

Modern Practice in the Seismic Response Analysis of Embankment Dams

W.D. Liam Finn¹, R.H. Ledbetter² and W.F. Marcuson III²

The evolution of practice in the seismic response analysis of embankment dams is traced from 1960 to 1994. This background provides a reference framework for understanding modern engineering practice in seismic design and analysis. Comprehensive methods of analysis based on plasticity theory or nonlinear hysteretic models of soil response, which have found their way into practice, are presented. Some important historical cases illustrating the state-of-the-art of modern practice are presented which include the deformations of rockfill dams, the large deformation analysis of a dam with liquefied materials and the analysis of multi-remediation measures for a large embankment dam on a potentially liquefiable foundation.

INTRODUCTION

Major contributions in analysis leading to a more fundamental understanding of the seismic response of embankment dams were made by Ambraseys [1-3]. He assumed that soil was a viscoelastic material and treated the dams as one-dimensional (1-D) and two-dimensional (2-D) shear beams in his analyses. He demonstrated how the incoming motions were amplified throughout the dam, the contributions of the different modes of vibration of the dam to the global response, and how the seismic coefficient varied along the height of the dam. Ambraseys [3] studied the elastic response of dams in both wide and narrow rectangular valleys and showed that if the ratio of the width of the valley to the height of the dam was less than three, the seismic response changed significantly. This confirmed earlier results by Hatanaka [4,5]. Despite the limitations of the

viscoelastic model of soil behavior, this analysis captured many of the important characteristics of seismic response and provided the starting point for subsequent developments. Seed and Martin [6] carried out similar analyses for a variety of dam sizes and material properties and provided a comprehensive database for selecting appropriate values of seismic coefficients. They also drew attention to the deficiencies in the seismic coefficient method, should the materials in the dam lose strength during an earthquake.

Newmark [7] clarified many aspects of the problem of seismic stability of slopes. He pointed out that although the factor of safety in an equilibrium analysis incorporating the seismic coefficient might show a factor of safety less than one, this need not imply that the performance of an embankment dam would be unsatisfactory or its stability compromised. The factor of safety was less than one only for short

1. University of British Columbia, Vancouver, Canada.

2. U.S. Army Corps of Engineers, Vicksburg, MS, USA.

intervals during which the dam underwent some deformation. Newmark stressed that what counted, was whether the deformations that the dam suffered during the earthquake were tolerable or not. Obviously, large deformations that resulted in loss of freeboard and extensive cracking of the dam were not acceptable. The level of tolerable deformation should be based on the particular characteristics of the dam under study, judgement of experienced dam designers and an appreciation of the reliability with which the deformations can be estimated.

One of the most significant events which contributed to the rapid development of geotechnical earthquake engineering and the estimation of seismic displacement was the application of finite element methods to the analysis of embankment dams, for the first time, by Clough and Chopra [8]. This was followed by the seismic response analysis of slopes by Finn [9,10] and the analysis of central and sloping core dams by Finn and Khanna [11]. The latter study demonstrated the effects of the stress transfer between core and shell. All these analyses were conducted using a viscoelastic constitutive model of the soil and, therefore, were not capable of modelling the porewater pressure development or permanent deformations. To overcome this problem Finn [12] outlined a procedure for interpreting the effects of the dynamic stresses computed by the viscoelastic analysis with the help of data on porewater pressures and strains from laboratory cyclic loading tests.

A major improvement in analysis occurred in 1972 when Seed and his colleagues at the University of California at Berkeley developed the equivalent linear method of analysis for approximating nonlinear behavior. This method was incorporated in the 1-D shear wave propagation program SHAKE by Schnabel [13]. The technique was extended to 2-D finite element analysis by Idriss et al. [14] and Lysmer et al. [15] in the programs QUAD-4 and FLUSH, respectively. These programs took into account the strain dependence of damping and shear modulus. However, the analysis was still elastic and, as a result, permanent deformations could

not be estimated directly. Despite the limitation of elastic behavior, these programs led to more realistic analyses of embankment dams under earthquake loading and have remained the backbone of engineering practice to the present day.

By 1975, geotechnical engineers seemed to have many of the analytical and laboratory capabilities necessary for realistic assessments of the seismic safety and deformation behavior of embankment dams. These methods were put to the test when Seed et al. [16-18] undertook a comprehensive study of the liquefaction induced slide in the Lower San Fernando Dam which occurred as a result of the San Fernando earthquake of 1971. The analyses predicted that the dam would undergo large deformations upstream during the earthquake. In fact, the dam did not deform significantly until some 20 to 30 seconds after the earthquake [19]. This post-earthquake slide was attributed later by Seed [19] to porewater pressure redistribution.

The equivalent linear method of analysis used in the study of the San Fernando Dam is a total stress analysis and does not take into account the effect of porewater pressures on soil properties and dynamic response during the earthquake. Therefore, the total stress analysis tends to predict a stronger response than that actually occurs. Because the strains in the dam were estimated from triaxial tests simulating the estimated loading, the stronger response leads to greater deformations during the earthquake. Another major factor resulting in very different strains in the triaxial test compared to those in the dam, is the radically different boundary conditions on soil elements in the test compared to the corresponding elements in the dam. The strains deduced from the triaxial tests are incompatible with conditions in the dam. The difference between predicted performance and field performance of the San Fernando Dam provided the stimulus for the development of both nonlinear and effective stress methods of dynamic analysis which could take nonlinear response and the effects of porewater pressures into account directly. The Martin-Finn-Seed (MFS) model for generating

porewater pressures during earthquake loading based on the strain response of the soil was developed by Martin et al. [20] and paved the way for dynamic effective stress analysis and the direct estimation of displacements.

The first nonlinear dynamic effective stress analysis based on the MFS porewater pressure model was developed by Finn et al. [21,22] and was incorporated in the 1-D program DESRA-2 by Lee and Finn [23]. An updated version of the program, DESRA-2R, incorporating a more convenient form of the MFS porewater pressure model by Byrne [24], a modified nonlinear stress-strain law incorporating yield, and a joint element for simulating water accumulation beneath impermeable layers was developed by Finn and Yoshida [25]. A rudimentary 2-D version of this program was developed by Siddharthan and Finn [26]. An updated comprehensive program TARA-3 was developed by Finn et al. [27]. TARA-3 has the capability to conduct both static and dynamic analysis under total stress or effective stress conditions and can compute permanent deformations directly. The program uses properties that are normally measured in connection with important engineering projects.

Since the mid 1980's, other nonlinear effective stress programs have been developed mostly based on some version of plasticity theory and Biot's consolidation equation. These programs are mathematically and analytically quite powerful but use some properties which are not routinely measured in the laboratory or the field. Detailed presentations of some of these programs may be found in Pande and Zienkiewicz [28] and in comprehensive critical reviews in Finn [29,30] and Marcuson III et al. [31]. A number of these models were recently used in numerical predictions of seismic centrifuge tests on soil models [32]. The evaluation of these experiments is still under way and is too early to draw conclusions about the relative merits of the various models.

