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Preloaded High-Temperature Constitutive Models and Relationships for Concrete

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Abstract. *The behavior of concrete structures that are exposed to extreme thermo-mechanical loading is an issue of great importance in nuclear engineering. The structural fire-safety capacity of concrete is very complicated because concrete materials have considerable variations. Constitutive models and relationships for preloaded Normal and High Strength Concrete (NSC and HSC) subjected to fire are needed, which are intended to provide efficient modeling and to specify the fire-performance criteria of the behavior of preloaded concrete structures exposed to fire. In this paper, formulations for estimating the parameters affecting the behavior of unconfined preloaded concrete at high temperatures are proposed. These formulations include residual compression strength, initial modulus of elasticity, peak strain, thermal strain, transient creep strain and the compressive stress-strain relationship at elevated temperatures. The proposed constitutive models and relationships are verified with available experimental data and existing models. The proposed models and relationships are general and rational, and have good agreement with the experimental data. More tests are needed to further verify and improve the proposed constitutive models and relationships.*

Keywords: *Preloaded concrete; Constitutive models and relationships; Normal and high strength concrete; Fire; High temperature.*

INTRODUCTION

Concrete is a heterogeneous material with a wide variety of usage in structural design [1]. Modeling of the mechanical behavior of concrete at temperatures above ambient is necessary in the analysis of hypothetical accidental situations in a nuclear reactor or in other accidental fire situations. The design of fire resistant structural elements requires realistic knowledge on the behavior of concrete at high temperatures. When concrete is exposed to high temperatures, there may be considerable variations in physical and mechanical properties with an irreversible loss of stiffness and strength including the possibility of increased ductility in the post-peak regime. The fire resistance of concrete can be determined by three test methods, available for finding the residual compressive strength of con-

crete at elevated temperatures: stressed, unstressed, and unstressed residual strength tests. The stressed and unstressed tests are compatible for assessing the strength of concrete during high temperatures, while the unstressed residual strength test is excellent for finding residual properties after elevated temperatures. In the stressed test, specimens are restrained by a preload prior to, and throughout, the heating process. In the unstressed test, the specimens are heated without restraint. Both stressed and unstressed specimens are loaded to failure under uni-axial compression when the steady-state temperature is reached at the target temperature. The unstressed residual property test method is designed to provide the property data of concrete at room temperature after exposure to elevated temperatures [2,3].

At present, using prescriptive approaches is generally established for the fire resistance of Reinforced Concrete (RC) members, which are based on either empirical calculation methods or standard fire resistance tests. These approaches do not provide a rational and realistic fire safety assessment and have

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major drawbacks. New codes are moving towards performance-based design, and temperature-dependent calculations are expected to be required to satisfy certain performance criteria. There is an increased focus on the use of numerical methods for evaluating the fire performance of structural members, which depends on the properties of the constituent materials. Knowledge of the high-temperature properties of concrete is critical for fire resistance assessment under performance-based codes [4].

The parameters that control concrete behavior are: compressive strength, tensile strength, peak strain, modulus of elasticity, creep strain, thermal conductivity and thermal strain, which are nonlinear functions of temperature. Also, aggregate types of concrete influence concrete behavior exposed to fire [5]. Many compressive and tensile constitutive models for concrete at normal temperatures are proposed. The constitutive laws of concrete materials under fire conditions are complicated, and knowledge of current thermal properties is based on the limited material properties. There are either limited test data for some elevated temperature properties, or there are considerable differences and inconsistencies in the elevated temperature test data for other properties of concrete [2,6,7]. These differences and inconsistencies are mainly due to the differences in test methods, condition of procedures, and the environmental parameters accompanying the tests [8]. Thus, at present, there are no well-founded constitutive relationships in codes and standards for many of the high-temperature properties of concrete [2]. Although the computational methods and techniques for estimating the fire performance of structural members of buildings are proposed, research studies that provide inputting data, such as the constitutive laws of concrete materials, into these computational methods, has not kept pace [9]. Much of the information in ACI216R [10] is based on experimental test results undertaken during the 1950s and 1960s that contain no comprehensive constitutive relationships [4].

In this study, constitutive models and relationships are proposed for preloaded Normal and High Strength Concrete (NSC and HSC) at elevated temperatures, which are compared to available ones and verified with pervious experimental data. Regression analyses are conducted on existing experimental data to propose residual compression strength, initial modulus of elasticity, peak strain, thermal strain and transient creep strain. In the present paper, at first, the models that were proposed for residual compression strength, initial modulus of elasticity, peak strain, thermal strain and creep strain are verified with experimental data. Secondly, compressive stress-strain relationships for NSC and HSC at elevated temperatures are proposed and verified with the experimental data.

COMPRESSIVE STRENGTH OF PRELOADED NSC AND HSC AT HIGH TEMPERATURES

Several models have been proposed to estimate unloaded concrete compressive strength at high temperatures. The model for the preloaded concrete compressive strength of concrete at high temperatures is the Hertz [11] model. Hertz [11] proposed a model (Equation 1) that recognizes the variation of f'_{cT} with the type of aggregate:

$$f'_{cT} = f'_c \left[1 \bigg/ \left(1 + \frac{T}{T_1} + \left(\frac{T}{T_2}\right)^2 + \left(\frac{T}{T_8}\right)^8 + \left(\frac{T}{T_{64}}\right)^{64} \right) \right].$$ (1)

Siliceous aggregate:

$$T_1 = 15000, \quad T_2 = 800,$$

$$T_8 = 570, \qquad T_{64} = 100000.$$

Lightweight aggregate:

$$T_1 = 100000, \quad T_2 = 1100,$$

$$T_8 = 800, \qquad T_{64} = 940.$$

Other aggregates:

$$T_1 = 100000, \quad T_2 = 1080,$$

$$T_8 = 690, \qquad T_{64} = 1000.$$

In this study, a model is proposed for the preloaded compressive strength of normal, high strength (siliceous aggregate), carbonate and lightweight aggregate preloaded concretes at elevated temperatures, regression analyses for which are proposed and conducted on existing experimental data, as expressed in Equations 2 to 6, respectively.

Normal strength concrete (Siliceous aggregate):

$$f'_{cT} = f'_c \begin{cases} 1.0 \\ 1.06 + 0.00025\,T - 2.235 \times 10^{-6}\,T^2 \\ \qquad + 8 \times 10^{-10}\,T^3 \\ 0.44 - 0.0004\,T \\ 0 \end{cases}$$

$$\left. \begin{array}{l} 20^\circ\mathrm{C} \leq T \leq 200^\circ\mathrm{C} \\ 200^\circ\mathrm{C} < T \leq 800^\circ\mathrm{C} \\ 900^\circ\mathrm{C} \leq T \leq 1000^\circ\mathrm{C} \\ T > 1000^\circ\mathrm{C} \end{array} \right\}.$$ (2)

High strength concrete (Siliceous aggregate):

$$f'_{cT} = f'_c \begin{cases} 1.0 \\ 0.83 + 0.0019\,T - 5.2 \times 10^{-6}\,T^2 \\ \qquad + 3 \times 10^{-9}\,T^3 \\ 0 \end{cases}$$

$$\left. \begin{matrix} 20^\circ\mathrm{C} \le T \le 100^\circ\mathrm{C} \\ 100^\circ\mathrm{C} < T \le 800^\circ\mathrm{C} \\ T > 800^\circ\mathrm{C} \end{matrix} \right\}. \qquad (3)$$

Carbonate aggregate concrete:

$$f'_{cT} = f'_c \begin{cases} 1.00537 - 2.9 \times 10^{-4}\,T \le 1.0 \\ 1.05 - 0.0017\,T + 5 \times 10^{-6}\,T^2 \\ \qquad - 5 \times 10^{-9}\,T^3 \\ 0 \end{cases}$$

$$\left. \begin{matrix} 20^\circ\mathrm{C} \le T \le 400^\circ\mathrm{C} \\ 400^\circ\mathrm{C} < T < 900^\circ\mathrm{C} \\ 900^\circ\mathrm{C} \le T \end{matrix} \right\}. \qquad (4)$$

