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Research Note

Time-Domain Analysis of Sandwich Shells with Passive Constrained Viscoelastic Layers

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Damping effects on THE vibration behavior of a sandwich cylindrical shell with a passive constrained viscoelastic layer are investigated in the time-domain. Equations of motion in terms of transverse modal coordinates in the frequency-domain are obtained by means of an energy method and the Lagrange equation, and they are solved by the assumed-mode method. The viscoelastic behavior is represented by the frequency complex modulus model. The equations of motion are transferred from the frequency-domain to the time-domain by the Inverse Fast Fourier Transform, (IFFT). Thickness effects of constrained and viscoelastic layers are investigated by a transient external load response and evaluation of the damping factor and settling time.

INTRODUCTION

Constrained Layer Damping (CLD) treatments are used to suppress noise and vibration in a complex structural system. These treatments provide effective suppression by the dissipation of energy in a soft, heavily damped, viscoelastic core (VEC) sandwiched between the two face sheets of a composite panel in flexure.

The use of constrained viscoelastic treatment for improving the damping in structures is a very popular method in cases of structures made of conventional materials (such as steel) which possess little material damping. The effect of the outer elastic layer (the constraining layer) is to increase the deformation in the VEC, thus, resulting in higher energy dissipation in the viscoelastic material. There are two primary methods for dissipating energy in the VEC of a constrained layer damping treatment: Shear deformation and compressional deformation. Shear deformation results when the constrained layer and base structure move parallel to each other, acting to shear the viscoelastic core. Compression deformation results when the upper and lower layers move perpendicular to each other, acting to compress (or stretch) the viscoelastic material.

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Kerwin and DiTaranto [1,2] focused on the mathematical modeling of long, simply supported beams with soft VEC and thin stiff constraining layers. Mead and Markus [3] developed a sixth order differential equation of motion, in terms of the transverse displacement of the beam for arbitrary boundary conditions. The analytical work presented in that paper used the fundamental assumption that shearing of the VEC is the only cause of energy dissipation and that compressional damping does not occur in this system. Douglas and Yang [4] modified or extended the model for different applications. Ramesh and Ganesan [5] used the finite element method to solve a cylindrical system with thin axial strips, which bonded to the cylinder with a viscoelastic material. Recently, Chen and Huang [6] developed a generic theory for the CLD treated shell. The time-domain behavior of a hysteretically damped structure was obtained from the frequency-domain response by Lunden and Dahlberg [7] and Karlsson [8], using the Fourier Transform (FT) technique.

To the authors' knowledge, available literature, regarding the time-domain analysis of the constrained layer damping treatment of sandwich cylindrical shells with core viscoelastic material, is limited. The objective of the present study is to obtain the transient external load response of a constrained sandwich shell with viscoelastic core material, in order to investigate the damping effects of viscoelastic and constraint layers.

To obtain equations of motion in the frequency domain, the Lagrange equation is applied and the governing equation between the layers and boundary condition is yielded. Then, the equations of motion are transferred from the frequency domain to the time domain by an Inverse Fast Fourier Transform (IFFT).

ANALYTICAL MODEL FOR SANDWICH CYLINDRICAL SHELLS

Kinematic Relations

The configuration of a 3-layered sandwich shell is illustrated in Figure 1. Layers 1 and 3 are assumed to be thin, homogenous and isotropic; therefore, the transverse shear deformation of the shell and Constrained Layer (CL) can be neglected. The core is of linear viscoelastic material, with a frequency dependent complex shear modulus, $G(\omega)$.

