Predicting the Production Performance of Gas Reservoirs

S. Ameri, K. Aminian and S. Mohaghegh[1]

Production performance and recovery under a variety of conditions are the primary requirements for optimization of natural gas reservoirs. The conventional methods, as well as numerical reservoir models, fall short of providing long-term gas reservoir production performance when the necessary data are lacking. In the absence of complete data, production type curves can be used as a simple alternative for production forecasting.

Production type curves are dimensionless constant pressure solutions that are plotted on a log-log scale. A number of production type curves for gas reservoirs are available in the literature. The published type curves, however, have ignored several factors that influence production performance. This paper discusses the development and application of three sets of production type curves for: (a) single-phase gas flow; (b) two-phase flow of gas and water; and (c) single-phase gas flow in horizontal wells. The influence of important factors such as the pressure dependency of gas properties, non-Darcy flow, drainage area shape, horizontal well penetration and relative permeability have also been discussed.

The type curves have been evaluated against both simulated and actual gas well production history data and found to be very reliable. The type curves can be utilized, through graphical history matching techniques, for predicting gas reservoir production performance under a variety of conditions and for evaluating the key reservoir parameters.

INTRODUCTION

Accurate prediction of future production rates under various operating conditions is the primary requirement for the natural gas reservoirs feasibility evaluation and performance optimization. The conventional method of utilizing deliverability and material balance equations to predict the production performance of the gas reservoirs cannot be often utilized when the complete reservoir data are lacking. It should be also noted that a large number of gas reservoirs, especially in the Middle East and Europe, produce substantial amounts of water together with gas. Furthermore, the interest in horizontal wells as an effective alternative to improve the economics of natural gas production has continued to increase in recent years. The conventional method of predicting the production performance for gas reservoirs generally does not lend itself to more complex cases involving wet sands or horizontal wellbores.

The finite-difference numerical reservoir simulators can be also utilized to predict the production performance of gas reservoirs, especially in more complex cases. However, to use numerical reservoir simulators is costly and

1. Department of Petroleum and Natural Gas Engineering., West Virginia University, Morgantown, WV, USA.

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time consuming. In the absence of complete reservoir data, production decline type curves have been used as a quick and reliable alternative for predicting production performance of gas reservoirs. This paper introduces accurate and reliable production-decline type curves that can be utilized to predict the production performance of the reservoir containing either vertical or horizontal wells that are producing single-phase gas, as well as reservoirs producing both gas and water. The analytical and numerical solutions that have been utilized to generate these type curves are also discussed.

Fundamentally, production decline type curves are a family of the dimensionless constant terminal pressure solutions that are plotted on a log-log scale. Production type curves can be employed for gas reservoirs production forecasting in two different manners. First, they can be utilized to predict the production performance during preliminary evaluations. In this case, the reservoir parameters are estimated and the type curves are utilized as a quick and accurate tool for production forecasting. This is particularly useful when dealing with more complex cases involving two-phase flow and horizontal wells, thus avoiding costly and time-consuming computer simulations.

The second application of type curves is for production history analysis. The production history (production rate versus time) is often the only readily available performance data from many gas wells. This is particularly the case with marginal gas wells, where it becomes impractical and uneconomical for an operator to conduct pressure surveys and deliverability tests in order to obtain the necessary data for conventional methods of production forecasting. To utilize the production decline type curves for production history matching, first, a log-log plot of actual data is made. Then, the plot of actual data is overlaid on the same size type curve and a single type curve is found that most nearly has the same shape as the plot of the actual data. The matched type curve differs from the plot of actual data only by a shift in coordinates. Once a best fit has been found, a match point can be selected to evaluate the reservoir parameters from the shift in coordinates. Finally, the matched type curve and the determined reservoir parameters are utilized to predict the future production rates.

BACKGROUND

The idea of using type curves for production forecasting was first introduced by Fetkovich [1]. The graphical type curve matching, however, has had widespread application for pressure transient data analysis. There has been a number of attempts to develop production type curves for gas reservoirs in the literature. Fetkovich [1] used an empirical model to develop a set of type curves for gas wells producing against constant back pressure. However, his model ignored the dependency of gas properties on pressure. Carter [2] generated a set of type curves with a finite-difference reservoir model. Carter showed that during the pseudo-steady-state flow, the type curves were influenced by the pressure due to dependency of gas properties on the pressure. Fraim and Wattenburger [3] also have illustrated the effect of gas properties on the behavior of the gas production decline. Both Carter and Fraim and Wattenburger, however, neglected the effect of non-Darcy flow in the development of their type curves.

