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Evaluation of Energy-Based Modal Pushover Analysis in Reinforced Concrete Frames with Elevation Irregularity

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Abstract. *In nonlinear static (pushover) methods of analysis as an alternative to time history analysis, the capacity curve of the structure is established with respect to the roof displacement. Disproportionate increases in the roof displacement and even outright reversals of the higher modes can distort the capacity curve of the equivalent single degree of freedom system in these kinds of method, including MPA. To overcome this problem, recently, "Energy-Based" the Modal Pushover Analysis (Energy-Based MPA) method has been introduced. In this method, the absorbed energy and/or the external work in the pushover analysis is considered. Accordingly, the assessment of the Energy-Based MPA method is important in the seismic analysis of asymmetrical and tall buildings. In this paper, the seismic demands of concrete structures with irregularity in elevation are determined, using Energy-Based MPA. For assessment of the presented technique, the results are compared with those from the Non-Linear Time History Analysis (NL-THA). Seven examples including a 2-D simulation of a 12-story building are modeled, using the Opensees Code. For each case, different types of irregularity, such as mass, geometry and variations due to the difference in elevation are considered. Story-drifts and floor-displacements are used as the main parameters for assessment of the results. Based on a study of the structural performance of the models, it has been made clear that different types of the above-mentioned irregularity in elevation do not have any significant effect on the Energy-Based MPA method. Consequently, this method can be considered as an accurate alternative technique for NL-THA, to fairly estimate the seismic demands of structures.*

Keywords: *Energy-Based MPA; Time history analysis; Elevation irregularity.*

INTRODUCTION

The nonlinear time history analysis can be regarded as the most accurate method of seismic demand prediction and performance evaluation of structures. However, this method requires the selection of an appropriate set of ground motions and also a numerical tool to handle the analysis of the data which is in many cases computationally expensive. In this way, the nonlinear static analysis (pushover) can simply be introduced as an effective alternative technique. In this method, structural performance is evaluated using static nonlinear analysis and estimation of the strength and deformation capacities of the structure. The results are compared with the demands at the corresponding performance levels [1,2]. The use of the nonlinear static analysis, named hereafter, pushover analysis, dates back to the 1970's, but only after gaining importance during the last 10-15 years have dedicated publications started to appear on the subject. Initially, the majority concentrated on discussing the range of applicability of the method and its advantages and disadvantages, compared to elastic or non-linear dynamic procedures [3-6]. Nonlinear static pushover analysis has some limitations, such as the inability to include higher mode effects. The importance of higher modes was discussed in the ATC-40 publication and is recognized in various references [1,2,7-10]. In an attempt to consider higher mode effects, Paret et al. [9] and Sasaki et al. [10] suggested the simple, yet efficient, Multi-Mode Pushover procedure (MMP). This method comprises several pushover analyses under forcing vectors representing the various modes deemed

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to be excited in the dynamic response. In 2001, Chopra and Goel [7] developed a method called multimode pushover analysis, primarily for estimating inter-story drifts in framed structures. Later, the ATC-55 project attempted to apply this procedure toward estimating story-shears and overturning moments, in addition to floor-displacements and inter-story drifts. The project encountered some problems, pointing out reversals in the third mode pushover of a three-story steel moment-resistant frame building (Figure 1). This setback indicates that increments in roof displacement are in a direction opposite to the base-shear, which may happen depending on the mechanism that gets developed within the structure. Recognizing that the roof displacement may not always be the best index as a basis for establishing the properties of so-called "Equivalent Single Degree of Freedom" (ESDF) systems, Hernández-Montes et al. [11] developed an alternative index, known as an Energy-Based Displacement. Subsequently, Tjhin et al. [12] showed that using energy-based displacement instead of roof displacement, improves peak roof displacement estimation to establish the properties of the first mode ESDF system.

Meanwhile, experiences from recent earthquakes show that the behavior of irregular buildings is significantly different from that of regular ones. Accordingly, the UBC code started to differentiate between irregular and regular buildings. Hence, parameters such as resistance ratio, stiffness, mass and geometrical irregularities of a story, with respect to neighboring stories, were considered [13]. Also, as mentioned in the SEAOC code, another type of irregularity is irregularities due to the difference in story elevation, which is sometimes extremely essential for architectural reasons [14].

Evaluation of the seismic demands of structures using Energy-Based MPA for concrete frames with irregularity in elevation is the main objective of this

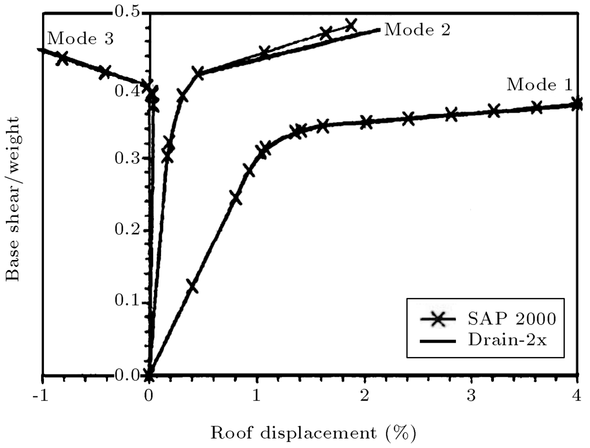

Figure 1. Reversal of third mode pushover curve for three-story steel moment-resistant frame [11].

research. As case studies, several frames are modeled using the Opensees code. For each frame, a different type of irregularity, such as mass, geometry and/or irregularities due to the difference in elevation, is considered. Story-drifts and floor-displacements are used as the main parameters for assessment of the results. The accuracy of Energy-Based MPA and especially its performance in vertically irregular frames is fairly investigated through comparison versus the method of Non-Linear Time History Analysis (NL-THA).

ENERY-BASED MODAL PUSHOVER ANALYSIS

In the energy-based pushover approach, the capacity curve associated with each modal pushover is determined based on the work done in the analysis. The work is computed incrementally, typically, for each step in the pushover analysis [11].