Love's assumptions [9] are applied and corresponding displacements for the three layers are given by the following equation:

$$U_x = u_x(x,\theta) + z\beta_x(x,\theta),$$
$$U_\theta = u_\theta(x,\theta) + z\beta_\theta(x,\theta),$$
$$U_z = u_z(x,\theta). \tag{1}$$

The stress-strain relationship in the cylindrical shell and in the constrained layer is described by:

$$\sigma^i_{xx} = \frac{E_i}{1-\upsilon_i^2}(\varepsilon_{xx} + \upsilon_i\varepsilon_{\theta\theta}),$$
$$\sigma^i_{\theta\theta} = \frac{E_i}{1-\upsilon_i^2}(\upsilon_i\varepsilon_{xx} + \varepsilon_{\theta\theta}), \qquad i = c, s,$$
$$\sigma^i_{x\theta} = \frac{E_i}{1-\upsilon_i^2}\left(\frac{1-\upsilon_i}{2}\varepsilon_{x\theta}\right). \tag{2}$$

For the viscoelastic layer, the stress relation is [5] as follows:

$$\sigma^v_{xz} = G_{v(\omega)}\varepsilon^v_{xz},$$
$$\sigma^v_{\theta z} = G_{v(\omega)}\varepsilon^v_{\theta z}. \tag{3}$$

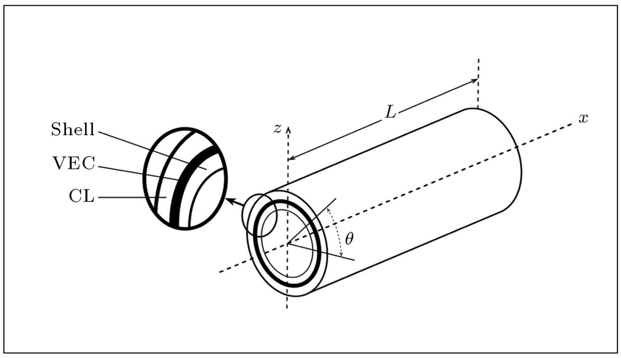

Figure 1. Simply supported cylindrical shell with viscoelastic core (VEC).

The cylinder is approximated by a thin shell, therefore, the displacements in x and θ directions are assumed to vary linearly through the shell thickness. The displacement in the transverse direction is independent of z and the shear strains in the face layers are negligible. Thus, the strain displacement relations are given by:

$$\varepsilon^i_{xx} = \frac{\partial u^i_x}{\partial x} - z\frac{\partial^2 u_z}{\partial x^2},$$
$$\varepsilon^i_{\chi\theta} = \frac{1}{r_i}\frac{\partial u^i_\theta}{\partial\theta} + \frac{u_z}{r_i} - \frac{z}{r_i^2}\frac{\partial^2 u_z}{\partial\theta^2}, \qquad i = s, c,$$
$$\varepsilon^i_{x\theta} = \frac{\partial u^i_\theta}{\partial x} + \frac{1}{r_i}\frac{\partial u^i_x}{\partial\theta} - 2\frac{z}{r_i}\frac{\partial^2 u_z}{\partial x\partial\theta}. \tag{4}$$

While its strain-displacement relations of the VEC are given by:

$$\varepsilon^v_{xz} = \beta^v_x + \frac{\partial u_z}{\partial x},$$
$$\varepsilon^v_{\theta z} = \beta^v_\theta - \frac{u^v_\theta}{r_v} + \frac{1}{r_v}\frac{\partial u_z}{\partial\theta}. \tag{5}$$

The deformation pattern of three layers in the axial direction is shown in Figure 2. Taking into consideration Love's simplification, the assumption of a no-slip condition between layers yields the following:

$$u^s_j(x,\theta) + \frac{h_s}{2}\beta^s_j(x,\theta) = u^v_j(x,\theta) - \frac{h_v}{2}\beta^v_j(x,\theta),$$
$$u^c_j(x,\theta) - \frac{h_c}{2}\beta^s_j(x,\theta) = u^v_j(x,\theta) + \frac{h_v}{2}\beta^v_j(x,\theta). \tag{6}$$