Schmidt et al. [4] and Aminian et al. [5] developed similar theoretical models that correctly accounted for both non-Darcy flow and dependency of gas properties on pressure and numerically generated more representative type curves for gas wells producing against constant back pressure. However, these type curves were found to be sensitive to the initial reservoir pressure and formation permeability [4] as well as the skin factor [5].

One of the problems associated with the type curve application stems from the fact that each type curve has been generated by keeping the reservoir parameters and the operating conditions constant during the entire life of the reservoir. This implies that the production rates predicted by type curves are based on the assumption that the factors which have influenced the production in the past will not change in the future. Aminian et al. [6] have discussed the violation of this assumption in practice due to changes in well spacing owing to infill drilling, backpressure changes due to compressor installation and well stimulation, all of which will alter one or more of the reservoir and operating conditions.

More recently, an analytical constant pressure pseudo-steady state solution has been introduced by Aminian et al. [7]. This solution has been derived by utilizing pseudo-pressure and pseudo-time to account for the dependency of gas properties on pressure. The important factors such as non-Darcy effects, well stimulation (or damage), shape of the drainage area and well interference that influence the performance of the gas wells are also taken into account. The general solution can predict the production rates when the reservoir parameters or operating conditions are altered through proper modification of the dimensionless terms. This solution has been utilized in this paper for generating a reliable set of production decline type curves for single-phase gas reservoirs.

The use of single-phase gas flow type curves to predict the performance of gas reservoirs producing under two-phase flow conditions will lead to over-estimation of the production performance of the reservoir [8]. At the same time, the conventional production decline type curve cannot provide accurate production predictions for the reservoirs containing horizontal wells due to significantly different flow geometry associated with horizontal wells [9]. Therefore, two separate sets of production type curves have been developed with the aid of a numerical reservoir simulator for horizontal wells and two-phase flow of gas and water. The description of each model and the resulting type curves are provided in the following sections.

TYPE CURVES FOR SINGLE-PHASE GAS RESERVOIR

The constant pressure solution for single-phase gas reservoirs producing under a pseudo-steady-state flow regime [7], given below, can be used to generate production-decline type curves for a variety of conditions.

$$\ln q_D + 2\,(1 - F_{ND_i})(q_D - 1) + \frac{F_{ND_i}\, F_{taD}\, t_D}{L[1 - \frac{1}{X_i}]} = 0 \, . \qquad (1)$$

The description of various dimensionless terms are given in Appendix A. The pressure dependency of the gas properties is represented by F_{taD}, which contains the pseudo-time, and L, which contains pseudo-pressure. The non-Darcy flow effects are quantified by F_{ND_i}, which includes both shape and skin factors.

It should be mentioned that for a reservoir producing against constant back pressure F_{ND_i}, L and X_i are constant because they are only dependent on the limiting values of the pressure (i.e., p_i and p_{wf}). F_{taD}, on the other hand, varies throughout the life of the reservoir, since it depends on the pressure. To generate a type curve from Equation 1, it is necessary to determine F_{taD} for each pressure or, in other words, for each point on the type curve. Two approaches are presented in the literature for this purpose. The first approach (the indirect method) utilizes a stepwise method of solving material balance and deliverability equations simultaneously to determine rate versus time and converts the results to dimensionless rate and time [5,8]. The second approach, introduced by Abidi [10], solves Equation 1 directly by utilizing polynomial approximations for F_{taD} as a function of t_D.

It is interesting to note that if both the non-Darcy and pressure dependency of the gas properties are ignored (i.e., $F_{taD} = 1$, $L = 1.0$ and $F_{ND_i} = 1$), Equation 1 reduces to the familiar exponential decline. This is, of course, only true for single-phase liquid flow. If only the non-Darcy flow is ignored (i.e., $F_{ND_i} = 1$), then Equation 1 reduces to exponential decline against normalized time [3]. Therefore, Equation 1 is the most general and accurate form of the constant pressure pseudo-steady-state solution for single-phase gas flow.

