An Optimal Fuzzy Approach to the Control of Predator-Prey Equations

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The use of fuzzy logic in control system theory has had widespread success and has stimulated high interest. With this success, new theories and methodologies are proposed for analysis and control of nonlinear systems. In this paper a case study is performed over the Volterra equations. In contrast to some of the well-known classical methods and so-called modern techniques for nonlinear control design which are based on exact mathematical modeling, this approach originates from human-like approximate reasoning and imprecision. This approach is a knowledge-based control in which membership functions of system variables are used to cope with the nonlinearity of the mathematical model. The design is initially developed to form an approximate or incomplete knowledge-base for control strategy. A new type of methodology is presented for optimization and reduction of the membership function's parameters. The proposed approach is implemented on a simulated model of predator-prey equations, and the results are compared with the one obtained by feedback linearization technique. It is shown that the performance of the proposed method is superior to that of the modern nonlinear control design.

INTRODUCTION

The concept of fuzzy logic is widely used in many different fields today, ranging from control applications, robotics, biological and medical science, image and speech processing to applied operation research, decision making and expert systems [1]. Fuzzy logic has been very successful in solving problems, where a conventional model-based approach is very difficult or inefficient and/or expensive to be implemented. This reality is illustrated by a case study of the Volterra equations.

Since the most critical side of a fuzzy controller design lies beyond the number and the shape of membership functions [2,3], a new methodology based on optimization and sensitivity analysis is presented through which all of the parameters of the membership functions are obtained and those of less sensitivity are discarded. Furthermore, the proposed approach is also compared to the feedback linearization technique. It is shown that its shortcomings can be recovered by utilizing the proposed method.

The predator-prey model which is adaptable to many biological or physical systems is described by the following differential equations:

$$\dot{x}(t) = -ax(t) + bx(t)y(t), \tag{1a}$$

$$\dot{y}(t) = cy(t) - dx(t)y(t), \tag{1b}$$

where $x(t)$ and $y(t)$ are the states denoting the

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number of predators and preys (of course in continuous domain), respectively.

The system consists of nonlinearities in both equations and two stationary points for which the balance of the system may be served. Without any loss of generality, hereafter it will be assumed that all the parameters are one. The two points which form the set of stationary points become:

$$\underline{x}_1 \triangleq [x,y]' = [1,1]' \ , \quad \underline{x}_2 = \underline{0} \ .$$

The first one is a center to which none of the system trajectories terminate and the other is a saddle point. These are plotted in Figure 1 for a free system.

It should be noted that a center cannot be achieved except when it is at the initial position and the a saddle point is reachable only in one direction. It is obvious that a little perturbation from each one leads to a periodic variation around the center or instability towards the infinity in one of the states. Therefore, the system requests a robust control which may be imposed to each one of the states as an input signal.

First, the feedback linearization method and then the fuzzy control theory are studied, trying to keep the system stable on its nontrivial steady state set point. Finally, the regulation problem for this system will be considered. To attain such a goal, the following model is considered, where $u(t)$ is regarded as

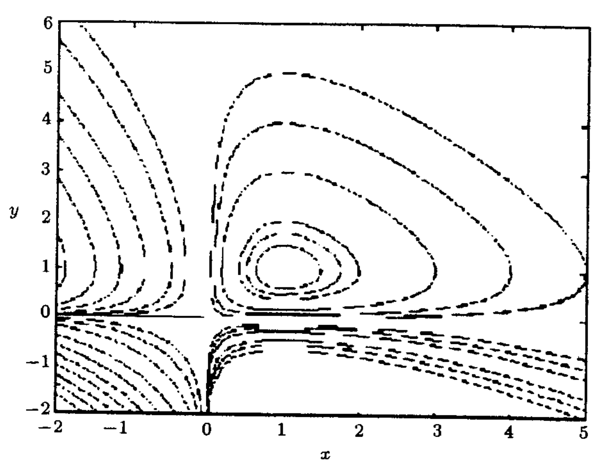

Figure 1. Free system trajectories.

a control signal in the second equation. Let us assume that the number of the preys are controllable by either slaughtering or providing more food. Therefore, Equations 1a and 1b can be described as follows:

$$\dot{x}(t) = -x(t) + x(t)y(t) \ , \tag{2a}$$

$$\dot{y}(t) = y(t) - x(t)y(t) + u(t) \ , \tag{2b}$$

where, in the coming sections, the regulation problem for this system will be investigated.

FEEDBACK LINEARIZATION

The philosophy behind this approach is transferring a nonlinear system into a linear one through some appropriate calculations and then applying the linear control techniques [4].

Input-State Linearization

Using input state linearization, an auxiliary state variable is defined, namely $z(t) = x(t)y(t)$, and the system equations are rewritten (dropping the time symbol for simplicity) as:

$$\dot{x} = -x + z \ ,$$

$$\dot{y} = y - z + u \ .$$

Substituting for y and $\dot{y}$ using z and $\dot{z} = x\dot{y} + \dot{x}y$, the following is obtained:

$$\dot{x} = -x + z \ , \tag{3a}$$

$$\dot{z} = \frac{z^2}{x} - xz + xu \ . \tag{3b}$$

Now, by choosing:

$$u = \frac{v + xz - z^2/x}{x} \ , \quad x \neq 0 \ , \tag{4}$$

a simple linear system of the following form is obtained:

$$\dot{\underline{z}} = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \underline{z} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \ ; \quad \underline{z} = [x,z]' \ , \tag{5}$$

which is easily stabilizable by a proper choice of a state-feedback given as:

$$v = -\underline{k}'\underline{z} \ ; \ \underline{k} = [\alpha, \beta]' \ . \tag{6}$$

To regulate the states about $\underline{z} = [1,1]'$, the above control law is written as:

$$v = -\underline{k}'\underline{z} + \alpha r \ , \qquad (7)$$

where r is the desired reference input. Replacing the linear controller in Equation 4, the nonlinear controller becomes:

$$u = z - \alpha - \frac{\beta z}{x} - \frac{z^2}{x^2} + \frac{\alpha+\beta}{x} \ . \qquad (8)$$

Note that the inverse value of DC gain of the linear controller, $\frac{\alpha+\beta}{\alpha}$, is substituted for r to ensure a totally unit DC gain.