Substituting Equation 6 into Equation 5 yields [6]:

$$\varepsilon^v_{xz} = \frac{1}{h_v}(u^C_x - u^S_x) + \frac{1}{2h_v}(h_c + h_s + 2h_v)\frac{\partial u_z}{\partial x}, \tag{7}$$

$$\varepsilon^v_{\theta z} = \left(\frac{1}{h_v} - \frac{1}{2r_v}\right)u^c_\theta - \left(\frac{1}{h_v} + \frac{1}{2r_v}\right)u^s_\theta + \left(\frac{h_c}{2h_vr_c} + \frac{h_s}{2h_vr_s} - \frac{h_c}{4r_vr_c} + \frac{h_s}{4r_vr_s} + \frac{1}{r_v}\right)\frac{\partial u_z}{\partial\theta}. \tag{8}$$

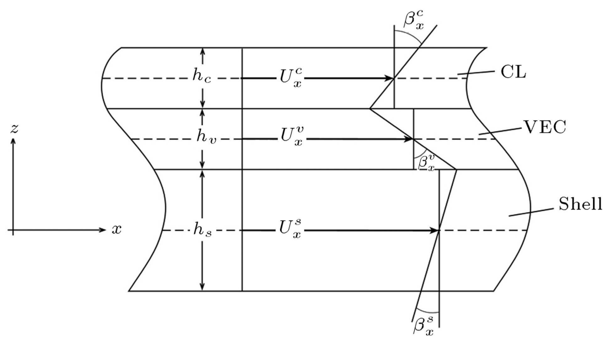

Figure 2. The deformation pattern of the layers in x direction.

Energy Expression

The kinetic energy of the layers with the in-plane inertia neglected (Donell-Mushtari-Valso assumptions) [9] are:

$$T_s = \frac{1}{2}\rho_0 h_s r_s \int_0^{2\pi} \int_0^L \left(\frac{du_z}{dt}\right)^2 dx d\theta, \tag{9}$$

$$T_v = \frac{1}{2}\rho_0 h_v r_v \int_0^{2\pi} \int_0^L \left(\frac{du_z}{dt}\right)^2 dx d\theta, \tag{10}$$

$$T_c = \frac{1}{2}\rho_0 h_c r_c \int_0^{2\pi} \int_0^L \left(\frac{du_z}{dt}\right)^2 dx d\theta. \tag{11}$$

The strain energy of the three layers is:

$$U_S = \frac{r_s}{2} \int_0^{2\pi} \int_0^L \int_{-h/2}^{h/2} [\sigma_{xx}\varepsilon_{xx} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{x\theta}\varepsilon_{x\theta}] dz dx d\theta, \tag{12}$$

$$U_C = \frac{r_c}{2} \int_0^{2\pi} \int_0^L \int_{-h/2}^{h/2} [\sigma_{xx}\varepsilon_{xx} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{x\theta}\varepsilon_{x\theta}] dz dx d\theta, \tag{13}$$

$$U_V = \frac{r_v}{2} \int_0^{2\pi} \int_0^L \int_{-h/2}^{h/2} [G_v \varepsilon_{xz}^2 + G_V \varepsilon_{\theta z}^2] dz dx d\theta. \tag{14}$$

Assuming an external transverse point load of $F(x, \theta, t)$ applied on the cylindrical surface, the work done by this force can be expressed as:

$$Q_Z = \int_0^{2\pi} \int_0^L r_S F(x, \theta, t) w(x, \theta, t) dx d\theta. \tag{15}$$

Equations of Motion

The governing equations of a sandwich shell with a viscoelastic core excited by an external transverse load are derived via the Lagrange equation.

$$\frac{d}{dt}\left(\frac{\partial T_i}{\partial q_i}\right) - \frac{\partial T_i}{\partial q_i} + \frac{\partial U_i}{\partial q_i} = Q_i, \tag{16}$$

where q_i represents the ith generalized coordinate and Q_i is the ith generalized force. T and U are the kinetic and strain energies of the whole system, respectively, which are expressed as:

$$T = T_s + T_v + T_c, \qquad U = U_s + U_v + U_c. \tag{17}$$

For a cylinder, the displacements can be approximated by:

$$u_Z(x, \theta, t) = \sum_{j=1}^{\infty} W_j(x, \theta)\zeta(t) = [W]^T\{\zeta\}, \tag{18}$$

