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Multi-Criteria Unit Commitment with Flexible Constraints and Uncertain Coefficients

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In this paper, the short-term multi-criteria thermal unit commitment problem in power systems is considered. Minimizing the two objectives of fuel cost and environmental emission has been regarded as the multiple programming criteria. The generating units are subjected to flexibly bounded regions. These soft limitations are modeled as fuzzy constraints defined by proper membership functions. The main policy for selecting the generating units is first derived based on a fuzzy dynamic programming approach. Having obtained the main optimal policy, the optimal share of each of the selected units is then determined based on a fuzzy linear programming method where flexible constraints and uncertain coefficients in objective functions are considered. Furthermore, the suggested algorithm is tested and verified on a sample power system.

INTRODUCTION

The optimal operation of power systems is often achieved by developing a program which predicts the generation scheduling of the power system usually for a period of one day to almost two weeks. Such a program determines the optimal combination and share of the generating units such that the hourly forecasted loads are maintained and the system operational constraints are satisfied. The most usual objective function in this optimization problem, i.e., the total cost of the fuel consumed during the programming period, is to be minimized. However, minimizing the environmental impact measured as the pollutants emitted by the power plants, or maximizing the power system security indicated by specific security indices and so on, are other usual objectives which can be considered in multi-criteria generation scheduling of power systems.

After the introduction of fuzzy sets theory [1] and decision making in a fuzzy environment [2], a number of papers on fuzzy linear programming method applied to the vector-max problem [3] have been reported. There have been also several reports about the successful application of the fuzzy systems in modeling and solution to some power system problems [4]. Some of these reports are concerned with the fuzzy load forecasting [5], fuzzy optimal power flow [6], fuzzy voltage and reactive power control [7] and fuzzy unit commitment [8]. In the latter report [8], a fuzzy dynamic programming method is used for generation scheduling of power systems with the single criteria of minimizing the total fuel cost. In [9], a fuzzy logic based procedure was suggested to coordinate multiple objectives, though, the start up and shut down costs of the generating units were not included.

In this paper, the total fuel cost and the total emission produced by the power plants are considered as the multiple objective functions of the unit commitment problem. The generating units are considered to operate within specified flexible regions which are modeled as fuzzy constraints.

The optimal policy of the program, i.e., the desired selected list of the generating units, is first determined by a new algorithm based on the fuzzy dynamic programming approach. The off-line costs and emissions are also included.

The second part of the paper presents a method used for optimal dispatch of the forecasted load between the selected units in all of the time intervals or stages of the developed program. Analysis of the results obtained in a case study on a sample power system is also presented.

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THE MULTI-CRITERIA OPTIMAL POLICY

The conventional short-term unit commitment problem in power systems is usually accomplished based on the single criterion of minimizing the total fuel cost.

Several methods have been used to solve this problem among which the mixed integer linear programming and the dynamic programming methods are more efficient and mostly used [10]. The dynamic programming method has the capability of handling different types of mathematical objective functions or even discretely tabular formed data [11]. In this method, the total programming period is divided into equal time intervals or stages of usually one hour duration and an optimal path or policy is derived stage by stage. Solution of the problem of optimal generation scheduling of power systems using a single objective of cost function is well adapted to this method and a large number of papers have been published in this field [11]. However, the fuzzy dynamic programming method of solution for the multiple criteria optimal generation scheduling is the core of the suggested algorithm introduced in this work.

Before presenting the details of the proposed fuzzy dynamic programming approach, modeling of the fuzzy constraints of the generation range of the units is first considered. Every generating unit can generate within specific amounts of power of $P_i^{\min}$ and $P_i^{\max}$ which are not rigidly fixed but rather flexible. The constraint on the generation range of unit i can be modeled by a fuzzy number with a trapezoidal membership function shown in Figure 1, where $(1-d)P_i^{\min}$ and $(1+d)P_i^{\max}$ are the minimal and maximal permitted deviated values of the generating power of unit i, $d \in [0,1]$ and is specified by the decision maker.