The estimation of post-liquefaction deformations is an important part of assessing the consequences of liquefaction in embankment dams. Finn and Yogendrakumar [33] developed

the program TARA-3FL to track large post-liquefaction deformations using an updated Lagrangian technique for coping with the large strains and deformations.

EMPIRICAL & SEMI-EMPIRICAL METHODS FOR SEISMIC RESPONSE ANALYSIS OF EMBANKMENT DAMS

The Newmark Method

Newmark [7] developed a method based on a sliding block analogy for estimating earthquake induced relative displacements. It is interesting to note that on a project for the U.S. Army Corps of Engineers, the late D.W. Taylor [34] appears to have independently developed a similar model for a similar purpose. The Taylor model was first implemented by R.V. Whitman at M.I.T. in 1953. A remarkable sentence is taken from the May 20 letter, "The procedure therefore cannot be expected to have much validity if, as in the writer's opinion, the threat of damage from earthquake action lies not in an increase of activative force but in a progressive decrease in shearing resistance as a result of many cycles of application of the activating force."

Deformation of a dam is modelled as the displacement of a rigid block sliding on an assumed failure surface under the action of the ground motions at the site. Various potential sliding surfaces in the embankment are analyzed statically to find the inertia force $F_I = (W/g)a_y$ required to cause failure (Figure 1). The average yield acceleration a_y is then deduced from this force. The sliding block is assumed to have the same acceleration time history as the ground. The yield acceleration is deducted from the acceleration time history, and the net acceleration (the shaded area in Figure 1) is available to generate permanent displacements. The analysis is conducted on the equivalent model of a horizontal sliding block on a plane with only one-way motions allowed.

Makdisi and Seed [35] modified the Newmark method by taking the flexibility of the dam into account. The average accel-

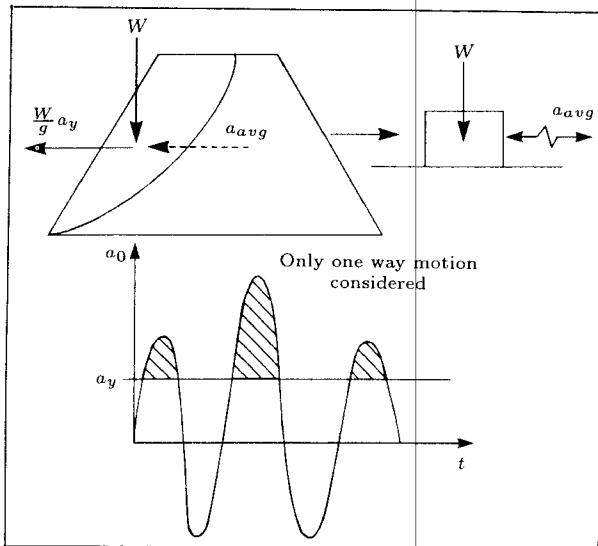


Figure 1. Elements of Newmark's deformation analysis.

ation time record of the sliding block is obtained usually from a QUAD-4 analysis. The method differs from the Newmark approach in generating relative displacements by the net accelerations above the sliding surface, whereas Newmark used the net accelerations below the sliding surface. The QUAD-4 accelerations in the sliding block are determined without taking the yield accelerations into account. Therefore, in many cases of strong motions, estimates of displacements would probably be conservative.

Byrne [36,37] modified the Newmark approach by deriving an "equivalent" seismic coefficient which allows the pattern of displacements to be estimated by a 2-D finite element analysis.

The Newmark method was introduced at a time when there were no direct methods of computing permanent deformations. It is still widely used despite all the evidence that the sliding block model is not a very good representation of how embankment dams deform, especially embankment dams with low factors of safety. This was shown as early as 1958 by Clough and Pirtz [38] in their shake table tests on models of Kenny Dam. The method is useful in comparing the deformation potential of alternative design proposals, or in comparing a design with that of an existing dam. However,

in many cases, similar results to those of the Newmark method can be achieved by using Makdisi and Seed's [35] simplified approach. Using current technology and finite element analysis, permanent deformations can be calculated directly without restrictive assumptions about the mode of deformation.

METHODS FOR NONLINEAR DYNAMIC RESPONSE ANALYSIS

The state of the art of earthquake analysis procedures for concrete and embankment dams was summarized in Bulletin 52 of the International Commission on Large Dams [39]. The bulletin outlined a general framework for analysis in both total and effective stress modes applicable to embankment dams using equations which coupled the response of soil and water. It recommended three levels of analyses:

1. Simple total stress methods including pseudostatic analysis using seismic coefficients when porewater pressures are negligible and no significant degradation in soil properties occurs.
2. The equivalent linear method of analysis coupled with the use of laboratory data [16,19] when substantial porewater pressures are generated.
3. Effective stress analysis conducted in "a direct and fundamental manner".

Pseudostatic analysis with seismic coefficients might be used safely in areas where a long history of use has calibrated the seismic coefficients to reflect experience with dam behavior during earthquakes, such as Japan. It is not recommended where such direct experience is not available. The equivalent linear method is still the most widely used in practice, but "direct and fundamental" methods are finding increasing application. This is especially true in dealing with the complex problems that must be faced when evaluating the safety of existing dams which contain potentially liquefiable soils.

Equivalent Linear Analysis

The dynamic response of an earth dam is usually computed in engineering practice using an equivalent linear (EQL) method of 2-D analysis such as that incorporated in the computer programs QUAD-4 [14] or FLUSH [15]. The results may be corrected approximately for three dimensional (3-D) effects [40]. These corrections were used in the back analyses of Oroville Dam for the 1975 earthquake [41]. The correction is based on altering the shear modulus in the 2-D analysis so that the fundamental 2-D period matches the equivalent 3-D period.

Dakoulas and Gazetas [42,43] studied the problem again and Gazetas [44] points out that despite matching the fundamental period, the contributions of higher harmonics may be substantially underestimated. Therefore, assessing the seismic response of embankment dams in narrow valleys requires the exercise of engineering judgement, since the higher harmonics are likely to have their greatest effect at the crest of the dam.

The EQL analyses are conducted in terms of total stresses and the effects of seismically induced porewater pressures on elemental shear stiffness are not reflected in the computed strains, stresses, and accelerations. Since the analyses are elastic, they cannot predict the permanent deformations. Therefore, equivalent linear methods are used only to get the distribution of maximum accelerations and maximum shear stresses in the dam. Semi-empirical methods are often used to estimate the permanent deformations using either the acceleration or stress data from the equivalent linear analyses.

Deformations from Acceleration Data

Deformations are often estimated from the acceleration data using the Newmark method as modified by Makdisi and Seed [35]. The resulting deformations do not represent the deformation patterns of embankment dams under strong shaking, but they may provide a useful index of the potential level of deformation. If a sliding wedge can be found which undergoes large deformations, one would expect to es-

timate large deformations by an appropriate nonlinear finite element analysis. However, the deformations computed by the Newmark approach should not be used for estimating whether the seismic deformations will satisfy displacement criteria.