The production history of gas wells can be analyzed by graphical history matching with

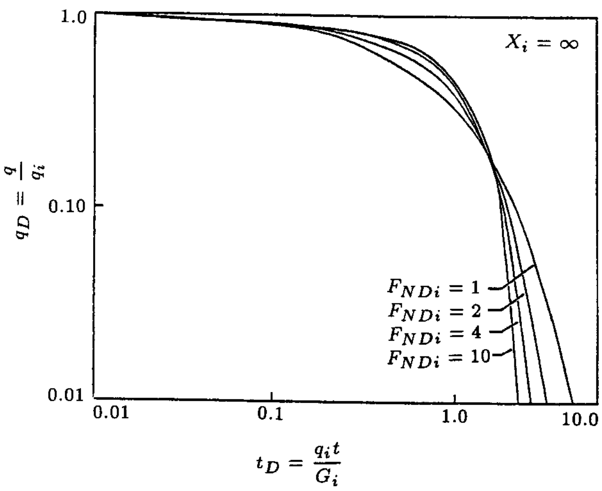

Figure 1. Production type curves for single-phase gas reservoirs ($p_i = 2000$ psia).

type curves generated with the aid of Equation 1. If the limiting values of the pressure (i.e., p_i and p_{wf}) are known, then L and X_i can be readily determined and a set of type curves can then be generated by varying F_{ND_i}. Figure 1 illustrates a set of type curves (for $p_i = 2,000$ psia and $p_{wf} = 0$). The effect of F_{ND_i} on the shape of the type curve for $F_{ND_i} > 10$ is not significant. Graphical history matching in this case can provide a reasonable estimate of F_{ND_i}. A more complicated case is when only one of the limiting values of pressure is known. In this case, multiple sets of type curves are generated for specific values of F_{ND_i} by varying X_i. Figure 2 illustrates a set of type curves (for $p_i = 2,000$ psia and $F_{ND_i} = 2$). Finally, the best combination of X_i and F_{ND_i} is estimated by graphical history matching.

When the best graphical match is obtained, the coordinates of the match point, as well as F_{ND_i} and X_i, can be used to determine G_i, a and b as follows:

$$q_i = \frac{q}{q_D} \quad \text{match point,} \tag{2}$$

$$G_i = \frac{q_i t}{t_D} \quad \text{match point,} \tag{3}$$

$$a = \frac{[P_P(p_i) - P_P(p_{wf})]}{q_i F_{ND_i}}, \tag{4}$$

$$b = \frac{(F_{ND_i} - 1)a}{q_i}. \tag{5}$$

Note that the future performance under a variety of conditions can be predicted easily after a, b and G_i are determined. If the future operating or reservoir conditions are different from the past, the values of a, p_{wf} and G_i may need to be modified appropriately. The accuracy and applicability of the type curves have been verified against simulated and actual production data from a number of gas wells [5-7].

When the presented type curves in this section are used to analyze the production history, care must be taken to assure the assumptions that are used to derive Equation 1 are not violated. Equation 1 has been derived for radial flow from a gas reservoir with no appreciable water drive against constant back pressure. Note that this solution is not valid for linear flow; however, inclusion of shape and skin factors in the gas flow equation (see Appendix A) allows application of this solution to fractured wells experiencing pseudo-radial flow. Transient data must not be included during type curve matching, because the solution is developed only for the pseudo-steady-state flow regime (stabilized flow). It should be realized that transient flow occurs when the well is put into production and generally lasts for a short period of time. Pseudo-

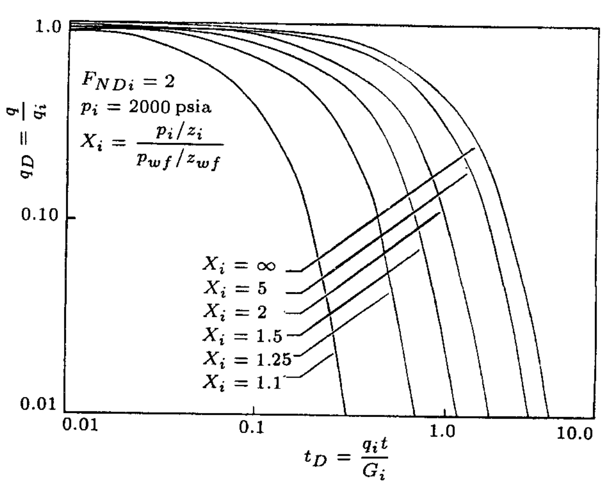

Figure 2. The effect of X_i on production type curves ($p_i = 2000$ psia).

steady-state flow regime prevails during most of the well's life. An exception to this rule is when extremely tight formations are involved. It is recommended in such cases to utilize production rates over longer periods of time, such as monthly, quarterly or annually, in order to minimize transient condition effects. Production data from reservoirs which have significantly different production mechanisms, such as a dual porosity system, two-phase flow or production owing to desorption, must not be analyzed with the type curves presented in this section.

