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Research Note

Comparison of Four Adaptive PID Controllers

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PID controllers have been used for many years in industry and if a controller is well tuned, its performance is acceptable for many industrial processes. When the operating point is changed, due to nonlinear behavior of most processes, the controller should be retuned. In this regard, several self-tuning PID controllers are proposed in the literature. In this paper, four such algorithms are compared through simulation and experimental studies. In the simulation study, the effects of factors such as process pole locations, non-minimum phase behavior and model changes on the performance of the schemes are investigated. Simulation and experimental results demonstrate that one of the schemes performs better than the others.

INTRODUCTION

PID controller is the most common control algorithm. The controller structure is simple and performs well when it is properly tuned. Many tuning schemes have been reported in the literature among which are techniques proposed by Ziegler and Nichols [1], Cohen and Coon [2] and Rivera et al. [3]. Although PID controllers are common and well-known, they are often poorly tuned. To overcome this problem, several autotuning approaches have been reported [4].

In an autotune PID, the controller parameters are tuned automatically on demand of the operator. When a process is time varying or has nonlinear dynamics, the controller adaptation should be done continuously. This type of PID is called adaptive or self-tuning. In this article, the performances of four such schemes are compared through computer simulation and experimental studies. In the next part, these algorithms are briefly reviewed.

COMPARED SELF-TUNING SCHEMES

In this article, four adaptive PID controllers proposed by Banyacz et al. [5], Vega et al. [6], Katende [7] and De Keyser [8] are compared. In all of the schemes, the following velocity form of PID is used:

$$u(t) = u(t-1) + G(q^{-1})e(t), \qquad (1)$$

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where $u(t)$ and $e(t)$ are plant input and output error, respectively, and:

$$G(q^{-1}) = g_0 + g_1 q^{-1} + g_2 q^{-2}.$$

In the first two schemes [5,6], the process is modeled by the following ARMA model:

$$y(t) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}u(t) + \nu(t), \qquad (2)$$

where,

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n},$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m},$$

$y(t)$ and $\nu(t)$ are plant output and unmeasured disturbances, respectively.

In these two schemes, the process model parameters are estimated by Recursive Least Squares (RLS) at each sampling time and based on these estimates, the PID parameters are updated. The schematic block diagram of these indirect adaptive schemes is presented in Figure 1.

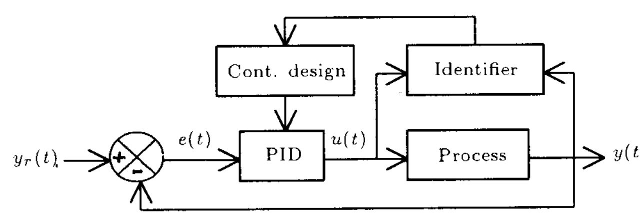

Figure 1. Indirect adaptive PID.

In the schemes proposed by Katende [7] and De Keyser [8], direct adaptive technique is utilized and, therefore, process model parameters are not estimated directly. For estimating the controller parameters, the RLS technique is used. The block diagram of these direct adaptive schemes is shown in Figure 2.

In what follows basic design principle governing each scheme is reviewed very briefly. In the first scheme proposed by Banyacz et al. [5], pole-cancellation technique is used for tuning the PID parameters. In this design, the process dynamics is modeled with a second order model ($n \leq 2$) and the controller parameters are designed to cancel the process model poles and achieve the desired phase-margin. A similar scheme is proposed by Tjokro and Shah [9]. In this design, the process model poles are cancelled also, however, the controller gain is adjusted to place the closed-loop pole at the desired location. This is equivalent to the former scheme and, therefore, only the first one is considered.

The second scheme which is called self-tuning predictive PID controller is proposed by Vega et al. [6]. The tuning strategy is based on the minimization of the following performance index:

$$I=\frac{1}{2}\sum_{j=0}^{t}\lambda^{t-j}[y_r(j)-\hat{y}(j+d/j)]^2+\alpha[G(q^{-1})e(j)]^2. \tag{3}$$

In the above equation, λ is a forgetting factor and α is the weighting factor for the control efforts. $y(j+d/j)$ denotes the optimal prediction of $y(j+d)$ in the least squares sense as a function of $y(j)$ and $u(j-1),\ldots$. In this design, there is no limitation on the system order and, therefore, higher order dynamics can be considered. Based on the above objective function, the controller parameters are calculated recursively.

