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# Multi-Input Multi-Output Direct Adaptive Control for a Distributed Parameter Flexible Rotating Arm

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In this paper, a Multi-Input Multi-Output (MIMO) Model Reference Adaptive Control (MRAC) scheme for a flexible rotating arm is developed. In order to construct a reference model to be followed by this distributed parameter system, a finite element method is used to approximate the behavior of the arm. An input error direct adaptive control algorithm is utilized as the control approach to account for parameter uncertainty. Assuming the same approximation and structure as the model for the actual system, the stability analysis of the proposed controller will be straightforward. Simulation results are provided to illustrate the performance of the proposed algorithm in the presence of disturbance and uncertainties. Also, the proposed algorithm results are compared with those of a conventional PD controller.

## INTRODUCTION

In many robotics applications where high motion speed, low energy consumption, and a wide operation range are required, long lightweight robot manipulators are commonly used. The combined effect of extended length and lightweight increases the structural flexibility of the manipulator; therefore, for accurate position tracking of a robot manipulator, the elasticity effect in the flexible structure cannot be neglected.

There is rich literature on the control of flexible robots and a number of different control approaches have been suggested, as for example, inverse dynamics, feedback linearization and inversion based techniques [1-6], sliding mode, $H_\infty$ and robust controllers [7-13], predictive control [14,15], hybrid force/position control [16,17], singular perturbation and two-time scale control [18-20], neural network and fuzzy controllers [21-26], active vibration control [26,27], optimal control [28,29], passivity-based control [30,31], command or input shaping methods [32] and many more contributions, which are not quoted here to save space.

When the parameters of the system are unknown, adaptive controllers can be used. One attractive feature of the adaptive controllers is that the control implementation does not require a priori knowledge of unknown system parameters. In a robotic system, some of the parameters, such as payload mass, link flexibility or friction coefficients, are difficult to compute or measure; therefore, adaptive controllers represent an important step in high-speed/precision robotic applications.

Most of the adaptive control schemes proposed for flexible robots are model-independent methods, in which fuzzy modeling, neural network approximation and energy-based approaches are used, in combination with adaptation mechanisms, to adjust neural network weights [33], to tune fuzzy controller gains [34], to generate reference models [35], to tune the internal model for unknown disturbances [36], or to suppress the vibrations [37]. A few direct adaptive control schemes have been proposed in which the modal frequencies [32,38,39] or controller gains [40] are updated on-line rather than adapting the model parameters.

To study alternative modeling and/or control schemes, theoretically or experimentally, for robot

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manipulators with flexible behavior, usually, first, a simple one-link flexible robot arm is considered and then the results are extended to more complex (multi-link) robots. Also, mostly, this one-link robot arm is considered with a rotational joint, a payload and a hub (e.g., see [1,3,8,23]).

The flexible link robot is governed by a set of partial differential equations and the dynamics of this robot can be modeled by assuming the flexible object to be a distributed parameter system or by assuming that it consists of lumped masses and springs (infinite or finite dimensional modeling). Usually, finite element approximation and mode summation procedures are used for finding the finite number of the mode shapes and natural frequencies of the robot.

The nontrivial extension of Single-Input Single-Output (SISO) adaptive control algorithms leads to MIMO adaptive control schemes, which may be achieved by using Matrix-Fraction Descriptions (MFDs). The use of MFDs allows one to develop a state space realization of the system [41]. The multi-port control of a distributed parameter system allows a more appropriate scheme, which is suitable for their distributed nature. Little work has been done in MIMO control of distributed parameter systems, although an indirect MIMO scheme with Auto Regressive (AR) model representation of the distributed parameter systems was developed to dampen the vibration of a cantilever beam and a Recursive Least Square (RLS) model was used for the estimation algorithm [42].

In this paper, an input error direct model reference adaptive control scheme with unknown model parameters is developed for a flexible rotating arm. The flexible arm is assumed to be fixed in a rigid hub at one end and carrying a payload in its tip on the other end. The reference model structure is obtained by FEM approximation and by dividing the actual system into N elements, where each element has two nodes and each node has two degrees of freedom, i.e., its transverse displacement and slope. The control scheme is developed from the idea of the MIMO adaptive control scheme, described in [43,44]. The controller applies input signals (forces and torques) on nodes. Assuming the same structure for the actual system and the reference model, the stability analysis of the proposed controller will be the same as that for linear MIMO systems. Simulation results are provided to demonstrate the performance of the proposed algorithm in the presence of disturbance and uncertainties.