Lightweight aggregate concrete:

$$f'_{cT} = f'_c \begin{cases} 1.003158 - 1.57 \times 10^{-4}\,T \le 1.0 \\ 1.035 - 0.0015\,T + 5 \times 10^{-6}\,T^2 \\ \qquad - 5 \times 10^{-9}\,T^3 \\ 0 \end{cases}$$

$$\left. \begin{matrix} 20^\circ\mathrm{C} \le T \le 200^\circ\mathrm{C} \\ 200^\circ\mathrm{C} < T \le 900^\circ\mathrm{C} \\ T \ge 1000^\circ\mathrm{C} \end{matrix} \right\}. \qquad (5)$$
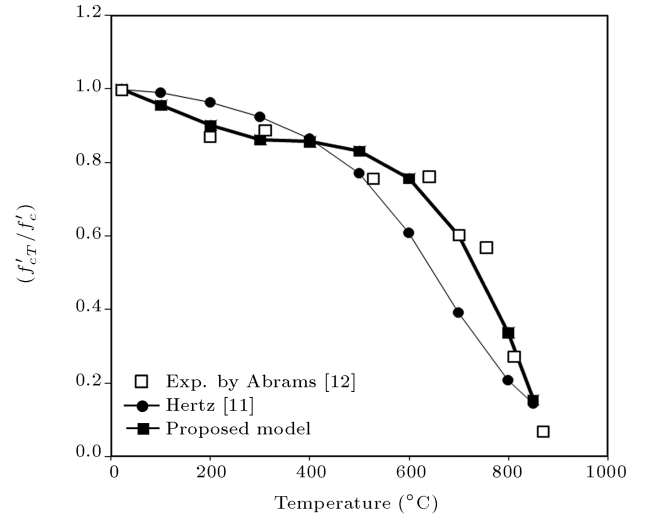
The proposed models at elevated temperatures are compared separately with experimental data and the Hertz [11] model as shown in Figures 1 to 4. Figure 1 makes a comparison between the Hertz [11] model
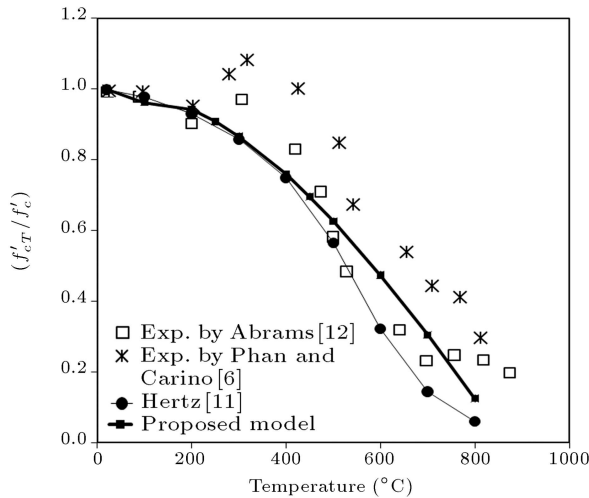
Figure 1. Comparison between compressive strength models of preloaded NSC at elevated temperatures with experimental data.
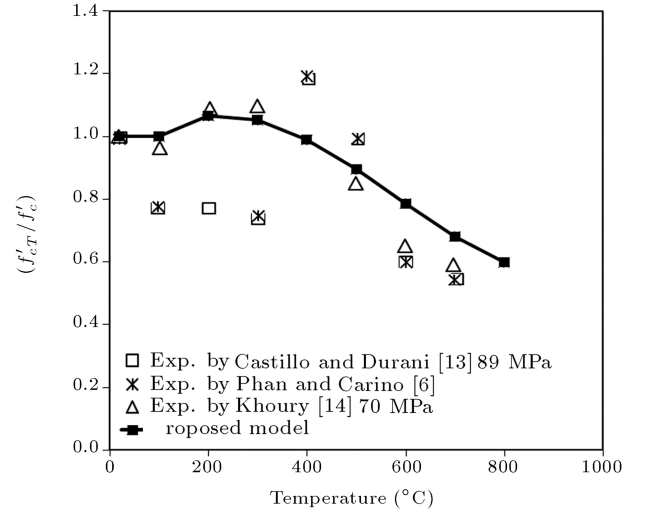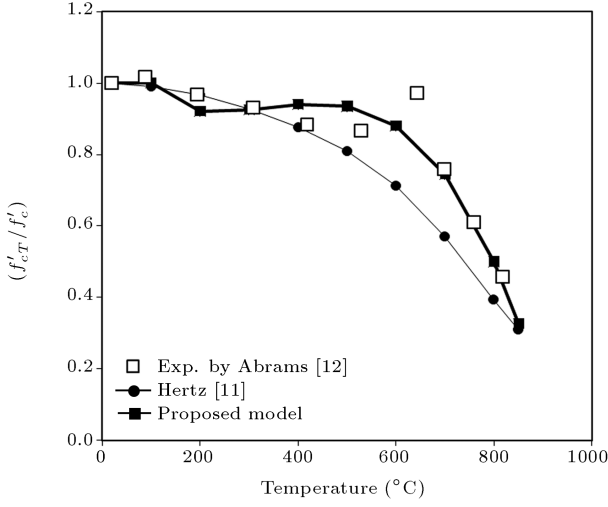

Figure 2. Comparison between compressive strength models of preloaded HSC at elevated temperatures with experimental data.

Figure 3. Comparison between compressive strength models of carbonate aggregate preloaded concrete at elevated temperatures with experimental data.

and the proposed model for preloaded NSC at different temperatures, against the experimental results of Abrams [12] and Phan and Carino [6]. The proposed model has good accuracy with the experimental results. Figure 2 shows the proposed model for preloaded HSC at different temperatures, against the experimental results of Castillo and Durani [13], Khoury et al. [14] and Phan and Carino [6], which indicates that the model fits well with experimental results. The model is unique and no other model has been found for HSC, which is very important due to the widespread usage of HSC around the world especially in high-rise buildings that are faced with structural fire-safety problems. The compressive strength of HSC varies differently and more unfavorably with temperature compared to

Figure 4. Comparison between compressive strength models of lightweight aggregate preloaded concrete at elevated temperatures with experimental data.

that of NSC. The differences are more pronounced in the temperature range between 25°C to about 400°C, where HSC sustains markedly higher strength loss than NSC. Differences become less significant at temperatures above 400°C. The variations of compressive strength with temperature may be characterized by an initial stage of strength loss (25°C to approximately 100°C), followed by a stage of stabilized strength and recovery (100°C to approximately 400°C), and a stage above 400°C characterized by a monotonic decrease in strength with increasing temperature. The strength recovery stage of HSC occurs at higher temperatures than NSC [2]. Figure 3 shows a comparison of the Hertz [11] model and the proposed model for preloaded carbonate aggregate concrete against the experimental results of Abrams [12]. The proposed model has good accuracy with the experimental results in comparison to the Hertz [11] model. Fig-

ure 4 represents a comparison between the proposed model for preloaded lightweight aggregate concrete, the Hertz [11] model and the experimental results of Abrams [12]. The proposed model is more rational with the experimental results in comparison with the Hertz [11] model especially for temperatures above 400°C.

## INITIAL MODULUS OF ELASTICITY AT ELEVATED TEMPERATURES (PRELOADED CONCRETE)

The elastic modulus of concrete could be affected primarily by the same factors influencing its compressive strength [15,16]. The most important available models for the elastic modulus of preloaded concrete at high temperatures are summarized in Table 1. In this paper, a model for the elasticity modulus of concrete at elevated temperatures is proposed, regression analyses for which are conducted on existing experimental data to propose it, and which is expressed as Equation 6:

$$E_{crT} = E_c \left\{ \begin{array}{l} 1.0 \\ 1.03 - 0.00025\,T - 9 \times 10^{-7}\,T^2 \\ 0 \end{array} \right. \quad \left. \begin{array}{l} 20°C \\ 100°C < T \le 800°C \\ T > 800°C \end{array} \right\}. \qquad (6)$$

Figure 5 provides a comparison between Table 1 models and the proposed model for the elasticity modulus of preloaded concrete at elevated temperatures against the experimental results of Anderberg and Thelandersson [17] and Khoury et al. [14]. The proposed model fitted well with most of the experimental results.

Table 1. Elastic modulus models at elevated temperatures (preloaded concrete).