Analysis Procedure

Step-by-step implementation of the Energy-Based MPA procedure is presented as follows:

1. Compute natural frequencies, ω_n, and modes, φ_n, for linear-elastic vibration of the structure.
2. Define the force distribution: $\mathbf{s}_n^* = \mathbf{m}\varphi_n$.
3. Apply the force distribution characterized in step 2 incrementally, and record the base-shears and associated story-displacements. The structure should be pushed just beyond the expected targeted roof displacement, in the selected mode. Since the targeted roof displacement may not be known at the start of the procedure, iterations may get necessary. This step can conveniently be implemented in the finite element code.
4. Employ the energy-based pushover approach presented by Hernández-Montes et al. [11], the capacity curve associated with each modal pushover analysis is determined based on the work done in the analysis. The work is computed incrementally typically for each step in the pushover analysis. The increment in the energy-based displacement of the nth mode ESDOF system, $\Delta D_{e,n}$, is obtained as:

$$\Delta D_{e,n} = \frac{\Delta E_n}{V_{b,n}}, \tag{1}$$

where ΔE_n = increment of work done by the lateral forces acting through the displacement increment associated with one step of the nth mode pushover analysis and $V_{b,n}$ = base-shear at that step of the pushover analysis, which is equal to the sum of the lateral forces at that step. The incremental displacements, $\Delta D_{e,n}$, are accumulated (summed) to obtain the displacement, $D_{e,n}$, of the ESDOF system at any given step in the modal pushover analysis.

5. Idealize the pushover curve as a bilinear curve using the FEMA-273 procedure.
6. According to the previously defined nonlinear behavior and using a specific damping value, analyze the equivalent SDOF system of each mode through the NL-THA method. Subsequently, obtain the maximum Energy-Based displacement response, $D_{e,n}$, of each record.
7. Compute the average maximum displacement responses ($\hat{D}$), using Equation 2 in which n is the number of records.

$$\hat{D} = \exp\left[\frac{\sum_{i=1}^{n} \ln((D_{e,n})_i)}{n}\right]. \tag{2}$$

It should be mentioned that the exponential average method has been used for analysis. In this method, at first, energy-based displacements were obtained from each record. Then, the target displacement, which is necessary for Energy-Based MPA, was calculated using Equations 1 and 2. This procedure was repeated for each mode separately. Finally, the desired responses were extracted from pushover database values at the target displacement. There is an alternative method for computing responses in which energy-based MPA is implemented for each mode, similar to the aforementioned method. However, in this process, each energy-based displacement obtained from each record is a target displacement, separately for Energy-Based MPA. The desired responses were extracted from pushover database values at every target displacement. The second method is more accurate, although huge amounts of calculation are needed.

8. Combine the peak modal responses, according to the Square-Root-of-Sum-of-Squares (SRSS) or other forms of modal response combination.

The above mentioned Energy-Based MPA Step-by-Step procedure is also illustrated in Figure 2. Since the computation of pick deformation demands a separate procedure, which is not necessary here, it is advisable to pick a high-value pick deformation, D_n, in step 3 of the flowchart and improve it by iteration, which is considered in the procedure.

DESCRIPTION OF CASE STUDIES

In order to assess Energy-Based MPA methodologies for irregularity in elevation, seven concrete frames with different types of irregularity (i.e., mass, geometrical and irregularities due to the difference in elevation) have been selected. Subsequently, a detailed description of the analytical tool used, the modeling approach, as well as nonlinear dynamic analyses is presented.

Analytical Tool

Opensees [15] is utilized to run Energy-Based MPA, NL-THA analysis and the structural design of concrete frames, accordingly. In this design approach, structural members have been discretized using a beam-column model based on a distributed fiber element approach. In the fiber model, the sectional stress-strain state of the beam-column elements is obtained through integration of the nonlinear uni-axial stress-strain response of the individual fibers into which the section has been subdivided. If a sufficient number of fibers are employed, the distribution of material nonlinearity across the section area is accurately modeled (Figure 3). Five integration Gauss points per element are used for the numerical integration.

Modeling of the Frames

In order to evaluate Energy-Based MPA for irregularity in elevation, seven 2-D reinforced concrete frames have been selected; one 12 story regular frame and six 12 story irregular frames.

For each irregular case, a different type of irregularity, such as mass, geometry and changes due to the difference in elevation, is considered. The geometric characteristics of the structural system are illustrated in Figure 4. In the case of mass irregularities, it has appropriately been considered 150% more than the neighboring stories located in the middle and/or top of the frames. Each frame has a total width of 20 meters with equal bay widths of 3 meters and a total height of 36 meters with equal story heights of 3 meters, each. Also, effective slab widths of 5 meters were adopted for the frames; all located within a high seismic risk area with type II soil, designed according to ACI 2005 codes [16]. The lateral force resisting system is a special moment frame with importance factor = 1. All columns and beams have been selected with dimensions of 60×60, 55×55 and 50×50 cm with proper reinforcements.

Records

Seven input time-histories were employed for dynamic analysis of the seven above-mentioned concrete structures. The selection of these seven records aimed at guaranteeing that all models would be subjected to a wide-ranging type of earthquake action. The characteristics of the records are summarized in Table 1. All records were scaled to PGA = 0.35 g and type II soils, for better comparison and uniformity in the results.