The main weakness of this approach lies beyond the above solution. It can be shown analytically that in the linear system formed by Equations 4 and 5, the hidden state, y, always tends to one regardless of x and z, i.e. :

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} \frac{z(t)}{x(t)} = \lim_{s\to 0} \frac{sZ(s)}{X(s)} = 1 \ .$$

However, as shown in Figure 2, the time response of $y(t)$ is of a certain shape which for small settling times, takes large overshoots in transient, i.e. the smaller the rise time is, the larger overshoot is caused and the smaller the overshoot is, the more time it takes to settle. According to this infirmity, there is not enough flexibility in the controller design.

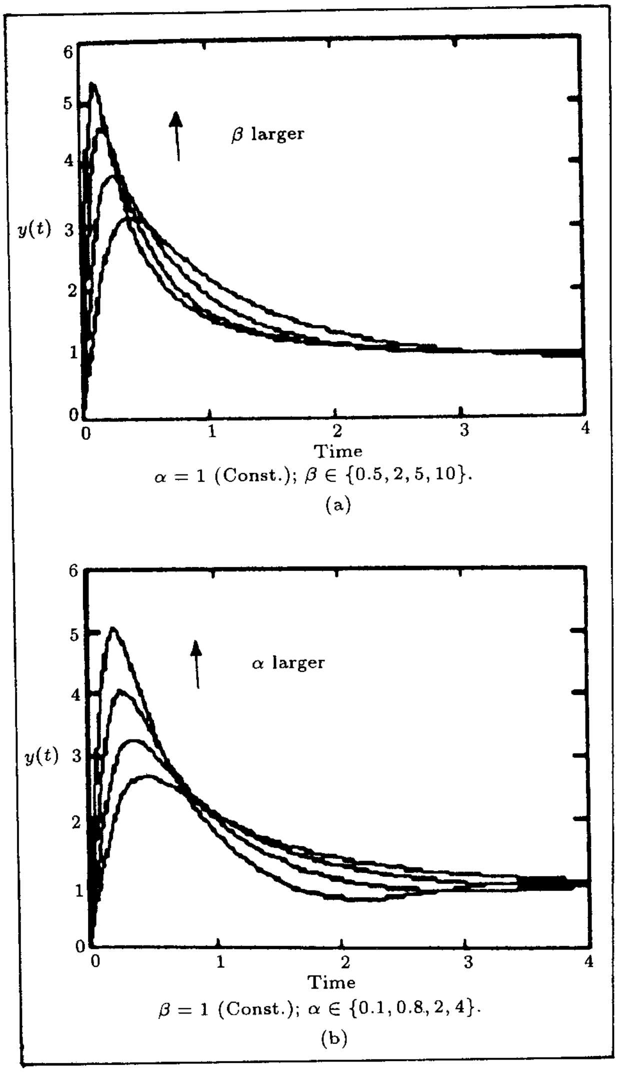

Figure 2. Time response for input-state feedback linearization with the control signal, $v = -\alpha[x - (\alpha+\beta)/\alpha] - \beta z$

Input-Output Linearization

The weakness mentioned previously is recovered by this approach. Choosing an input of the following form:

$$\begin{aligned} u(t) = & -\alpha[y(t) - y_d(t)] \\ & + x(t)y(t) + \dot{y}_d(t) - y_d(t) \ , \end{aligned} \qquad (9)$$

the output error dynamics, $\dot{y}(t) - \dot{y}_d(t)$, will have a single ordered equation which can arbitrarily be designed by the parameter α, given as:

$$\dot{e}(t) = (1-\alpha)e(t) \ .$$

The output then varies according to the following form:

$$\dot{y}(t) = (1-\alpha)y(t) + (\alpha - 1)y_d(t) + \dot{y}_d(t) \ ,$$

which has an analytical solution given by:

$$y(t) = c\hat{e}^{(1-\alpha)t} + \hat{e}^{(\alpha-1)t} \cdot \int_0^t [(\alpha-1)y_d(\tau) + \dot{y}_d(\tau)]\, \hat{e}^{(1-\alpha)\tau} d\tau \ , \qquad (10)$$

where $\hat{e}$ denotes the exponential term. Assuming that $y(0) = y_0$, $y_d(t) = \bar{y}_d$ and $\dot{y}_d = 0$, the following equation is obtained:

$$y(t) = (y_0 - \bar{y}_d)\hat{e}^{(1-\alpha)t} + \bar{y}_d \ . \qquad (11)$$

This method may also face other problems produced by the internal dynamics. Substituting for $y(t)$ from Equation 11 in Equation 2a, the internal state becomes:

$$x(t) = x_0 \exp \left\{ \frac{y_0 - \bar{y}_d}{1-\alpha} \left[\hat{e}^{(1-\alpha)t} - 1 \right] + (\bar{y}_d - 1)t \right\} . \tag{12}$$

Since:

$$\lim_{t \to \infty} x(t) = x_0 \exp \left[\frac{1 - y_0}{\alpha - 1} \right] ,$$

the stability of the system is warranted for $\bar{y}_d = 1$ and any $\alpha > 1, y_0 \leq 1$. But the steady state of $x(t)$ is not desirable, since it does not satisfy $x = 0$. Therefore, the controller costs an infinite resource of energy:

$$\lim_{t \to \infty} u(t) = x(\infty) - 1 \neq 0 .$$

Figure 3 shows the system response to the input, given in Equation 9, for various α's and x_0's.

As a conclusion, the feedback linearization method is not a satisfactory solution and this convinces the designer to utilize a more suitable control method.