Reference	Elastic Modulus at Elevated Temperatures
Anderberg and Thelandersson [17]	$E_{crT} = \frac{2f'_{cT}}{\varepsilon'_{cT}},$
Schneider [18]	Normal weight concrete: $E_{crT} = (-0.001552\,T + 1.03104)gE_c, \quad 20°C \le T \le 600°C,$ $E_{crT} = (-0.00025\,T + 0.25)gE_c, \quad 600°C \le T \le 1000°C,$ Lightweight concrete: $g = 1 + \frac{f_{ci}}{f'_c}\frac{T-20}{100}, \qquad \frac{f_{ci}}{f'_c} \le 3.0,$ $E_{crT} = (-0.00102\,T + 1.0204)gE_c, \quad 20°C \le T \le 1000°C,$
Khennane and Baker [19]	For preloaded concrete: $E_{crT} = (-0.000634\,T + 1.012673)E_c$ if $20°C \le T \le 525°C,$ $E_{crT} = (-0.002036\,T + 1.749091)E_c$ if $525°C \le T \le 800°C,$
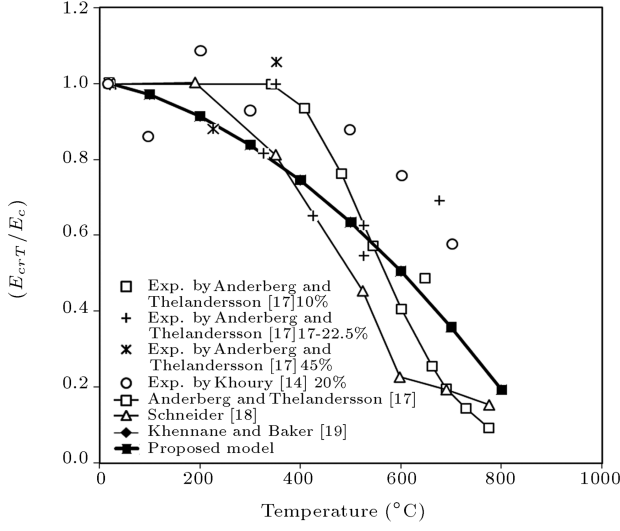
Figure 5. Comparison between elastic modulus models of preloaded concrete at elevated temperatures with experimental data.

PEAK STRAIN AT HIGH TEMPERATURES (PRELOADED CONCRETE)

The most important models for the peak strain of preloaded concrete at high temperatures are the Khennane and Baker [19] and Terro [20] models. Khennane and Baker [19] studied the experimental results provided by Anderberg and Thelandersson [17] and proposed the following equation for the peak strain of concrete, having an initial compressive stress during the heating process.

$$\varepsilon_{\max} = 0.00000167\,T + 0.002666 \geq 0.003,$$

$$\text{if } T \leq 800°\text{C}. \tag{7}$$

Terro [20] proposed the following equation for the peak strain of concrete that accounts for the initial compressive stress level.

$$\varepsilon_{\max} = (50\lambda_L^2 - 15\lambda_L + 1)\varepsilon_{c1}' + 20(\lambda_L - 5\lambda_L^2)\varepsilon_{c2}'$$

$$+ 5(10\lambda_L^2 - \lambda_L)\varepsilon_{c3}',$$

$$\varepsilon_{c1}' = 2.05 \times 10^{-3} + 3.08 \times 10^{-6}\,T + 6.17\times10^{-9}\,T^2$$

$$+ 6.58 \times 10^{-12}\,T^3,$$

$$\varepsilon_{c2}' = 2.03 \times 10^{-3} + 1.27 \times 10^{-6}\,T + 2.17\times10^{-9}\,T^2$$

$$+ 1.64 \times 10^{-12}\,T^3,$$

$$\varepsilon_{c3}' = 0.002. \tag{8}$$

In this paper, a much simpler model for the peak strain of preloaded concrete at elevated temperatures is proposed, regression analyses for which are conducted on existing experimental data in order to propose it, and which is expressed as Equation 9:

$$\varepsilon_{\max} = 0.0028 + 2 \times 10^{-6}\,T,$$

$$20°\text{C} \leq T \leq 800°\text{C}. \tag{9}$$

Figure 6 provides a comparison between Khennane and Baker [19] and Terro [20] models and the proposed model for peak strain preloaded concrete, against the experimental results of Anderberg and Thelandersson [17]. The proposed model has good accuracy with experimental results in comparison with others.

THERMAL STRAIN (UNLOADED AND PRELOADED CONCRETE)

The free thermal expansion is predominantly affected by the aggregate type. The free thermal expansion is not linear with respect to temperature. The presence of free moisture will affect the result below 150°C, since the water being driven off may cause net shrinkage. Traditionally, it is expressed by a linear function of temperature by employing a thermal expansion coefficient, α [21].

$$\varepsilon_{th} = \alpha(T - 20°\text{C}). \tag{10}$$

For concrete with siliceous or carbonate aggregates, α can be taken equal to 18×10^{-6} or 12×10^{-6} per °C [22]. The most important available models for the thermal strain of unloaded concrete at high temperatures are summarized in Table 2. In this study, models for the thermal strain of unloaded siliceous, carbonate and
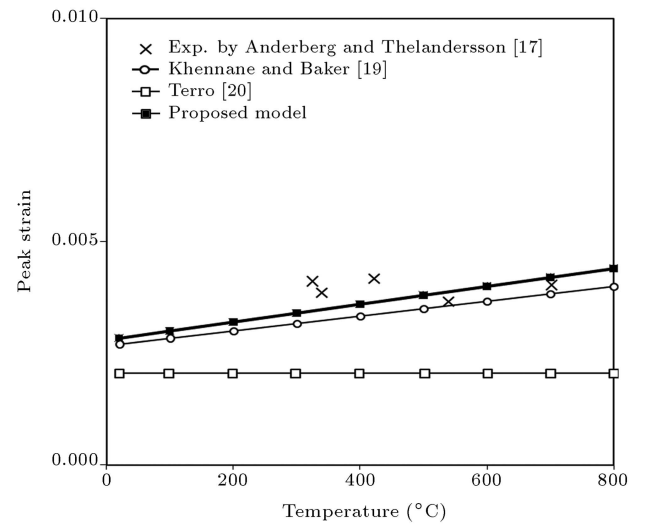
Figure 6. Comparison between peak strain models of preloaded concrete at elevated temperatures with experimental data.

Table 2. Thermal strain models (unloaded concrete).

Reference	Thermal Strain Models (Unloaded Concrete)
Lie [23]	Siliceous and Carbonate Aggregate: $\varepsilon_{th} = [0.004(T^2 - 400) + 6(T - 20)] \times 10^{-6}$, Lighweight: $\varepsilon_{th} = 7.5 \times 10^{-6}(T - 20°\text{C})$,
EN 1992-1-2 [24]	Siliceous Sggregate: $\varepsilon_{th} = \begin{cases} -1.8 \times 10^{-4} + 9 \times 10^{-6}\ T + 2.3 \times 10^{-11}(T)^3 & 20°\text{C} \le T \le 700°\text{C} \\ 14 - tiems 10^{-3} & 700°\text{C} \le T \le 1200°\text{C} \end{cases}$, Carbonate Aggregate: $\varepsilon_{th} = \begin{cases} -1.2 \times 10^{-4} + 6 \times 10^{-6}\ T + 1.4 \times 10^{-11}(T)^3 & 20°\text{C} \le T \le 805°\text{C} \\ 12 \times 10^{-3} & 805°\text{C} \le T \le 1200°\text{C} \end{cases}$, Lighweight: $\varepsilon_{th} = 8 \times 10^{-6}(T - 20°\text{C})$,

lightweight aggregate concretes at elevated temperatures are proposed, regression analysis for which are conducted on the existing experimental data in order to propose it, and which are expressed as Equations 11 to 14:

Siliceous aggregate concrete:

$$\varepsilon_{th} = 0.00045 + 1 \times 10^{-6}\ T + 2 \times 10^{-8}\ T^2,$$

$$100°\text{C} \le T \le 800°\text{C}, \tag{11}$$

$$\varepsilon_{th} = \alpha(0.00045 + 1 \times 10^{-6}\ T + 2 \times 10^{-8}\ T^2),$$

$$100°\text{C} \le T \le 800°\text{C}. \tag{12}$$

Compressive Strength range:

$$20C - 60C \quad \alpha = 1,$$

$$C70 \quad \alpha = 0.85,$$

$$C80 \quad \alpha = 0.75,$$

$$C90 \quad \alpha = 0.65,$$

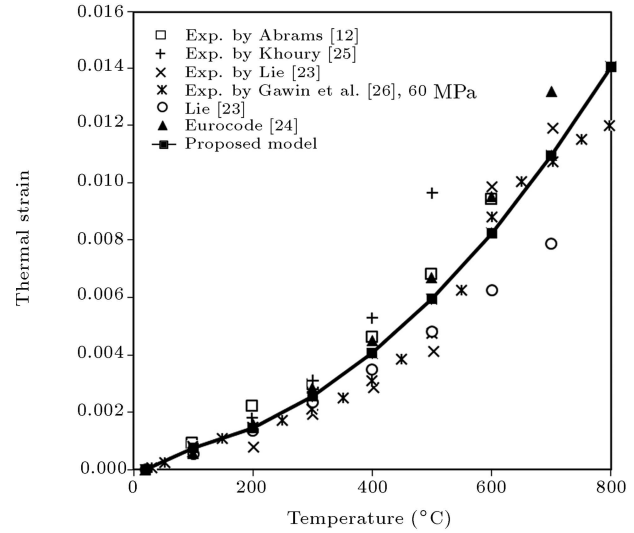$$C100 \quad \alpha = 0.5.$$

Carbonate aggregate concrete:

$$\varepsilon_{th} = 0.0001 + 5 \times 10^{-7}\ T + 2 \times 10^{-8}\ T^2,$$

$$100°\text{C} \le T \le 800°\text{C}. \tag{13}$$

Lightweight aggregate concrete:

$$\varepsilon_{th} = -0.00045 + 8 \times 10^{-6}\ T,$$
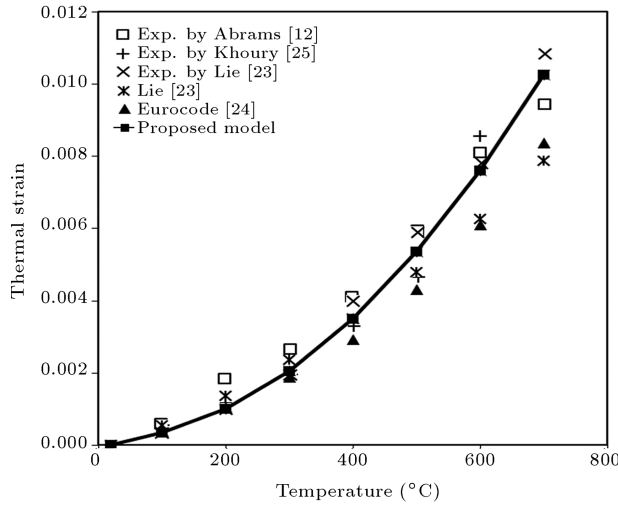
$$100°\text{C} \le T \le 800°\text{C}. \tag{14}$$

Figure 7. Comparison between thermal strain models of normal strength unloaded siliceous aggregate concrete at elevated temperatures with experimental data.

Figure 7 provides a comparison between Lie [23] and EN 1992-1-2 [24] models and the proposed model for the thermal strain of normal strength unloaded siliceous aggregate concrete against the experimental results of Abrams [12], Khoury et al. [25], Lie [23], and Gawin et al., 60MPa [26]. Figure 8 represents a comparison between Lie [23] and EN 1992-1-2 [24] models and the proposed model for the thermal strain of unloaded siliceous aggregate concrete against the experimental results of Gawin et al., 90 MPa [26]. Figures 9 and 10 provide a comparison between Lie [23] and EN 1992-1-2 [24] models and the proposed model for the thermal strain of unloaded carbonate and lightweight aggregate concretes separately against the experimental results of Abrams [12], Khoury et al. [25] and Lie [23]. The proposed models have good accu-
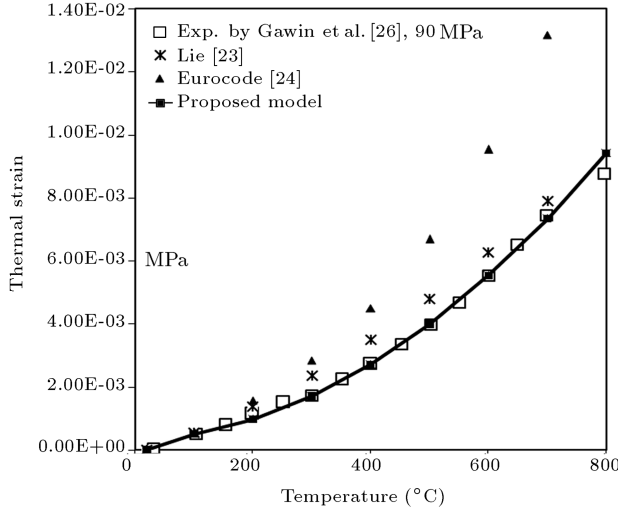
Figure 8. Comparison between thermal strain models of high strength unloaded siliceous aggregate concrete at elevated temperatures with experimental data.
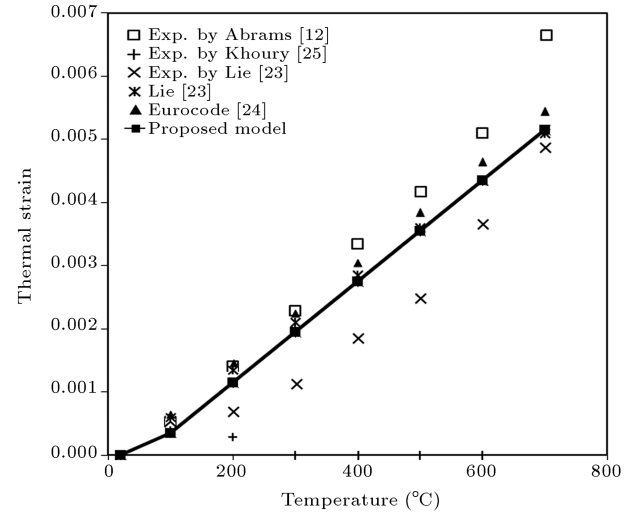

Figure 9. Comparison between thermal strain models of unloaded carbonate aggregate concrete at elevated temperatures with experimental data.

racy with the experimental results in comparison with others.

In this study, also models for the thermal strain of preloaded concrete at elevated temperatures are proposed as functions of preloading percentages, regression analyses for which are conducted on existing experimental data in order to propose it, and which are expressed as Equations 15-18:

For 10%-15% preloaded:

$$\varepsilon_{th} = -0.0002 - 5 \times 10^{-7} \, T + 2.6 \times 10^{-8} \, T^2$$

$$- 4 \times 10^{-11} \, T^3,$$

$$100°C \leq T \leq 800°C. \tag{15}$$

Figure 10. Comparison between thermal strain models of unloaded lightweight aggregate concrete at elevated temperatures with experimental data.

For 15%-30% preloaded:

$$\varepsilon_{th} = -0.00047 - 7.2 \times 10^{-6} \, T + 6 \times 10^{-8} \, T^2$$

$$- 10^{-10} \, T^3,$$

$$100°C \leq T \leq 800°C. \tag{16}$$

For 30%-45% preloaded:

$$\varepsilon_{th} = -0.0001 - 1.8 \times 10^{-5} \, T + 6 \times 10^{-8} \, T^2$$

$$- 9 \times 10^{-11} \, T^3,$$

$$100°C \leq T \leq 800°C. \tag{17}$$

For 45%-60% preloaded:

$$\varepsilon_{th} = -0.0007 - 10^{-5} \, T + 5 \times 10^{-8} \, T^2 - 8 \times 10^{-11} \, T^3,$$
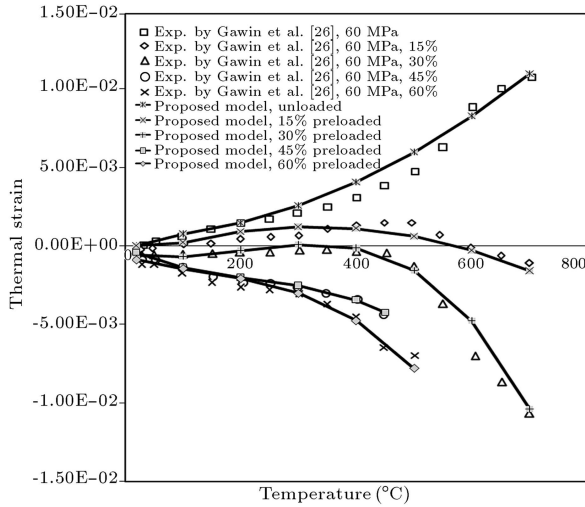
$$100°C \leq T \leq 800°C. \tag{18}$$

Figure 11 provides a comparison between the proposed models for the thermal strain of preloaded concrete against the experimental results of Gawin et al. [26] for unloading 15%, 30%, 45% and 60% preloading cases, respectively. The proposed models fit very well with most of the experimental results, which indicate more preloading percentage and more reduction of thermal strain. This figure also indicates the trend of thermal strain-temperature curves, which is entirely a function of the preloading percentage.

CREEEP STRAIN AT ELEVATED TEMPERATURES

It was observed that preloaded concrete elements experience a characteristic marked increase in strains during

Table 3. Creep strains models.