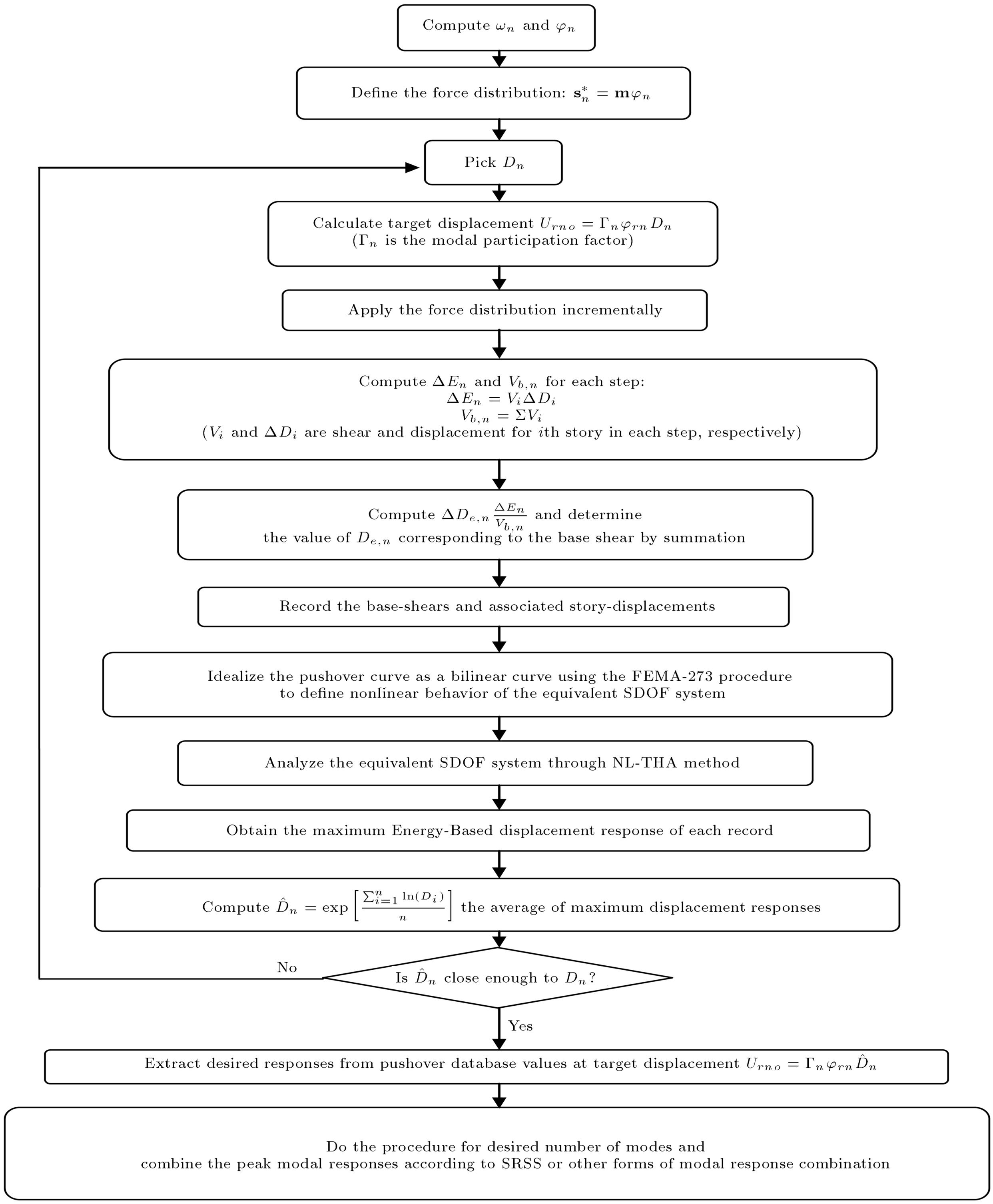

Figure 2. Energy-Based MPA procedure.

RESULTS AND DISCUSSION

The Energy-Based MPA procedure was implemented for a regular frame and six irregular frames subjected to the selected ground motion. To estimate seismic demands, the contributions of the first three "modes" were included in the analysis of the frames. The first mode pushover curves for each model ($V_{bn}/\Gamma_n L_n$ versus $D_{e,n}$) were illustrated in Figure 5 indicating the peak response obtained for the ESDF system. As you see, each pushover curve idealizes as a bilinear curve; using the FEMA-273 procedure to define the nonlinear

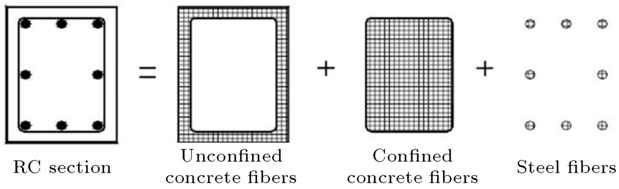

Figure 3. Fiber discretization in a reinforced concrete section.

behavior of the ESDOF system. The same procedure has been employed for higher modes. The results of Energy-Based MPA implementation to each model and for each mode are given in Table 2.

The combined values of floor-displacements and story-drifts were computed including one, two and three modes. Figures 6 and 7 show the floor-displacements and their associated errors, respectively. Figures 8 and 9 demonstrate the story-drifts and their errors. The accuracy of the Energy-Based MPA in the prediction of displacement and story-drift demands was determined using the following criteria:

$$\text{Error} = \frac{r_{\text{EB-MPA}} - r_{NL\text{-THA}}}{r_{\text{NL-THA}}} \times 100, \tag{3}$$

where $r_{\text{EB-MPA}}$ and $r_{\text{NL-THA}}$ are the responses obtained from Energy-Based MPA and NL-THA, respectively.

As expected, just the first mode results were inadequate for estimation of the seismic demands of the frames. In case of displacements, the results in the lower stories were underestimated, and for the upper stories were close to NL-THA. As for story-drifts, in the lower stories, the results were underestimated in the middle stories (the results were close to NL-THA) and in the upper stories, the results were overestimated with the contribution of higher modes.