$$u_X^s(x, \theta, t) = \sum_{j=1}^{\infty} U_j^s(x, \theta)\eta_j^s(t) = [U^s]^T\{\eta^S\}, \tag{19}$$

$$u_\theta^s(x, \theta, t) = \sum_{j=1}^{\infty} V_j^s(x, \theta)\xi_j^s(t) = [V^s]^T\{\xi^S\}, \tag{20}$$

$$u_X^c(x, \theta, t) = \sum_{j=1}^{\infty} U_j^c(x, \theta)\eta_j^c(t) = [U^c]^T\{\eta^c\}, \tag{21}$$

$$u_\theta^c(x, \theta, t) = \sum_{j=1}^{\infty} V_j^c(x, \theta)\xi_j^c(t) = [V^c]^T\{\xi^c\}. \tag{22}$$

The shell is assumed to be simply supported at two ends, and a unit harmonic point load, applied at $x^* = 0.1$ and $\theta^* = 0$, is as follows:

$$F_z(t) = \delta(x - x^*)\delta(\theta - \theta^*)e^{i\omega t}. \tag{23}$$

The mode shape functions are selected to be as follows:

$$W_{mn} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \sin\left(\frac{m\pi x}{L}\right)\cos n(\theta - \varphi),$$

$$U_{mn}^i = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \cos\left(\frac{m\pi x}{L}\right)\cos n(\theta - \varphi), \quad (i = s, c),$$

$$V_{mn}^i = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \sin\left(\frac{m\pi x}{L}\right)\sin n(\theta - \varphi). \tag{24}$$

By using the above shape functions and substituting Equations 9 to 14 into the Lagrange Equation 16, the equation of motion of the cylinder is yielded in the following form:

$$[M]\{\ddot{\varsigma}_i\} + [K]\{\varsigma_i\} = \{Q_z(t)\}, \tag{25}$$

where $[M] = [M^s] + [M^v] + [M^c]$ is the mass matrix and $[K] = [K^s] + [K^v] + [K^c]$ is the stiffness matrix of the sandwich cylinder. $\{Q_z(t)\}$ is a column of generalized force and vector $\{\varsigma_i\} = [\zeta, \xi^s, \eta^s, \xi^c, \eta^c]$ is a column vector containing the modal coefficients. Note that the in-plane inertia terms are neglected in the equations of motion. Consequently, the in-plane equations are in static equilibrium, such that the in-plane generalized coordinates are eventually replaced by the following transverse generalized coordinates:

$$[M]\{\ddot{\zeta}\} + [K^*(\omega)]\{\zeta\} = \{Q_z(t)\}. \tag{26}$$

TIME DOMAIN EQUATION

In the equation of motion (Equation 26) for the vicoelastic material, a complex shear modulus model is used. In this equation, $K^*(\omega)$ is a complex stiffness matrix and is defined by:

$$[K^*(\omega)] = [K^s + K^c] + [K^{'V}(\omega) + iK^{''V}(\omega)], \quad (27)$$

where, K^s and K^c are stiffness matrices for the shell and CL layer. $K^{'V}(\omega)$ is determined using the shear modulus, $G'(\omega)$, while $K^{''V}(\omega)$ is found using the imaginary part of the complex modulus of the viscoelastic, $G''(\omega) = \eta_G(\omega)G'(\omega)$, where $\eta_G(\omega)$ is the viscoelastic loss factor in the shear and ω is the frequency. The transient response of the system cannot be obtained effectively by applying direct integration or by modal methods, because, in this case, it is not possible to determine the variation of the material properties, $G'(\omega)$, with respect to time. The time-domain behavior of a structure may be obtained from the frequency-domain response by the FT technique. Incidentally, it is necessary to solve the following system of complex linear equation:

$$\{[K^*(\omega_j)] - \omega_j^2[M]\}\{\zeta(\omega_j)\} = \{Q(\omega_j)\}. \quad (28)$$

Using the Fourier transform, the frequency spectra of excitation can be defined as $F(\omega_j) = \Im[F(t_k)]$, where t_k is a set of discrete time for the excitation, $F(t)$, and the displacement of the structure in the time domain can be obtained from the inverse Fourier transform of $\zeta(t_k) = \Im^{-1}\{\zeta(\omega_j)\}$.