## FLEXIBLE ROTATING ARM (PLANT) MODELING

Consider a flexible beam, one end of which is clamped to a control motor shaft and rotated by its rotor. It is assumed that the beam satisfies the Euler-Bernoulli hy-

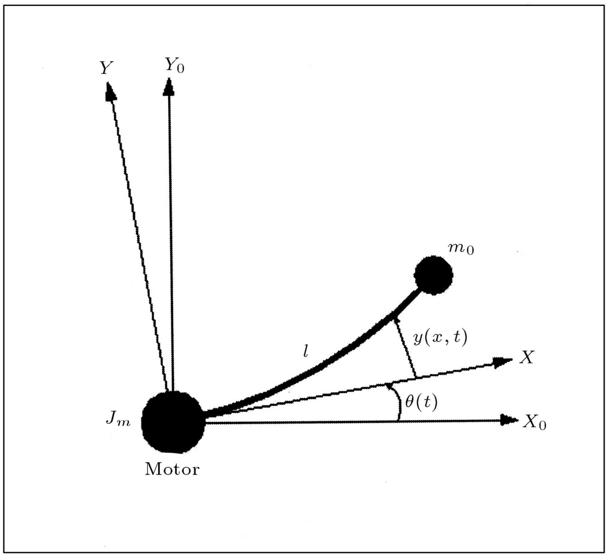


**Figure 1.** Flexible rotating arm with payload, $m_o$, hub's moment, $J_M$ of inertia, $J_m$ and length $l$.

pothesis [45] and the shear deformations are negligible. Also, the tip body of the flexible beam is considered as a concentrated mass, as shown in Figure 1. So, the dynamic equations of motion for the system, assuming no damping effects, can be modeled as follows [46]:

$$\frac{\partial^2 y(x,t)}{\partial t^2}+\frac{EI}{\rho}\frac{\partial^4 y(x,t)}{\partial x^4}=-x\ddot{\theta}(t),$$

$$0<x<l,\ t>0,$$

$$y(0,t)=\frac{\partial y}{\partial x}(0,t)=0,$$

$$EI\frac{\partial^2 y}{\partial^2 x}(l,t)=0,$$

$$m\ddot{y}(l,t)-EI\frac{\partial^3 y}{\partial x^3}(l,t)=-ml\ddot{\theta}(t),$$

$$y(x,0)=y_0(x),$$

$$\dot{y}(x,0)=y_1(x), \tag{1}$$

where $l$, EI, $\rho$ and $m$ are the length, uniform flexural rigidity, uniform mass density and the mass attached to the tip, respectively. Also, $y=y(x,t)$ is the transverse displacement of the beam, with respect to the floating frame, $XY$ and $\theta(t)$, shown in Figure 1, is the floating frame's rotation angle, with respect to the inertial frame, $X_0Y_0$ [46].

Finally, the equation of motion of the control motor can be written as:as:

$$J_m\ddot{\theta}(t)=\tau(t)+EI\frac{\partial^2 y}{\partial x^2}(0,t), \tag{2}$$

where $J_m$ and $\tau(t)$ are, respectively, the rotational mass moment of inertia and the input torque of the motor.

For simulation purposes, one may use finite difference or finite element or any other approximation methods to obtain the time description of the above equations. The finite element method has been used for this, since using FEM approximation is practically acceptable and efficient. One of the most important problems with approximation of the partial differential equations of motion of a flexible link robot is ignoring higher frequencies, which can be avoided by dividing the link into more elements.

## CONTROL SCHEME

### Model Reference Adaptive Control

A model reference adaptive control can be regarded as an adaptive servo system, in which the desired performance is expressed in terms of a reference model. It is assumed that the plant parameters are unknown and the controller parameters are updated recursively using an identifier. In a direct adaptive control scheme, which will be used, an identification scheme is designed that directly identifies the controller parameters. Internal signals in the system, called auxiliary signals, construct the system output. Output error and input error approaches can be used for error estimation. In MIMO systems, the same approach is used, although the scheme is much more complicated [43,44].

### Reference Model Dynamics

A reference model is used to specify the ideal behavior of the adaptive control system to the external commands. To construct the appropriate reference model, a finite element approximation of the governing equations of the system is utilized.

#### *Finite Element Method (FEM) Approximation*

For FEM approximation of the above system, it is assumed that the beam is divided into N elements, each of which has two nodes at its ends. The weak formulation for this equation is obtained using variational methods and, then, by inserting boundary conditions, the domain may be discretized. Since two degrees of freedom are assumed in each node, due to transverse displacement and slope, the Hermite family of interpolation functions may be used for approximation as follows:

$$y^e(x) = \sum u_j^e . \Phi_j^e(x), \tag{3}$$

$$\begin{aligned}
\Phi_1^e(x) &= 1 - 3\left(\frac{x}{h_e}\right)^2 + 2\left(\frac{x}{h_e}\right)^3, \\
\Phi_2^e(x) &= -x.\left(1 - \frac{x}{h_e}\right)^2, \\
\Phi_3^e(x) &= 3\left(\frac{x}{h_e}\right)^2 - 2\left(\frac{x}{h_e}\right)^3, \\
\Phi_4^e(x) &= -x.\left(\left(\frac{x}{h_e}\right)^2 - \frac{x}{h_e}\right),
\end{aligned} \tag{4}$$

where $h_e$ is the length of each element [47,48].