Generally, the response contributions of higher modes have significantly improved the story drift results, especially for upper stories in tall buildings. However, the floor displacements were not sensitive to the contributions of higher modes. The seismic demands were computed more accurately for VGA-I,

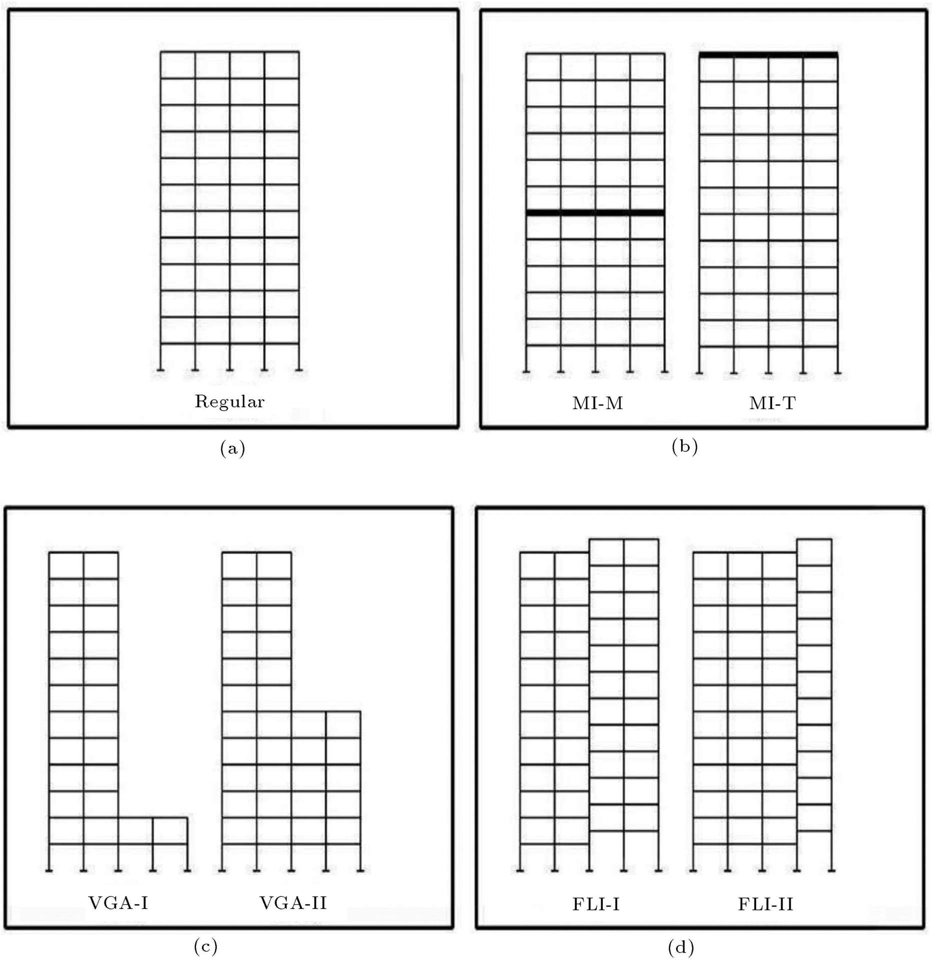

Figure 4. (a) Regular Frame; (b) Mass irregularities; (c) Geometrical irregularities; (d) Irregularities due to the difference in elevation.

Table 1. Characteristics of the records employed.

	Record	Date	Acc. (cm/s^2)	Duration (sec)
1	Imperial valley	10/15/79	357.1	40
2	Kobe	01/16/95	51.4	50
3	Loma Prieta	10/18/89	95.74	30
4	Northridge	1/17/94	133.41	30
5	Parkfield	06/28/66	466.95	10
6	San Fernando	02/09/71	206	35
7	Tabas	09/16/78	820.11	40

Table 2. The results of Energy-Based MPA implementation.

Model	Mode No.	M (Ton-s^2/m)	Γ_1	M^*	T_{Opensees} (sec)	$T_{\text{Normalized Curve}}$ (sec)	$D_{e,n}$	Pick Roof Disp. (cm)
Regular	1st	111.34	1.3647	83.4831	1.4970	1.4579	10.9665	15
	2nd	111.34	0.5623	13.8782	0.5403	0.4384	3.5795	2.02
	3rd	111.34	0.3185	5.5795	0.3150	0.3661	2.4412	1.07
MI-M	1st	128.13	1.3744	97.5848	1.5956	1.5671	11.6525	16.02
	2nd	128.13	0.5597	14.7272	0.5870	0.5239	4.2444	2.38
	3rd	128.13	0.3196	6.3668	0.3308	0.3946	2.9534	1.29
MI-T	1st	128.63	1.3449	96.0375	1.6670	1.5960	11.7565	15.82
	2nd	128.63	0.5108	16.4298	0.5986	0.4082	3.3536	1.72
	3rd	128.63	0.2582	6.5988	0.3435	0.4325	3.5041	1.56
VGA-I	1st	74.23	1.4295	47.6995	1.4279	1.3806	10.3191	14.75
	2nd	74.23	0.6719	9.6991	0.5282	0.4681	3.8486	2.59
	3rd	74.23	0.4462	7.0073	0.2967	0.3876	2.6752	1.67
VGA-II	1st	93.52	1.6034	59.1037	1.3276	1.3583	10.2218	16.39
	2nd	93.52	0.8373	20.1065	0.6030	0.7430	6.2619	5.24
	3rd	93.52	0.4169	4.5657	0.3201	0.9846	7.5872	3.16
FLI-I	1st	111.65	1.3723	86.5411	1.4933	1.4481	11.0355	15.14
	2nd	111.65	0.5787	13.9675	0.5422	0.4008	3.0585	1.77
	3rd	111.65	0.3357	5.0272	0.3170	0.3909	2.8761	0.97
FLI-II	1st	111.27	1.3777	84.8849	1.5956	1.5671	10.9041	15.02
	2nd	111.27	0.5913	13.8486	0.5331	0.4082	3.2007	1.89
	3rd	111.27	0.3555	5.1804	0.3111	0.3627	2.4666	0.88

FLI-I and MI-M frames, compared to VGA-II, FL-II and MI-T, respectively. The sources of these errors can be explained as follows.