The discrete Fourier transform pair will be in the following form:

$$F(\omega_j) = \Im[F(t_k)] = \frac{\Delta\omega}{2\pi}\sum_{k=0}^{N-1} F(t_k)e^{-i(2\pi jk/N)},$$

$$\zeta(t_k) = \Im^{-1}[\zeta(\omega_j)] = \Delta t\sum_{j=0}^{N-1} \zeta(\omega_j)e^{i(2\pi jk/N)}, \quad (29)$$

where, N is the number of samples. Obviously, the accuracy of the discrete Fourier transform depends on the number of samples, N, and the sampling interval, Δt. The choice of $\Delta\omega$ and N depends on the frequency response shape. The frequency interval, $\Delta\omega$, for the inverse transform must be the reciprocal of the total time record length and computed from $\Delta\omega = 2\pi/N\Delta t$. It is necessary to note that the value of function at a discontinuity must be defined as the mid-value, if the inverse Fourier transform is held. Moreover, using the discrete Fourier transform, it is necessary to remember that it is based on the assumption of periodicity of the load applied. For a periodic function with a known period, it is necessary to choose an $N\Delta t$ interval equal to a period or an integer multiple of a period.

NUMERICAL RESULTS

Numerical examples are given to compare the three treatments. The geometric and material properties for the illustrated examples are as follows:

Shell : $E_S = 70$ GPa, $L = 0.35$ m,

$\rho_S = 2710$ Kg/m^3, $h_S = 0.002$ m,

$r_S = 0.1$ m,

CL : $E_C = 49$ GPa, $\rho_C = 7500$ Kg/m^3;

VEC : $\rho_V = 1104$ Kg/m^3.

The behavior of the viscoelastic material in this analysis is the complex modulus in the frequency domain representation, given by Douglas [4], as:

$$G(\omega) = 0.142(\omega/2\pi)^{0.494}(1 + 1.46i) \text{ MPa}. \quad (30)$$

The Frequency Response Function (FRF) approach has been used in calculating the steady-state response of viscoelastic material in the following form:

$$\text{FRF} = \{Q\}^T[K - M\omega^2]^{-1}\{Q\}. \quad (31)$$

Figure 3 shows the FRF diagram for the CLD ($h_v = 0.5$, $h_c = 0.5$) and bare shell. As expected, for the CLD, the amplitude of FRF is decreased, due to the increase of damping. In the numerical time treatment, N and Δt are chosen to be 10000 and 0.001 sec., respectively, resulting in $\Delta\omega = 0.2\pi s^{-1}$. This time interval is selected in order for the response of the system vanishes at this time. The results of the transient response analysis for one thickness of CL and one thickness of VEC are shown in Figures 4 and 5.

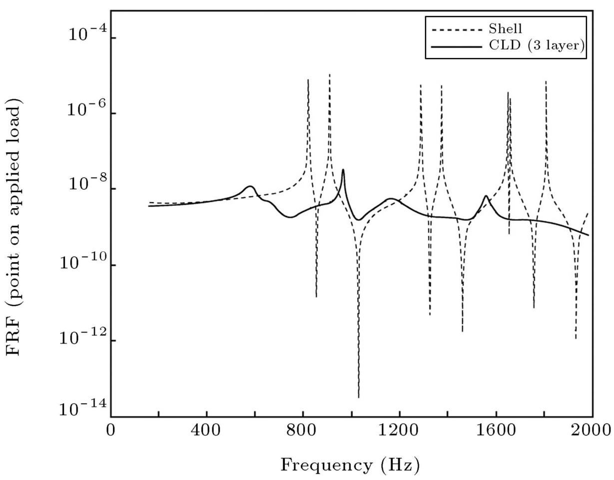

Figure 3. FRF with CLD treatment ($h_v = 0.5$, $h_c = 0.5$) and bare shell.