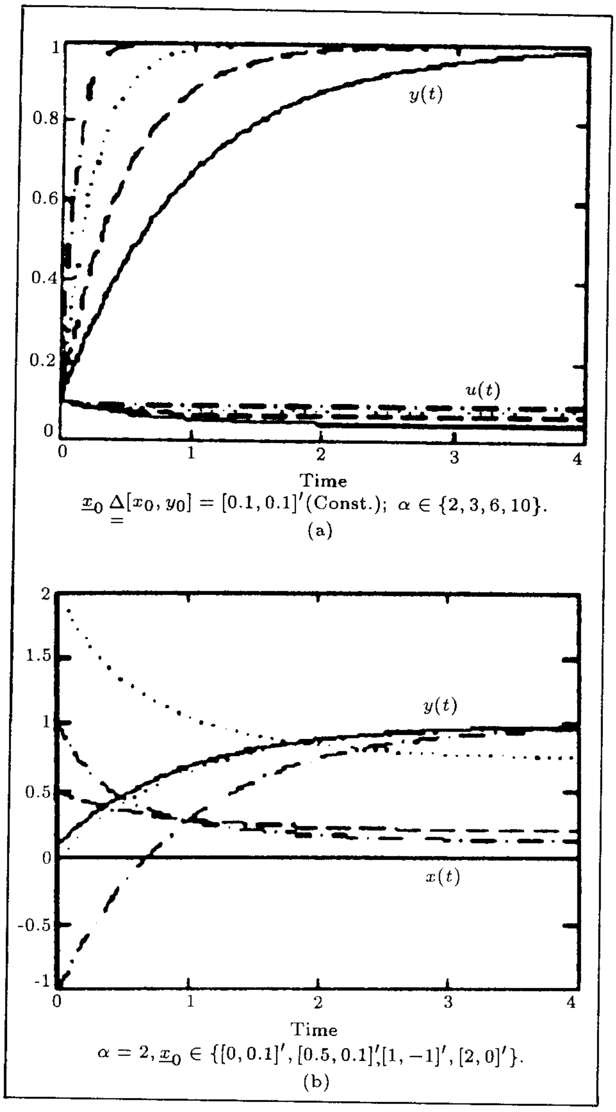

Figure 3. Input-output feedback linearization results.

FUZZY CONTROLLER STRUCTURE

In this section, the designing procedure of a fuzzy controller for the system based on the measurements of the error and its rate of change is considered. Again, assuming r as the desired set point for $y(t)$, the following relations hold:

$$e(t) = y(t) - r ,$$

$$\dot{e}(t) = \dot{y}(t) .$$

The controller requires a rule base with two inputs determining the control signal, $u(t)$. The structure of the controller depends on the fuzzy sets used for the fuzzification of the measured signals, $e(t)$ and $\dot{e}(t)$, and also the defuzzification sets of the controller output. Thus, the associated membership functions that can be used for this purpose are pointed out.

Applied Fuzzy Sets

Many forms of fuzzy membership functions have been suggested by the researchers. The two following functions have been studied, each characterized by two parameters:

Type 1 (Trapezoidal):

$$f_t(\alpha) = \begin{cases} 2 - \frac{\alpha - \mu}{\sigma}; & \mu + \sigma < \alpha < \mu + 2\sigma \\ 1; & \mu - \sigma \leq \alpha \leq \mu + \sigma \\ 2 + \frac{\alpha - \mu}{\sigma}; & \mu - 2\sigma < \alpha < \mu - \sigma \\ 0; & \text{otherwise} . \end{cases}$$

Type 2 (Exponential):

$$f_e(\alpha) = 1 - e^{-(\frac{\sigma}{\alpha-\mu})^4}$$

The corresponding shapes for both memberships are plotted in Figure 4. For the purpose of comparison, note that $f_t(2\sigma) = 0$, $f_e(2\sigma) \simeq 0.06$ and $f_t(\sigma) = 1$ but $f_e(\sigma) \simeq 0.6$. So, it is clear that the sharpness of the membership function, Type 1, at the edges is smoothed. Since a limited interval should be assigned for each of the signals in which the fuzzy sets are to be distributed, two parameters specifying the limits for each signal are determined. Note that the number of the fuzzy sets, n, defined in the interval, can vary. Starting with three sets, namely Zero, Positive and Negative and assuming symmetry for all of them with respect to Zero, n parameters determine the required fuzzy sets. These parameters are collected in an n vector $\underline{p} \triangleq [\sigma_Z, \mu_P, \sigma_P]$.

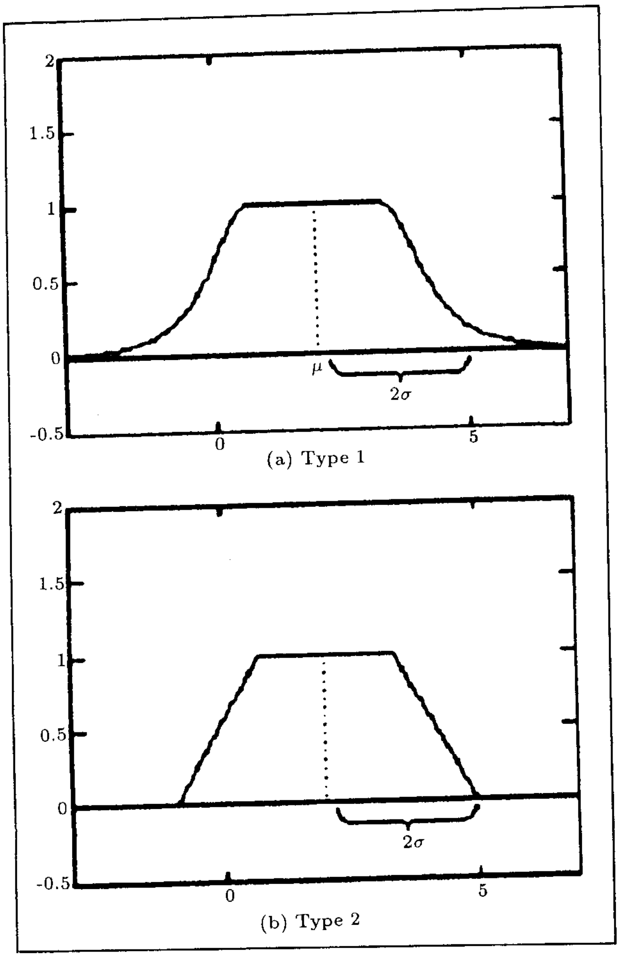

Figure 4. Applied membership functions.

Table 1. Rule base 1.

e \ $\dot{e}$	N	Z	P
N	P	P	Z
Z	Z	Z	Z
P	Z	N	N

Table 2. Rule base 2.

e \ $\dot{e}$	NL	NS	ZE	PS	PL
NL	PL	PL	PS	PS	ZE
NS	PS	PS	PS	ZE	ZE
ZE	ZE	ZE	ZE	ZE	ZE
PS	ZE	ZE	NS	NS	NS
PL	ZE	NS	NS	NL	NL

Fuzzy Rule Bases

An intuitive inference rule base, based on human-like reasoning, can be easily formed by associating a linguistic output variable to each combination of the measurements. First, the rule base is constructed according to Table 1.

As it is shown, all signals are arranged in three linguistic variables: "Zero", "Positive" and "Negative". Adding two other linguistic variables to each group, a more complete rule base can be formed as it is shown in Table 2.