Reference	Creep Strains
Anderberg and Thelandersson [17]	$\varepsilon_{tr} = k_{tr}\left(\frac{\sigma_c T}{f'_c}\right)\varepsilon_{th}, \quad T \leq 550°\text{C}, \quad \frac{\partial \varepsilon_{tr}}{\partial T} = 0.0001\left(\frac{\sigma_c T}{f'_c}\right),$ $T \geq 550°\text{C}, \quad 1.8 \leq k_{tr} \leq 2.35,$
Schneider [18]	$\varepsilon_{tr} = \frac{\phi}{g}\frac{f_c}{E_{cr}}, \quad \phi = g\{C_1 \tanh[\gamma_{w(T-20)}] + C_2 \tanh[\gamma_o(T - T_g)] + C_3\} + \frac{\sigma_c T}{f'_{cT}}\frac{T-20}{100},$ $\frac{\sigma_c T}{f'_{cT}} \leq 3.0, \quad \gamma_w = (0.3w + 2.2) \times 10^{-3},$ where w is the moisture content and C_1, C_2, C_3, γ_o and T_g are constants with values equal to 2.60, 1.40, 1.40, 0.0075 and 700 for concrete with siliceous aggregates, 2.60, 2.40, 2.40, 0.0075 and 650 for concrete with carbonate aggregates, and 2.60, 3.00, 3.00, 0.0075 and 600 for concrete with lightweight aggregates.
Diederichs [27]	$\varepsilon_{tr} = \frac{\sigma_c T}{f'_c}[3.3 \times 10^{-10}(T - 20)^3 - 1.72 \times 10^{-7}(T - 20)^2 + 0.0412 \times 10^{-3}(T - 20)],$
Terro [20]	$\varepsilon_{tr} = \varepsilon_{0.3} \times \left(0.032 + 3.226\frac{f_{ci}}{f'_c}\right)\frac{V_a}{0.65}, \quad \frac{f_{ci}}{f'_{cT}} \leq 3.0,$ $\varepsilon_{0.3} = -43.87 \times 10^{-6} + 2.73 \times 10^{-8} \ T + 6.35 \times 10^{-8} \ T^2 - 2.19 \times 10^{-10} \ T^3 + 2.77$ $\times 10^{-13} \ T^4,$ $\varepsilon_{0.3} = -1625.78 \times 10^{-6} + 58.03 \times 10^{-6} \ T - 0.6364 \times 10^{-6} \ T^2 + 3.6112 \times 10^{-9} \ T^3$ $-9.2796 \times 10^{-12} \ T^4 + 8.806 \times 10^{-15} \ T^5,$
Nielsen et al. [28]	$\varepsilon_{tr} = 0.000038(\sigma_c T/f'_c)T.$

Figure 11. Comparison between thermal strain models of preloaded concrete at elevated temperatures with experimental data.

initial heating [25,29]. This increase significantly exceeds expected creep strains and was termed as a transient creep strain [22,23,30]. The most important models for the creep strain of concrete at high temperatures are summarized in Table 3. In this study also models for the creep strain of preloaded concrete at elevated temperatures are proposed, regression analyses for which are conducted on existing experimental data in order to propose it, and which are expressed as Equations 19-21:
For 10%-20% preloaded:

$$\varepsilon_{tr} = 0.0006 - 3.8 \times 10^{-6} \ T + 2.25 \times 10^{-8} \ T^2,$$

$$100°\text{C} \leq T \leq 600°\text{C}. \tag{19}$$

For 20%-40% preloaded:

$$\varepsilon_{tr} = 0.00095 - 6 \times 10^{-6} \ T + 3 \times 10^{-8} \ T^2,$$

$$100°\text{C} \leq T \leq 600°\text{C}. \tag{20}$$

For 40%-60% preloaded:

$$\varepsilon_{tr} = -8 \times 10^{-5} + 4.1 \times 10^{-6} \ T + 3 \times 10^{-8} \ T^2,$$

$$100°\text{C} \leq T \leq 600°\text{C}. \tag{21}$$

Figures 12 to 14 show the proposed models fit well with most of the experimental results, as well as Nielsen's model [28], which is linear and rational with the experimental results of temperatures less than 500°C. It can be used if simplified calculations are required. Schneider's model [18] provides a lower bound for the experimental results.

COMPRESSIVE STRESS-STRAIN RELATIONSHIPS AT ELEVATED TEMPERATURES

The most important available compressive stress-strain relationships for concrete at high temperatures are summarized in Table 4. In this study, a compressive

Table 4. Compressive stress-strain relationships at elevated temperatures.

Reference	Compressive Stress-Strain Relationships at Elevated Temperatures
Anderberg and Thelandersson [17]	$\sigma_{cT} = f_1 - 880(\varepsilon_{cT} - \varepsilon_1), \quad \varepsilon_{cT} \geq \varepsilon_1, \quad \sigma_{cT} = E_{crT}\left[\varepsilon_{cT} - \frac{\varepsilon_{cT}^2}{2\varepsilon'_{cT}}\right], \quad \varepsilon_{cT} \leq \varepsilon_1;$ $\varepsilon_1 = \varepsilon'_{cT}\left(1 - \frac{880}{E_{crT}}\right), \qquad f_1 = E_{crT}\left(\varepsilon_1 - \frac{\varepsilon_1^2}{2\varepsilon'_{cT}}\right);$
Lie and Lin [31]	$\sigma_{cT} = f'_{cT}\left[1 - \left(\frac{\varepsilon_{cT} - \varepsilon_{\max}}{3\varepsilon_{\max}}\right)^2\right], \quad \varepsilon_{cT} > \varepsilon_{\max}$ $\sigma_{cT} = f'_{cT}\left[1 - \left(\frac{\varepsilon_{\max} - \varepsilon_{cT}}{\varepsilon_{\max}}\right)^2\right], \qquad \varepsilon_{cT} \leq \varepsilon_{\max}$
Schneider [18]	$\begin{array}{l} n = 2.5 \text{ for lightweight concrete} \\ n = 3.0 \text{ for normal-weight concrete} \end{array} \qquad \varepsilon_{cT} = \left[1 + \frac{1}{n-1}\left(\frac{\varepsilon_{cT}}{\varepsilon'_{cT}}\right)^n\right]\frac{\sigma_{cT}}{E_{crT}};$
Terro [20]	$n = 2.0, \quad \varepsilon_{cT} = \left[1 + \frac{1}{n-1}\left(\frac{\varepsilon_{cT}}{\varepsilon'_{cT}}\right)^n\right]\frac{\sigma_{cT}}{E_{crT}};$
EN1992-1-2 [24]	$\sigma_{cT} = \frac{3\varepsilon_{cT}f'_{cT}}{\varepsilon_{\max}\left(2 + \left(\frac{\varepsilon_c\,T}{\varepsilon_{\max}}\right)^3\right)}, \qquad \varepsilon_{cT} \leq \varepsilon_{cu},$
Kodur et al. [32]	$\sigma_{cT} = \left\{\begin{array}{ll} f'_{cT}\left[1 - \left(\frac{\varepsilon_{\max} - \varepsilon_{cT}}{\varepsilon_{\max}}\right)^H\right] & \varepsilon_{cT} \leq \varepsilon_{\max} \\ f'_{cT}\left[1 - \left(\frac{30(\varepsilon_{cT} - \varepsilon_{\max})}{(130 - f'_c)\varepsilon_{\max}}\right)^2\right] & \varepsilon_{cT} > \varepsilon_{\max} \end{array}\right\}$
Chang et al. [33]	$M = \frac{E_r}{E'_{cr}}, \quad M_o = \frac{E_c}{E'_c},$ $\frac{\sigma_{cT}}{f'_{cT}} = \frac{1}{1 + \left(M - \frac{n}{n-1}\right)\left(\frac{\varepsilon_{cT}}{\varepsilon_{or}}\right) + \left(\frac{1}{n-1}\right)\left(\frac{\varepsilon_{cT}}{\varepsilon_{or}}\right)^n}M\left(\frac{\varepsilon_{cT}}{\varepsilon_{or}}\right),$ $E_c = 5000\sqrt{f'_c}, \quad n_o = [f'_c/12] + 0.77 > 1.0, \quad n = n_o(M/M_o)^{1.014 - 0.0007T},$
Youssef and Moftah [34]	1. $\varepsilon_{cT} \leq \varepsilon_{oTc} + \varepsilon_{tr}, \quad \sigma_{cT} = K_{hT}f'_{cT}\left[2.0\left(\frac{\varepsilon_{cT}}{\varepsilon_{oTc} + \varepsilon_{tr}}\right) - \left(\frac{\varepsilon_{cT}}{\varepsilon_{oTc} + \varepsilon_{tr}}\right)^2\right],$ $\varepsilon_c \geq \varepsilon_{oTc} + \varepsilon_{tr} \quad \sigma_{cT} = K_{hT}f'_{cT}\left[1 - Z\left(\varepsilon_{cT} - \varepsilon_{oTc} - \varepsilon_{tr}\right)\right] \geq 0.2K_{hT}f'_{cT},$ $Z = 0.5/(\varepsilon_{50uT} + \varepsilon_{50h} - \varepsilon_{oTc} - \varepsilon_{tr}), \quad \varepsilon_{oTc} = \varepsilon_{oT} \times K_{hT},$ $K_{hT} = 1 + \frac{\rho_s f_{yT}}{f'_{cT}}, \quad \varepsilon_{50uT} = \frac{3 + 0.29f'_c}{145f'_c - 1000}\frac{\varepsilon_{oTc}}{\varepsilon'_c} + \varepsilon_{tr}.$ 2. $\varepsilon_{ocT} = \varepsilon_{oT}\left[1 + 5\left(\frac{f'_{ccT}}{f'_{cT}} - 1\right)\right],$ $f_c = 2f'_{ccT}\varepsilon_{cT}/\left[(\varepsilon_{ocT} + \varepsilon_{tr})\left(1 + \left(\frac{\varepsilon_{cT}}{\varepsilon_{ocT} + \varepsilon_{tr}}\right)^2\right)\right],$ $f'_{ccT} = f'_{cT}\left[-1.254 + 2.254\sqrt{1 + \frac{7.94f'_{lT}}{f'_{cT}}} - \frac{2f'_{lT}}{f'_{cT}}\right], \quad f'_{lT} = K_e\frac{2f_{yT}A_s}{d_s S_h}.$
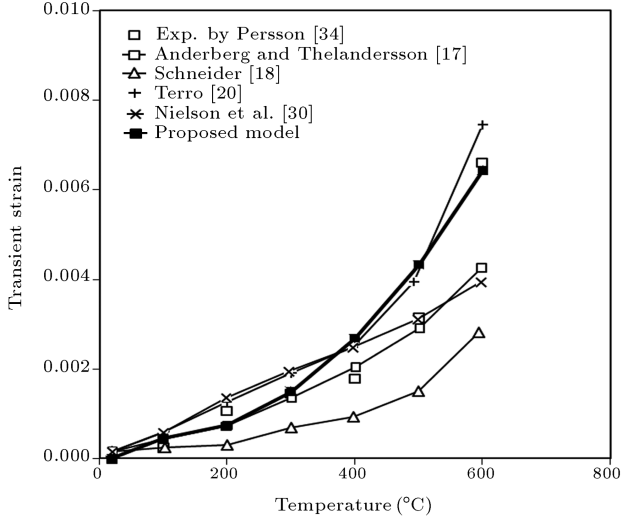
Figure 12. Relationship between transient creep strain and temperature for 16.7% preloading stress level.
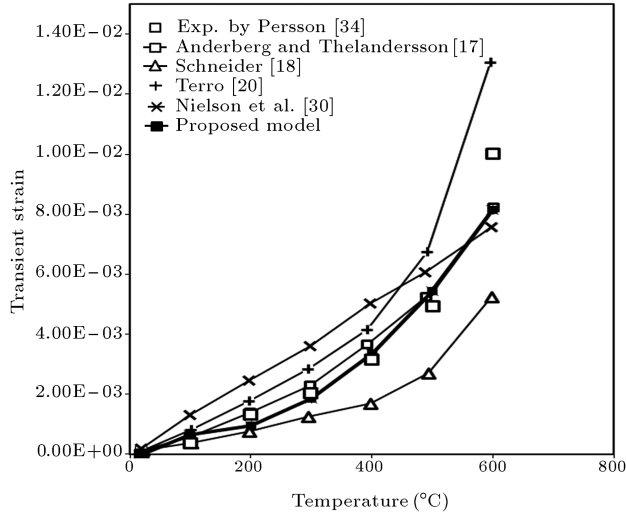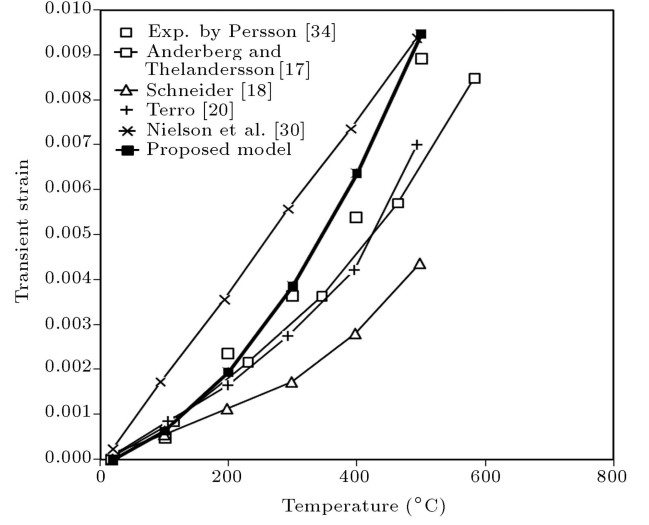