It should be noted that finding a unique match is not always simple. However, if other information besides the production history is available then the history matching process can be enhanced. Figures 1 and 2 show little difference between the early parts ($t_D < 0.1$) of different type curves. As a result, it would be very difficult to obtain an accurate match if the available production history falls entirely on the early part of type curve. All data must be smoothed in order to achieve a reasonable graphical match. It is important to consider only the actual production time (days on line) and eliminate the down periods. Relatively minor fluctuations in the production data are generally minimized by utilizing longer time intervals in the analysis.

TYPE CURVES FOR TWO-PHASE GAS RESERVOIRS

A large number of gas reservoirs are wet sands which produce water as well as gas. There are cases in which initial water saturation is less than the critical water saturation of the formation and water is immobile. In this case, in order to take the presence of water phase into consideration in reservoir engineering calculations, one needs to decrease formation porosity by subtracting the portion of the pore volume occupied by the immobile water phase.

However, in most of the wet sands, especially in the Middle East and Europe, initial water saturation exceeds the formation critical water saturation and water becomes a dynamic phase. It is thus produced simultaneously with gas. In a two-phase gas reservoir, relative permeability characteristics of the formation control the flow of fluids through the formation. Therefore, a dry sand reservoir and a wet sand reservoir with similar properties exhibit different production and depletion characteristics. Use of single-phase production type curves to predict the performance of gas reservoirs undergoing two-phase flow conditions will lead to over–estimation of the capabilities of the field in hand. The purpose of type curves presented in this section is to provide a practical tool by which performance prediction and reservoir characterization of the wet gas sands can be achieved with good accuracy.

Mohaghegh and Ertekin [8] derived a single expression, starting with the gas and water equations, which represents the two-phase flow in wet-gas sands. After some mathematical manipulation, which includes linearization, this equation may be expressed as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p^2}{\partial r}\right) = \frac{\phi c_t}{\lambda_t}\frac{\partial p^2}{\partial t} . \tag{6}$$

The formulation of the Equation 6 and description of the dimensionless groups are given in Appendix B.

Mohaghegh [11] showed that it is necessary to generate separate sets of production type curves for different p_{wf}/p_i, however, for a given p_{wf}/p_i, the type curves are unique for different values of p_i. A numerical reservoir simulator [11] was utilized to generate production type curves from Equation 6. Figure 3 shows gas production type curves for $p_{wf}/p_i = 0.1$ and $p_i = 2000$ psia. As can be seen, the type curves are constructed for different r_e/r_w values. The corresponding water production type curves are shown in Figure 4. The type curves were extensively tested against a wide range of fluid and formation properties to assure their reliability and uniqueness [11].

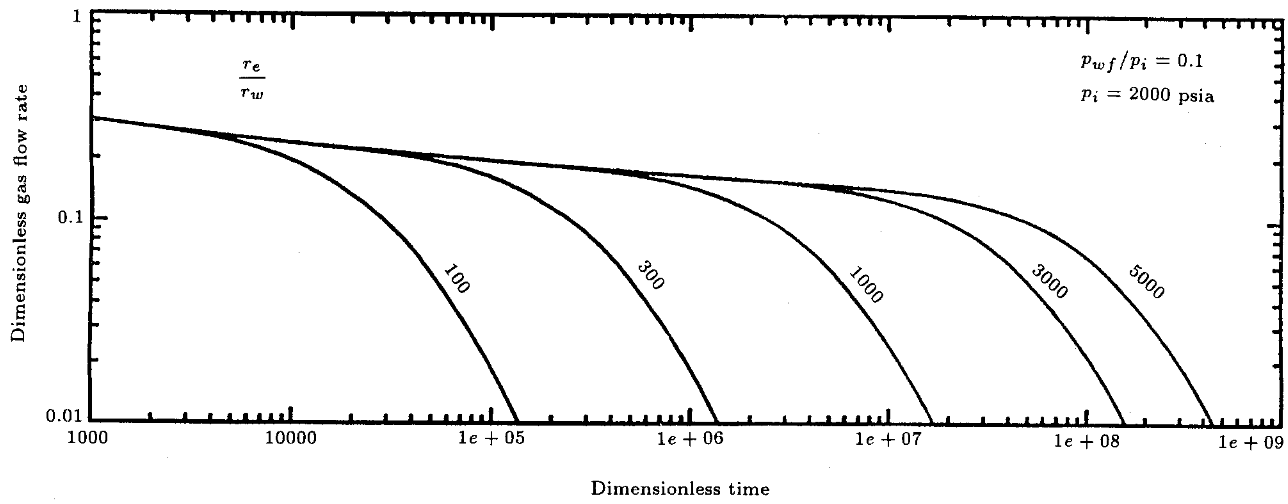

Figure 3. Gas production type curves for two-phase gas reservoirs ($p_i = 2000$ psia).