To discuss details of the proposed algorithm, consider the following notations:

1. Nint: Number of time intervals or program stages.
2. S: A feasible state as a combination of the on-units which can maintain the forecasted load of a time interval.
3. $\{S\}$: Set of the feasible states at Jth time interval. This set is supposed to have s members.
4. $\{L\}$: Set of the feasible states at the previous time interval, $J-1$. This set is supposed to have l members.
5. Cost(J,S,L): The minimum fuel cost consumed to arrive at and stay within the state S of the interval J, via state L of the previous stage.
6. Pcost(J,S): The minimum fuel cost for one hour production at interval J, determined by economic load dispatch between the "on" units in the state S.
7. Scost(L,S): Sum of the start up costs of units turned on when going from state L to state S.
8. Mincost(J,S): The least possible fuel cost consumed to arrive at and stay within the state S in the interval J.

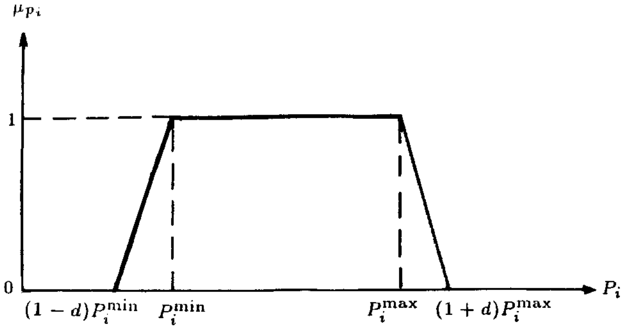

Figure 1. The membership function of the fuzzy generation range of unit i.

Considering the environmental objective function alone, as for the economic criteria, similar items would be defined and called Emission(J,S,L), Pemis(J,S), Semis(L,S) and Minemis(J,S) respectively.

Suppose that the solution is at state S in stage J coming via state L of the previous stage. The total consumed fuel cost related to this path can be determined by:

$$\text{Cost}(J,S,L) = \text{Pcost}(J,S) + \text{Scost}(L,S) + \text{Mincost}(J-1,L). \tag{1}$$

There are l possible ways to come to state S of stage J. The least of these l values, as the cost of the best path to S, is determined by:

$$\text{Mincost}(J,S) = \min_{\{L\}}[\text{Cost}(J,S,L)]. \tag{2}$$

Considering the environmental objective function alone, similar relations are defined as follows:

$$\text{Emission}(J,S,L) = \text{Pemis}(J,S) + \text{Semis}(L,S) + \text{Minemis}(J-1,L), \tag{3}$$

$$\text{Minemis}(J,S) = \min_{\{L\}}[\text{Emission}(J,S,L)]. \tag{4}$$

Not considering the "Crew Constraint" [10], there are $l \times s$ feasible paths to arrive at stage J via stage $J-1$. Based on fuel cost values Cost(J,S,L) of Equation 1, a fuzzy set with a membership function μ_{Cost} can be defined such that its membership values are equal to 1 for the least value and 0 for the greatest value of Cost(J,S,L) and other cost values have membership values distributed linearly between 1 and 0.

Based on the $l \times s$ values for Emission(J,S,L) of Equation 3, in a similar way, a fuzzy set μ_{Emission} is defined.

Notice that μ_{Cost} and μ_{Emission} determined in stage J show the individual degree of satisfaction of the economic and environmental objective functions, respectively, for their related path L to S at stage J.

