the best choice and based on I_3, N is better than M. As a result, indices I_1 and I_4 only consider expected travel time (the center of the PTT MF); however, the other two indices I_2 and I_3 also account for PTT uncertainties and are, respectively, appropriate for risk-seeking and risk-averse drivers. Henn points out how the indices are sensitive to some parts of the MFs [19]. Figure 4 indicates that index I_2 is used to compare the left sides of the MFs and I_3 is used to compare the right sides. This is intuitive when one knows that risk-averse decision makers choose an alternative in a pessimistic manner and use the right part of cost MFs in making their decision. Similarly, risk-seeking decision makers make choices based on the left part of the cost MF.

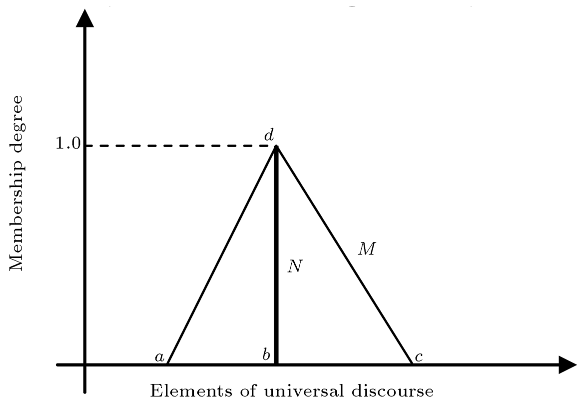

Figure 3. Fuzzy numbers "M" and "N".

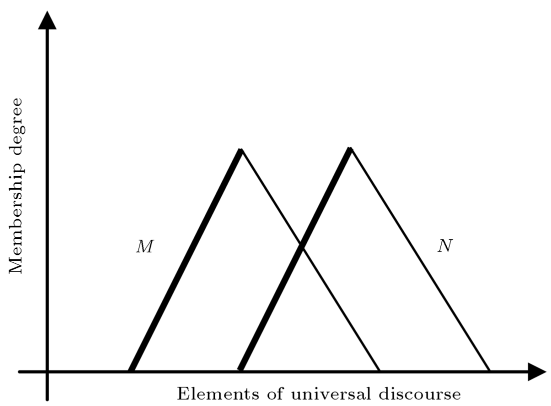

Figure 4a. Effective MF parts based on index I_2.

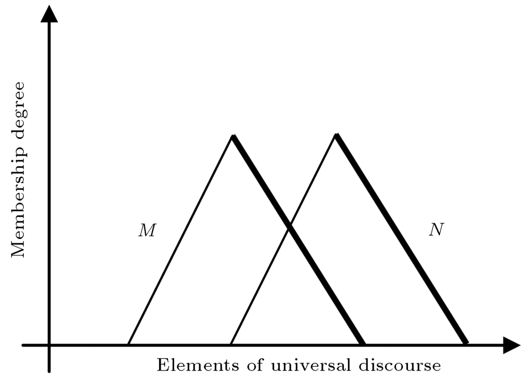

Figure 4b. Effective MF parts based on index I_3.

In reality, if a path's PTT uncertainty increases, then the proportion of traffic using this path decreases [16]. Because I_3 describes a person who avoids uncertainty at the right part of travel time MF, it turns out that the index I_3 can be appropriately used to compare the PTT MFs of certain paths in order to select the best one.

A combination of index I_3 and the way a MF is constructed will eventually lead to a case in which drivers choose, from between two links, the link with a lower degree of saturation when the expected travel times (centers of the MF) are equal for both. According to the PTT MF definition in the previous section, if two links have equal expected travel times (i.e. the same center value in their MFs) but different degrees of saturation, then the links' MF limits will be different. The link with the higher degree of saturation has a lower left limit and a higher right limit. On the other hand, a person who is in the I_3 category will not prefer the choice with the higher right limit or, in other words, the link with higher degree of saturation, when the expected travel times are the same. For a person with an I_2 index, the path with more congestion is preferable; therefore, this is another indication that the I_3 index should be used for the purpose of shortest path determination in a congested network.