PRODUCTION DECLINE TYPE CURVES FOR HORIZONTAL WELLS

The use of horizontal wells to produce oil and natural gas has gained a great deal of interest in recent years. The use of horizontal well technology to stimulate production is not a new idea. However, significant advances in drilling technology during the last decade have made it possible to drill and complete horizontal wells commercially. The horizontal wells are currently being considered for production enhancement in a variety of reservoirs. In general, horizontal wells are believed to enhance the production in naturally fractured reservoirs and reservoirs with good vertical permeability. In low permeability reservoirs, a horizontal well can be considered as a long, controlled infinite-conductivity vertical fracture and has significant potential to increase the gas recovery. The selection of horizontal wells over vertical wells must be based on technical and economical feasibility evaluations. Production performance is the key variable for economic feasibility evaluation. Therefore, it is desirable to predict the production performance for preliminary evaluations to avoid costly and time consuming computer simulations.

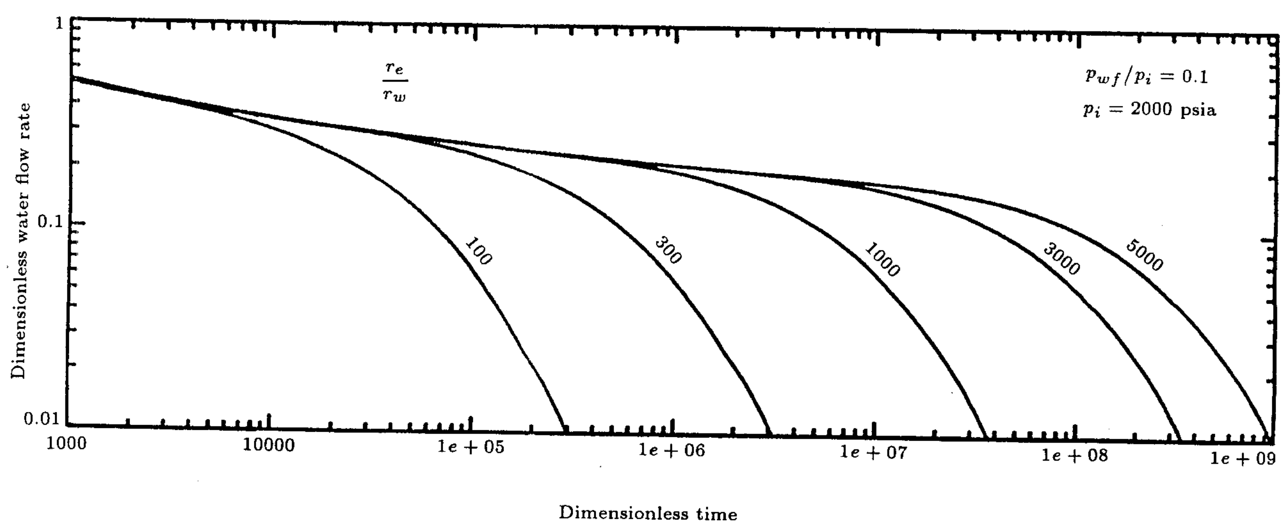

Figure 4. Water production type curves for two-phase gas reservoirs.

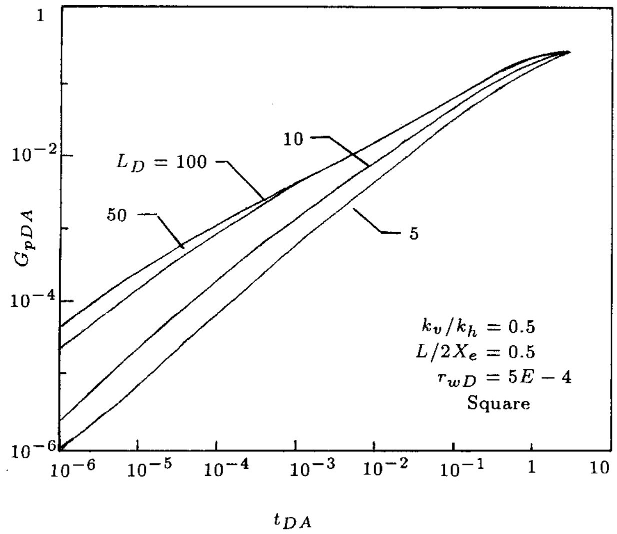

Figure 5. Gas production type curves for horizontal wells in square drainage area.