The third scheme is proposed by Katende and Jutan [7]. This design is based on the minimization of the following cost function:

$$J=E\{e(t+d)+\beta\Delta u(t)\}^2. \tag{4}$$

After several simplifying assumptions, a recursive updating formula is obtained for direct estimation of the controller parameters.

The fourth scheme, which is proposed by De Keyser [8], is a direct adaptive model reference controller. In a special case, where the controller has three parameters, it becomes a PID controller. The reference model is a first order model with time delay:

$$R(q^{-1})=q^{-d}\frac{1-\gamma}{1-\gamma q^{-1}}. \tag{5}$$

Similar to the former scheme, the recursive least squares technique is used for updating the controller parameters.

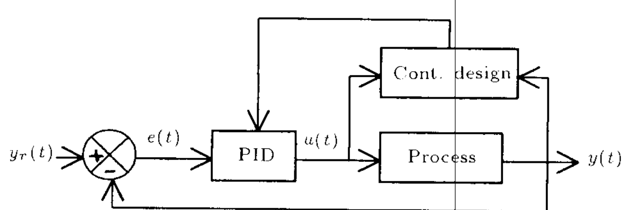

Figure 2. Direct adaptive PID.

Table 1. Design parameter for different schemes.

Scheme No.	Design Parameter
I [5]	ϕ_m (phase margin)
II [6]	α
III [7]	β
IV [8]	γ

SIMULATION RESULTS

In this section, the performances of the four aforementioned schemes are compared through simulations. The effects of the following factors are investigated:

a) Pole locations of the discrete model,

b) Non-minimum phase behavior,

c) Process model changes.

In all simulations, recursive least squares with variable forgetting factor [10] is used. The initial values of the covariance matrix and parameter estimates are chosen as follows:

$$\mathbf{P}(0)=100\mathbf{I},$$

$$\mathbf{G}(0)=\mathbf{0},$$

where **I** is the identity matrix. Every scheme has one design parameter which is shown in Table 1. The value of this parameter for each scheme is so selected to minimize the sum of the absolute value of the error.

Effects of Pole Locations and Non-Minimum Phase Behavior

Simulation results demonstrate that as model poles approach the center of the unit circle, the difference in performances of the four schemes becomes less. Therefore, systems having poles near the unit circle are chosen for simulations. Sampling time is selected based on the following rule [11]:

$$\frac{T_r}{h}=4-10, \tag{6}$$

where T_r and h are rise time and sampling time respectively.

The continuous and discrete models of the simulated examples are given in Table 2. The first example is a second order model plus lag and it should be noted that the dynamic behavior of many real processes can be modeled by such a transfer function. The second system used for simulation which has complex poles is an example of systems with oscillatory responses. The last example is selected to evaluate the performances of the control schemes for non-minimum phase processes.

In simulations, a white noise with zero mean and variance of 0.01 is added to the output of the model. In each simulation, a sequence of set-point changes with magnitude of two is provided with switching every 25 samples. After four changes in the set-point, a step change in load is applied at $k = 150$. For schemes I and II, an PRBS input with height of two is applied to the open loop system during the first 25 samples. Since applying such an input does not improve the performances of schemes III and IV, it is not considered for these schemes.

The simulation results are shown in Figures 3 to 14. The results indicate that the best performance belongs to scheme I. Schemes II, III and IV do not perform well in the case of non-minimum phase systems, especially when the right hand side zero is close

Table 2. Transfer functions of the simulated models.

Process No.	Continuous Model	Discrete Model
I	$\frac{e^{-S}}{(3S+1)(3.53S+1)}$	$\frac{q^{-2}(0.0386+0.0314q^{-1})}{1-1.47q^{-1}+0.54q^{-2}}$
II	$\frac{0.14e^{-S}}{S^2+0.223S+0.139}$	$\frac{q^{-2}(0.064+0.059q^{-1})}{1-1.68q^{-1}+0.8q^{-2}}$
III	$\frac{(1-3S)e^{-S}}{(2S+1)(1.73S+1)}$	$\frac{q^{-2}(-0.4+0.57q^{-1})}{1-1.17q^{-1}+0.34q^{-2}}$