Figure 13. Relationship between transient creep strain and temperature for 33% preloading stress level.

stress-strain relationship for NSC and HSC at elevated temperatures (which is based on Carreira and Chu's [35] model with several modifications, and is developed by using the proposed residual compression strength, the initial modulus of elasticity, peak strain, thermal strain and transient creep strain) is expressed as Equation 22:

$$\frac{\sigma_c\,T}{f'_{cT}} = \frac{\beta_{mT}\left(\frac{\varepsilon_{cT}}{\varepsilon_{\max}}\right)}{\beta_{mT} - 1 + \left(\frac{\varepsilon_{cT}}{\varepsilon_{\max}}\right)^{\beta_{mT}}},$$

$$\beta_{mT} = \beta_{mT,a}(\text{fitted}) = [1.02 - 1.17(E_p/E_c)]^{-0.74},$$

if $\varepsilon_{cT} \leq \varepsilon_{\max}$,

**Figure 14.** Relationship between transient creep strain and temperature for 50% preloading stress level.

$$\beta_{mT} = \beta_{mT,d}(\text{fitted}) = \beta_{mT,a}(\text{fitted}) + (a + bt),$$

if $\varepsilon_{cT} \geq \varepsilon_{\max}$,

$$a = 2.7 \times (12.4 - 1.66 \times 10^{-2}\,f'_{cT})^{-0.46},$$
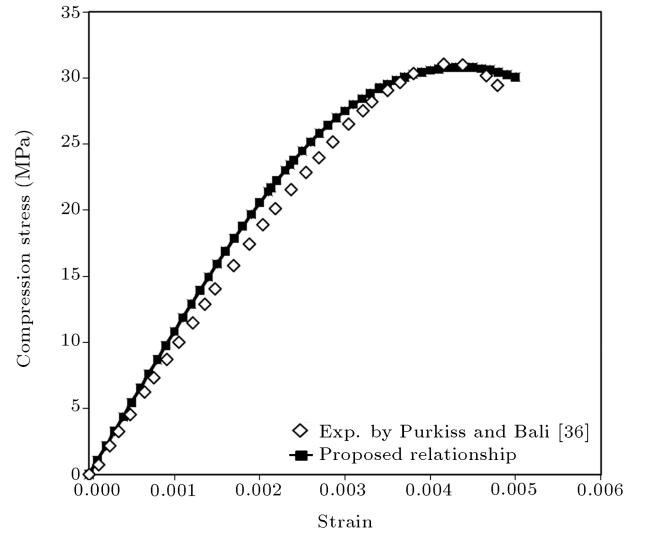
$$b = 0.83\exp(-911/f'_{cT}). \qquad (22)$$

Figures 15 to 17 provide a comparison between the proposed relationship for 20% preloaded NSC against the experimental results of Purkiss and Bali [36] at 200°C, 550°C and 700°C, which indicates the proposed model has good agreement with experimental results. Figures 18 to 20 provide a comparison between the proposed relationship for 60% preloaded NSC against the experimental results of Purkiss and Bali [36] at
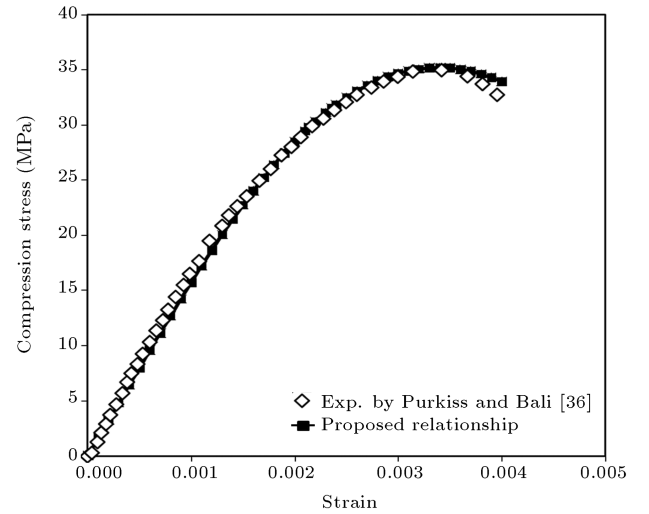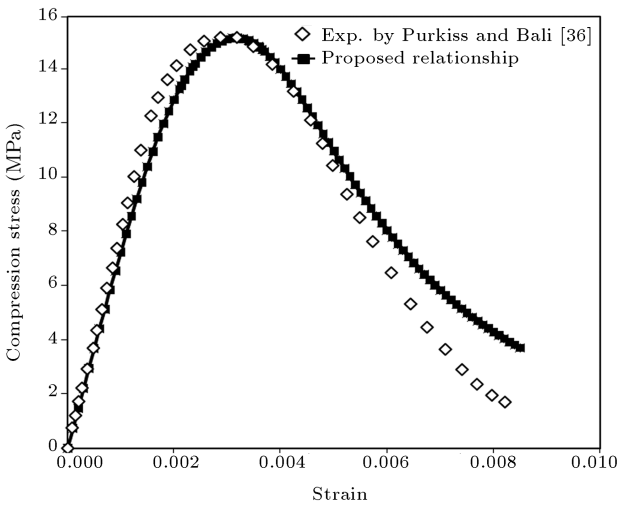