In accordance with the solutions of the preceding optimization problem, it is experimented that not only extra rules will not be significant, but also the simplest rule base is sufficient for a quite successful optimal control.

OPTIMAL FUZZY CONTROL

Similar to the fundamentals of the optimal control methods, a possible objective function is defined by:

$$J(\underline{p}) = \|\underline{y}(N) - \underline{y}_d(N)\|_2^2 ,$$

where $\underline{y}(N)$ and $\underline{y}_d(N)$ are the two vectors containing sampled signals of the real output and the desired one. Both are defined as follows:

$$\underline{y}(N) = [y(1), y(2), \dots, y(N)]' ,$$

$$\underline{y}_d(N) = [y_d(1), y_d(2), \ldots, y_d(N)]' .$$

Note that it has been assumed that $x(t)$ is not measurable, otherwise it would have been included in the objective function. Now, the problem of designing the fuzzy controller reduces to the solution of the following optimization problem:

$$\min J(\underline{p}) = \sum_{t=1}^{N}[y(t) - y_d(t)]^2, \tag{13}$$

subject to the system equations, where $u(t)$ is obtained through the rule base 1 or 2 by Center of Gravity defuzzification method.

Required Input – Desired Output

By considering the output alone, there is no doubt that the best desired trajectory for the output is to jump from the initial condition to the desired set point in a few steps. However, there are always some constraints in practice that limit such a rapid jump. The main restriction is the input energy of the system. A simple calculation that can aid in finding the required input is proposed as follows:

Assume that the maximum value of $u(t)$ which should be tolerated by the system is taken at the starting time. Furthermore, suppose that the desired transient, $y_d(t)$, be a constant slope that conveys the ouput from y_0 to y_d in a few steps, N, as it is represented in Figure 5. This implies that:

$$y_1 = y_0 + \frac{1}{N}(\bar{y}_d - y_0) , \tag{14}$$

where $\bar{y}_d \triangleq y_d(N)$ and $y_1 \triangleq y(1)$. Also from Equation 2b, for a proper sampling period T_s, it can be obtained that:

$$y_1 = y_0 + T_s(y_0 - x_0 y_0 + u_0) . \tag{15}$$

Solving Equations 14 and 15 for u_0, one obtains:

$$u_0 = \frac{1}{T_s N}[\bar{y}_d - (1 + T_s N)y_0] + x_0 y_0 , \tag{16}$$

e.g. for $T_s = 0.05, N = 10, y_d = 1$ and $y_0 = x_0 = 0.1, u_0$ becomes 1.71.

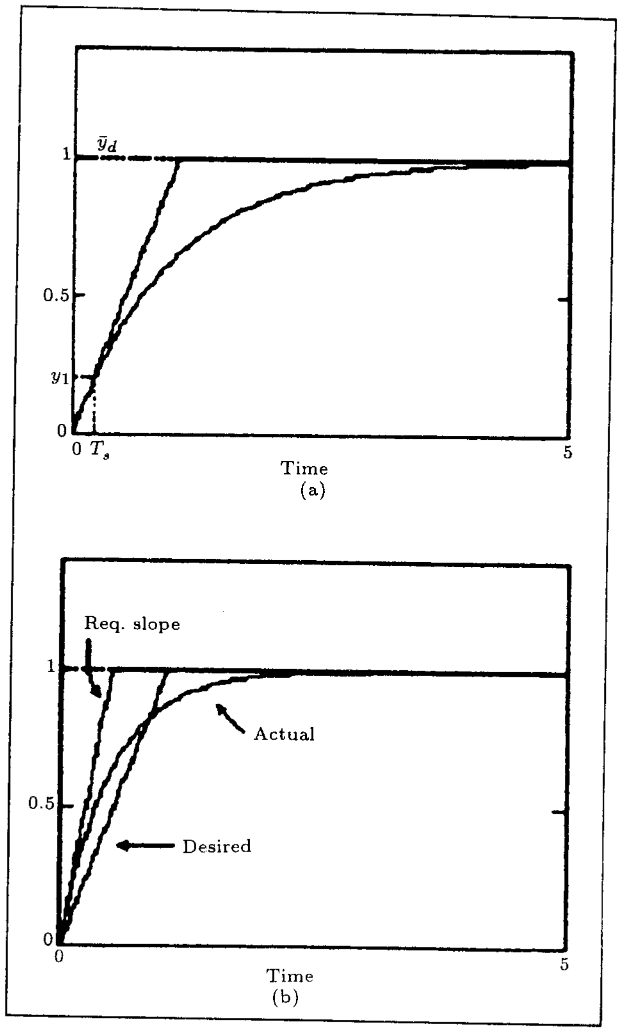

Figure 5. A sample output transient.

Similar results can be deduced utilizing the analytic solution introduced before. Each of the Equations 8 or 9 with appropriate design parameters (α, β), depending on N and T_s, may be used for this purpose. For instance, choosing $\alpha = \frac{1+T_s N}{T_s N}$ and using Equation 9, the required initial input can be expected to be:

$$u_0 = (\alpha - 1)\bar{y}_d - \alpha y_0 + x_0 y_0 . \tag{17}$$

Also, other information about u_0 can be obtained by similar approximations based on Equation 8. Selection of a suitable transient, from Figures 2 and 3, can guide us to calculate for the required u_0, where:

$$u_0 = x_0 y_0 - \alpha - y_0(\beta + y_0) + \frac{\alpha + \beta}{x_0} . \tag{18}$$

So, the final choice should be considered as the largest one for which the above approximations predict.

Another fact is perceptible from Figure 5b. It is evident that to minimize the distance between $\underline{y}(N)$ and $\underline{y}_d(N)$, the actual output may start with a larger slope and thus requiring larger input. Therefore, it is better to consider $u(0)$ larger than what has been obtained in the preceding section.

Optimization – Parameter Reduction

Many experiments show that a solution can always be found for Equation 13, given various initial conditions. To illustrate the process of optimal design, let us examine a numerical example for which the results are collected in the next table. It should be mentioned that any initial condition, closer to the vertical axis, causes larger overshoot in the time response. Therefore, $\underline{x}(0) = [0.1, 0.1]'$ can be chosen as an ill initial condition. The other required parameters are the same as the example given for Equation 16.