Deformations from Stress Data

A more detailed picture of potential strains and deformations is obtained using Seed's semi-empirical method [16]. The computed dynamic stresses in soil elements in the dam are converted to equivalent uniform stress cycles and are applied to laboratory specimens in consolidated states similar to corresponding elements in the dam. The resulting strains in the laboratory specimens are assigned to the corresponding elements in the dam. This procedure gives an incompatible set of strains which are an indication of the potential for straining at selected locations within the dam.

These procedures were used to investigate the slide in the Lower San Fernando Dam during the 1971 earthquake [16-18]. Large upstream displacements were predicted to occur during the earthquake. In fact, the failure occurred under static loading conditions shortly after the earthquake shaking had ceased. This case history was a major motivation for the development of more general constitutive relations for modelling nonlinear behavior in terms of effective stresses and providing reliable estimates of porewater pressures and permanent deformations under seismic loading.

Nonlinear Methods of Analysis

A hierarchy of constitutive models is available for the direct and fundamental analysis of the dynamic response of embankment dams to earthquake loading. The models range from the relatively simple equivalent linear model to complex elastic-kinematic hardening plasticity models. Detailed critical assessments of these models may be found in Finn [29,30] and Marcuson III et al. [31]. This review presents only the methods used in current practice and outlines their advantages and limitations.

Elastic-Plastic Methods

It is generally recognized that elastic-plastic models of soil behavior under cyclic loading should be based on a kinematic hardening theory of plasticity using either multi-yield surfaces or a boundary surface theory with a hardening law giving the evolution of the plastic modulus. These constitutive models are complex and incorporate some parameters not usually measured in field or laboratory testing. Soil is treated as a two-phase material using coupled equations for the soil and water phases. The coupled equations and the more complex constitutive models make heavy demands on computing time [30].

Validation studies of the elastic-plastic models suggest that, despite their theoretical generality, the quality of response predictions is strongly stress path dependent [30,45]. When loading paths are similar to the stress paths used in calibrating the models, the predictions may be good. As the loading path deviates from the calibration path, the prediction becomes less reliable. In particular, the usual method of calibrating these models, using data from static triaxial compression and extension tests, does not seem adequate to ensure reliable estimates of response for the dynamic cyclic shear loading paths that are important in many kinds of seismic response studies. It is recommended that calibration studies of elastic-plastic models for dynamic response analysis should include appropriate cyclic loading tests, such as triaxial, torsional shear, or simple shear tests. The accuracy of pore pressure prediction in the coupled models is highly dependent on the accurate characterization of the soil properties. It is difficult to characterize the volume change characteristics of loose sands and silts which control porewater pressure development because of the problems of obtaining and testing undisturbed samples representative of the field conditions. As a check on the capability of these models to predict porewater pressure adequately, it is advisable to use them to predict the field liquefaction resistance curve as derived from normalized Standard Penetration Test data [46].

Typical elastic-plastic methods used in current engineering practice to evaluate the seismic response of embankment dams are DYNAFLOW [47], DIANA [48], DSAGE [49], DYNARD [50], FLAC [51], DYSAC2 [52,53] and SWANDYNE 4 [54,55]. There is no published information on the current version of DIANA which is an extensive modification of the earlier program. Programs DSAGE, DYNARD, and FLAC are proprietary to their developers.

The constitutive model of DYNA-FLOW is based on the concept of multi-yield surface plasticity. The initial load and unload (skeleton) stress-strain curve obtained from laboratory test data is approximated by linear segments and the curves for loading, unloading and reloading follow the Masing criteria [56]. The procedure can include anisotropy. The program allows dissipation and redistribution of porewater pressures during shaking. Validation of the program has been by data from centrifuge tests. The computational requirements of the code are quite intensive.

DSAGE is predecessor of the program FLAC. The latter is a microcomputer implemented code based on the explicit finite difference method for modelling nonlinear static and dynamic problems. The program uses an updated Lagrangian procedure for coping with large deformations.

DYNARD uses an explicit finite difference method for Lagrangian nonlinear analysis allowing large strains and displacements. It analyzes the deformation and response of earth structures to the simultaneous effects of gravity and seismic shaking using undrained strength and degradable undrained soil moduli. The cyclic and nonlinear behaviour of soils is incorporated in the analysis by a 2-D bounding surface model, similar to that of Cundall [57] and Dafalias and Hermann [58].

DYSAC2 is a fully coupled nonlinear dynamic analysis procedure. The constitutive model is also based on bounding surface plasticity. The program has been validated in a preliminary way using the results of centrifuge model tests [59].

SWANDYNE 4 is a general purpose

elastic-plastic computer code which permits a unified treatment of such problems as the static and dynamic nonlinear drained and undrained response analyses of saturated and partially saturated soils to earthquake loading. The formulations and solution procedures, upon which the computer code is based, are presented in Zienkiewicz et al. [54,55].

Direct Nonlinear Analysis

The direct nonlinear approach is based on direct modelling of the soil nonlinear hysteretic stress-strain response. The Waterways Experiment Station (WES) has been working with the direct nonlinear dynamic effective stress analysis methods of Finn for more than ten years. This approach is represented here by the program TARA-3 [27], which is proprietary.

WES has extensive experience using this method in practice and a number of field studies are available. Some of these studies are used in the remainder of this paper to simply illustrate the use of dynamic effective stress and seismic deformation analyses in evaluating and/or remediating the seismic safety of embankment dams.

The objective during analysis is to follow the stress-strain curve of the soil in shear during both loading and unloading. Checks are built into the TARA-3 program to determine whether or not a calculated stress-strain point is on the stress-strain curve and corrective forces are applied to bring the point back on the curve if necessary. To simplify the computations, the stress-strain curve is assumed to be hyperbolic. This curve is defined by two parameters which are fundamental soil properties, the strength, τ_{max} , and the in situ small strain shear modulus, G_{max} . The response of the soil to uniform all round pressure is assumed to be nonlinearly elastic and dependent on the mean normal effective stress.

The response of the soil to an increment in load, either static or dynamic, is controlled by the tangent shear and tangent bulk moduli appropriate to the current stress-strain state of the soil. The moduli are functions of the level of effective stress, and therefore, excess porewater

pressures must be continually updated during analysis and their effects on the moduli taken progressively into account.

During seismic shaking, two kinds of porewater pressures are generated in saturated soils, transient and residual. The residual porewater pressures are due to plastic deformations in the sand skeleton. These persist until dissipated by drainage or diffusion and, therefore, they exert a major influence on the strength and stiffness of the soil skeleton. These pressures are modelled in TARA-3 using the MFS porewater pressure model [20].

VALIDATION OF CONSTITUTIVE MODELS

Element Tests

Constitutive models are normally validated by their usage in predicting the response in single element tests such as the static or cyclic triaxial test. However, single element tests may be a necessary, but are not sufficient because they do not provide an adequate validation of the predictive capability of a model. The stresses or the strains are known a priori and there is no need to solve the boundary value problem using the constitutive model to predict the response. All practical applications involve the solution of the equilibrium equations and the continuity equations under a prescribed set of boundary conditions and a prescribed input. Therefore, adequate model validation requires an inhomogeneous stress field which is not the case in the element test.