Although I_3 is selected for use in the FSPA for a network with congestion, the FSPA is also applicable to a risk-seeking individual; therefore, the values of indices I_2 and I_3 for two triangular fuzzy numbers are computed. Suppose that $M = (m^L, m, m^R)$ and $N = (n^L, n, n^R)$ are two fuzzy numbers. According to Figure 2, $I_2(M)$ is the cross-point of $(-\infty, N[$ and the left side of the "M" MF. The membership degree of $(-\infty, N[$ and left side of the "M" MF are determined as follows:

$$\mu_{(-\infty,N[} = \begin{cases} 1 & \text{if } x \leq n^L \\ \frac{n-x}{n-n^L} & \text{if } n^L < x < n \\ 0 & \text{if } x \geq n \end{cases} \tag{12}$$

$$\mu_M(x < m) = \begin{cases} 0 & \text{if } x < m^L \\ \frac{x-m^L}{m-m^L} & \text{if } m^L < x < m \end{cases} \tag{13}$$

Index I_2 is computed as follows:

$$I_2(M) = \frac{n - m^L}{(m+n) - (m^L + n^L)}. \tag{14}$$

Similar to the above computation, the values of index I_3 for "M" and "N" are determined as follows:

$$I_3(M) = \frac{n^R - m}{(m^R + n^R) - (m+n)}. \tag{15}$$

It is assumed that when the indices for the two fuzzy numbers "M" and "N" are equal, "M" and "N" are also equivalent. For example, if $I_2(M) = I_2(N)$, then based on the value of index I_2, "M" is as preferable as "N". For two fuzzy numbers, "M" and "N", we also have [19] the following:

$$I_2(M) + I_2(N) = 1,$$

$$I_3(M) + I_3(N) = 1. \tag{16}$$

Thus, we can say that if "M" and "N" are equivalent, then:

$$I_2(M) = I_2(N) = 0.5,$$

and:

$$I_3(M) = I_3(N) = 0.5.$$

Based on the index definitions, if the left side cross-point of "M" and "N" is equal to 0.5, then the two numbers are also equivalent by index I_2. A similar deduction for the right cross-point of "M" with "N" is true for index I_3. Figure 5 shows the MFs of "M" and "N" for the two situations.

So far, I_3 has been selected for the comparison of fuzzy numbers, the numerical value of I_3 has been computed for a general case, and it was shown that this index takes the degree of saturation in route choice into consideration when expected travel times of the routes are equal. The rest of this section describes how I_3 affects the PTTs of several parallel paths where both the degree of saturation and expected travel time of the routes are different. To accurately identify how index I_3 is used to compare different MFs, an example is presented. Consider Figure 6, which shows the right

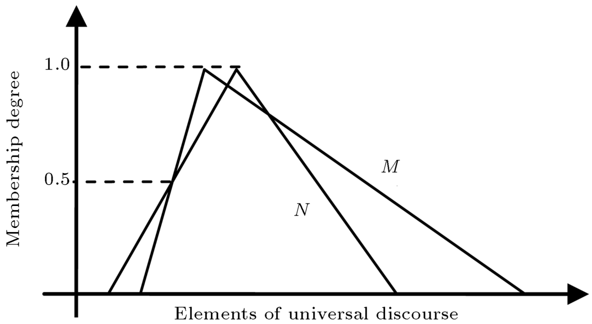

Figure 5a. Two equivalent fuzzy numbers based on index I_2.

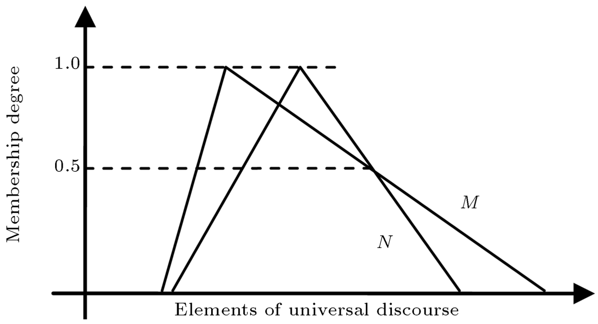

Figure 5b. Two equivalent fuzzy numbers based on index I_3.

sides of the MFs for four fuzzy numbers A, B, C, and D. Despite their differing right limits, these fuzzy numbers are considered equal based on I_3, because the cross point of the right hand sides of these MFs is the same and equal to 0.5.

However, the expected travel time (MF center) and the travel time uncertainty (the MF right limit) increase and decrease, respectively, as one examines the MFs for "A" to "D" sequentially. It is obvious that I_3 simultaneously takes into consideration the expected travel time and uncertainty in the risk-averse situation when comparing two fuzzy numbers.

The computational view of two equivalent fuzzy numbers based on I_3 is as follows.