For the case of mass irregularity, as expected, the adverse effect of mass irregularity on the results increases, as the irregularity is placed on higher stories.

For the case of geometrical irregularity, when the irregularity is in the lower stories (i.e., VGA-I), its effects can be neglected, since the frame behaves like a regular 2 span frame. As the irregularity is placed on upper stories, its adverse effect increases (i.e., VGA-II), but when the irregularity is placed

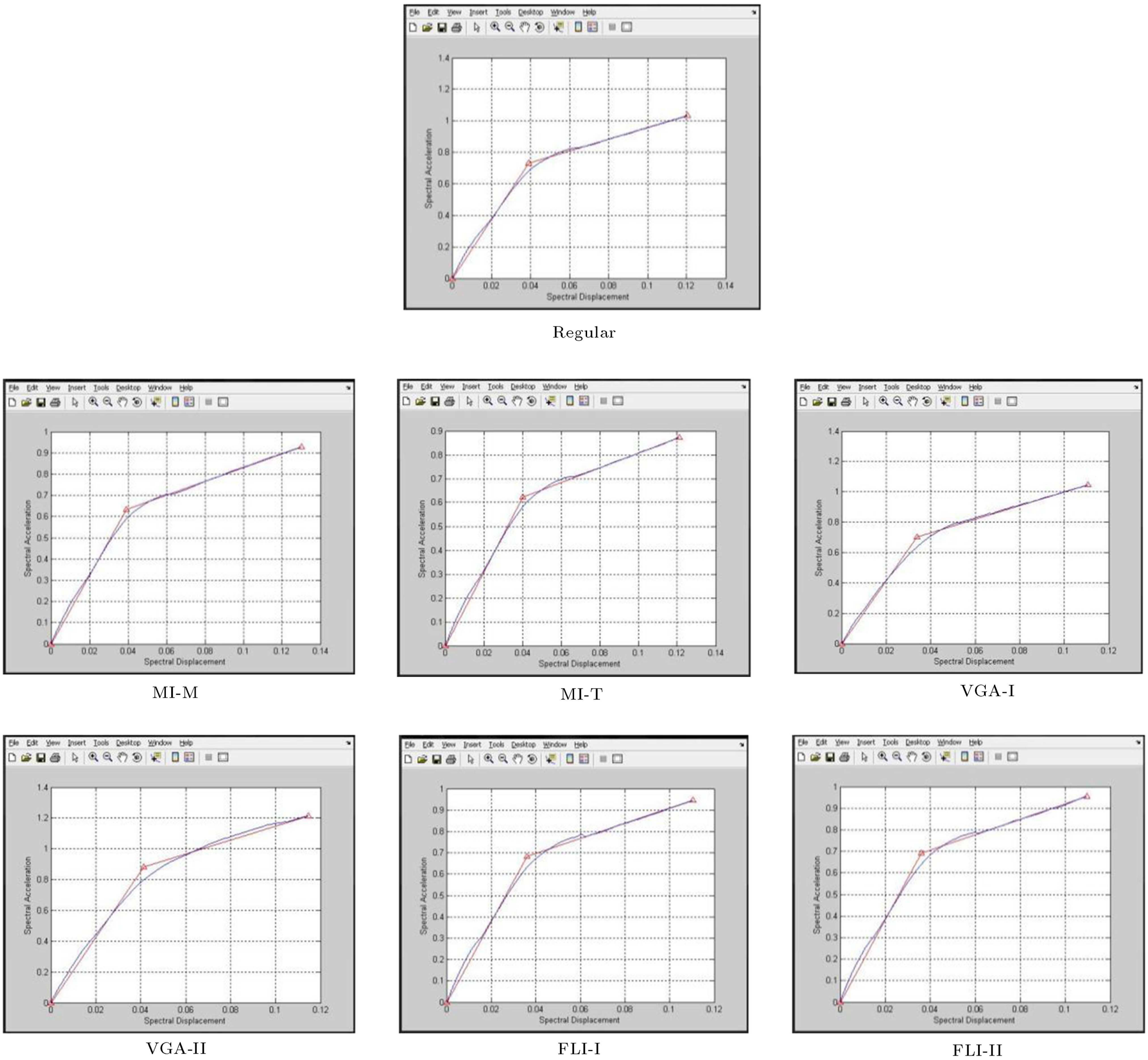

Figure 5. The first mode pushover curves for each model ($V_{bn}/\Gamma_n L_n$ versus $D_{e,n}$).

near the roof, the frame behavior approaches a 4 span regular frame. In other words, the adverse effect of geometrical irregularity is maximized when it is placed in the middle stories.

For the case of irregularities due to differences in elevation, the FLI-I may be considered more symmetrical compared to FLI-II, which results in less error.

Generally, comparison of results reveals that with the inclusion of a sufficient number of modes estimated story-drifts using Energy-Based MPA are in good agreement with NL-THA results (see Table 3).

CONCLUSION

The Energy-Based MPA procedure for estimating the seismic demands of concrete structures has been applied to frames with irregularity on elevation. Seven concrete frames including one regular and six irregular frames having different types of irregularity (i.e., mass, geometry, and irregularities due to the difference in elevation) were studied. Although Energy-Based MPA is not as accurate as the NL-THA method, it has the power of estimating seismic demands to a useful degree of accuracy for practical purposes.