Table 3 contains the optimum parameters of the membership functions of Type 1 for the fuzzification of $u(t), e(t)$ and $\dot{e}(t)$. Starting with various initial guesses for the vector $\underline{p} = [\sigma_{zu}, \mu_u, \sigma_u, \sigma_{ze}, \mu_e, \sigma_e, \sigma_{zr}, \mu_r, \sigma_r]'$ and using the Gauss-Newton method [5], the nine unknown parameters are found. The subscripts are self-defining and the symbol r denotes the rate of changes in error, i.e. $\dot{e}$. As it is clear from $\underline{p}$, Table 1 is first considered as a fundamental rule base. It is easy to conclude that many different points can be accepted as solutions to the optimization problem . Of course, they may cause some differences in the time responses, as they are shown in Figure 6.

Also it is observed that some of the parameters do not change so much during the optimization process. This means that they are not so sensitive with respect to the optimization process.

Investigation of the sensitivity for the parameters with respect to the objective function $J(\underline{p})$ encourages us to abandon those with less sensitivity. An algorithmic decision is to choose the parameters for which the objective function shows the highest sensitivity and the optimization procedure causes the most displacement from their initial values. According to Table 3, sensitivity levels of the parameters (with respect to the optimization process) are given in Table 4.

It is clear that the vector of parameters $\underline{\theta} = [\mu_u, \sigma_{zu}, \sigma_{ze}, \sigma_e]'$ has shown to be the most important factor for almost all the experiments in a weighted average sense. The second step is to reduce the parameters vector $\underline{p}$ to $\underline{\theta}$.

Table 3. Optimization results of the first step.
Trapezoidal membership functions (9 parameters for 3 variables).

Par. No.	1	2	3	4	5	6	7	8	9	
Case No.	σ_{zu}	μ_u	σ_u	σ_{ze}	μ_e	σ_e	σ_{zr}	μ_r	σ_r	$J(\underline{p})$
1	1.00	8.00	4.00	0.60	6.00	3.00	2.00	10.00	4.00	0.4802
-*	0.15	7.80	4.04	0.45	5.91	2.97	2.48	10.00	4.00	0.0119
2	0.20	10.00	5.00	0.20	6.00	4.00	0.50	10.00	5.00	3.9021
-	1.44	9.87	4.94	0.18	6.35	3.28	0.50	10.01	4.96	0.0568
3	0.20	10.00	4.00	0.20	6.00	3.00	0.50	10.00	5.00	0.0422
-	0.22	10.00	4.30	0.07	5.99	3.01	0.50	9.84	5.32	0.0296
4	0.50	12.00	8.00	0.40	10.00	7.00	5.00	10.00	6.00	1.4379
-	0.40	9.78	6.45	0.02	10.71	5.45	5.00	10.03	5.96	0.0158
5	2.00	8.00	3.00	0.80	8.00	4.00	5.00	15.00	4.00	2.3764
-	1.02	7.35	3.01	0.10	7.71	4.27	5.00	15.00	4.00	0.0469

* The first row in each case is the initial guess and the second is the optimal design.

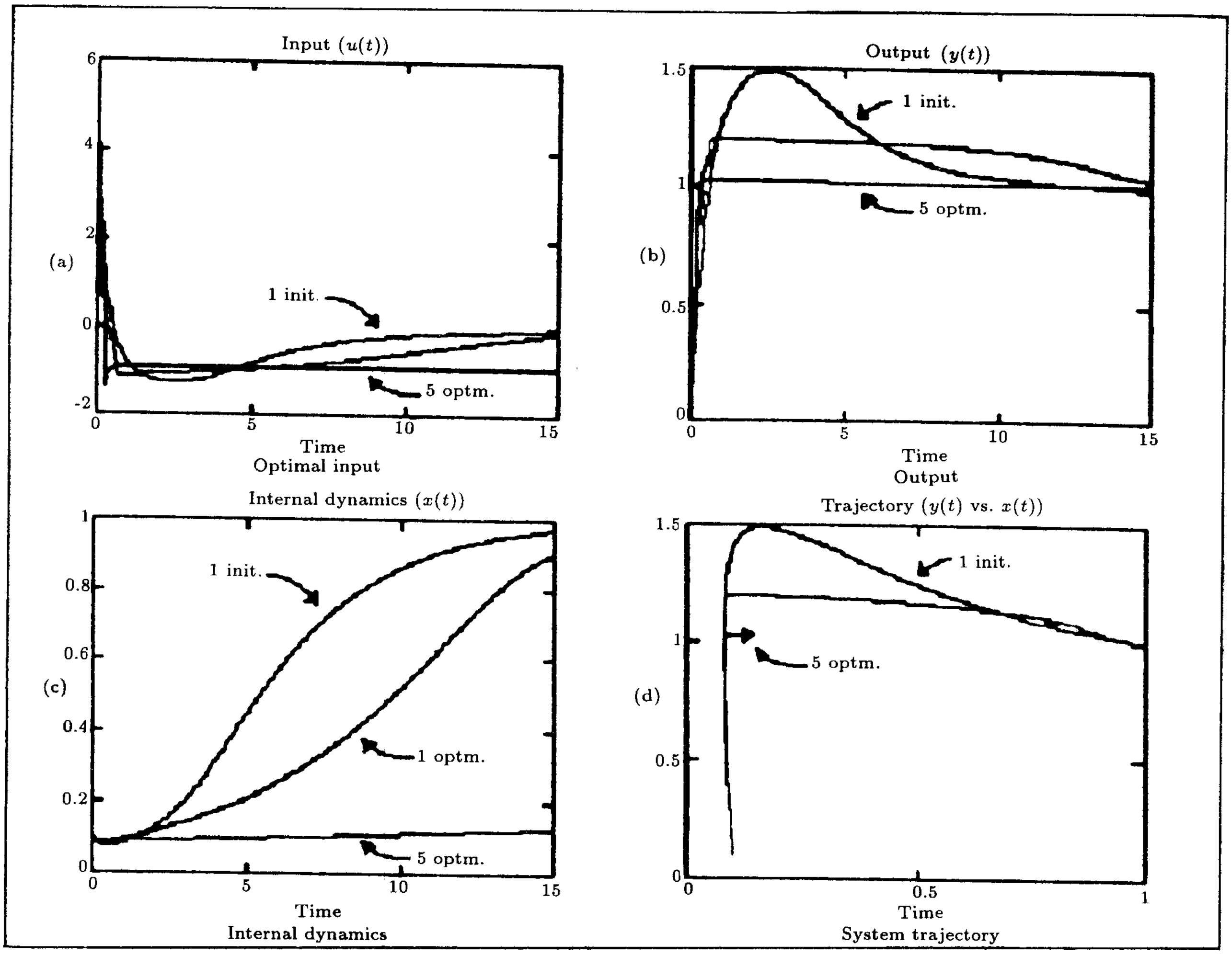

Figure 6. Time responses due to the optimized membership functions listed in Table 3.