Using Equations 15 and 16, we can compute $I_3(N)$:

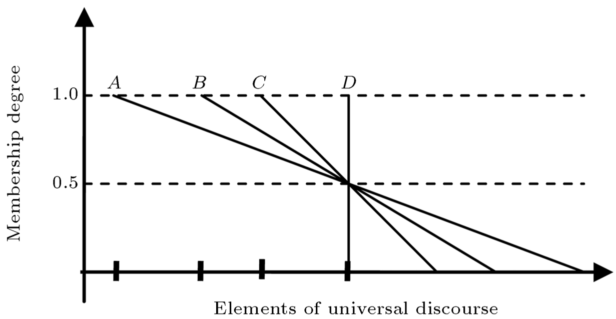

Figure 6. Four equivalent fuzzy numbers based on I_3.

$$I_3(N) = 1 - I_3(M) = 1 - \frac{n^R - m}{(m^R + n^R) - (m + n)}$$

$$= \frac{m^R - n}{(m^R + n^R) - (m + n)}. \tag{17}$$

Two given fuzzy numbers "M" and "N" are equivalent, if $I_3(M) = I_3(N)$, therefore, the following holds:

$$I_3(M) = I_3(N) \Leftrightarrow \frac{n^R - m}{(m^R + n^R) - (m + n)}$$

$$= \frac{m^R - n}{(m^R+n^R)-(m+n)} \Leftrightarrow n^R+n=m^R+m. \tag{18}$$

Similar to the above reasoning, we can also say, based on I_3, that "N" is less than "M" if and only if $n^R+n < m^R + m$.

FUZZY SHORTEST PATH ALGORITHM (FSPA)

The proposed FSPA is similar to the Dijkstra algorithm, which uses a labeling procedure [20]. The FSPA is capable of finding the shortest path in an urban transportation network, in which the PTT is a fuzzy number assigned to each link.

The FSPA uses the I_3 index described in the previous section to compare parallel link PTTs and to choose the shortest path. First, it is necessary to define the FSPA parameters:

$V =$	the set of all nodes,
$S =$	the set of labeled nodes,
$\bar{S} =$	the set of unlabeled nodes,
$\tilde{\phi}(zv) =$	the fuzzy length (fuzzy travel time) of link (z, v),
$\tilde{d}(v) =$	the fuzzy length from the origin to the node v (current node),
$l(v) =$	the node before v in the shortest path from the origin (previous node),
$i =$	algorithm step counter,
$T =$	the tree Graph from s,
$s =$	the origin,
$z_i =$	$(i + 1)$th labeled node,
neighbor(z_i) =	the set of nodes that connect to node z_i with only one link.

The algorithm is as follows:

Step 1- Initialization:

Set:

$$\tilde{d}(s) = (0, 0, 0), \qquad S = \{s\},$$

$$\bar{S} = V - \{S\}, \qquad z_0 = s, \qquad i = 0.$$

Set:

$$\tilde{d}(v) = \infty, \qquad l(v) = s, \qquad \forall v \neq s.$$

Step 2- Update S, $\bar{S}$ and $\tilde{d}(v)$:

2-1- $\tilde{d}(v) = \overline{\min}\{\tilde{d}(v), \tilde{d}(z_i) + \tilde{\phi}(z_i v)\}$

$$\forall v \in \bar{S} \cap \text{neighbor}(z_i).$$

If:

$$\tilde{d}(v) = \tilde{d}(z_i) + \tilde{\phi}(z_i v),$$

then:

$$l(v) = z_i.$$

2-2- Find z_{i+1} such that:

$$\overline{\min_{v \in S}}\{\tilde{d}(v)\} = \tilde{d}(z_{i+1}).$$

2-3- $S = S \cup \{z_{i+1}\}$, $\bar{S} = \bar{S} - \{z_{i+1}\}$.

Step 3- If $i = n - 1$ stop, else $i = i + 1$ and go to step 2.

Note: The function $\overline{\min}$ computes the minimum of the fuzzy numbers according to index I_3.

There are two points that need to be made about this algorithm if we intend to use it to find the solution for a network with fuzzy travel time. First, all links coming to a labeled node, except for the shortest, will be ignored in future steps. For example, take the sample network shown in Figure 7 in which node "A" is a labeled node, and assume that link "1" travel time is less than link "2" travel time.

Using the conventional Dijkstra algorithm, which has a deterministic travel time for each link, we might claim that link "2" is ignored until the end of the algorithm because we can easily show that the shortest path from origin "O" to destination "D" does not contain link "2". This characteristic should also hold true for routes with a fuzzy travel time and a triangular MF to ensure that the labeling algorithm gives the best path between each origin and destination. Second, the proposed FSPA similar to those from other references [1,17] assumes that users include a shared link's travel time in the total travel time of a path when

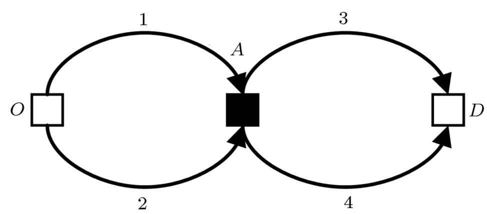

Figure 7. Routes from origin "O" to destination "D".