Figure 15. Comparison between the proposed relationship for 20% preloaded NSC against experimental results of Purkiss and Bali [36] at 200°C.
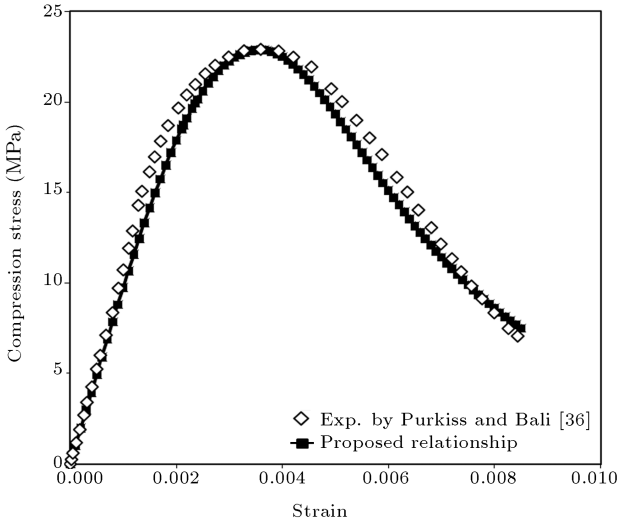
Figure 16. Comparison between the proposed relationship for 20% preloaded NSC against experimental results of Purkiss and Bali [36] at 550°C.

Figure 18. Comparison between the proposed relationship for 60% preloaded NSC against experimental results of Purkiss and Bali [36] at 200°C.
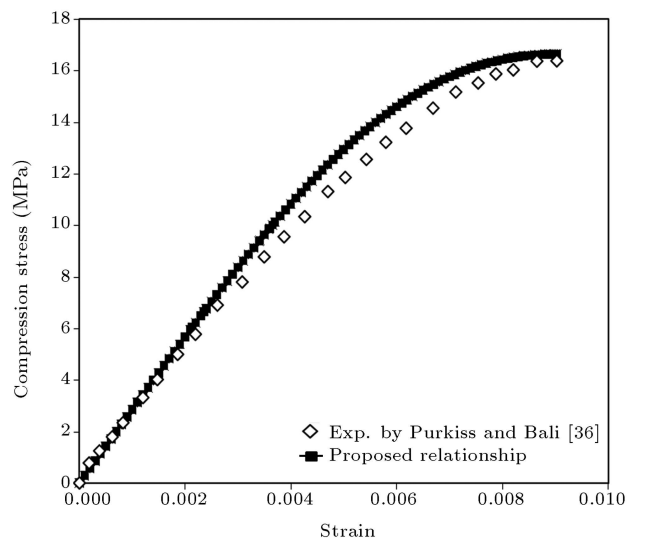

Figure 17. Comparison between the proposed relationship for 20% preloaded NSC against experimental results of Purkiss and Bali [36] at 700°C.
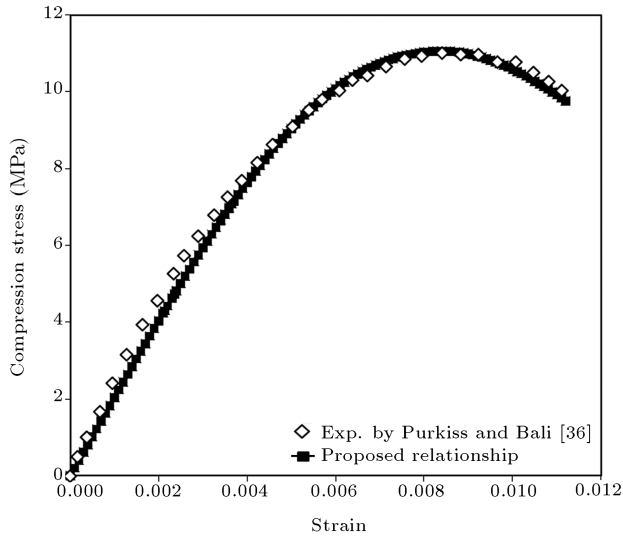

Figure 19. Comparison between the proposed relationship for 60% preloaded NSC against experimental results of Purkiss and Bali [36] at 575°C.

200°C, 575°C and 700°C. The proposed relationship has good accuracy with the experimental results at elevated temperatures. Figures 21 to 25 provide a comparison between the proposed relationship for 20% preloaded HSC against the experimental results of Khoury et al. [14] at 20°C, 100°C, 300°C, 500°C and 600°C. The proposed relationship is rational and fits well with the experimental results.
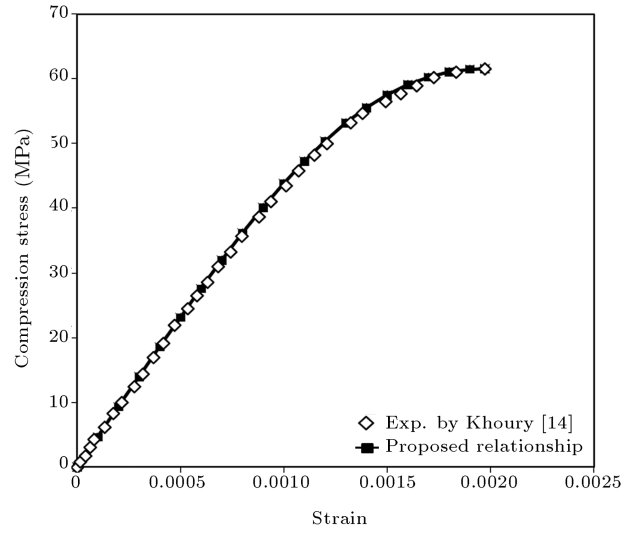
CONCLUSIONS

The fire performance of RC structural members is affected by the high-temperature properties of concrete materials. There is a major changeability in the reported test data on the high-temperature properties
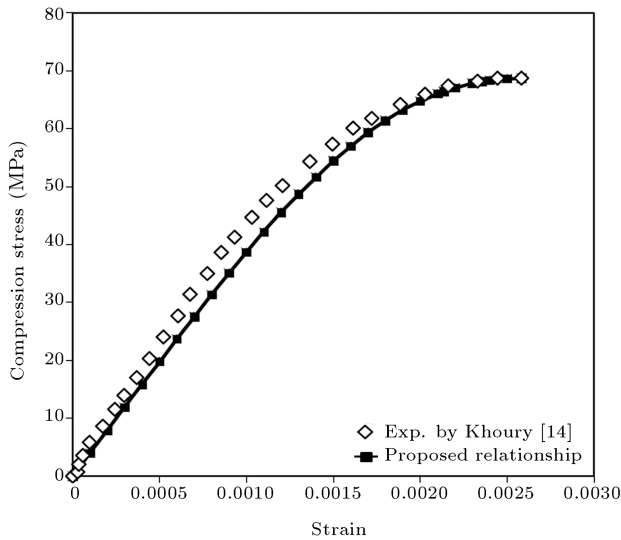
of concrete. Therefore, there are large discrepancies in the current constitutive models for the thermal and mechanical properties of concrete. The main parameters of concrete that affect the stress-strain relationship of preloaded concrete at high temperatures are: residual compression strength, initial modulus of elasticity, peak strain, thermal strain and transient creep strain. Initial compressive stresses will cause a reduction in the concrete compressive strength and peak strain at high temperatures while increasing transient creep strains. In this paper, constitutive models and relationships for preloaded NSC and HSC subjected to fire are proposed, which are intended to provide efficient modeling and specify the fire-performance criteria of the behavior
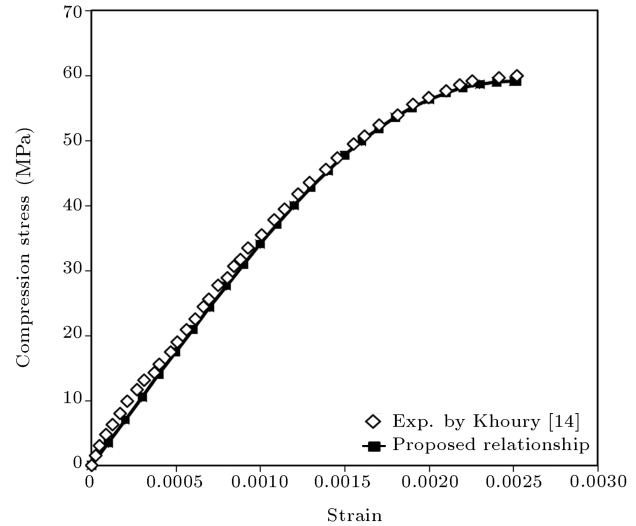
Figure 20. Comparison between the proposed relationship for 60% preloaded NSC against experimental results of Purkiss and Bali [36] at 700°C.