Considering two objective functions simultaneously, using the "and" operator defined by Zadeh [1,2], the overall degree of satisfaction of a path L to S at stage J, called $\mu_{\text{Path}}(J, S, L)$, can be determined by:

$$\mu_{\text{Path}}(J, S, L) = \min_{\{L\}}[\mu_{\text{Cost}}(J, S, L), \mu_{\text{Emission}}(J, S, L), \mu_{\text{Optimal}}(J-1, L)], \quad (5)$$

where $\mu_{\text{Optimal}}(J-1, L)$ is the largest value of the overall degree of satisfaction of both objective functions achieved in state L of the previous time interval. At the present state J, $\mu_{\text{Optimal}}(J, S)$ is easily determined by:

$$\mu_{\text{Optimal}}(J, S) = \max_{\{L\}}[\mu_{\text{Path}}(J, S, L)] . \quad (6)$$

To clarify further, one can describe the suggested algorithm used for finding the optimal policy of the program as follows:

1. For evaluation of Pcost and Pemis, all generating units are assumed to be permitted to operate within their widest generation range, i.e., $(1-d)P_i^{\min} \le P_i \le (1+d)P_i^{\max}$. Also, the nonfuzzy coefficients of the objective functions, i.e., α_i, β_i, a_i and b_i, defined in the next section, are used. As such, a single objective linear programming problem is solved to determine Pcost and Pemis separately. Programming starts from the specified initial conditions at the first time interval and proceeds stage by stage forward to the last time interval. At every stage J, using relations defined by Equations 1 through 5, the optimal degree of satisfaction $\mu_{\text{Optimal}}(J, S)$ is determined for all members of $\{S\}$.
2. At the last time interval the largest value of $\mu_{\text{Optimal}}(\text{Nint}, S)$ is selected and the corresponding state, called S^*, and its related passing state in the previous stage, called L^*, are identified.
3. Back tracing towards the first stage determines the optimal path, or policy, of the program.

OPTIMAL LOAD DISPATCH IN THE OPTIMAL PATH

Having obtained the optimal path of the program, the optimal generation share of the selected units at every stage should be determined. This is done by multi-criteria fuzzy constrained optimal dispatching of the forecasted load of a stage as discussed in this section.

Suppose that the solution is at stage J, passing state S^* which contains n selected on-units. The objective function of the hourly production fuel cost and hourly produced emission, denoted by $\tilde{C}(P)$ and $\tilde{E}(P)$, are supposed to be linear, or approximately linear, functions of the unit generated power P_i, defined by:

$$\tilde{C}(P) = \sum(\tilde{\alpha}_i P_i + \tilde{\beta}_i) \qquad \text{units of cash / Mwh} \quad (7)$$

$$\tilde{E}(P) = \sum(\tilde{a}_i P_i + \tilde{b}_i) \qquad \text{units of emission index / Mwh}, \quad (8)$$

where the fuzzy coefficients $\tilde{\alpha}_i, \tilde{\beta}_i, \tilde{a}_i$ and $\tilde{b}_i$ are fuzzy numbers with triangular membership functions shown in Figure 2.

Based on the objective functions defined by Equations 7 and 8, for evaluation of the optimal share of the production units, the following fuzzy optimization problem can be defined:

$$\min : \begin{array}{c}\tilde{C}(P) \\ \tilde{E}(P)\end{array}$$

subject to:

$$P_i = \tilde{P}_i^{\text{nominal}}, \qquad i = 1, \dots, n$$

$$\sum P_i = \text{Demand}. \quad (9)$$

Several methods have been reported to solve optimization problems of the type specified by Problem 9. In almost all of the methods, using a fuzzy ordering relation [10–15], the fuzzy optimization problem is converted to a nonfuzzy parametric programming model. In [14], the possibility of dominance (PD) of a fuzzy number $\tilde{a}$ over $\tilde{b}$ has been defined and used for converting the fuzzy programming problem to a multicriteria nonfuzzy optimization model.