Centrifuge Tests

The centrifuge test offers the best opportunity for validating models by the solution of boundary value problems. Centrifuge models can be extensively instrumented, prepared under controlled conditions and shaken by prescribed input. Constitutive models, numerical procedures, and finite element models can be clearly tested by seeing how well the performance of the centrifuge model can be predicted. Also, numerical models and procedures can be calibrated and improved or modified for

phenomena that may not have been adequately accounted for in a model.

The TARA-3 model has been subjected to validation studies on the centrifuge over a three-year period, 1984–86. The tests were conducted at Cambridge University on behalf of the Nuclear Regulatory Commission of the United States. Results from the validation studies have been reported by Finn [30,60,61]. Many examples of applications of TARA-3 have been reported by Finn et al. [62].

A validation program based on centrifuge tests has recently been conducted in the United States under the auspices of the National Science Foundation called the VELACS program. The acronym arose from the title Verification of Liquefaction Analysis by Centrifuge Studies. A conference on predictions made under this program was held at the University of California at Davis in October 1993 [32].

Case History from Field

Opportunities for quantitative validation by case histories in the field are quite limited, primarily because structures are not generally adequately instrumented and earthquakes are rare. The 1987 Edgecumbe earthquake in New Zealand, $M = 6.7$, provided an opportunity to see whether the acceleration response and the permanent deformations could be adequately modelled in Matahina Dam.

The dam is located on the Rangatake

River in the eastern Bay of Plenty Region of New Zealand about 23 km from the earthquake epicenter and about 11 km from the main surface rupture.

Founded on rock, the dam is 86 m high and has a crest length of 400 m (Figure 2). The core is low plasticity weathered greywacke and slopes upstream. Dam shells are compacted rockfill of hard ignimbrite. The transition zones adjacent to the core are the fines and soft ignimbrite stripping from the rockfill quarry and left abutment excavation [63,64].

Matahina Dam was instrumented to measure accelerations at three locations along the crest, at the mid point between the crest and the base and at the base. Lateral and vertical displacements of the downstream slope have been monitored consistently at many locations since the dam was constructed. Readings were taken shortly before the earthquake and immediately afterward, and the earthquake induced permanent deformations were determined as shown in Figure 2. These deformations provided the basis for checking the capability of TARA-3 for estimating permanent deformations.

A study was conducted to simulate the performance of the dam during the Edgecumbe earthquake using the TARA-3 program preparatory to calculate how the dam might behave under the design earthquake which was substantially larger than the Edgecumbe earth-

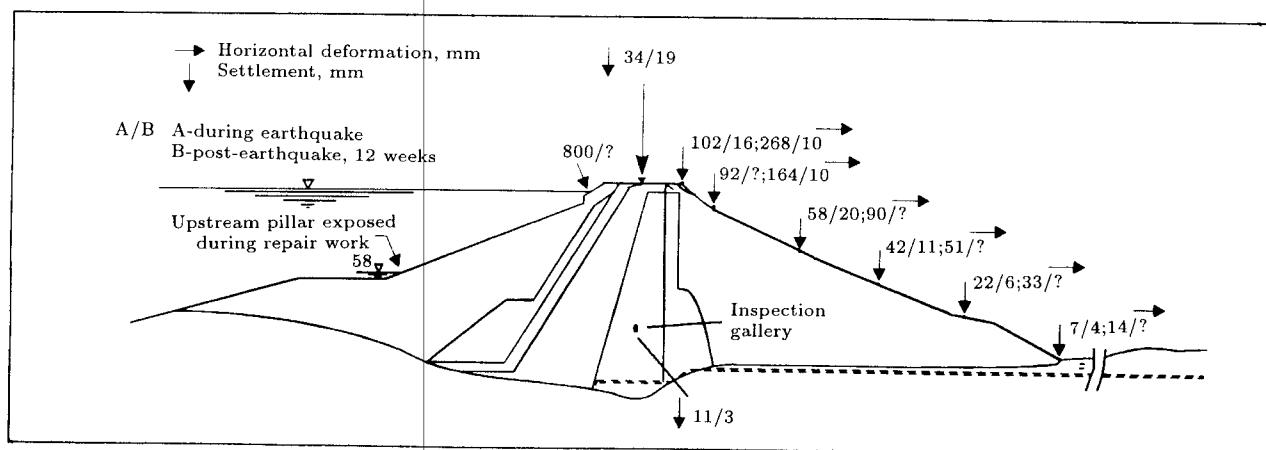


Figure 2. Cross-section of Matahina Dam showing the locations of displacement measurements. The vertical and horizontal components are given for the estimated displacements during the earthquake.

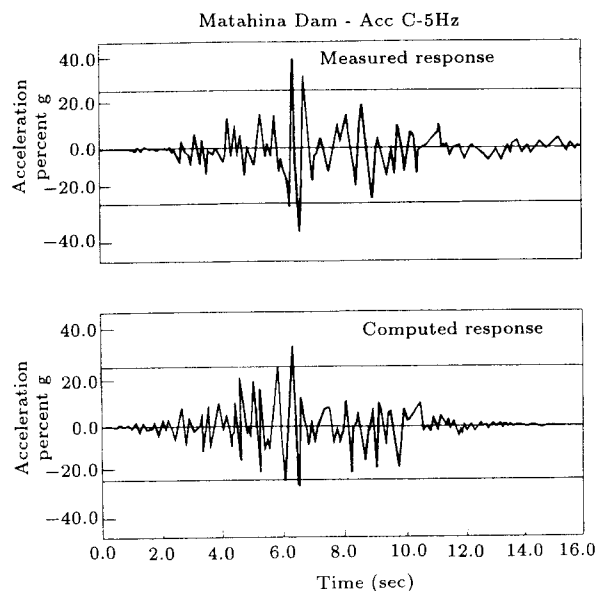


Figure 3. Measured and computed accelerations at the crest of Matahina Dam.

quake [61]. Analyses assumed that no porewater pressure developed in the rockfill, and the core deformed under undrained conditions during the earthquake. The properties of the clay core were obtained by laboratory testing and in situ measurements. Stiffness of the rockfill was estimated by measuring average shear wave velocities at various locations. Strength was conservatively taken from the literature and, as a first step, volume change properties of the rockfill were estimated by inverse analysis from the measured deformations.

The computed and recorded accelerations at the crest for the Edgecumbe earthquake are shown in Figure 3. Recorded and computed deformations during the earthquake at points on the downstream slope are shown in Table 1.

There is good agreement except for the node at the crest. The discrepancy here may be due to the fact that appurtenant structures on the crest were not modelled. The model was considered to be satisfactory for estimating the response under the design earthquake.