Table 4. Sensitivity levels of the parameters.

Par. No.	8	7	9	3	5	2	6	1	4
Sens.	.0838	.1038	.1261	.1980	.2054	.2288	.2677	.3381	.3511

Since the effect of μ_u appears specially in the value of u_0, this section is followed with an attempt to determine that parameter large enough to ensure that the desired output transient can be attained. However, Table 5 includes the results for the optimization with respect to these four parameters, $\underline{\theta}$, fixing all the others with the same predetermined values.

Moreover, these results admit that μ_u is fixable too, but this step is skipped and left for another case study. Some algorithms can be suggested based on the analytical calculations, which is helpful in choosing a proper value for μ_u . As expected, it can be shown that an analytical approach leads to complicated formulae. For the sake of simplicity and to improve our confidence about the existence of the solution for Equation 13, considering those four parameters (and by fixing μ_u, even with only three), it is better to let μ_u be much larger than:

$$2 \lim_{\sigma_{xu} \to 0} u(0) \approx \mu_{\min} ,$$

which is approximately the least needed value,

Table 5. Optimization results of the second step.
Trapezoidal membership functions (4 parameters for 2 variables.)

Par. No.	**1**	**2**	**3**	**4**	
Case No.	σ_{zu}	μ_u	σ_{ze}	μ_e	$J(\underline{\theta})$
1	0.1000	8.0000	0.4000	6.0000	0.1502
-*	0.0473	7.9750	0.5737	5.9991	0.0079
2	1.0000	8.0000	0.6000	6.0000	0.8579
-	0.7767	7.4592	0.0089	5.2226	0.0792
3	0.2000	10.0000	0.5000	5.0000	0.1428
-	0.1025	9.9049	0.1213	4.9663	0.0416
4	0.5000	12.0000	0.4000	7.0000	0.0785
-	0.0870	12.0719	0.1536	6.9764	0.0371
5	2.0000	8.0000	0.8000	4.0000	2.3913
-	0.0498	7.2221	0.2754	3.3919	0.0321

$\sigma_u = \frac{1}{2}\mu_u, \sigma_e = \frac{1}{2}\mu_e, \sigma_{zr} = 2, \mu_r = 2$ and$\sigma_r = 10$.
* The first row in each case is the initial guess and the second is the optimal design.

say twice, as a rough guess. The next figure represents a sample set of the optimally produced outputs. Note that quite various sets of the results have been selected to plot which may have some problems (like chattering or overshoot) and they will be discussed in the next section. It means that some optimal responses may have not satisfied the designer as being acceptable, but, as mentioned, there are always many solutions and one can choose a desired one. It should be noted that this is because of the short interval (0 to N) in which the optimization is done, but there is no gurantee to get a desired response after the Nth sample. In return, the noticeable advantage of this method is the way it speeds up the procedure. It can be seen, as in these results, that the desired output transient can be even coincided thoroughly by the optimized parameters.

IMPROVEMENT (ERROR ATTENUATION-RESPONSE SMOOTHNESS)

The proposed fuzzy controller is compared with the feedback linearization approach. The simulations show the performance of the optimal controller. There is enough evidence to conclude that the fuzzy controller produces better results. In Figures 6 and 7, it is seen that the fuzzy controller can give an ideal response with almost no overshoot, coinciding with the desired transient. Even for some cases of non-optimal parameters (initial values, e.g. Case 1 in Figure 6, or Case 2 in Figure 7), almost similar responses compared to what has been shown in Figure 2 are observed. However, much less settling time or overshoot are found. This very small error (see Figure 7a, 7c or 8a, where it is less than 3%), which reduces slowly, is proportional to the optimal choice of σ_z.

Some distinct solutions are suggested to improve performance and lessen the sensitivity of the controller to the initial conditions and disturbances which may lead to chattering or even instability. A possible solution is to assign a symmetric triangular membership function to Zero linguistic term, that is:

Type 3 (Triangular):

$$f_g(\alpha) = \begin{cases} 1 + \frac{\alpha}{2\sigma}; & -2\sigma \leq \alpha \leq 0 \\ 1 - \frac{\alpha}{2\sigma}; & 0 \leq \alpha \leq 2\sigma \\ 0; & \text{elsewhere.} \end{cases}$$