Based on this approach, Problem 9 can be re-

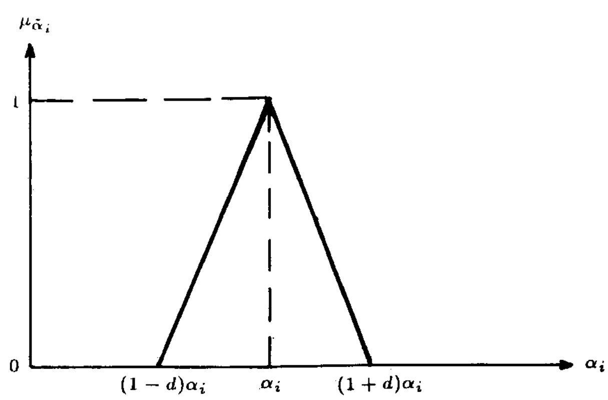

Figure 2. The membership function of $\tilde{\alpha}_i$.

placed by:

$$\max \quad \begin{matrix} PD(\tilde{\text{Cost}}, \tilde{C}(P)) \\ PD(\tilde{\text{Emission}}, \tilde{E}(P)) \end{matrix}$$

Subject to:

$$PD(P_i, \tilde{P}_i^{\text{nominal}}) \geq \theta \qquad i = 1, \ldots, n$$

$$PD(\tilde{P}_i^{\text{nominal}}, P_i) \geq \theta \qquad i = 1, \ldots, n$$

$$\sum P_i = \text{ Demand}, \tag{10}$$

where $\tilde{\text{Cost}}$ and $\tilde{\text{Emission}}$ are the corresponding fuzzy goals of the objective functions $\tilde{C}(P)$ and $\tilde{E}(P)$, which can be defined by the decision maker (DM) or determined as follows.

$\tilde{\text{Cost}}$ can be considered as a fuzzy number shown in Figure 3, where $\underline{\text{Cost}}$ is the minimal value of $C(P)$ of the following linear programming problem:

$$\begin{aligned} \min: \quad & C(P) = \sum(\alpha_i P_i + \beta_i) \\ \text{S.T.}: \quad & (1-d)P_i^{\min} \leq P_i \leq (1+d)P_i^{\max} \\ & \sum P_i = \text{ Demand}. \end{aligned} \tag{11}$$

Also, $\underline{\text{Emission}}$ is defined in a similar way, using the following problem.

$$\begin{aligned} \min: \quad & E(P) = \sum(a_i P_i + b_i) \\ \text{S.T.}: \quad & (1-d)P_i^{\min} \leq P_i \leq (1+d)P_i^{\max} \\ & \sum P_i = \text{ Demand}. \end{aligned} \tag{12}$$

σ_{Cost} may be considered as:

$$\sigma_{\text{Cost}} = \text{ Cost}^* - \underline{\text{Cost}}, \tag{13}$$

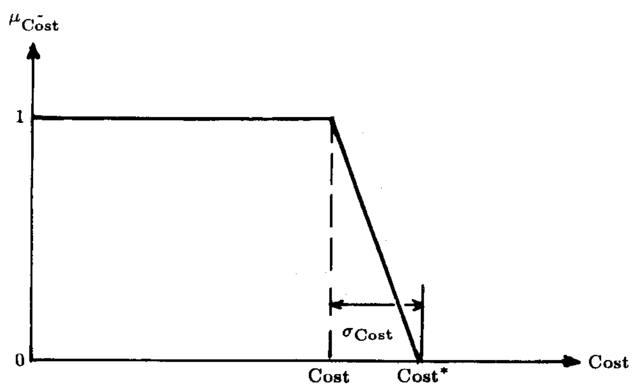

Figure 3. The fuzzy goal of $\tilde{\text{Cost}}$.

where Cost* is the value of $C(P)$ using the optimal solution $[P_1^*, \ldots, P_n^*]^T$ of Problem 12. Similarly, σ_{Emission} may be defined as:

$$\sigma_{\text{Emission}} = \text{ Emission}^* - \underline{\text{Emission}}, \tag{14}$$

where Emission* is the value of $E(P)$ using the optimal solution of Problem 11.