EVALUATION OF POST-LIQUEFACTION BEHAVIOR

A challenging technical problem for geotech-

Table 1. Measured and computed seismic displacements of Matahina Dam in mm.

Node	X_{means}	X_{comp}	Y_{means}	Y_{comp}
215	-	85	-34	-44
235	268	234	-102	-99
271	164	153	-92	-88
323	90	98	-58	-53
340	51	54	-42	-41
366	33	33	-22	-22
202	-	10	-11	-5

nical earthquake engineers involves the post-liquefaction behaviour of existing dams with potentially liquefiable zones in the structure or foundation. Two major challenges are: (1) estimating the post-liquefaction behavior of the dam, and (2) planning cost-effective remedial measures.

In the context of this section, liquefaction is synonymous with strain softening of sand in undrained shear as illustrated by curve 1 in Figure 4. When the sand is strained beyond the point of peak strength, the undrained strength drops to a value that is maintained more-or-less constant over a large range in strain. This is called the undrained steady state or residual strength.

If the driving shear stresses due to gravity on a potential slip surface in an embankment are greater than the undrained steady state strength, deformations will occur until the driving stresses are reduced to values compatible with static equilibrium. The more the driving

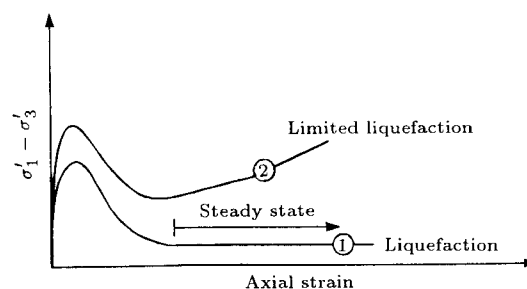


Figure 4. Types of liquefaction behavior.

stresses exceed the steady state strength the greater the deformations are needed to achieve equilibrium. Clearly, the residual strength is a key parameter controlling the post-liquefaction behavior.

If the strength increases after passing through a minimum value, the phenomenon is often called limited liquefaction and is illustrated by curve 2 in Figure 4. Limited liquefaction may also result in significant deformations because of the strains necessary to develop the strength to restore stability.

Procedures for assessing post-liquefaction behavior will be presented here and illustrated by a number of case histories.

Post-Liquefaction Response

Major difficulties are associated with estimating reliably what will happen after liquefaction in order to plan for cost-effective remediation. The most basic approach to the problem is to investigate the stability of the embankment by limiting equilibrium analysis which incorporates the residual strength of the liquefied soils. Usually a factor of safety of 1.1 to 1.2 is considered acceptable. Reliance on acceptable factors of safety alone is not adequate. Test data [65] shows that large strains may be necessary to mobilize the residual strength or a significant level of post-liquefaction shearing resistance. The associated deformations can result in unsatisfactory behavior of the dam despite adequate factors of safety. Additionally, as the thickness of a liquefied soil deposit increases, the assumption of well-defined failure surfaces becomes less reliable, and the dam may significantly deform from bearing capacity failure in the deposit. Therefore, it is necessary to conduct post-liquefaction deformation analyses to investigate the full consequences of liquefaction.

Deformation Analysis

Analysis of post-liquefaction deformation is an essential adjunct to stability analysis. The global picture of dam behavior provided by such an analysis allows the designer to adopt deformation criteria for evaluating dam performance

in addition to factors of safety. Deformation analysis (a) suggests the failure mode that is likely to develop and (b) makes clear where in the cross-section is the best intervene to remediate the structure and foundation. The factor of safety is not a discriminating tool for deciding on the type or extent of remedial measures. A factor of safety of 1.2 can have different connotations depending on the dam geometry and the extent and location of liquefied zones. But the displacement can be interpreted in the light of dam-specific criteria about the allowable potential loss of freeboard or the tolerable extent of potential horizontal deformation. Engineers can make sounder and more cost-effective decisions based on both factor of safety and deformation data than using the factor of safety alone.

An independent assessment of the equilibrium of the final position should be conducted using a conventional static stability analysis. The factor of safety determined in this way should be unity or greater depending on whether the deformations occurred relatively slowly after the earthquake or during it when inertia forces were acting.

When liquefaction is triggered, the undrained shear strength will drop to the residual strength. The post-liquefaction stress-strain curve cannot now sustain the pre-earthquake stress-strain condition and the unbalanced shear stress are redistributed throughout the dam. This process leads to progressive deformation of the dam until equilibrium is reached.

A computer program, TARA-3FL, which is a variation of the general computer program TARA-3, has been developed by Finn and Yogendrakumar [33] for estimating large post-liquefaction deformations based on the above concepts.

Since the deformations may become large, it is necessary to update progressively the finite element mesh. Each calculation of incremental deformation is based on the current shape of the dam, not the initial shape as in conventional finite element analysis. In essence, an updated Lagrangian procedure is used.

The application of post-liquefaction anal-

ysis in practice will be illustrated by two historical cases: Sardis Dam and the Upper San Fernando Dam. A detailed study of the Sardis Dam will be presented in the next section. This example illustrates how to evaluate the consequences of liquefaction, select proposed remediation measures and assess the performance of the remediated dam.

CASE HISTORIES OF REMEDIATION ANALYSIS

Sardis Dam, Mississippi

Sardis Dam is a U.S. Army Corps of Engineer dam constructed in the late 1930's located in northwestern Mississippi, 16 km southeast of the town of Sardis on the Little Tallahatchie River. The dam is approximately 4,600 m long with a maximum height of 36 m. It was constructed by hydraulic filling and consists of predominantly a silt core surrounded by a sand shell (Figure 5).

The foundation consists of a 3 to 6 m thick zone of natural silty clay called the topstratum clay as shown in Figure 5. The top stratum clay is underlain by previous alluvial sands (substratum sands) approximately 12 m thick which in turn are underlain by Tertiary silts and clays.

The U.S. Army Engineer District, Vicksburg, evaluated the seismic stability of Sardis Dam for a maximum credible earthquake having a peak horizontal acceleration of 0.20 g. Field and laboratory testing and seismic stability analyses indicated that significant strength loss or liquefaction which threatens upstream stability may occur in (1) the hydraulically placed silt core, (2) a discontinuous layer (1.5 to 4.5 m thick) of clayey silt located in the topstratum clay and (3) the upper 3 to 9 m of sand shell along the lower portion of the upstream slope [66-68].

The liquefaction or strength loss potential of the clayey silt was judged on the basis of a modification [67] to the Chinese criteria developed by Wang [69]. The residual strength (S_{ur}) of the clayey silt was estimated from field vane tests and laboratory investigations to be 0.075 times the effective overburden pressure (p'), $S_{ur} = 0.075p'$ [67]. The following discussions are related to a section where the weak clayey silt layer is 1.5 m thick.