Generally, the response contributions of higher modes significantly improve the story drift results, especially for upper stories in tall buildings, but the floor displacements are negligibly affected. Based on the study of the structural performance of the models, it is concluded that different types of irregularity in elevation does not have any significant effect on the Energy-Based MPA method. Consequently, this

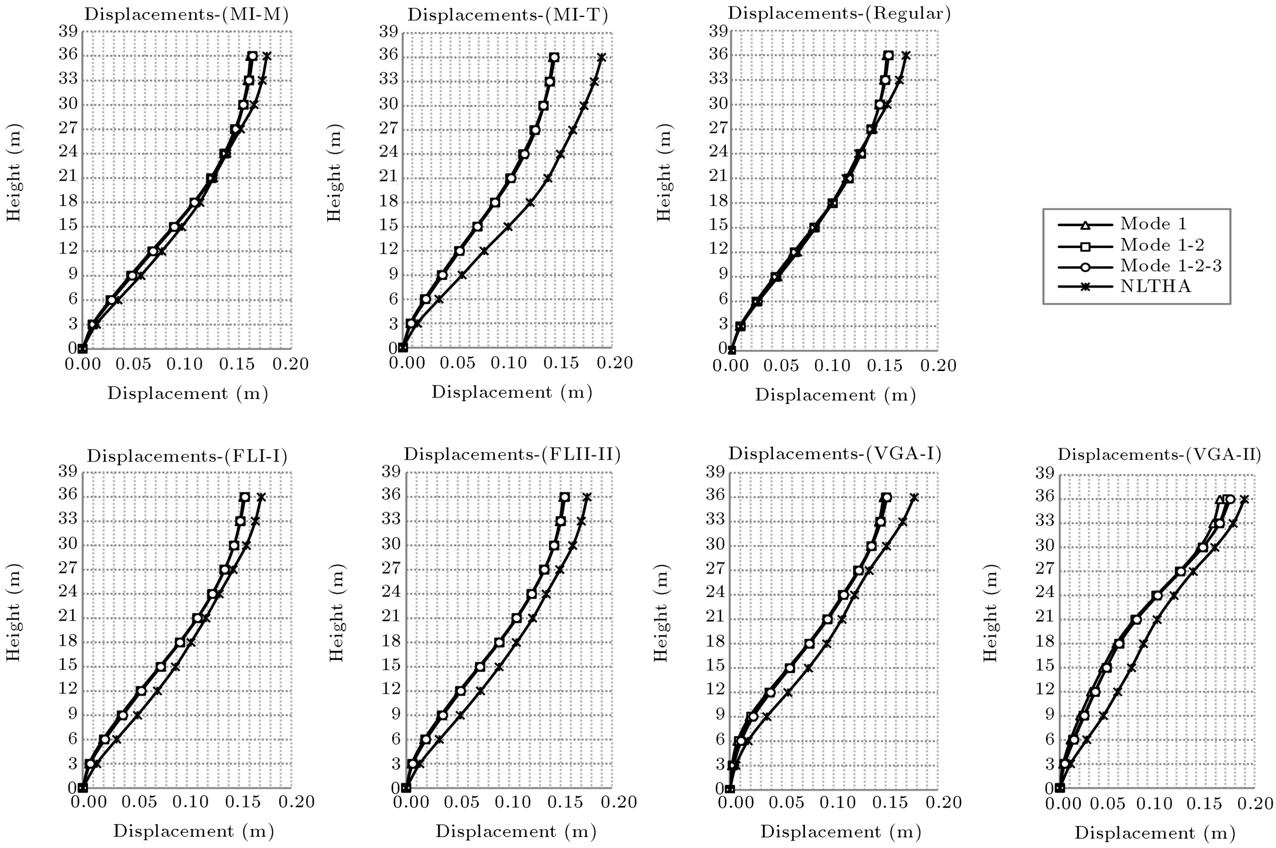

Figure 6. Floor-displacements determined through Energy-Based MPA in comparison with NL-THA.

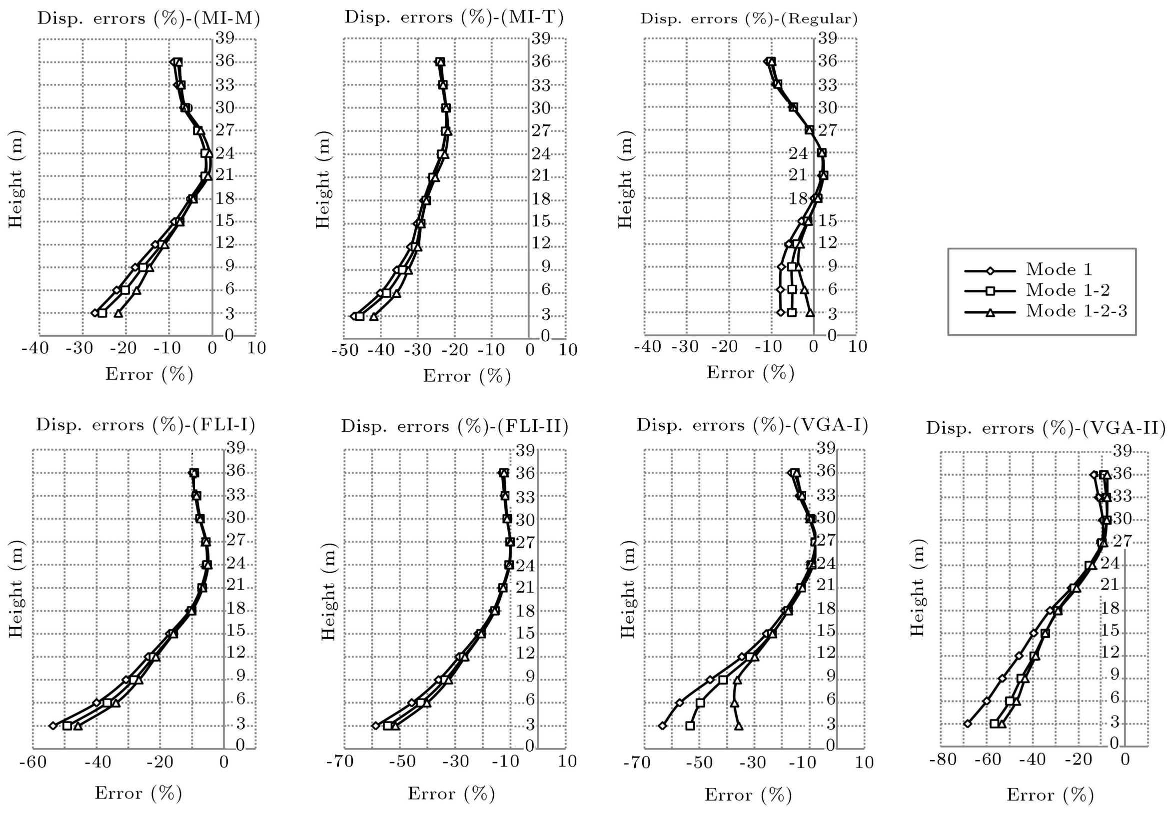

Figure 7. Floor displacement errors determined through Energy-Based MPA in comparison with NL-THA.