Therefore, the equation of motion of each element can be obtained as follows, by substituting Equation 3 in Equation 1, multiplying by a test function and, then, integrating in the domain [47]:

$$[M]_e\{\ddot{y}\}_e + [K]_e\{y\}_e = \{F\}_e, \tag{5}$$

where $[M]_e$ and $[K]_e$ are the mass and stiffness matrices of each element, respectively as follows:

$$M_e = c\frac{h_e}{420}\begin{bmatrix} 156 & -22h & 54 & 13h \\ -22h & 4h^2 & -13h & -3h^2 \\ 54 & -13h & 156 & 22h \\ 13h & -3h^2 & 22h & 4h^2 \end{bmatrix}, \tag{6}$$

$$K_e = \frac{(EI)_e}{h_e^3}\begin{bmatrix} 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix}, \tag{7}$$

where $c$ and $(EI)_e$ are the longitudinal density and stiffness of each element.

The equations of motion of each element may be assembled into one matrix equation, of the following form:

$$[M_m]\{\ddot{\eta}_m\} + [K_m]\{\eta_m\} = \{F_m\}, \tag{8}$$

where:

$$\eta_m = \{y_{1_m} \;\; \theta_{1_m} \;\; y_{2_m} \;\; \theta_{2_m} \cdots \; y_{N+1_m} \;\; \theta_{N+1_m}\}^T_{2N+2}, \tag{9}$$

in which, $y_{i_m}$ and $\theta_{i_m}$ are the displacement and slope for each node of the reference model, respectively.

The Newmark method has been used for time solution approximation [47,48]. The input to the reference model is $r(t)$ as shown in Figure 2.

### Model Reference MIMO Adaptive Control for a Flexible Arm

The objective of the present work is to train a control scheme for the trajectory tracking of a flexible rotating

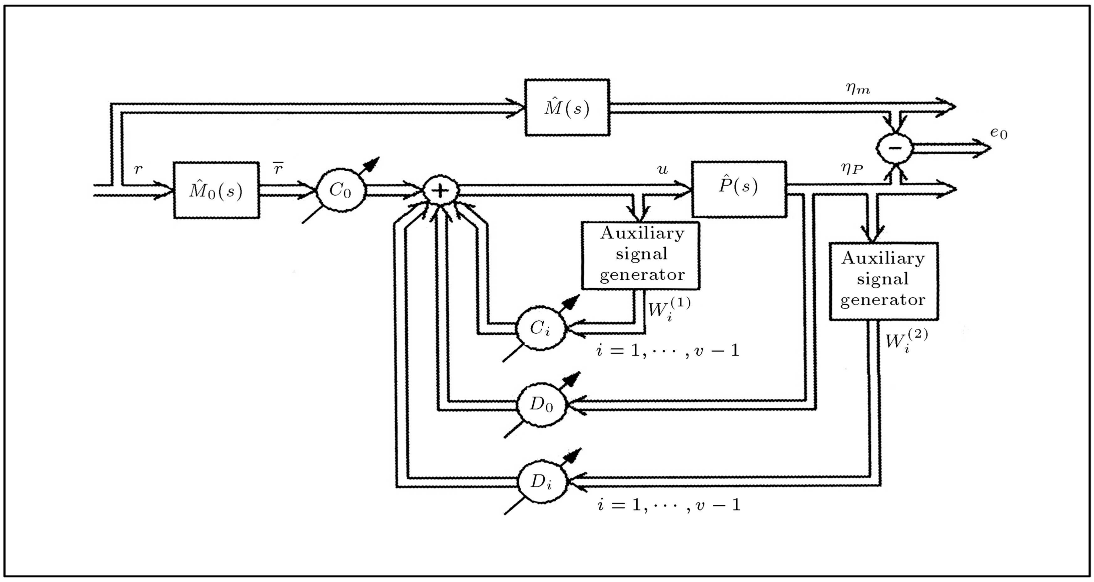


**Figure 2.** MIMO controller structure-adaptive form.

arm. With respect to the dynamic equations of the motion of the arm and by using FEM approximation, the following vector shows the degrees of freedom of the system:

$$\eta_p = \{y_{1_p}\ \theta_{1p}\ y_{2_p}\ \theta_{2_p} \cdots\ y_{N+1_p}\ \theta_{N+1_p}\}^T_{2N+2}, \quad (10)$$

where $y_{i_p}$ and $\theta_{i_p}$, respectively, are the displacement and slope for each node of the actual system on which the measurement instruments should be mounted (compare $\eta_p$ with the definition of Equation 9).

To construct the model reference MIMO adaptive control, the reference model transfer matrix and the Hermite normal form should be obtained [41].

The reference model transfer matrix may be found using a modal matrix, $\{\phi_m\}$ [43], (which is the eigenvectors square matrix of the arm motion) for Equation 8:

$$\phi_m^T[M_m]\phi_m\{\ddot{x}_m\} + \phi_m^T[K_m]\phi_m\{x_m\} = \phi_m^T\{F_m\},$$

$$\eta_m = \phi_m.x. \quad (11)$$

The products $\phi_m^T M_m \phi_m$ and $\phi_m^T K_m \phi_m$ are diagonal matrices due to orthogonality and are called generalized mass and stiffness matrices, respectively, so the model transfer matrix in this new coordinate system can be presented, using generalized mass and stiffness matrices [45].