Further conversion of Problem 12, using the techniques proposed in [14], leads to the following parametric linear programming problem.

$$\begin{aligned} \max: \quad & \theta \\ \text{S.T.}: \quad & \sum(\alpha_i P_i + \beta_i) - (1-\theta) \cdot d \cdot \sum(\alpha_i P_i + \beta_i) \\ & \leq \underline{\text{Cost}} + (1-\theta) \cdot \sigma_{\text{Cost}} \\ & \sum(a_i P_i + b_i) - (1-\theta) \cdot d \cdot \sum(a_i P_i + b_i) \\ & \leq \underline{\text{Emission}} + (1-\theta) \cdot \sigma_{\text{Emission}} \\ & P_i \geq P_i^{\min} - (1-\theta) \cdot d \cdot P_i^{\min}, \quad i = 1, \ldots, n \\ & P_i \leq P_i^{\max} + (1-\theta) \cdot d \cdot P_i^{\max}, \quad i = 1, \ldots, n \\ & \sum P_i = \text{ Demand}. \end{aligned} \tag{15}$$

CASE STUDY

To test and verify the suggested unit commitment algorithm, a sample power system consisting of eight generating units has been considered. These units are regarded to have cost and emission functions as some quadratic functions of their generating power P_i, derived from [16] and modified as:

$$C_i(P_i) = K_{ci}(K_{1i}P_i + K_{2i}P_i^2), \tag{16}$$

$$E_i(P_i) = K_{ei}(K_{1i}P_i + K_{2i}P_i^2). \tag{17}$$

Considering the total hourly produced NO_x pollutants as the emission index, the test system of the same reference, [16], was used and modified to establish the input data for the developed computer program, as presented in Table 1.

To use the introduced unit commitment algorithm, it is required to adapt Equations 16 and 17 in the linear form, defined as follows:

$$c_i(P_i) = (\alpha_i P_i + \beta_i), \tag{18}$$

$$e_i(P_i) = (a_i P_i + b_i). \tag{19}$$

The coefficients of above approximate linear equations, using parameters of Table 1, are given in Table 2.

To compensate for the approximation involved and the uncertainty in the coefficients of Equations 18

Table 1. Characteristics of the generation units.

Unit No	P_i^{min} Mw	P_i^{max} Mw	K_{1i} Gj/Mw	K_{2i} Gj/Mw	K_{ci} S/Gj	K_{ei} Kg/Gj	Sc_i S/h	Se_i Kg/h
1	75	300	10.5343	0.0001	1.42	0.166	1200	400
2	60	250	10.5343	0.0001	1.42	0.166	1300	500
3	25	80	10.1887	0.0008	2.56	0.259	1500	900
4	20	60	10.1887	0.0008	2.56	0.259	400	800
5	95	400	10.1887	0.0008	2.56	0.259	200	700
6	80	350	10.1887	0.0008	2.98	0.124	1200	200
7	45	180	10.1887	0.0008	2.98	0.124	500	800
8	40	160	8.5841	0.0040	2.61	0.111	1400	60

Table 2. The coefficients of linear objective functions.

Unit No.	α_i	β_i	a_i	b_i
1	14.9928	-2.0448	1.7527	-0.2390
2	14.9869	-1.3632	1.7520	-0.1594
3	26.2973	-4.0806	2.6605	-0.4128
4	26.2463	-2.4484	2.6554	-0.2474
5	27.0930	-77.5322	2.7411	-7.8441
6	31.3836	-66.5017	1.3059	-2.7672
7	30.8967	-19.2380	1.2856	0.8005

and 19, it will be more appropriate to consider α_i, β_i, a_i and b_i as fuzzy numbers with membership functions shown in Figure 2. Also in accordance with the physical characteristics of the real world, a flexible generation range is considered to contain the thermal generating units, as used in Equation 9 and shown in Figure 1. In this case study, a ten percent deviation from the maximum and minimum nominal values for generating power of units, shown in Table 1, and also objective functions coefficients, i.e. $d = 0.1$, is allowed.