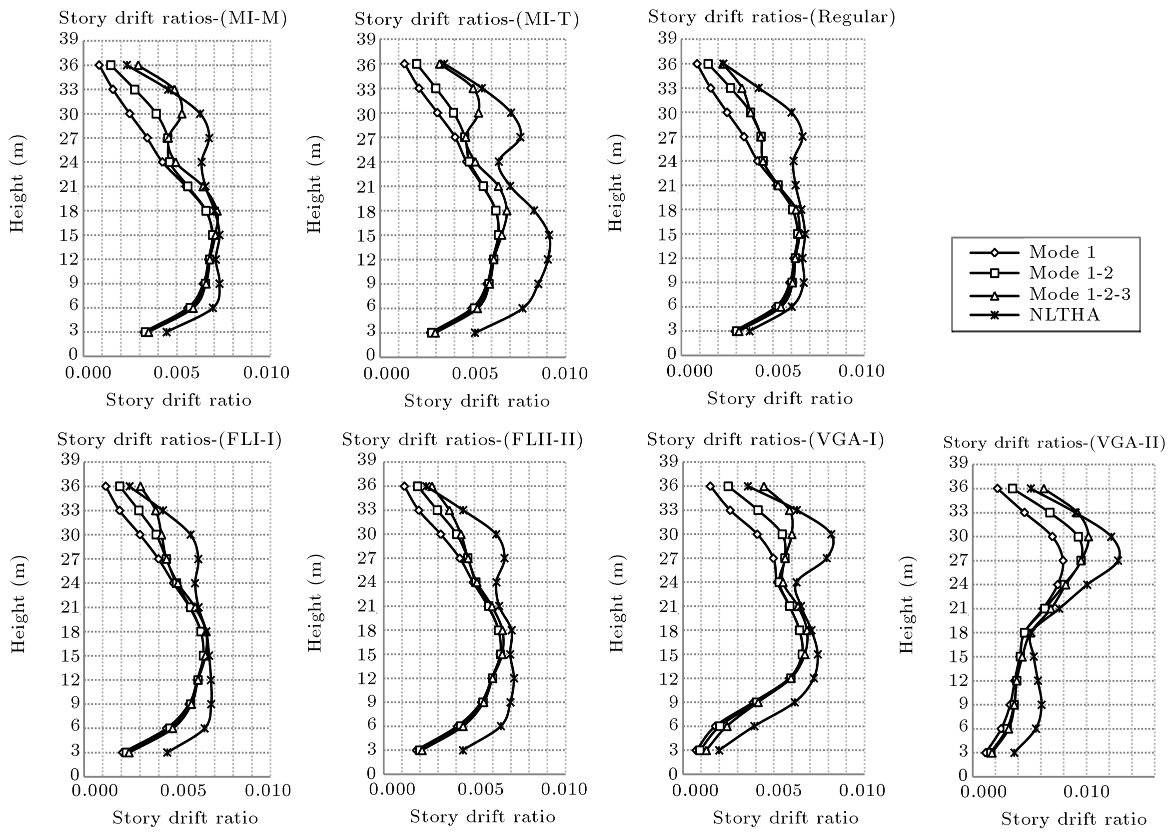

Figure 8. Story-drifts determined through Energy-Based MPA in comparison with NL-THA.