Deformation Analysis of Sardis Dam

Deformation analyses by TARA-3FL supplemented by slope stability analyses were used to investigate the post-liquefaction response of the dam and to develop the remediation

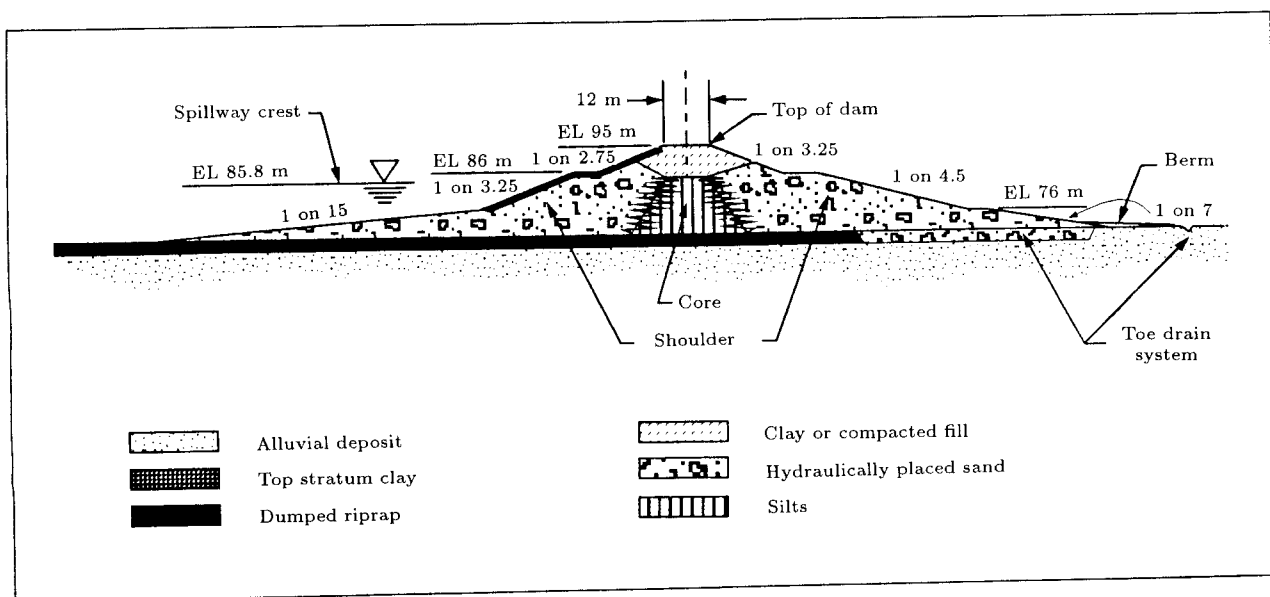


Figure 5. Cross-section of Sardis Dam.

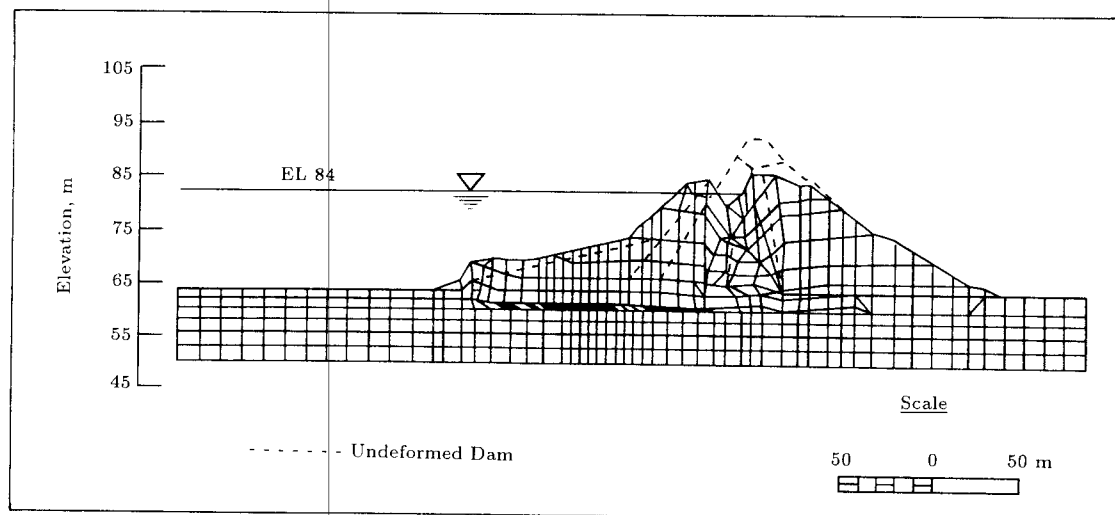


Figure 6. Deformed cross-section of Sardis Dam.

requirements.

The initial and final deformed shapes of the dam are shown in Figure 6 for a particular distribution of residual strengths. Substantial vertical and horizontal deformations may be noted, together with intense shear straining in the weak thin layer. Different deformed shapes resulted from different assumptions about the distribution of residual strengths.

The static stability of each of these deformed shapes was analyzed by Spencer's method [70] using the program UTEXAS2 [71]. In the clearly unstable region defined by a factor of safety less than one for the undeformed section, computed factors of safety for deformed sections were in the range of 1.0 ± 0.05 (Figure 7).

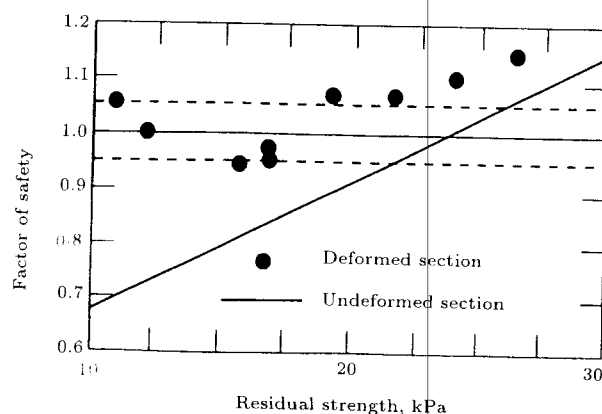


Figure 7. Factors of safety for Sardis Dam.

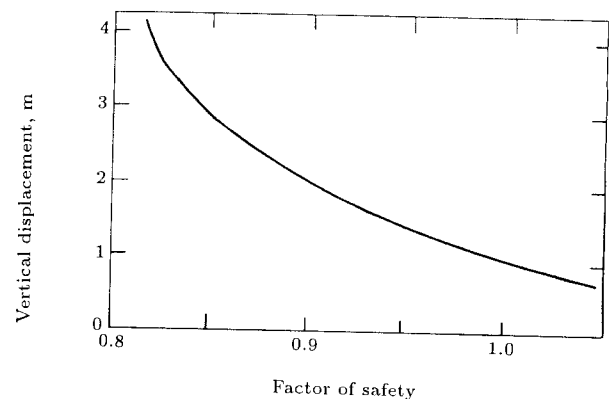


Figure 8. Variation of loss of freeboard with factor of safety of undeformed dam.

The variation of vertical crest displacement with factor of safety of the undeformed dam is shown in Figure 8. This type of plot gives much more meaning to the factor of safety by associating an index of overall critical displacement such as loss of freeboard with each factor.