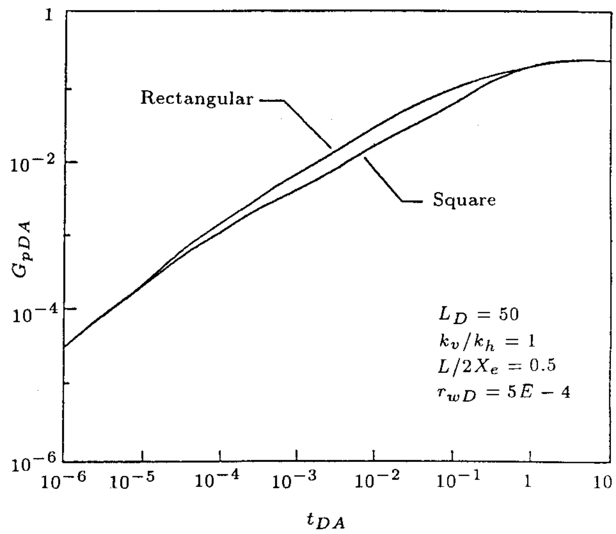

Figure 6. The influence of drainage area shape on horizontal well type curves.

General constant pressure solutions [9] that are presented in the form of type curves can be utilized for this purpose. Figure 5 illustrates the type curves for finite reservoirs with square drainage area. A finite-difference, three-dimensional, single-phase gas reservoir model was utilized to generate the type curves. The description of the model and validation of the results are presented by Duda [12]. Two dimensionless groups have been found to influence the behavior of horizontal wells. They are dimensionless wellbore radius, r_{wD}, and dimensionless well length, L_D. The Jefinition of various dimensionless groups associated with horizontal type curves are given below:

$$r_{wD} = \frac{2r_w}{L}, \tag{7}$$

$$L_D = (\frac{L}{2h})\sqrt{\frac{k_v}{k_h}}, \tag{8}$$

$$G_{P_{DA}} = [\frac{36\,T}{h\,\phi\,c_t\,A\,\Delta\,P_P\,(p)}]\,G_p, \tag{9}$$

$$t_{DA} = [\frac{0.02532\;k}{\phi\,c_t\,\mu\,A}]\,t. \tag{10}$$

In addition, the shape of drainage area and horizontal well penetration (L/X_e) have been found to influence the shape of the type curves. Figure 6 compares the production performance of a horizontal well in square and rectangular drainage areas. Figure 7 illustrates the effect of horizontal well penetration on the type curves. The reliability of the type curve predictions as compared to actual production data from horizontal wells has been also documented [9,12].

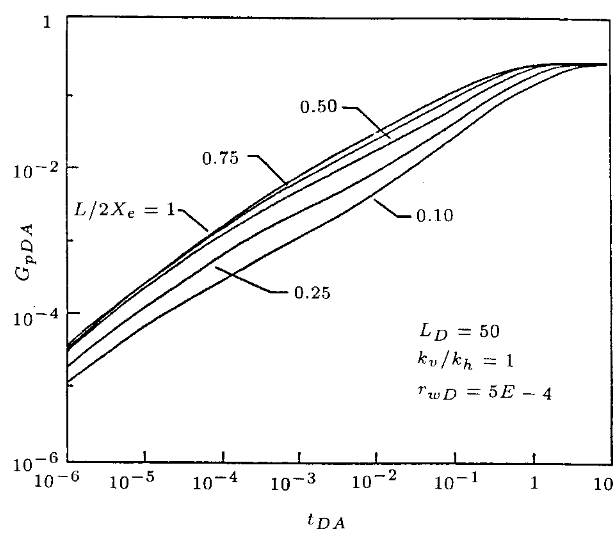

Figure 7. The influence of horizontal well penetration on type curves.