The forecasted hourly aggregate load, as a twenty four hour profile is shown in Figure 4.

The start up fuel costs containing, the shut down costs of the generating units, called Sc_i, and the start up emission index, containing the shut down emission index of units, called Se_i, are also given in Table 1.

Based on the above data, the suggested multi-criteria unit commitment algorithm has been programmed and executed for the following cases:

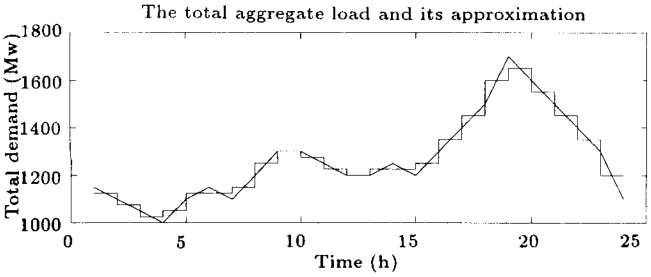

Figure 4. The forecasted hourly aggregate load and its approximation.

1. A single criterion of cost.
2. A single criterion of emission.
3. Both above criteria.
4. Both criteria with flexible constraints and fuzzy coefficients.

The obtained results for the total programming costs and the total programming produced emission are shown in Figures 5 and 6, respectively.

As can be observed in Figure 5, the total programming costs of the Cases 3 and 4 obtained from multi-criteria unit commitment, as expected, are located between the two extremes of single criterion cases. Incidentally, Case 4 has a better result compared with Case 3, due to the flexibility of constraints and the principle of preference applied to the coefficients.

Similar results are obtained for the total product emission, as shown in Figure 6. For clarity, results of only the first 12 hours are presented in Figure 5. However, Figure 6 presents the complete results.

Further results investigations of the case study indicated in Figures 5 and 6 are outlined in the following remarks:

1. In Case 1, where only the total cost is minimized, the least production cost and the most produced emission are obtained. This is the conventional short-term unit commitment problem in power systems.
2. In Case 2, the other objective function, emission, is minimized. In this case, the highest cost and the lowest produced emission are obtained.
3. Case 3 considers both above objectives simultaneously. This is the case of multi-criteria unit commitment problem which is solved by the proposed fuzzy dynamic programming algorithm of the paper. All of the values of cost and emission objectives are located between the two extreme cases of 1 and 2. This justifies the obtained results since the solution of the problem is a compromised solution.

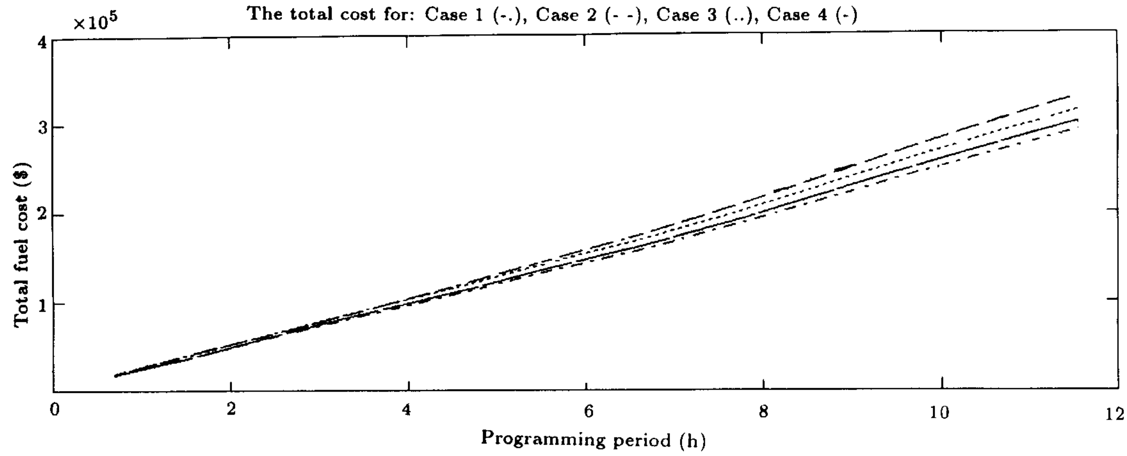

Figure 5. The total production cost versus the programming period in various cases.