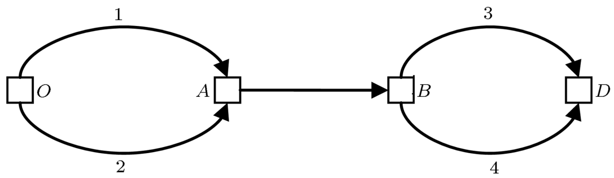

Figure 8. Routes from origin "O" to destination "D" with a shared link.

comparing all paths between an origin/destination pair. For example, consider the network shown in Figure 8.

It seems that in reality users only compare routes based on the unique links within the routes. In other words, shared links should not have any effect on the final decision. According to the above example, this means that a driver chooses the shortest path from "O" to "D" by comparing the travel time of the possible routes and ignoring link "AB" because it is commonly shared by all routes from "O" to "D". However, in Dijkstra's shortest path algorithm, the total travel time from origin to destination is computed at each step and includes the time spent in shared links for all routes. The shortest path obtained by users (when not considering shared links) and the Dijkstra algorithm will be the same if the travel time is assumed to be deterministic, however, in fuzzy circumstances, this result should be further reviewed.

In order to show that the above two points hold true under fuzzy circumstances, it is sufficient to prove that if fuzzy travel time "A" is less than "B", then the summation of "A" with another fuzzy number, such as "C", is also less than the sum of "B" and "C". In other words, the following relationship (Lemma 2) should be proven:

$$A \leq B \Leftrightarrow A + C \leq B + C. \tag{19}$$

It is possible to prove that this relationship is true when the I_3 index is used to compare fuzzy travel times. Before presenting the proof for Lemma 2, we need to first prove Lemma 1.

Lemma 1

The alternative "A_1" is the best alternative between n alternatives if, and only if, $I_3(A_1) \geq 0.5$.

Without a loss of generality, assume that the best alternative corresponds to the shortest path.

Proof

Proof by contradiction:

(a) Suppose that "A_1" is the best alternative, but $I_3(A_1) < 0.5$ (contradiction hypothesis). $I_3(A_1)$ is equal to:

$$\min\{N(A_1 \leq A_2), N(A_1 \leq A_3), \cdots, N(A_1 \leq A_i), \cdots, N(A_1 \leq A_n)\}.$$

According to the contradiction hypothesis, at least one alternative like "A_i" exists, such that:

$$I_3(A_1) = N(A_1 \leq A_i) < 0.5. \tag{20}$$

Because:

$$N(A_1 \leq A_i) + N(A_i \leq A_1) = 1,$$

then:

$$N(A_1 \leq A_i) = 1 - N(A_i \leq A_1) < 0.5$$
$$\Rightarrow N(A_i \leq A_1) > 0.5 > N(A_1 \leq A_i). \tag{21}$$

The above equation shows that there is an alternative named "A_i" that is better than "A_1". This result is in clear contradiction with the first hypothesis, so that the contradiction hypothesis is invalid.

(b) Conversely, suppose $I_3(A_1) \geq 0.5$ but the alternative, "A_1", is not the best alternative (contradiction hypothesis). Thus there is an alternative with $i \neq 1$, such that "A_i" is the best alternative. Therefore:

$$N(A_i \leq A_1) > 0.5 \Rightarrow N(A_1 \leq A_i) < 0.5. \tag{22}$$

We know that:

$$I_3(A_1) = \min\{N(A_1 \leq A_2), N(A_1 \leq A_3), \cdots, N(A_1 \leq A_i), \cdots, N(A_1 \leq A_n)\}, \tag{23}$$

$$I_3(A_1) = N(A_1 \leq A_i) < 0.5. \tag{24}$$

Therefore, $I_3(A_1) < 0.5$. This result is in clear contradiction to the first assumption ($I_3(A_1) \geq 0.5$), so it follows that the alternative, "A_1", is the best one.

Figure 9 clarifies Lemma 1. If A_2 shifts to the left on the X-axis, then I_3 will decrease and, when $I_3(A_1) < 0.5$, the fuzzy number, A_2, will be smaller than A_1.