The Hermite normal form, $\hat{H}$, for this model is selected to have the following form [43]:

$$\hat{H} = \begin{bmatrix} \frac{1}{(s+a)^2} & 0 & \cdots \\ 0 & \ddots & \vdots \\ \vdots & \cdots & \frac{1}{(s+a)^2} \end{bmatrix}. \quad (12)$$

Therefore, the reference model can be described by:

$$\hat{M} = \hat{H}\hat{M}_0 \in R^{(2N+2)\times(2N+2)}_{(2N+2),0}(s), \quad (13)$$

where $\hat{M}_0$ is a proper stable transfer matrix shown in Figure 2.

Then, the model state space representation will be:

$$\hat{M} = \frac{\phi_m^T\{F_m\}(s)}{\{x_m\}(s)}$$

$$= \begin{bmatrix} \frac{1}{m_{gm11}s^2+K_{gm11}} & 0 & \cdots \\ 0 & \ddots & \vdots \\ \vdots & \cdots & \frac{1}{m_{gm2N+2}s^2+K_{gm2N+2}} \end{bmatrix}, \quad (14)$$

and:

$$\hat{M} = \phi_m^{-T}\hat{M}'\phi_m^{-1}, \quad (15)$$

where $m_{gmii}$ and $K_{gmii}$ are elements of generalized diagonal mass and stiffness matrices, respectively [43]. So, $\overline{r} = \hat{M}_0(r)$ will be the input to the plant (Figure 2).

### Identifier Structure

Figure 2 shows the block diagram for the input error model reference MIMO adaptive control algorithm. $W_i^{(1)}$ and $W_i^{(2)}$ are the auxiliary signals generated by filtering the plant input and $\eta_p$ (defined in Equation 10), respectively. $C_0$, $C_i$, $D_0$ and $D_i$ are $(2N+2)\times(2N+2)$ matrices, which include controller parameters and are defined in order to make the plant behave similar to the model; so the matrix of the controller parameters can be described as follows:

$$\Theta = \begin{Bmatrix} [C_0]_{(2N+2)\times(2N+2)} \\ [C_1]_{(2N+2)\times(2N+2)} \\ \vdots \\ [C_{v-1}]_{(2N+2)\times(2N+2)} \\ [D_0]_{(2N+2)\times(2N+2)} \\ [D_1]_{(2N+2)\times(2N+2)} \\ \vdots \\ [D_{v-1}]_{(2N+2)\times(2N+2)} \end{Bmatrix}_{(4v(N+1))\times(2N+2)} \tag{16}$$

By defining $\hat{\lambda}(s)$ as a monic, Hurwitz polynomial of degree $v-1$ (where $v$ is the observability index, which is the maximum of the row degree indices in the left fraction of $\hat{P}$ [41] set equal to 2 for mechanical systems governed by Newton's second law) and defining $\hat{\Lambda}(s) \in R_{2N+2}^{2N+2}(s)$, such that:

$$\hat{\Lambda}(s) = \text{diag}(\hat{\lambda}(s)), \tag{17}$$

the auxiliary filtered signals will be:

$$W_i^{(1)} = \frac{s^{i-1}}{\hat{\lambda}}(u),$$

$$W_i^{(2)} = \frac{s^{i-1}}{\hat{\lambda}}(\eta_p), \qquad i = 1 \cdots v-1. \tag{18}$$

So, the regressor vector is defined as follows, and consists of internal signals:

$$W = \left\{ \overline{r} \quad W_1^{(1)} \quad \cdots \quad W_{v-1}^{(1)} \quad \eta_p \quad W_1^{(2)} \quad \cdots \quad W_{v-1}^{(2)} \right\}^T$$
$$= \begin{Bmatrix} \overline{r}_{(2N+2)\times 1} \\ \overline{W}_{2(2v-1)(N+1)\times 1} \end{Bmatrix}_{(4v(N+1))\times 1}. \tag{19}$$

The input vector to the controller would be [43]:

$$u = \Theta^T W. \tag{20}$$

Now, $\hat{L}(s)$ can be defined, as follows:

$$\hat{L}(s) = \text{diag}[\hat{l}(s)] \in R_{2N+2}^{2N+2}(s), \tag{21}$$

where $\hat{l}(s)$ is a monic, Hurwitz polynomial, such that its rank is equal to the rank of $H^{-1}$ [43]. This transfer function makes the control procedure capable of handling plants with relative degrees higher than one. So, by defining:

$$V = \begin{Bmatrix} (\hat{H}\hat{L})^{-1}\eta_p \\ \hat{L}^{-1}(\overline{W}) \end{Bmatrix}, \qquad \overline{V} = \hat{L}^{-1}(\overline{W}), \tag{22}$$

the input error will be:

$$e_2 = \Theta^T V - \hat{L}^{-1}(u). \tag{23}$$

Letting the controller parameter, $\Theta$, be updated with time, the scheme will be made adaptive. Extending the update laws from a SISO to a MIMO case, the normalized gradient algorithm may be used [43] as follows:

$$\dot{\overline{\Theta}} = -K_1 \frac{\hat{V}.e^T}{1 + K_2 \overline{V}^T \overline{V}}, \qquad K_1, K_2 > 0. \tag{24}$$

### Stability

To perform a stability analysis for the described system, first it is assumed that the actual system is approximated with the same FEM procedure, which is practically acceptable and efficient. In this case, the reference model and the system have the same structure but different parameters; then, the stability proof is straightforward and is similar to the SISO cases (see Lemma 3.6.2, Proposition 6.3.3 and Section 6.3 in [43]) in which:

- All states of the adaptive system are bounded functions of time;
- The output error $\in L_2$ and tends to zero as $t \to \infty$;
- -The regressor error $\in L_2$ and tends to zero as $t \to \infty$.