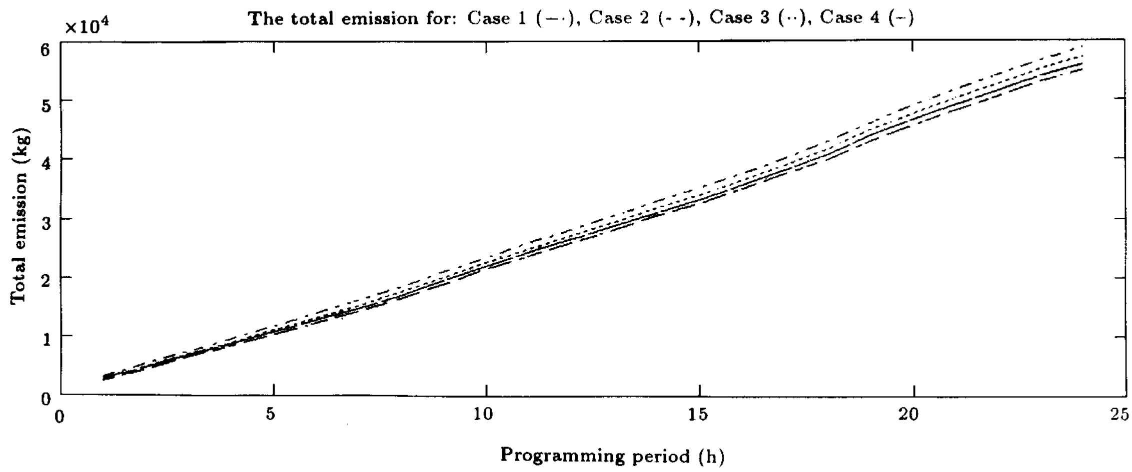

Figure 6. The total produced emission versus the programming period in various cases.

4. Case 4 considers the same objective functions as Case 3. However, flexibility in the nominal production boundaries of the generation units and uncertainties involved in modeling and linearization of objective functions, in terms of fuzzy sets, are considered. All of the values of cost and emission objectives are located under the corresponding values of Case 3. This is due to the flexibility of the constraints and the principle of preference used for modeling objective functions. As shown in Figures 5 and 6, the gain obtained in this case, compared to Case 3, for the total cost and the total emission of 12 hours programming are about $10,000 and 1000 kg, respectively.

CONCLUSIONS

The short-term multi-criteria thermal unit commitment problem in power systems containing some flexible constraints and fuzzy coefficients has been considered in this work. A fuzzy based dynamic programming approach has been used to find the compromised optimal policy, as the most possible close path to the individual optimal solution of objective functions simultaneously. The optimal policy is achieved by maximizing the overall degree of satisfaction of the feasible paths transferring through when going from one programming stage to the next.

Having obtained the optimal programming path, the optimal share of each committed unit is found by solving the appropriate fuzzy constrained multi-criteria optimal load dispatch problem at every optimal state in the optimal path by a fuzzy linear programming approach [17].

The suggested algorithm has been applied to an economic-environmental unit commitment problem on a sample power system. The total fuel cost and total emission of the generating units with fuzzy coefficients have been minimized such that a forecasted system load profile is maintained in all programming periods. The generated power of the units are fuzzily satisfied in their flexible production ranges which is a more realistic feature than to be viewed as crisp constraints. It has been demonstrated that the case of fuzzy model shows the most improvement in minimizing both objective functions.

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