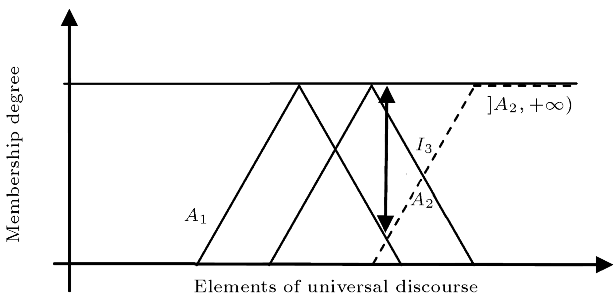

Figure 9. Illustration of $I_3(A_1)$ comparing A_1 and A_2.

Now, Equation 19 can be proven using the result from the lemma above.

Lemma 2

If "A", "B" and "C" are three fuzzy numbers with a triangular MF, then the following relation is true for index I_3:

$$A \leq B \Leftrightarrow A + C \leq B + C. \quad (25)$$

Proof

(a) $A \leq B \Rightarrow A + C \leq B + C$

The fuzzy numbers are shown as:

$$A = (a^L, a, a^R), \qquad B = (b^L, b, b^R),$$

$$C = (c^L, c, c^R).$$

It is known that:

$$N(A \leq B) = \frac{b^R - a}{(a^R + b^R) - (a + b)} \geq 0.5. \quad (26)$$

It should be shown that:

$$N(A + C \leq B + C)$$

$$= \frac{(b^R + c^R) - (a + c)}{(a^R + b^R + 2c^R) - (a + b + 2c)} \geq 0.5. \quad (27)$$

Rewriting Equation 26 yields:

$$N(A \leq B) = \frac{b^R - a}{(a^R + b^R) - (a + b)} \geq 0.5$$

$$\Rightarrow (b^R - a) \geq 0.5$$

$$\times [(a^R + b^R) - (a + b)]. \quad (28)$$

If we add the value $(c^R - c)$ to both sides of Equation 28, we obtain:

$$(b^R - a) + (c^R - c) \geq 0.5$$

$$\times \lfloor (a^R + b^R) - (a + b) \rfloor + (c^R - c) \Rightarrow \quad (29)$$

$$\Rightarrow (b^R + c^R) - (a + c) \geq 0.5$$

$$\times \lfloor (a^R + b^R + 2c^R) - (a + b + 2c) \rfloor \Rightarrow \quad (30)$$

$$\Rightarrow \frac{(b^R + c^R) - (a + c)}{(a^R + b^R + 2c^R) - (a + b + 2c)} \geq 0.5. \quad (31)$$

(b) $A + C \leq B + C \Rightarrow A \leq B$.

Because all of the above relationships are reversible, the inverse argument is simply proven. The proof is illustrated in Figure 10.

Finally, we should prove that the proposed FSPA truly finds the shortest path.

Proof of FSPA Truth

To show that the FSPA outcome is correct, we need to prove that $\tilde{d}(z_{i+1})$ is the shortest path with a fuzzy travel time from s to z_{i+1}. The induction method and contradiction are used to prove the algorithm. In the first step of the FSPA, the shortest path between s and s (which is zero) is obtained. Now, suppose in a middle step that the length of the shortest path from s to all nodes, available from $V - \bar{S}$, has been obtained and that z_{i+1} is a node labeled at the $i + 1$ step. The contradiction hypothesis shows that the minimum perceived travel time is not equal to the one obtained for the z_{i+1} node at the $i + 1$ step. In other words, in the true shortest path from the origin to z_{i+1} there is a node, say v, which has not yet been labeled. The mathematical translation of the contradiction hypothesis is below:

$$N\lfloor \tilde{d}(v) + \tilde{\phi}(vz_{i+1}) \leq \tilde{d}(z_{i+1}) \rfloor > 0.5. \quad (32)$$

Because a link travel time cannot be negative, we can write:

$$N[(0,0,0) \leq \tilde{\phi}(vz_{i+1})] \geq 0.5.$$

Using the Lemma 2 result, we can write:

$$N\lfloor \tilde{d}(v) \leq \tilde{d}(v) + \tilde{\phi}(vz_{i+1}) \rfloor \geq 0.5. \quad (33)$$

The contradiction hypothesis and Equation 33 result in:

$$N\lfloor \tilde{d}(v) \leq \tilde{d}(z_{i+1}) \rfloor > 0.5. \quad (34)$$

On the other hand, because z_{i+1} has been labeled before v:

$$N\lfloor \tilde{d}(z_{i+1}) \leq \tilde{d}(v) \rfloor > 0.5. \quad (35)$$

The two last equations (Equations 34 and 35) are in clear contradiction, so the contradiction hypothesis is invalid.