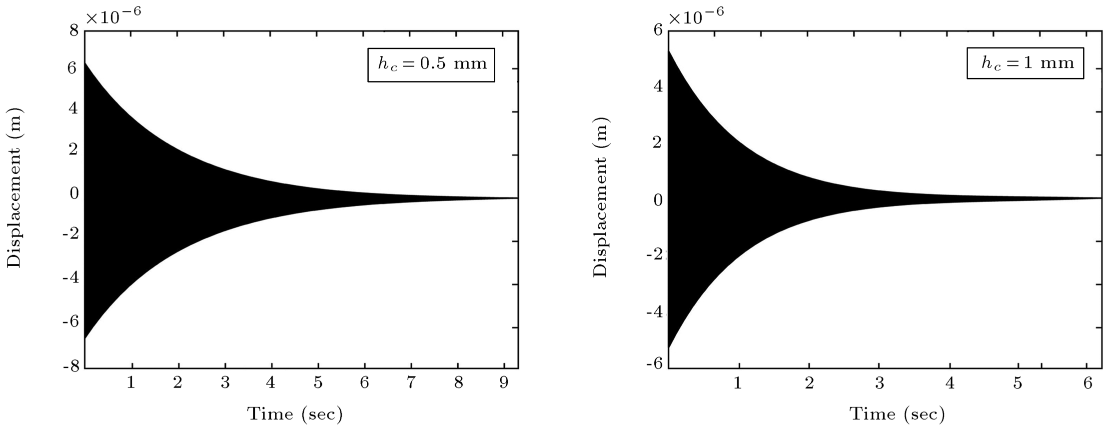

Figure 4. Transient response of sandwich shell with two thicknesses of CL ($hv = 0.2$ mm).

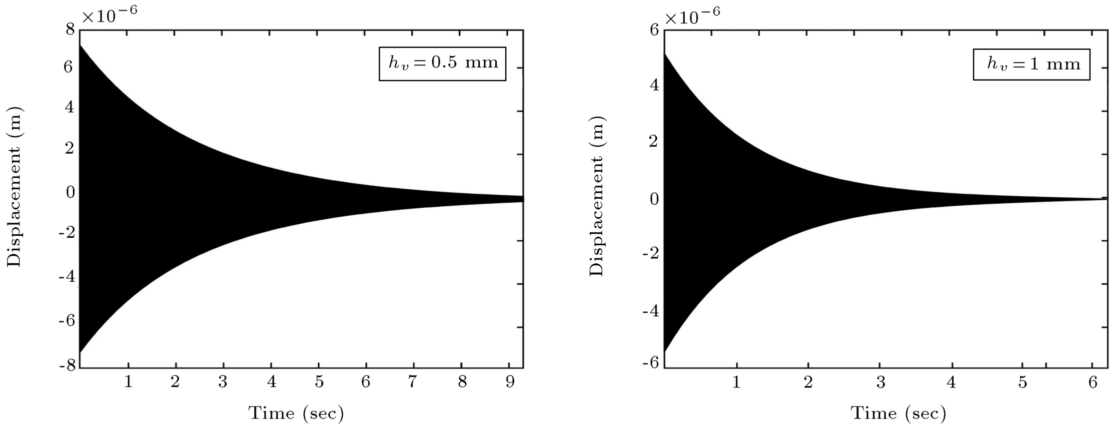

Figure 5. Transient response of sandwich shell with two thicknesses of VEC ($hc = 0.5$ mm).