## SIMULATION RESULTS

Simulation results will be presented for a flexible rotating arm with a payload attached to its end tip, to show the performance of applying an input error MRAC in five different cases. The system parameters are given for different cases in Table 1. With respect to these properties, the flexible link has a small bending stiffness, ($EI = 20$). To achieve control purposes, which are defined as following, a rigid link behavior and less deflection in the arm's tip, a flexible model with high bending stiffness ($EI = 200$) and different parameters

**Table 1.** Plant's material properties, length and inertia parameters.

| | Plant Parameters | | | | |
|---|---|---|---|---|---|
| **Case** | **1** | **2** | **3** | **4** | **5** |
| ***EI*** **(N/m$^2$)** | 20 | 20 | 20 | 20 | 20 |
| ***ρ*** **(kg/m$^3$)** | 2000 | 2000 | 2000 | 2000 | 2000 |
| ***l*** **(m)** | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| ***m$_o$*** **(gr)** | 10 | 10 | 10 | 10 | 10 |
| ***J$_m$*** **(kg.m$^2$)** | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

$EI$ = stiffness, $\rho$ = density, $l$ = length, $m_o$ = payload mass and $J_m$ = hub's moment of inertia.

**Table 2.** Reference model's material properties, length and inertia parameters.

| | Reference Model Parameters | | | | |
|---|---|---|---|---|---|
| **Case** | **1** | **2** | **3** | **4** | **5** |
| $\boldsymbol{EI}$ **(N/m$^2$)** | 200 | 200 | 200 | 200 | 200 |
| $\boldsymbol{\rho}$ **(kg/m$^3$)** | 3000 | 3000 | 3000 | 3000 | 3000 |
| $\boldsymbol{l}$ **(m)** | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| $\boldsymbol{m_o}$ **(gr)** | 10 | 30 | 10 | 10 | 10 |
| $\boldsymbol{J_m}$ **(kg.m$^2$)** | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

$EI$ = stiffness, $\rho$ = density, $l$ = length, $m_o$ = payload mass and $J_m$ = hub's moment of inertia.

are chosen (see Table 2). So, defining this model should lead to less deformation in the tip. The plant and reference model are simulated by an FEM model, in which the rotating link is divided into $N(10)$ elements with the same length and stiffness. The Newmark family of time integration scheme (Gallerkin method) is used for structural dynamics approximation [47,48]. Then, the controller and identifier structure, which are represented in state space form, together with FEM subfunctions, will be solved in the time domain.

The manner used in Figure 3 to show different node displacements and slopes is not repeated and assumed known for the other figures.

The system's response has been simulated for five different cases. In all, the number of elements is assumed to be 10 and a constant input, $r_i(t) = 0.02$, $i = 1, \cdots, 2N+2$, is applied to all degrees of freedom, except Case 2. The responses of the system to different disturbances and uncertainties are presented in the first four cases and, in the last one, a comparison between the controller and a simple PD controller is discussed.

1. Figure 4 illustrates a numerical comparison between the reference model and the plant deflections in local coordinates for constant inputs. These results show that the plant has a significant tracking performance of the reference model after 30 seconds (less than $\pi$ rad. rotation). It should be noted that the model has a high flexural stiffness, which will lead to less deflections in all its nodes, including its tip. Convergence performance of input and output errors is shown in Figures 5 and 6, respectively;
2. In this case, an uncertainty of 20 gr. in the payload mass is considered. The results are shown in Figure 7. It can be seen that the control system is able to handle this parameter uncertainty (twice as much as the plant payload mass) and the plant still follows the reference model output;

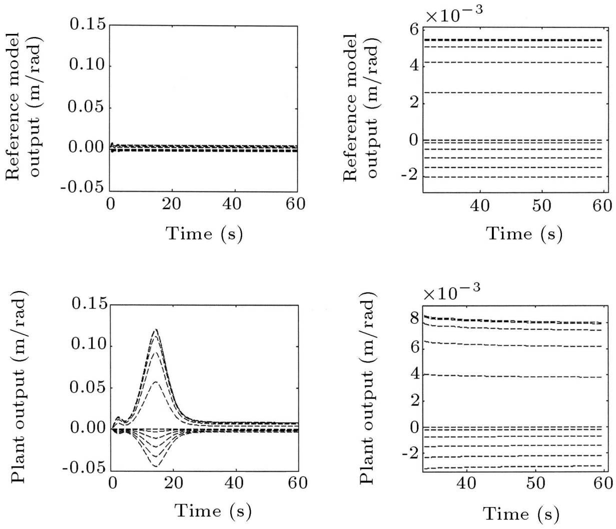


**Figure 4.** Reference model and plant's outputs ($\eta_m$ and $\eta_p$ (meter)) are plotted, respectively in 1st and 2nd rows. The left hand figures show the reference model and plant convergence significantly after about 30 seconds (Case 1).