FSPA APPLICATION FOR A REAL NETWORK

The FSPA was applied to traffic assignment on a real-world large-scale transportation network in Mashhad, one of the largest cities in Iran. Mashhad city is divided into 141 traffic zones and it has a street network with 935 nodes, 2538 links and 7157 origin-destination pairs with non-zero observed demand. An origin-destination survey was conducted through at-home interviewing. Data were gathered from 4% of the households and validated through the observation of several screen lines in the study area. The traffic volume for 118 links was recorded as part of the data gathering effort [21].

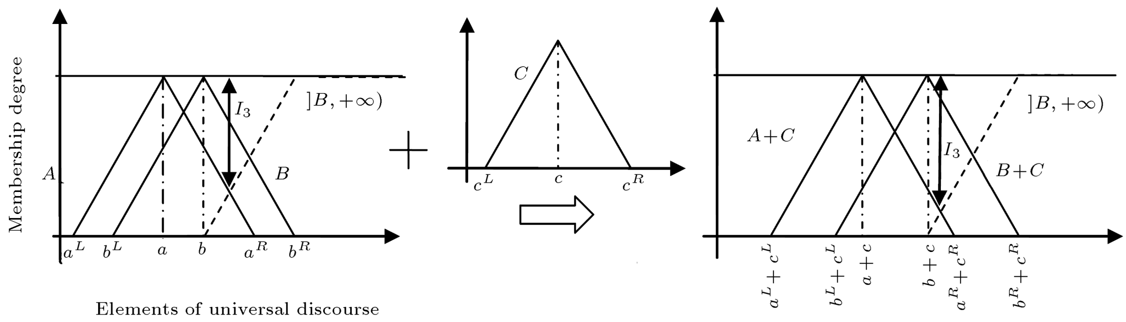

Figure 10. Illustration of Lemma 2.

The FSPA was implemented in a computer program using C++ language and run on a computer with a Pentium 4, 1.80GHz Central Processing Unit (CPU) PU and 512 megabytes (MB) of computer RAM memory. The shortest paths between all nodes in the Mashhad network were identified in less than one minute.

The link volumes were computed using an incremental assignment algorithm. For each step of the incremental assignment, the FSPA was used to find the shortest paths. Then, the results were compared with the observed volume of the links. Figure 11 illustrates the accuracy of the assignment results compared to the observations using an increasing α value.

The average Mean Square Error (MSE), which is used to compare the assignment results of the FSPA, using an increasing α value, was computed using the following equation:

$$\text{Average MSE} = \frac{\sum_{i=1}^{n}(OV_i - EV_i)^2}{n},$$

where:

$OV_i=$ observed volume of link "i",
$EV_i=$ estimated volume of link "i",
$n=$ number of links.

In this comparison, the volumes of 118 available observed links of the Mashhad network were compared to the assigned volumes. The assignment algorithm used the FSPA for different α values ranging from 0 to 2.6. For $\alpha = 0$, the assignment algorithm is the same as the traditional incremental assignment algorithm used by Dijkstra's shortest path algorithm. As Figure 11 indicates, as the α value increases, the average MSE decreases. The average MSE continues to decrease until $\alpha = 2.0$. To minimize the MSE, $\alpha = 2.0$ is optimal, therefore, it is used to define the membership functions for the Mashhad network link travel times. The travel time, for example link "a", will be shown using the following three parameters:

$$\tilde{t}_a = (t^l = t_a^0, t^c = t_a(x_a), t^r = t_a[3 \times x_a]),$$

where $\tilde{t}_a$, t_a^0, $t_a(x_a)$ are the PTT, free flow travel time, and travel time function for link "a", respectively. Because α is greater than 1 and the left boundary cannot be a negative number, it is assumed that the free flow travel time is equivalent to the minimum lower bound for the link travel time. The upper bound of the travel time is equal to the link travel time when the link volume is three times its observed volume.