SUMMARY AND CONCLUSIONS

1. Three sets of production decline type curves for single-phase gas flow in vertical wells, single-phase gas flow in horizontal wells and two-phase flow of gas and water have been reviewed and the influence of the important factors on the type curves have been discussed.

2. Type curves can be utilized for prediction of gas well production performance under a variety of conditions in order to optimize the reservoir performance.

3. Type curves can be utilized to evaluate important missing parameters such as recoverable reserves, rock properties and deliverability through graphical history matching techniques.

4. Type curves have been evaluated against both simulated and actual gas well production history data and found to be very reliable.

NOMENCLATURE

a	Darcy flow coefficient, psi^2/(cp)(Mcf/D)
A	drainage area, ft^2
b	non-Darcy flow coefficient, psi^2/(cp)(Mcf/D)^2
B_g	gas formation volume factor, ft^3/STB
B_w	water formation volume factor, bbl/STB
c_g	gas compressibility, psi^{-1}
c_{gi}	gas compressibility at p_i, psi^{-1}
c_t	total compressibility, psi^{-1}
G_i	gas in place at p_i, scf
G_p	cumulative gas production, scf
h	reservoir average thickness, ft
k_h	horizontal permeability, md
k_{rg}	relative permeability to gas, dimensionless
k_{rw}	relative permeability to water, dimensionless
k_v	vertical permeability, md
p_D	dimensionless pressure
p_i	average reservoir pressure at beginning of decline period, psia
$P_p(p)$	pseudo-pressure, (psi)^2/(cp)
p_{wf}	well flowing pressure (back pressure), psia
q	flow rate, Mcf/D
q_D	dimensionless flow rate
q_i	flow rate at p_i, Mcf/D
r_D	dimensionless radius
r_e	drainage area radius, ft
r_w	wellbore radius, ft
s	skin factor, dimensionless
s_{CA}	shape-skin factor, dimensionless
t	time, days
t_D	dimensionless time
t_{DA}	dimensionless time based on area
T	reservoir temperature, °R
z	gas deviation factor, dimensionless
β	turbulence factor, ft^{-1}
μ_g	gas viscosity, cp
μ_i	gas viscosity at p_i, cp
μ_w	water viscosity, cp
ϕ	porosity, fraction

REFERENCES

1. Fetkovich, M.J. "Decline curve analysis using type curves", *J. Pet. Technol.*, **37**(6), pp 1065–1077 (1980).

2. Carter, R.D. "Type curves for finite radial and linear gas flow systems: constant terminal-pressure case", *Soc. Pet. Eng. J.*, **25**(2), pp 719–728 (1985).

3. Fraim, M.L. and Wattenburger, R.A. "Gas reservoir decline curve analysis using type curves and real gas pseudo-time", *SPE FE*, **2**(4), pp 428–434 (1987).

4. Schmidt, S.H., Caudle, B.H. and Miller, M.A. "Gas well decline analysis incorporating real gas behavior and non-Darcy flow", *Proc. SPE Ann. Tech. Conf.* (1986).
5. Aminian, K., Ameri, S. and Hyman, M. "Production decline type curves for gas wells producing under pseudo steady state conditions", *Proc. SPE Eastern Reg. Conf.* (1986).
6. Aminian, K., Ameri, S. and Rieke, H.H. "Production history analysis in predicting dry gas-well performance", *J. Pet. Eng. & Sci*, **2**, pp 13–27 (1989).
7. Aminian, K., Ameri, S., Stark, J.J. and Yost, A.B. "Gas-well production decline in multiwell reservoirs", *J. Pet. Tech.*, **42**(12), pp 1573–1579 (1990).
8. Mohaghegh, S. and Ertekin, T. "Production decline curves for low pressure gas reservoirs undergoing simultaneous water production", *Proc. SPE Eastern Reg. Conf.* (1991).
9. Aminian, K., Duda, J.R. and Ameri, S. "Type curve development and application for predicting the production performance of horizontal wells in low permeability gas reservoirs", *Proc. SPE Ann. Conf.* (1989).
10. Abidi, H.R. "The effect of pressure dependent gas properties on the performance of gas wells", MS thesis, West Virginia University, WV, USA (1992).
11. Mohaghegh, S. "Pressure and production type curves as applied to wet sands and coal seams", PhD dissertation, The Pennsylvania State University, PA, USA (1992).
12. Duda, J.R. "Development of performance solutions for horizontal wells in natural gas reservoirs", MS thesis, West Virginia University, WV, USA (1991).
13. Lee, R.L., Logan, R.W. and Tek, M.R. "Effect of turbulence on transient flow of real gas through porous media", *SPE FE*, **2**(1), pp 108–120 (1987).