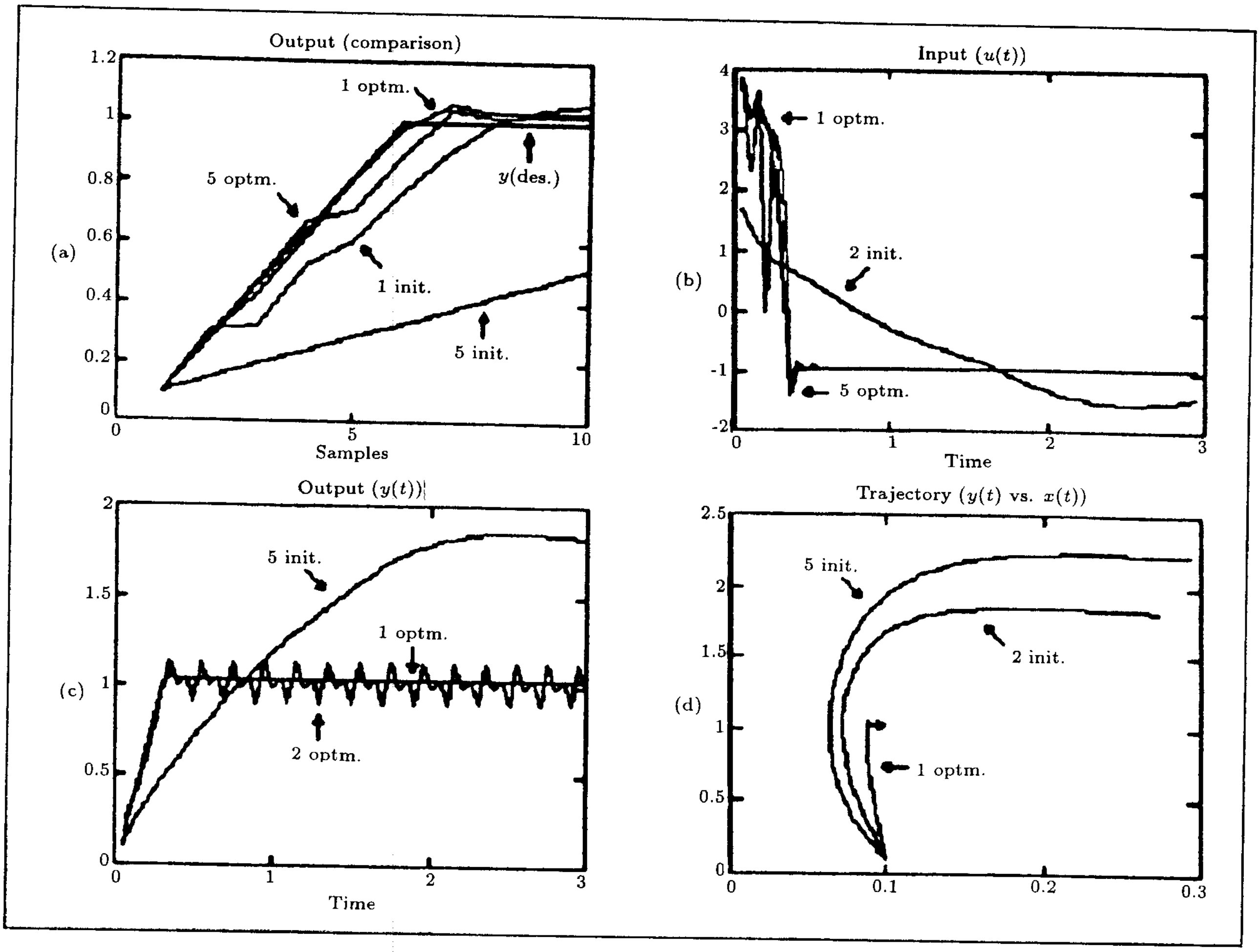

Figure 7. Time responses due to the optimized membership functions listed in Table 5. The optimal response for Cases 1, 2 and 5 and the initial result of Case 2 are plotted. The other optimal responses are quite similar to that of Case 1.

This choice, specially for $e(t)$, can help the error attenuation, but the increase of chattering probability should also be noticed. Paying attention to the second case of Table 5 notifies that small σ_{ze} (very narrow-singleton like membership functions for Zero) may lead to such high frequency oscillations known as chattering.

Finally, a more accomplished design is originated from the rule base 2. This will provide a desired condition to control the system much firmly, especially nearby the equilibrium point, coping with chattering and saving smoothness. Table 6 lists the optimization results assigning five exponential membership functions of Types 2 and 4 ($f(\alpha) = \exp -(\frac{|\alpha-\mu|}{2\sigma})$) for $u(t)$ and five other of Types 1 and 3 for $e(t)$ and $\dot{e}(t)$. This controller which is not so advantegous with respect to its previous conterparts (for its great complexity), produces the responses presented in Figure 8. The optimized membership functions of Case 1 in Table 6 are shown in Figure 9.

Here, another approach is referred to which is also applicable if the state variable $x(t)$ is measurable. From the results of the simulation, it is seen that the faster the $x(t)$ approaches the final value, the output $y(t)$ has the faster convergence too (see Figure 6 or 7). Therefore, it is possible to define the performance function in terms of both $x(t)$ and $y(t)$. This proposition is not discussed in this paper and can also be investigated.

Table 6. Optimization results due to the rule base 2.
Exponential membership functions for $u(t)$ and trapezoidal forms for $e(t)$ and $\dot{e}(t)$.

Par. No.	**1**	**2**	**3**	**4**	**5**	
Case No.	σ_{zu}	σ_{su}	μ_u	σ_{ze}	σ_{se}	$J(\underline{\theta})$
1	0.5000	1.0000	10.000	0.1000	0.8000	0.3480
-*	0.0780	1.0781	9.9864	0.0795	0.8805	0.0031
2	1.0000	1.0000	8.0000	0.8000	1.0000	0.5929
-	0.3225	1.8436	8.0245	0.6960	0.9855	0.0236
3	1.0000	1.0000	6.0000	1.0000	2.0000	1.3629
-	0.0165	2.5934	5.9960	0.7108	1.8371	0.0215

$\sigma_{lu} = \frac{1}{2}\mu_{su} = \sigma_{su}$, $\mu_{se} = 2\sigma_{se}$, $\sigma_{zr} = \sigma_{sr} = 2$ and $\mu_{lr} = 14$.
* The first row in each case is the initial guess and the second is the optimal design.

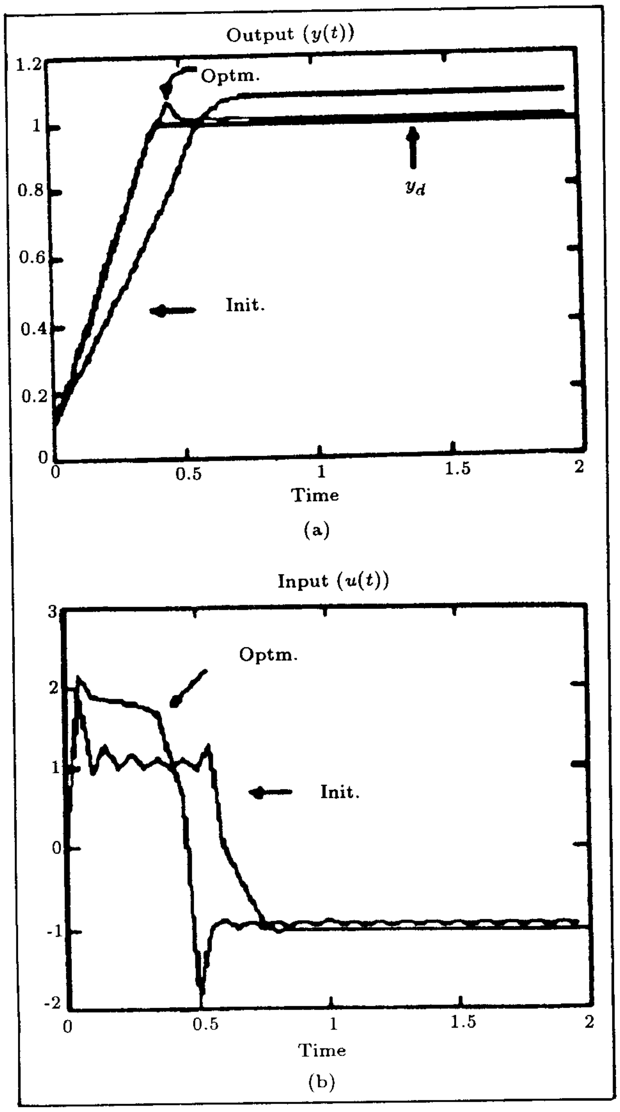

Figure 8. Time responses due to the optimized membership functions listed in Table 6.