To assess the applicability of the FSPA to traffic assignment, the results of the assignment using an incremental method with the FSPA are compared to the results of the UE assignment, as well as to a stochastic loading method, called Dial's assignment algorithm. Three graphs, shown in Figure 12, were used to compare the three assignment algorithms. The X-axis of these graphs corresponds to the estimated assigned volume and the Y-axis corresponds to the observed volumes. A trend line passes through the points, and the R^2 value, as well as the trend line equation, is included in the figures. As R^2 approaches 1, the accuracy of the estimated volume compared to the observed volume increases. It is expected that the trend line coefficient and constant will approach 1 and 0, respectively. The trend line equation formed using the assigned volumes from the FSPA through incremental assignment algorithm has a better fit to the theoretical values than the other two trend line equations. As shown in Figure 12, the volumes assigned using the

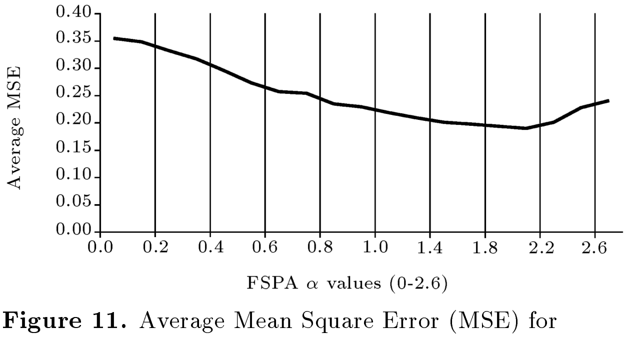

Figure 11. Average Mean Square Error (MSE) for different α values in FSPA.

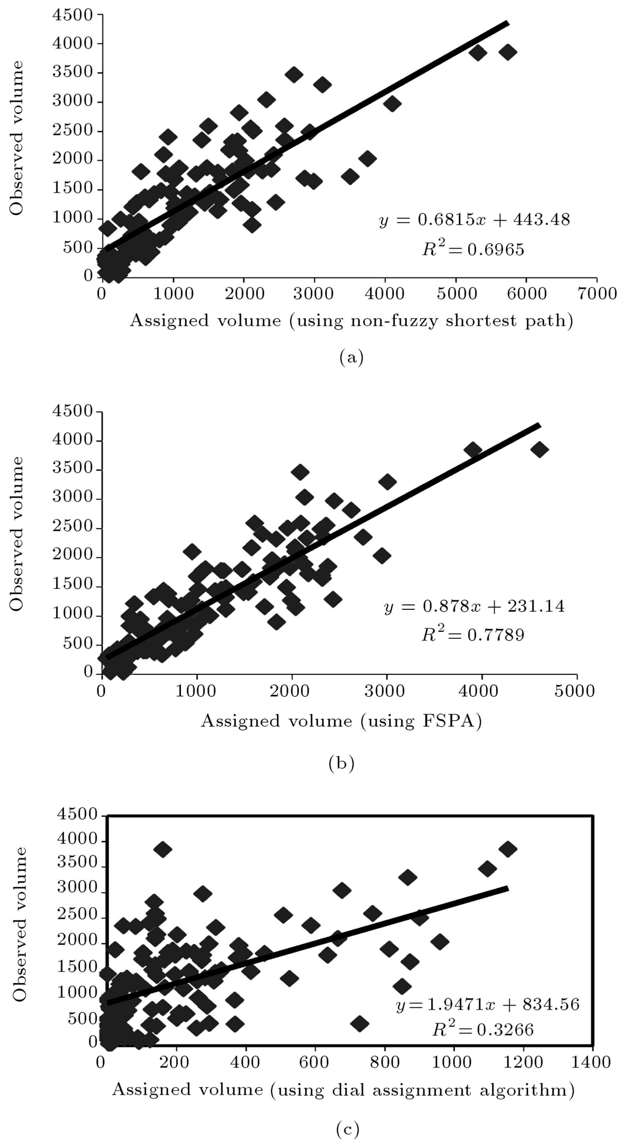

Figure 12. Comparison of the observed volume for links using (a) assigned volume by UE assignment with usual shortest path, (b) assigned volume by UE assignment with FSPA and (c) assigned volume by Dial's algorithm.

incremental assignment algorithm with the FSPA are the most accurate when compared to the observed volumes for 118 selected links. Consequently, the use of perceived travel times instead of definite travel times will increase the accuracy of the assignment model.

The computational platform for the three algorithms is the same. The assignment processing times were not considerably different between the algorithms. The results show the applicability of the FSPA for traffic assignment in real transportation networks.

CONCLUSION

This paper develops a FSPA for transportation networks, in which travelers' perceived travel times are assigned to links as fuzzy numbers defined using membership functions. Fuzzy theory appropriately takes into account the uncertainty embedded in travelers' perceptions of travel times; however, fuzzy logic and arithmetic are, to some extent, complicated. Because travelers use their perceived travel times for links in their route choice process, the chosen FSPA should be fine-tuned for traveler route choice modeling.