Remediation Requirements for Sardis Dam

The deformation analyses supplemented by slope stability analyses were used to investigate various proposals for remediation. Driven reinforced concrete piles were selected to control the post-liquefaction deformations of the dam (Figure 9). The location of the zone of reme-

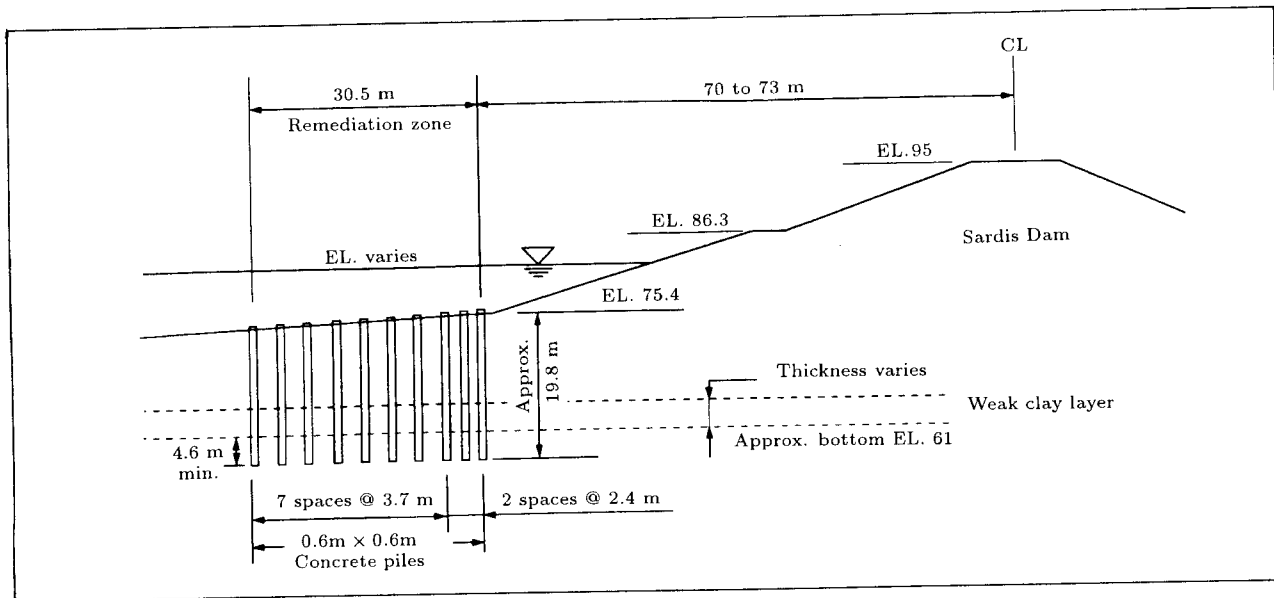


Figure 9. Cross-section of Sardis Dam showing location of remediation piles and weak thin layer (after Finn and Ledbetter [68]).

diation is controlled by the conservation level of the pool and a desire to avoid driving the piles through riprap on the upper slope above the slope break.

During shaking by the design earthquake, the saturated portion of the core and the weak layer of clayey silt outside the remediated zone are still expected to liquefy. This will result in increased lateral forces against the piles. Therefore, the piles must fulfill two functions: they must have sufficient strength to prevent shearing along the level of the weak layer and also have sufficient stiffness to prevent significant horizontal bending deformations that could lead to unacceptable loss of freeboard.

The static and dynamic loads for design of the piles were estimated by TARA-3FL and TARA-3 analyses. The time history of peak moments in the leading row of piles is shown in Figure 10 under the assumption that liquefaction occurred at the beginning of the earthquake. The final design of the pile installation was based on limiting vertical deformations of the crest to about 1.5 m. The steel reinforced concrete piles selected for remediation are 0.6 square meter placed 1.2 m on center perpendicular to the dam axis and 2.4 m on center parallel to the dam axis for the

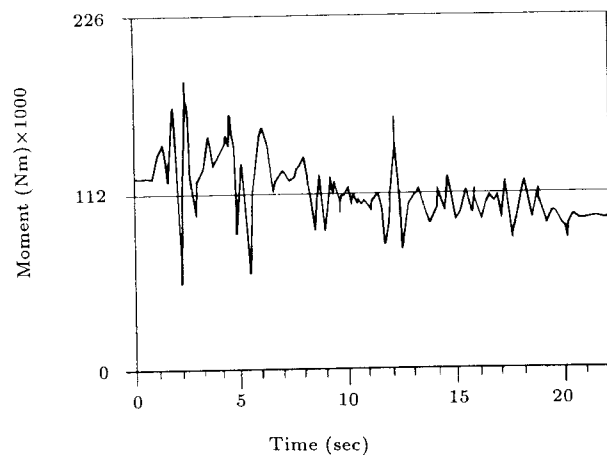


Figure 10. Variation in total moment during the earthquake.

first three rows closest to the dam center line and 3.7 m on center for the remaining seven rows.

Upper San Fernando Dam

Inel et al. [72] incorporated a simple soil model in the general purpose program FLAC [51] to investigate the deformations of the Upper San Fernando Dam during the 1971 San Fernando earthquake. The program uses an updated Lagrangian procedure similar to TARA-3FL for

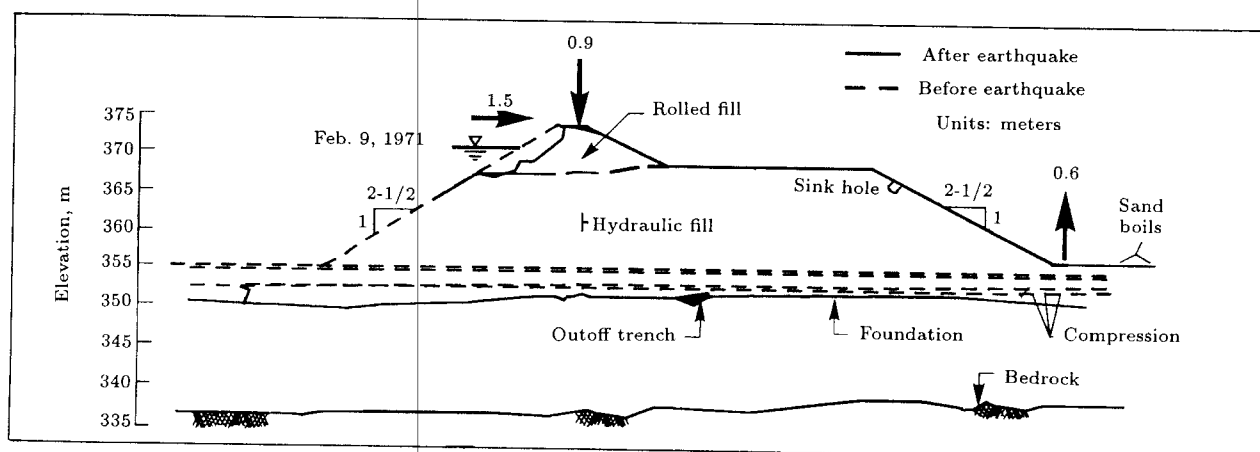


Figure 11. Upper San Fernando Dam.