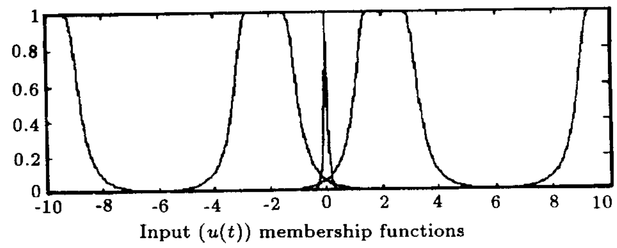

Figure 9. Membership functions of Case 1 in Table 6.

VALIDATION

The controller should be experimented in different cases rather than the design circumstances. Hence, the final optimal design (due to Table 6) is tested, setting various initial conditions and adding a disturbance term, $w(t)$, to Equation 2a. The controller shows a quite acceptable performance and trends the system towards the equilibrium point. An important point is that the objective considered for the system is to converge to its steady state slowly (see Figure 10). However, if there is an interest in the behavior of the other states, $x(t)$, or the input $u(t)$, extra information of each should be added to the performance index, Equation 13.

CONCLUSION

The major point, that has been addressed in this paper is a method which enables us to obtain the most proper membership function set and adequate range of its distribution that

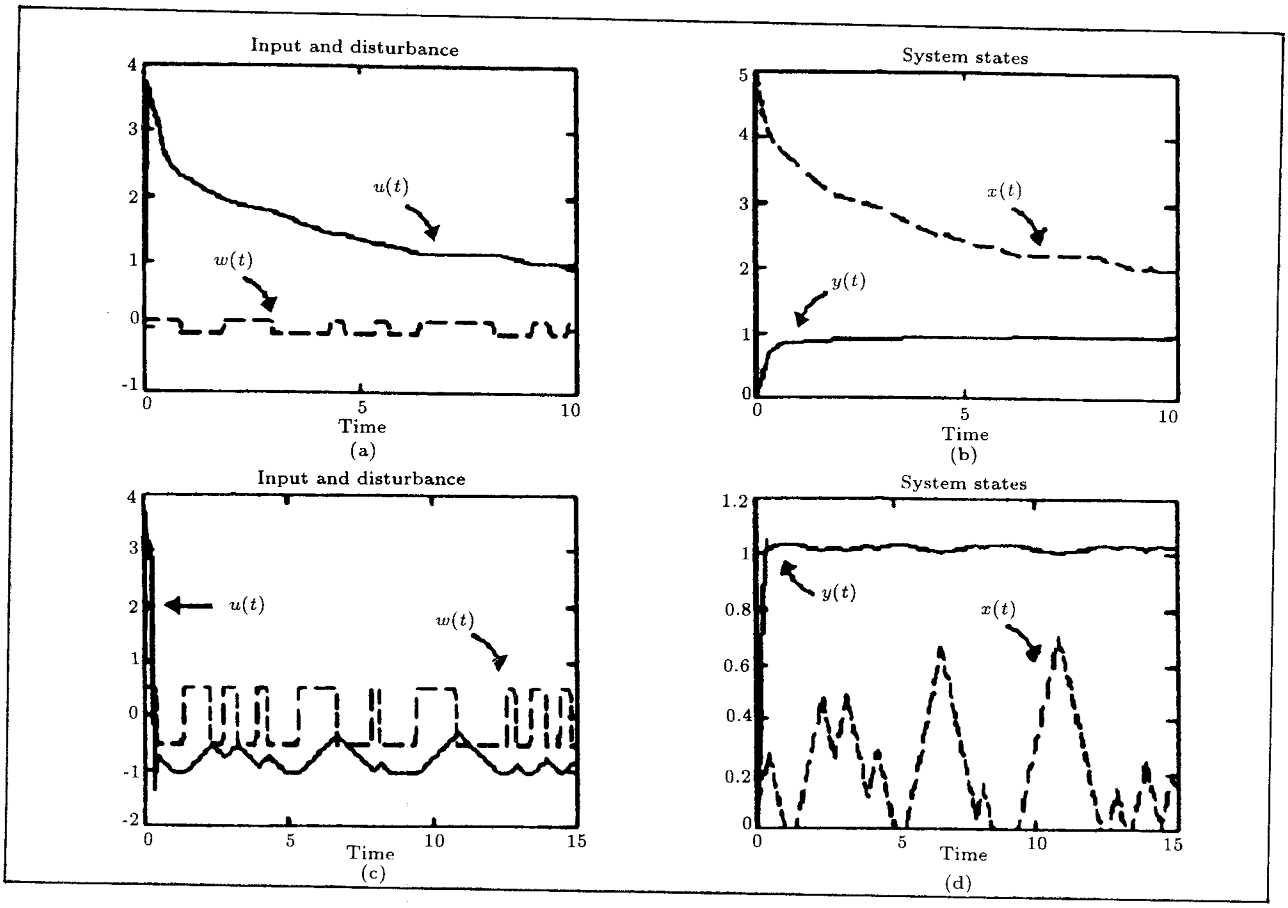

Figure 10. Performance tests for the Case 1 of Table 5 (under various initial conditions and disturbances).

offers the optimal control for a case study. It is concluded that despite some conventional triangular membership functions that partition the range of a variable entirely, a solution to the optimization problem may not often lead to uniformly distributed membership functions. Various types of these functions are examined for the aim of smoothness in response and/or reduction of settling time. Also, it is shown that the optimal fuzzy controller is relatively superior to the feedback linearization method and it overcomes the shortcomings of that nonlinear approach. Finally, using desired outputs in small duration makes it feasible to speed up the computation, also fixing the insensitive parameters improves the speed.

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