The aggregation of all travelers' route choices within a transportation network results in a traffic assignment that is traditionally computed using an assignment method such as a User Equilibrium (UE) or stochastic loading algorithm like Dial's assignment algorithm. In order to assess the applicability and performance of the resulting FSPA for traffic assignments, the results of the assignment, using an incremental method that incorporates FSPA, are compared to the results of an UE assignment, as well as to Dial's assignment algorithm for a large-scale real network. The comparison showed that an incremental assignment using our FSPA is the most accurate.

REFERENCES

1. Chang, P. and Lee, E. "Fuzzy decision networks and deconvolutions", *Computers and Mathematical with Applications*, **37**(11-12), pp. 53-63 (1999).
2. Okada, S. and Soper, T. "A shortest path problem on a network with fuzzy arc length", *Fuzzy Sets and Systems*, **109**(1), pp. 129-140 (2000).
3. Okada, S. "Fuzzy shortest path problems incorporating interactivity", *Fuzzy Sets and Systems*, **142**(3), pp. 335-357 (2004).
4. Blue, M., Bush, B. and Puckett, J. "Unified approach to fuzzy graph problems", *Fuzzy Sets and Systems*, **125**(3), pp. 355-368 (2002).
5. Chuang, T. and Kung, J. "The fuzzy shortest path length and the corresponding shortest path in a network", *Computers and Operation Research*, **32**(6), pp. 1409-1428 (2005).
6. Moazeni, S. "Fuzzy shortest path problem with finite fuzzy quantities", *Applied Mathematics and Computation*, **183**(1), pp. 160-169 (2006).
7. Ji, X., Iwamura, K. and Shao, Z. "New models for shortest path problem with fuzzy arc length", *Applied Mathematics Modeling*, **31**(2), pp. 259-269 (2007).
8. Hernandes, F., Teresa Lamata, M., Verdegay, J.L. and Yamakami, A. "The shortest path problem on networks with fuzzy parameters", *Fuzzy Sets and Systems*, **158**(14), pp. 1561-1570 (2007).
9. Teodorovic, D. and Kikuchi, S. "Transportation route choice model using fuzzy inference technique", *First International Symposium on Uncertainty Modelling and Analysis*, Ayyub, pp. 140-145, (1990).
10. Murat, Y.S. and Uludag, N. "Route choice modelling in urban transportation networks using fuzzy logic and

logistic regression methods", *Journal of Scientific and Industrial Research (JSIR)*, **67**(1), pp. 19-27 (2008).

11. Binetti, M. and De Mitri, M. "Traffic assignment model with fuzzy travel cost", *13th Mini EURO Conference Handling Uncertainty in the Analysis of Traffic and Transportation Systems*, pp. 805-812 (2002).
12. Arslan, T. and Khisty, C.J. "A rational approach to handling fuzzy perceptions in route choice", *European Journal of Operational Research*, **168**(2), pp. 571-583 (2006).
13. Zadeh, L. "Discussion: Probability theory and fuzzy logic are complementary rather than competitive", *Technometrics*, **37**(3), pp. 271-276 (1995).
14. Zadeh, L. "Fuzzy logic = computing with words", *IEEE Transaction on Fuzzy Systems*, **4**(2), pp. 103-111 (1996).
15. Ross, T.J., Booker, J.M. and Jerry Parkinson, W. "Fuzzy logic and probability applications: Bridging the gap", *Society for Industrial and Applied Mathematics*, 1st Ed., Philadelphia, pp. 18-20 (2002).
16. Henn, V. and Ottomanelli, M. "Handling uncertainty in route choice models: from probabilistic to possibilistic approaches", *European Journal of Operational Research*, **175**(3), pp. 1526-1538 (2006).
17. Sheffi, Y., *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*, Englewood Cliffs, NJ, Prentice-Hall, 1st Ed. (1985).
18. Dubois, D. and Prade, H. "Ranking fuzzy numbers in the settings of possibility theory", *Information Science*, **30**(2), pp. 183-224 (1983).
19. Henn, V. "Fuzzy route choice model for traffic assignment", *Fuzzy Sets and Systems*, **116**(1), pp. 77-101 (2000).
20. Xu, M., Liu, Y., Huang, Q., Zhang, Y. and Luan, G. "An improved dijkstra's shortest path algorithm for sparse network", *Applied Mathematics and Computation*, **185**(1), pp. 247-254 (2007).
21. Poorzahedi, H., Kermanshah, M., Ashtiani, H.Z. and Shafahi, Y. "Traffic assignment model and Mashhad transportation network performance in 1994", Sharif University of Technology, Institute of Transportation Studies and Research, submitted to Mashhad Municipality (1997).

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