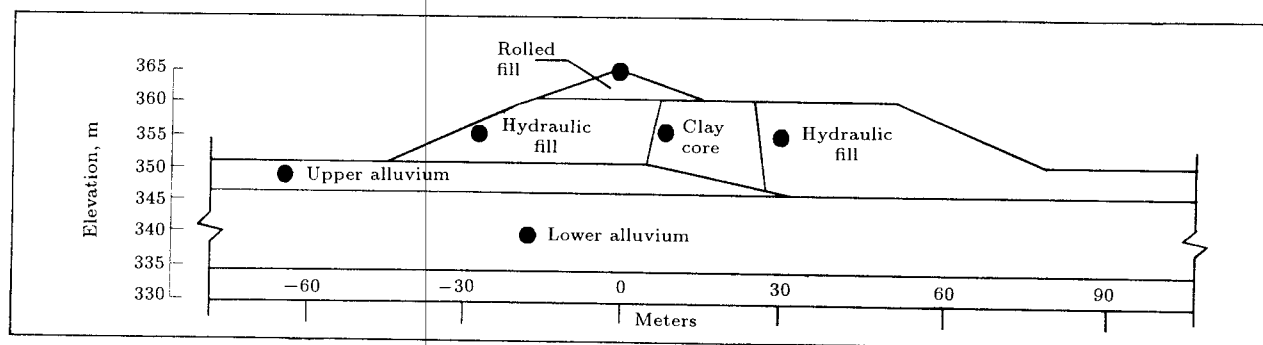


Figure 12. Upper San Fernando Dam representative cross-section [16].

coping with large deformations. The constitutive model incorporated the Mohr-Coulomb failure criterion and elastic shear and bulk moduli dependent on the mean normal effective stresses. Porewater pressures are generated by an incremental scheme [73,74].

The Upper San Fernando Dam (Figure 11) is an earth embankment with a maximum height of 24.4 m. During the 1971 San Fernando earthquake, $M_r = 6.6$, severe longitudinal cracks developed along the crest of the dam near the upstream slope. The crest moved downstream 1.5 m and settled 1 m. A pressure ridge, 0.6 m high, was created at the downstream toe. The pre-earthquake configuration is shown in Figure 11 as a dashed line.

The cross-section, soil properties and modified Pacoima Dam record used by Seed et al. [16] in a previous study of the dam were used in FLAC analysis. The cross-section used is shown in Figure 12.

As shown in Figure 13, the porewater

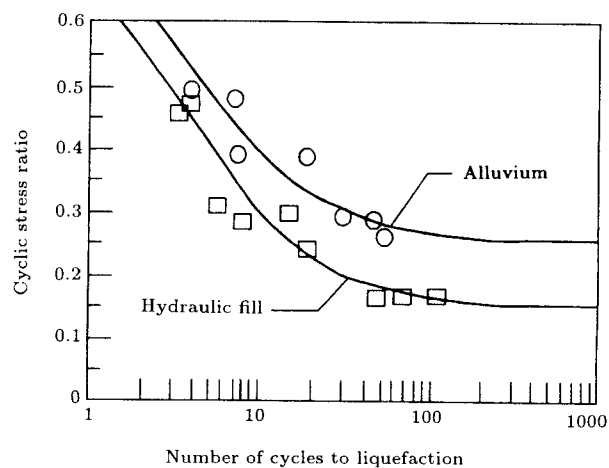


Figure 13. Upper San Fernando Dam cyclic shear strength curves, predicted and measured.

model satisfactorily predicted the laboratory-based liquefaction resistance curves. The deformed mesh at the end of shaking is shown in Figure 14. A settlement of the crest of about 1 m was predicted and a bulging of the down-

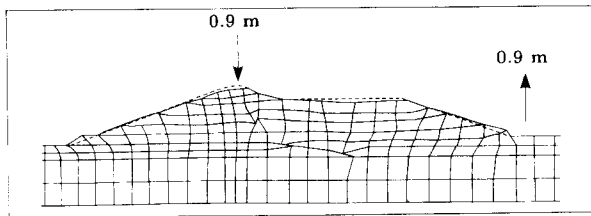


Figure 14. Deformed mesh of Upper San Fernando Dam.

stream toe of between 0.3 and 1 m. These agree well with the measured data. However, the computed overall deformation pattern differs significantly from the field pattern. The field data suggests that the entire upper portion of the dam moved downstream.

CONCLUSIONS AND RECOMMENDATIONS

The technology is available today for constructing safe embankment dams in any seismic environment. Indeed, even in the extreme cases where a major fault passes under the dam, it has been confidently asserted that "an embankment dam can be theoretically made safe against any feasible fault displacement" [75].

The major geotechnical problems facing dam designers in a seismic environment arise in the evaluation of the safety of existing dams. The most common factor leading to potential instability is the presence of loose saturated cohesionless soils in the dam itself and/or in the foundation which may liquefy during an earthquake. There are three difficult technical problems associated with potential instability induced by liquefaction. Will liquefaction be triggered? If so what will be the consequences? How can cost-effective remediation measures be designed to mitigate or prevent the consequences?

The post-liquefaction behavior of dams should be assessed using both limiting equilibrium analysis and deformation analysis. The extent and location of remediation should be determined primarily on the basis of calculated deformation patterns. For many dams, espe-

cially those with substantial freeboard, criteria based on factor of safety alone can result in unnecessary remediation costs.

Whether equilibrium or deformation procedures are used, the post-liquefaction undrained behavior of the liquefied material is the essential factor controlling the cost of remediation. It has two elements which should be well defined, the residual strength and the strain level required to reach it.

The dynamic response analyses of embankment dams are still largely based on technology developed in the 1970's which represent our first attempts to carry out nonlinear analyses by equivalent linear procedures. The stresses and accelerations determined in this way are input into other procedures for determining the performance of the dam. These procedures appear to work quite well provided the behavior of the dam is not strongly nonlinear and significant porewater pressures do not develop. More comprehensive methods are available which can deal with these problems directly, especially for evaluating the permanent displacements resulting from strong shaking with or without the presence of liquefaction. These procedures should be used when appropriate.

The Newmark procedure for estimating permanent deformations based on sliding block analysis is widely used despite all the evidence that deformations do not occur in this way. This method is particularly inappropriate when a large zone has liquefied in the embankment or foundation.

The seismic safety evaluation of dams has evolved from very empirical procedures in 1960 to a mature sophisticated professional practice in 1994. As pointed out above, the evaluation of well designed dams is well within the capabilities of the profession. The assessment of existing dams which may have potentially liquefiable zones can be done safely, but the uncertainties associated with the critical elements of the procedure are such that very conservative judgements are being made. The challenge for the profession in the immediate future is to reduce these uncertainties and thereby the extent and cost of the remediation of embankments.

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