APPENDIX A

The various dimensionless terms in Equation 1 are defined below:

$$q_D = \frac{q}{q_i}, \tag{A-1}$$

$$t_D = \frac{q_i\, t}{G_i}, \tag{A-2}$$

$$X_i = \frac{(\frac{p}{z})_i}{(\frac{p}{z})_{wf}}, \tag{A-3}$$

$$L = \frac{[P_p(p) - P_p(p_{wf})](\mu_{gi}\, c_{gi})}{2[\frac{(\frac{p}{z})_i}{(\frac{p}{z})_{wf}}]}, \tag{A-4}$$

$$F_{taD} = \frac{\int_0^t \frac{dt}{\mu_g\, c_g}}{\frac{t}{\mu_{gi}\, c_{gi}}}, \tag{A-5}$$

$$F_{NDi} = 1 + \frac{b}{a} qi\ , \tag{A-6}$$

where:

$$a = \frac{1422[1n(\frac{r_e}{r_w}) - 0.75 + s_{CA} + s]}{kh}, \tag{A-7}$$

and the non-Darcy coefficient, b, has been accurately described [13] as follows:

$$b = (\frac{1422\, T}{kh})\, C_1\, F_B\, . \tag{A-8}$$

For laminar flow ($0 < N_{FO} < 0.1$):

$$C_1 = 1 \quad , \quad F_B = 0\, ,$$

for transitional flow ($0.1 < N_{FO} < 1.0$):

$$C_1 = [1 - (\frac{r_e}{r_w})](\frac{\mu_{gi}}{\mu_g})\, ,$$

and for turbulent flow ($1.0 < N_{FO}$):

$$C_1 = [1 - (\frac{r_e}{r_w})](\frac{\mu_{gi}}{\mu_g})(N_{FO})^{-0.028}\, ,$$

where:

$$N_{FO} = \frac{2.224 \times 10^{-12}\, \beta\, \mu_g\, k\, q}{h\, \mu_{gi}\, r_w}, \tag{A-9}$$

$$F_B = \frac{2.224 \times 10^{-15}\, \beta\, \mu_g}{h\, \mu_{gi}\, r_w}, \tag{A-10}$$

$$P_p(p) = \int_0^p \frac{2pdp}{\mu_g z}. \tag{A-11}$$

APPENDIX B

Starting with gas and water equations which represent the two-phase flow in wet-gas sands, one can write:

$$\nabla . \left(\frac{\lambda_g}{B_g}\nabla p\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}\right), \tag{B-1}$$

$$\nabla . \left(\frac{\lambda_w}{B_w}p\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right), \tag{B-2}$$

where:

$$\lambda_g = \frac{k\, k_{rg}}{\mu_g}, \tag{B-3}$$

and

$$\lambda_w = \frac{k\, k_{rw}}{\mu_w}. \tag{B-4}$$

After some mathematical manipulation, which includes linearization, the above equations may be expressed as a single expression:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial p^2}{\partial r}) = \frac{\phi c_t}{\lambda_t}\frac{\partial p^2}{\partial t}. \tag{B-5}$$

Equation B-5 is similar to Equation 18 of [8]. The difference is that in Equation B-5, the pressure squared approach has been used instead of the pressure approach. Transforming the above equation into dimensionless form yields:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial \Delta p_D}{\partial r_D}\right) = \frac{\partial \Delta p_D}{\partial t_D}, \tag{B-6}$$

where:

$$\Delta p_D = \frac{p_i^2 - p^2}{(p_i^2 - p_{wf}^2)q_D}, \tag{B-7}$$

$$r_D = \frac{r}{r_w}, \tag{B-8}$$

$$t_D = \frac{2.637 \times 10^{-4}\lambda_t\, t}{\phi\, c_t\, r_w^2}. \tag{B-9}$$

To construct the type curves for gas and water phases, it is necessary to identify dimensionless gas production and water production rates explicitly:

$$q_{Dg} = \frac{14.24 \times 10^5 q_{gSC}\, z\, T}{\lambda_g\, h(p_i^2 - p_{wf}^2)}, \tag{B-10}$$

$$q_{Dw} = \frac{282.53\, q_{wSC}\, B_w\, p}{\lambda_w\, h(p_i^2 - p_{wf}^2)}. \tag{B-11}$$