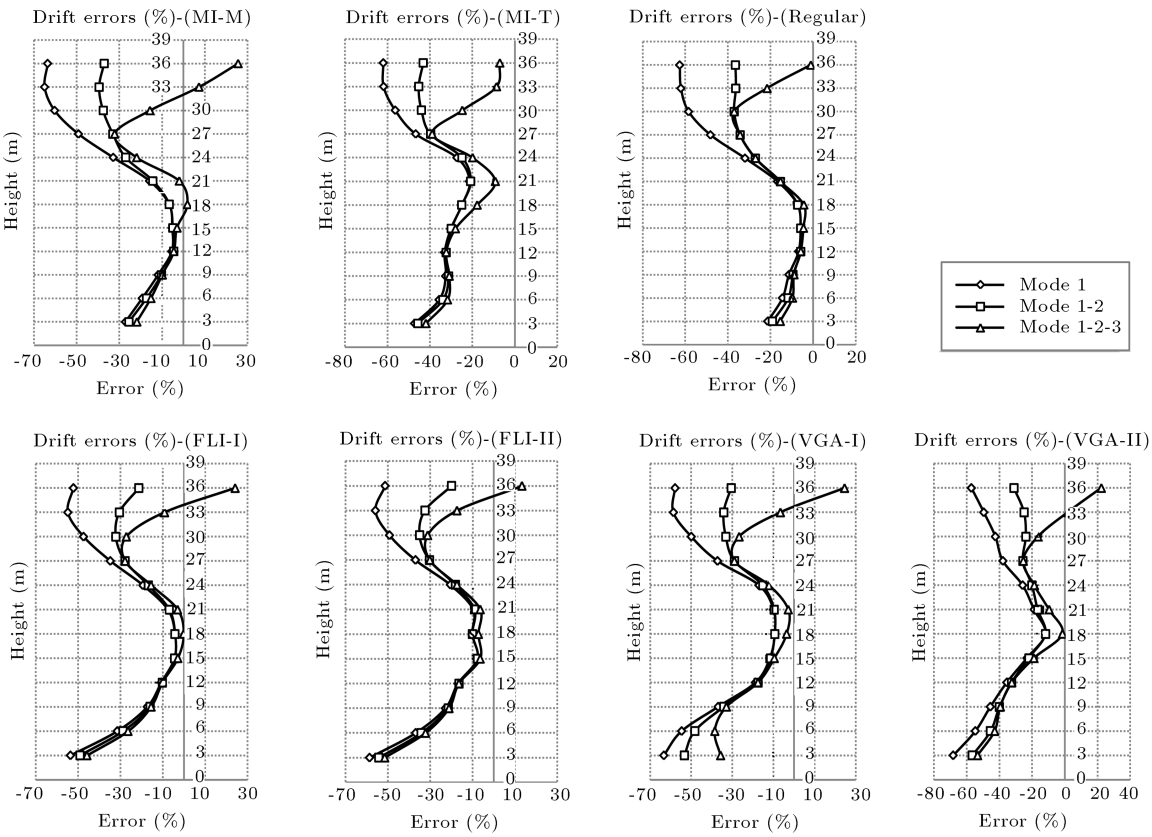

Figure 9. Story-drift errors determined through Energy-Based MPA in comparison with NL-THA.

Table 3. The evaluation of Energy-Based MPA implementation to models.

Irregularity Type	Energy-Based MPA Operation Location	Floor Displacement		Story Drift	
		Comparison with NL-THA	Effect of Contributions of Higher Modes	Comparison with NL-THA	Effect of Contributions of Higher Modes
Mass	Upper stories	Close	Negligible	Close-overestimated	Significant
	Middle stories	Close	Negligible	Close	Negligible
	Lower stories	Underestimated	Average	Underestimated	Average
Geometry	Upper stories	Close	Negligible	Close-overestimated	Significant
	Middle stories	Close	Negligible	Close	Negligible
	Lower stories	Underestimated	Average	Underestimated	Average
Difference in Elevation	Upper stories	Close	Negligible	Close-overestimated	Significant
	Middle stories	Close	Negligible	Close	Negligible
	Lower stories	Underestimated	Average	Underestimated	Average

method can be considered as an accurate alternative method for nonlinear time history analysis for estimating the seismic demands of structures.

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REFERENCES

1. Applied Technology Council "Seismic evaluation and retrofit of concrete buildings", Redwood City, Calif., USA, Report No. ATC-40 (1996).
2. Building Seismic Safety Council, NEHRP "Guidelines for the seismic rehabilitation of buildings", Federal Emergency Management Agency, Washington D.C., Report No. FEMA-273 (1997).
3. Bracci, J.M., Kunnath, S.K. and Reinhorn, A.M. "Seismic performance and retrofit evaluation of reinforced concrete structures", *Journal of Structural Engineering, ASCE*, **123**(1), pp. 3-10 (1995).
4. Eberhard, M.O. and Sozen, M.A. "Behavior-based method to determine design shear in earthquake resistant walls", *Journal of the Structural Division*, American Society of Civil Engineers, New York, **119**(2), pp. 619-640 (1993).
5. Fajfar, P. and Fischinger, M. "Nonlinear seismic analysis of R/C buildings: implications of a case study", *European Earthquake Engineering*, **1**, pp. 31-43 (1987).
6. Krawinkler, H. and Seneviratna, G.D.P.K. "Pros and cons of a pushover analysis of seismic performance evaluation", *Engineering Structures*, **20**(4-6), pp. 452-464 (1998).
7. Chopra, A.K. and Goel, R.K. "A modal pushover analysis procedure to estimate seismic demands for buildings: Theory and preliminary evaluation", PEER-2001/03, Pacific Earthquake Engineering Research Center, University of California, Berkeley, CA (2001).
8. Chopra, A.K., Goel, R.K. "Role of higher-mode pushover analyses in seismic analysis of buildings", *Earthquake Spectra*, **21**(4), pp. 1027-1041 (2005).
9. Paret, T.F., Sasaki, K.K., Elibeck, D.H. and Freeman, S.A. "Approximate inelastic procedures to identify failure mechanism from higher mode effects", *Proceedings of the Eleventh World Conference on Earthquake Engineering*, Paper 966, Pergamon, Elsevier Science Ltd, Acapulco, México (1996).
10. Sasaki, F., Freeman, S. and Paret, T. "Multi-Mode Pushover Procedure (MMP), a method to identify the effect of higher modes in a pushover analysis", *Proc. 6th U.S. National Conference on Earthquake Engineering*, Seattle, CD-ROM, EERI, Oakland (1998).
11. Hernández-Montes, E., Kwon, O.S. and Aschheim, M. 'An energy-based formulation for first- and multiple-

mode nonlinear static (pushover) analyses", *J. Earthquake Eng.*, **8**(1), pp. 69-88 (2004).

12. Tjhin, T., Aschheim, M. and Hernández-Montes, E. "Estimates of peak roof displacement using equivalent single degree of freedom systems", *J. Struct. Eng.*, **131**(3), pp. 517-522 (2005).

13. Uniform Building Code, *International Conference of Building Officials*, Whittier, California (1997).

14. SEAOC "Performance based seismic engineering of buildings", Vision 2000 Committee Structural Engineering Association of California, Sacramento, California (1995).

15. McKenna, F. and Fenves, G.L. "Open system for earthquake engineering simulation", http://opensees.berkeley.edu, Pacific Earthquake Engineering Center, University of California, Berkely.

16. American Concrete Institute, "Building code requirements for structural concrete ACI318-05", Detroit, MI, USA, Report No.ACI318-05 (2005).