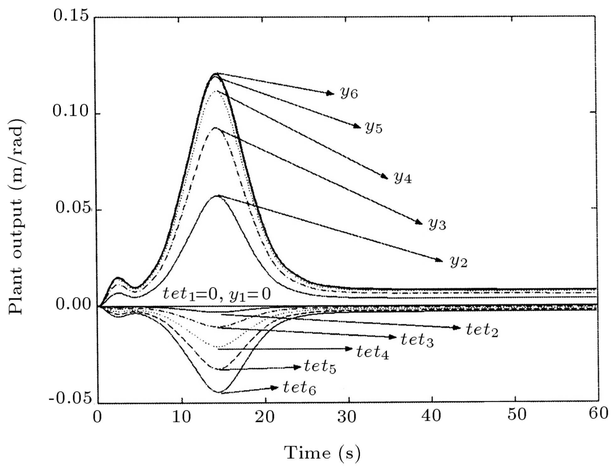


**Figure 3.** The outpu $\eta_p = [y, tet]^T$ where $y_i$ and $tet_i$ stand for transverse displacement and slope in node $i$, respectively.

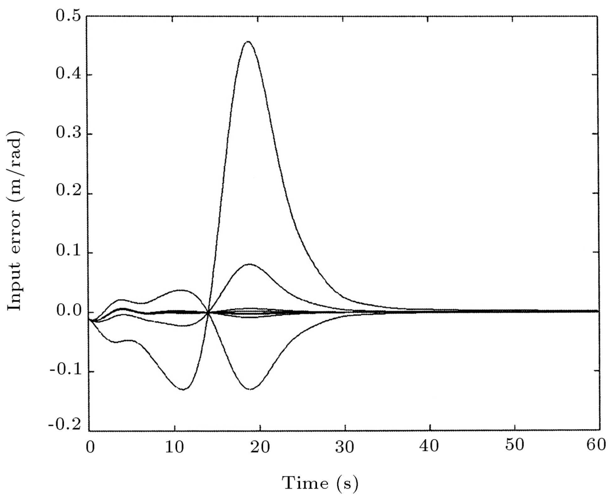


**Figure 5.** Input error $e_2 = \Theta^T V - \hat{L}^{-1}(u)$ vs. time (second) in Case 1.

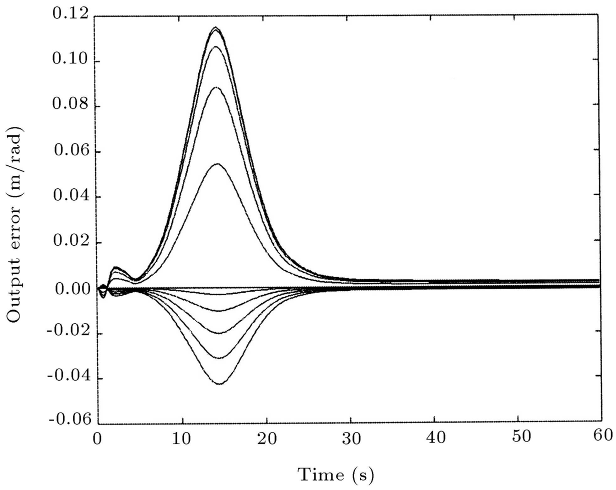


**Figure 6.** Output error convergence, $\eta_m - \eta_p$ (meter), vs. time (second) in Case 1.

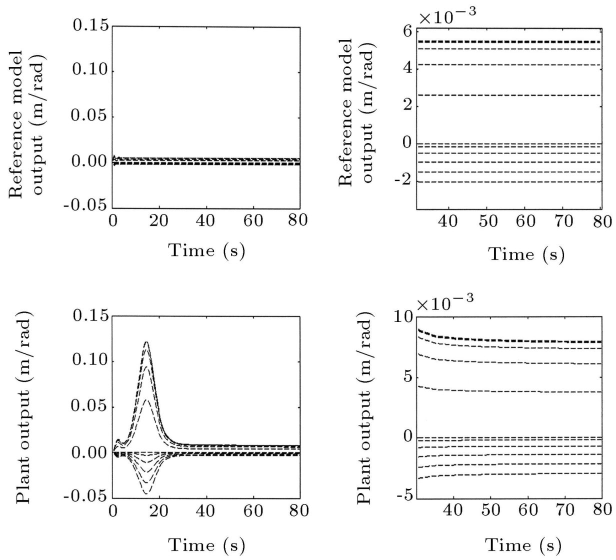


**Figure 7.** The uncertainties effect on parameter estimation ($m_o = 10$ gr and 30 gr in plant and reference model, respectively). $\eta_m$ and $\eta_p$ (meter) vs. time (second).