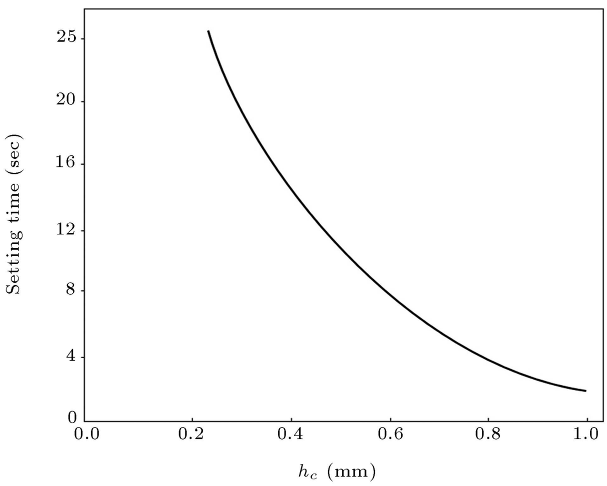

Figure 6. Thickness effect of CL on the settling time ($hv = 0.2$ mm).

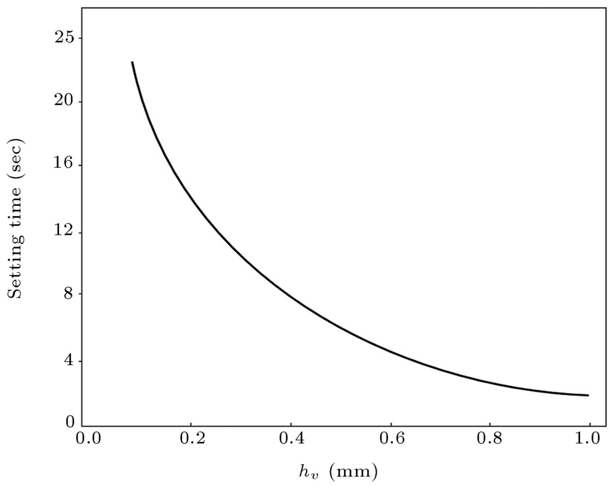

Figure 7. Thickness effect of VEC on the settling time ($hc = 0.5$ mm).

Figures 6 and 7 show settling times with a 5% criterion for various thicknesses of CL and VEC. Both figures confirm that the rate of the amplitude reduction is decreased with an increasing thickness of CL and viscoelastic layer. Figures 8 and 9 show the structural damping factor for different thicknesses of CL and VEC. Since polymer material is used as a damper in this structure, the ψ function may be used for

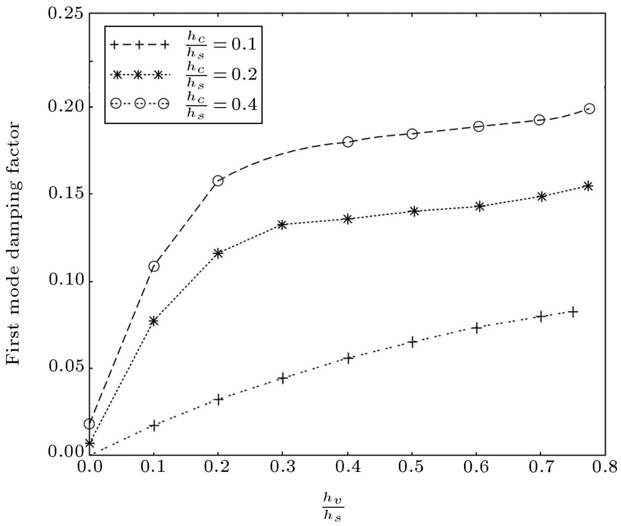

Figure 8. Structural damping factor with various thickness ratios of VEC to shell.

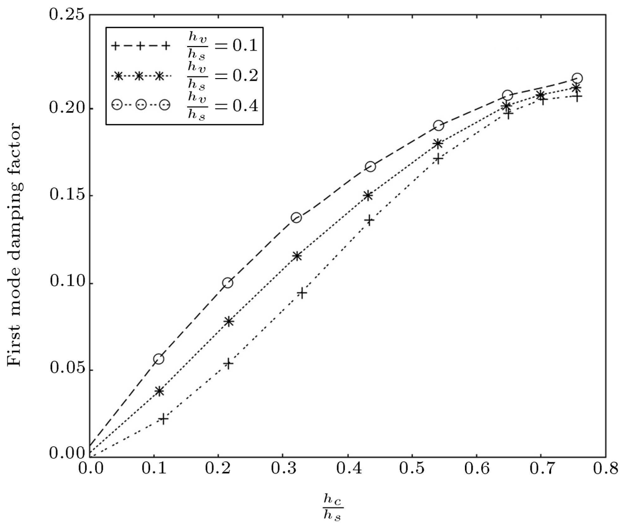

Figure 9. Structural damping factor with various thickness ratios of CL to shell.