Figure 21. Comparison between the proposed relationship for 20% preloaded HSC against experimental results of Khoury et al. [14] at 20°C.

Figure 22. Comparison between the proposed relationship for 20% preloaded HSC against experimental results of Khoury et al. [14] at 100°C.
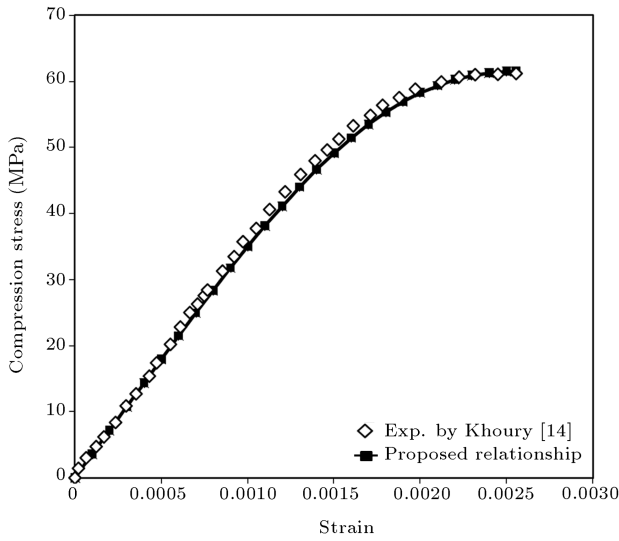

Figure 23. Comparison between the proposed relationship for 20% preloaded HSC against experimental results of Khoury et al. [14] at 300°C.

of concrete structures exposed to high temperatures. Attempts were made towards achieving rational and well-founded constitutive models and relationships for preloaded NSC and HSC at elevated temperatures. The major conclusions derived from the present work are:

1. The proposed models for compressive strength at elevated temperatures for preloaded NSC and HSC (siliceous), carbonate and lightweight aggregate concretes verify well the experimental results.

2. The proposed model for the elasticity modulus of preloaded concrete at elevated temperatures

is rational and compatible with the experimental results.

3. The proposed model for the peak strain of preloaded concrete at high temperatures has good agreement with experimental results, but further experimental tests are needed to further verify and improve the proposed model.

4. The free thermal strain models that are proposed for unloaded and preloaded concrete at high temperatures verify well with experimental results.

5. The creep strain models for preloaded concrete at high temperatures have good agreement with experimental results.

Figure 24. Comparison between the proposed relationship for 20% preloaded HSC against experimental results of Khoury et al. [14] at 500°C.

6. The proposed compressive stress-strain relationship for concrete is made, based on well-established relationships for concrete at elevated temperatures, which has a good conformity with the experimental test results of NSC and HSC at different high temperatures.

7. Additional tests at different temperatures are needed to study the influence of first compressive on the preloaded concrete compressive strength, concrete peak strain, and the initial modulus of elasticity of preloaded concrete. Tests are also required to assess the tension properties of preloaded concrete at elevated temperatures.
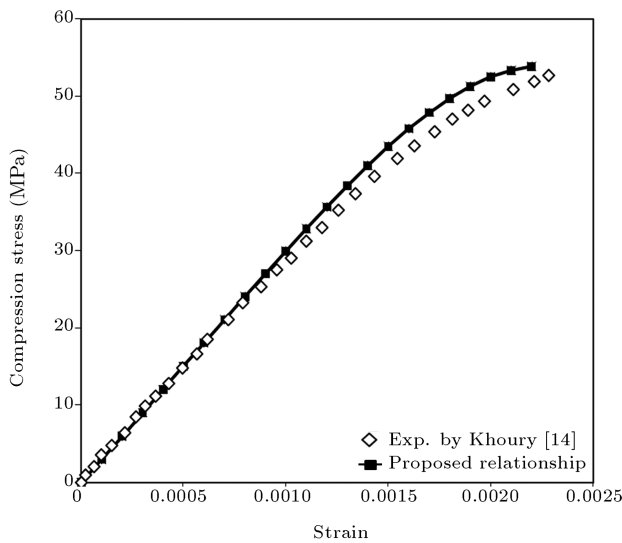
Figure 25. Comparison between the proposed relationship for 20% preloaded HSC against experimental results of Khoury et al. [14] at 600°C.

NOMENCLATURE

σ_c	concrete compressive stress at ambient temperature
f'_c	concrete compressive strength at ambient temperature
f_{ci}	initial compressive stress before heating
f_1	stress at the point of intersection of the two equations defining the stress strain curve of concrete
f_{yT}	yield strength of reinforcing bars at elevated temperature
σ_{cT}	concrete compressive stress at elevated temperature
f'_{cT}	concrete compressive stress at elevated temperature
f'_{ccT}	compressive strength of confined concrete at elevated temperature
f'_{lT}	effective lateral confining stress at elevated temperature
ε_c	concrete strain at ambient temperature
ε'_c	strain at maximum stress for concrete at ambient temperature
ε_{cu}	ultimate strain for concrete at ambient temperature
ε_0	strain at the elastic limit in compression
ε_{tu}	cracking strain
ε_1	strain at point of intersection of the two equations defining the stress strain curve of concrete
ε_{max}	strain at maximum stress of concrete at elevated temperature
ε_{oTc}	strain at maximum stress of confined concrete at elevated temperature
$\varepsilon'_{c1}, \varepsilon'_{c2}, \varepsilon'_{c3}$	strain at maximum stress as function of temperature for 0%, 10% and 20% initial stress level
ε_{tr}	transient creep strain
ε_{th}	unrestrained thermal strain
$\varepsilon_{0.3}$	transient creep strain for initial stress of 0.3 f'_c

ε_{50uT}	strain component that takes into account effect of concrete strength on the slope of the descending branch of unconfined concrete at elevated temperature
ε_{50h}	strain component that gives the additional ductility due to rectangular transverse reinforcement
E_c	initial modulus of elasticity at ambient temperature
E_{crT}	initial modulus of elasticity at elevated temperature
k_t	initial tangent stiffness to the stress-displacement curve
c	stiffening parameter
C_1, C_2, C_3	constants to account for aggregate type in evaluating transient creep strain
g	function to account for increase in modulus of elasticity due to external loads
n	a non-dimensional factor that accounts for effect of the weight of concrete on the shape of the stress-strain curve
T	fire temperature in degree Celsius ($\geq 20°C$)
T_1, T_2, T_8, T_{64}	constants describing the reduction in the concrete compressive strength for different aggregate types
Z	slope of the decaying branch of the concrete stress-strain curve
V_a	volume fraction of aggregate used to evaluate the transient creep strain
k_{tr}	constant (1.8 to 2.35) used to evaluate transient creep strain
K_{hT}	confinement factor at elevated temperature
K_e	confinement effectiveness coefficient
A_s	cross sectional area of transverse reinforcement
d_s	diameter of the transverse reinforcing bars
S_h	center-to-center spacing of the transverse reinforcement
λ_L	factor accounting for the initial compressive stress level
ϕ	function to evaluate transient creep strain
γ_o	constant to account for aggregate type in evaluating transient creep strain
γ_w	Function to account for the effect of moisture content on transient creep strain

$\beta_{m,a}$	modified material parameter at the ascending branch
$\beta_{m,d}$	modified material parameter at descending branch
β	material parameter that depends on the shape of the stress-strain curve
$\beta_{mT,a}$	modified material parameter at the ascending branch at elevated temperature
$\beta_{mT,d}$	modified material parameter at descending branch at elevated temperature

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