3. The effect of a more complex input, $r_i(t) = 0.02 + 0.02 \sin\left(\frac{10\pi.t}{T}\right), i = 1, \cdots, 2N+2$, to the reference model is shown in Figures 8 and 9, where $T$ is the period of the motion. The results again show a considerable similarity between the plant and reference model outputs;
4. In this case, a sinusoidal disturbance is considered in motor torque with an amplitude of about 10 percent of the actual input torque. Figures 10 and 11 show the effect of this disturbance in the system. The controller structure demonstrates a great robustness;
5. In the final case, the results of the presented control scheme have been compared with a simple PD controller [49,50]. The results (Figure 12) show that the proposed MIMO adaptive controller has a much better performance. Also, the PD controller is more sensitive to disturbance and controller gains and cannot overcome the plant uncertainties effects.

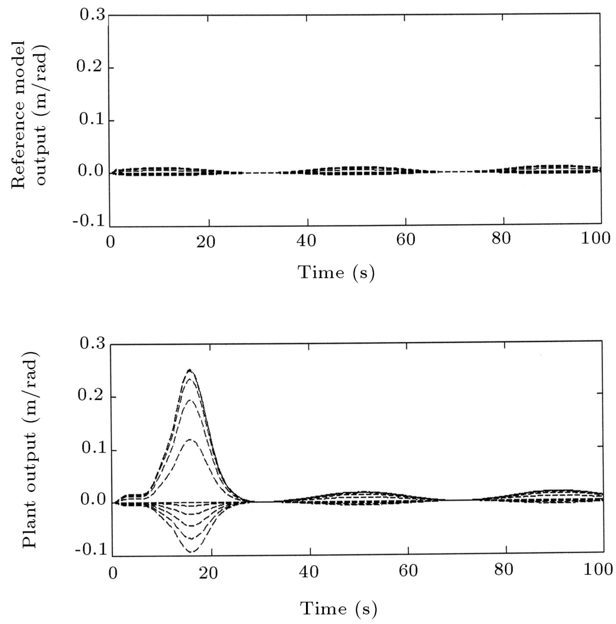


**Figure 8.** Effect of complex input presented in Case 3. 1st and 2nd row's figures show reference model and plant outputs, respectively. $\eta_m$ and $\eta_p$ (meter) vs. time (second).

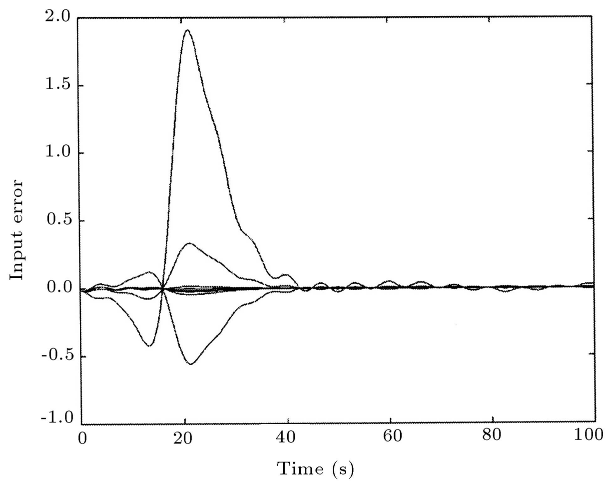


**Figure 9.** Input error $e_2 = \Theta^T V - \hat{L}^{-1}(u)$ vs. time (second) for Case 3.

The consequence of dividing a flexible link into more elements and having uncertainty in the motor hub inertia are also simulated, but not presented in this paper. The results show that using more elements will lead to better convergence, although it needs smaller time steps and more computational loads. So, the

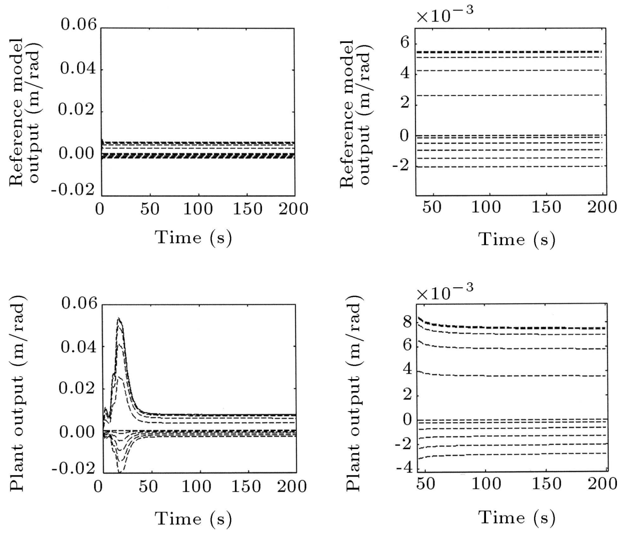


**Figure 10.** The disturbance effect on plant's inputs, where 1st and 2nd row's figures show reference model and plant outputs, respectively. $\eta_m$ and $\eta_p$ (meter) vs. time (second) for Case 4.