calculation of the damping value. If the external force on the structure is assumed to be harmonic, this function is defined as:

$$\mathrm{tag}(\psi) = \frac{\mathrm{Im}H(i\omega)}{\mathrm{Re}H(i\omega)}, \tag{32}$$

where $H(i\omega)$ =FRF is the frequency response function of the structure. If the viscoelastic core is modeled with a spring of constant stiffness, k, and a linear dashpot with a viscous damping coefficient, c, and elements combined in parallel, then one has:

$$\mathrm{tag}(\psi) = \frac{c\omega}{k} = 2\gamma\frac{\omega}{\omega n}, \tag{33}$$

where, ω_n is natural frequency and γ is the damping coefficient of the structure. The value of γ can be calculated by the Peak-Picking method [10] for the first vibration mode. Damping effects in a low thickness of CL and VEC are greater, as is evident from the curves in Figures 8 and 9. These results confirm the results of Figures 6 and 7.

CONCLUSION

In this paper, an approach to the vibration analysis of a cylindrical shell with Constrained Layer Damping (CLD) treatment is presented. Equations of motion, in terms of transverse modal coordinates, are obtained by means of an energy method, Love's assumptions and the assumed-mode methods. Using the Inverse Fast Fourier Transform (IFFT), the equations of motion are transferred from the frequency domain to the time domain. In the time-domain, the damping and settling time at different thicknesses of CL and viscoelastic layers are computed. Also, the Structural Damping Factor is calculated with various thicknesses of VEC and CL. All results confirm that stiffer CL with a larger h_c and thicker VEC with a given shear modulus always yield better damping, and that damping effects at smaller thicknesses of CL and VEC are greater. Also, increasing the VEC thickness by more than a specific value will not further affect the damping of the structure. In this case, the damping effect can be increased by increasing the thickness of CL.

NOMENCLATURE

s, c, v	cylindrical shell, the Constraining Layer (CL) and viscoelastic core (VEC),
E, G	Young's modulus and shear modulus,
G_v	complex shear modulus for VEC,
$\sigma^i_{xx}, \sigma^i_{x\theta}, \sigma^i_{\theta\theta}$	in plane stresses of cylindrical shell and CL,
$\varepsilon^i_{xx}, \varepsilon^i_{x\theta}, \varepsilon^i_{\theta\theta}$	in plane strains of cylindrical shell and CL,
$\sigma^V_{xz}, \sigma^V_{\theta z}, \varepsilon^V_{xz}, \varepsilon^V_{\theta z}$	transverse shear stresses and strains in viscoelastic layer,
u^i_x, u^i_θ, u_z	displacement in x, θ and transverse directions,
r_s, r_v, r_c	radius of mid-plane in cylindrical shell, VEC and CL,
h_s, h_v, h_c	thickness of cylindrical shell, VEC and CL,
$\beta^v_x, \beta^v_\theta$	angular displacement of VEC,
T_S, T_V, T_C	the kinetic energy of the layers,
U_S, U_V, U_C	the strain energy of the layers,
ρ_s, ρ_v, ρ_c	density of layers,
$\xi^s, \eta^s, \xi^c, \eta^c, \zeta$	generalized coordinate of layers,

Q_z	generalized force,
F	transverse point load,
$K^*(\omega)$	complex stiffness matrix,
K^s, K^v, K^c	stiffness matrix of layers,
$\eta_G(\omega), \eta_E(\omega)$	material loss factor in the shear and tension,
$H(i\omega)$	frequency response function,
ψ	damping value,
γ	damping coefficient.

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