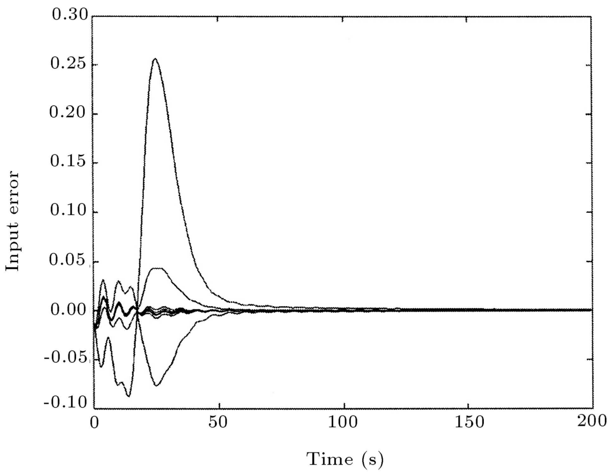


**Figure 11.** Input error $e_2 = \Theta^T V - \hat{L}^{-1}(u)$ vs. time (second) for Case 4.

convergence will be more dependent on controller gains, $\hat{\lambda}(s), \hat{L}(s)$ and the time step.

## CONCLUSION

A multi-input multi-output input error direct adaptive control scheme is presented to make a distributed parameter plant's output track the desired trajectories (reference model's output). The reference model is made by an FEM approximation of the distributed parameter system. It is desired that the plant behaves as a link with high stiffness (almost rigid). It appears that the proposed controller can handle uncertainties in estimated parameters $EI$, $\rho$ and $m_o$ in the reference model. The effects of a more complex input to the system and a disturbance in the motor torque are

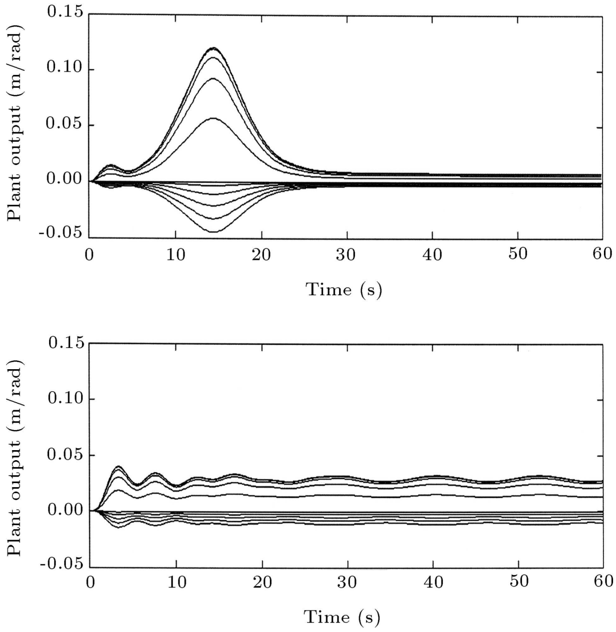


**Figure 12.** The comparison between MIMO adaptive (up) and PD (bottom) controllers. $\eta_p$ (meter) vs. time (second).

considered, too. The controller shows a significant robustness in these cases. Dividing a flexible link into more elements leads to more appropriate results, but it will be more sensitive to gains and time steps. The comparison between the MIMO adaptive and popular PD controller is also discussed and it can be stated that the proposed controller performs very well, especially in cases of parameter uncertainty and disturbance.

## NOMENCLATURE

| | |
|---|---|
| $l$ | length of the beam |
| $EI$ | uniform flexural rigidity of the beam |
| $\rho$ | uniform mass density of the beam |
| $m$ | the mass attached to the tip of the beam |
| $y = y(x,t)$ | beam transverse displacement with respect to the floating frame, $XY$ |
| $\theta(t)$ | floating frame's rotation angle with respect to the inertial frame, $X_0Y_0$ |
| $J_m$ | rotational mass moment of inertia of the motor |
| $\tau(t)$ | input torque of the motor |
| $h_e$ | length of each element |
| $\{\phi_m\}$ | the modal matrix |
| $[M]_e$ | mass matrix of each element |
| $[K]_e$ | stiffness matrix of each element |
| $c$ | longitudinal density of each element |
| $(EI)_e$ | stiffness of each element |

| | |
|---|---|
| $y_i$ | displacement of the $i$th node |
| $\theta_i$ | slope of the $i$th node |
| $r(t)$ | input to reference model |
| $m_{gmii}$ | an element of generalized diagonal mass matrix |
| $K_{gmii}$ | n element of generalized diagonal stiffness matrix |
| $W_i^{(1)}, W_i^{(2)}$ | auxiliary signals generated by filtering plant input and $\eta_p$ |
| $\eta_p$ | vector of measured degrees of freedom of the system |
| $\hat{l}(s)$ | a monic, Hurwitz polynomial such that its rank is equal to the rank of $H^{-1}$ |
| $e_2$ | input error |
| $C_0, C_i, D_0, D_i$ | the $(2N+2) \times (2N+2)$ matrices, which include controller parameters |
| $\Theta$ | vector of controller parameters |
| $\hat{P}$ | the plant |
| $\hat{M}$ | reference model transfer matrix |
| $v$ | observability index |
| $\hat{\lambda}(s)$ | a monic, Hurwitz polynomial of degree $v-1$ |

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