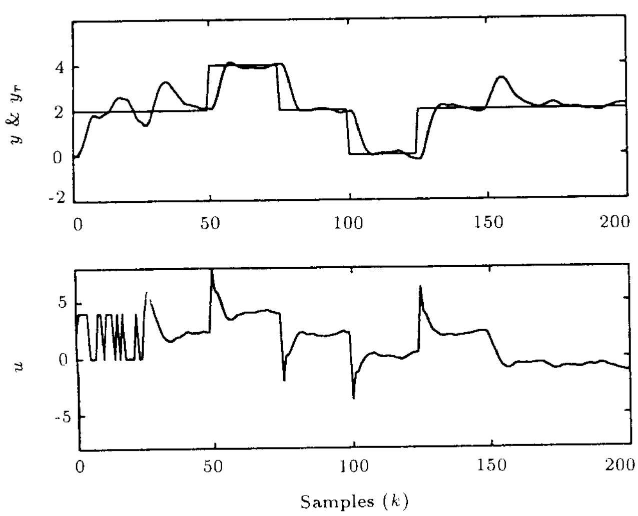

Figure 3. Response of process I using scheme I ($\phi_m = 68$).

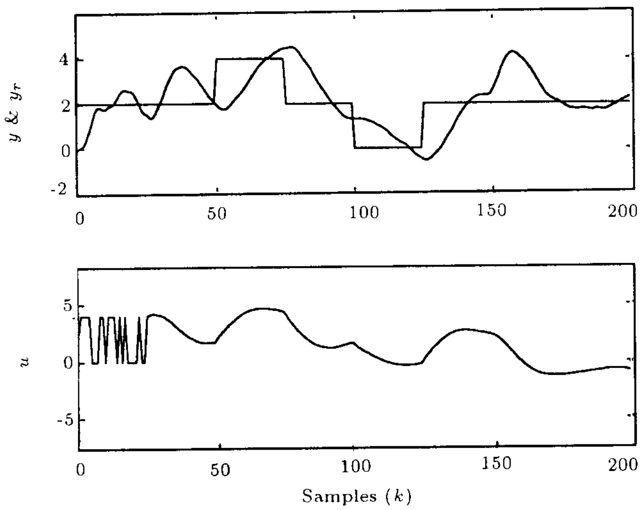

Figure 4. Response of process I using scheme II ($\alpha = 0.3$).

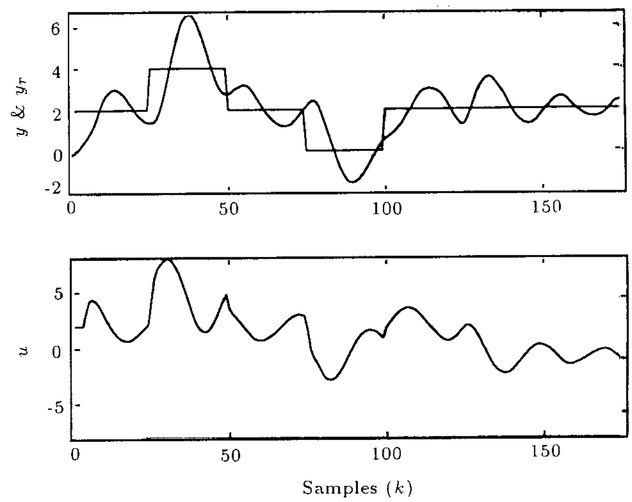

Figure 5. Response of process I using scheme III ($\beta = 1.2$).

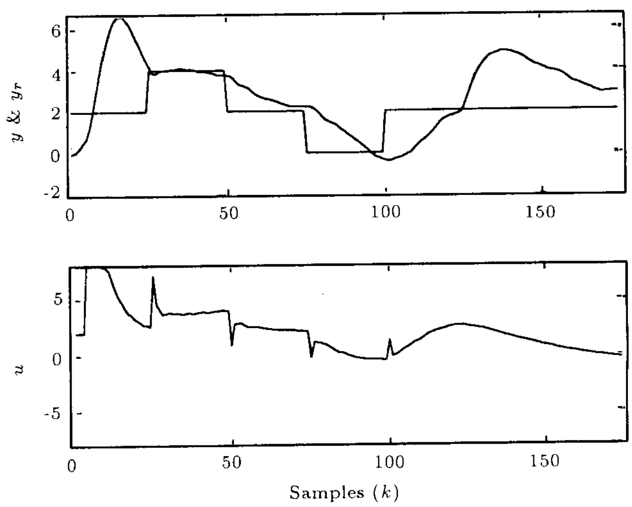

Figure 6. Response of process I using scheme IV ($\gamma = 0.78$).

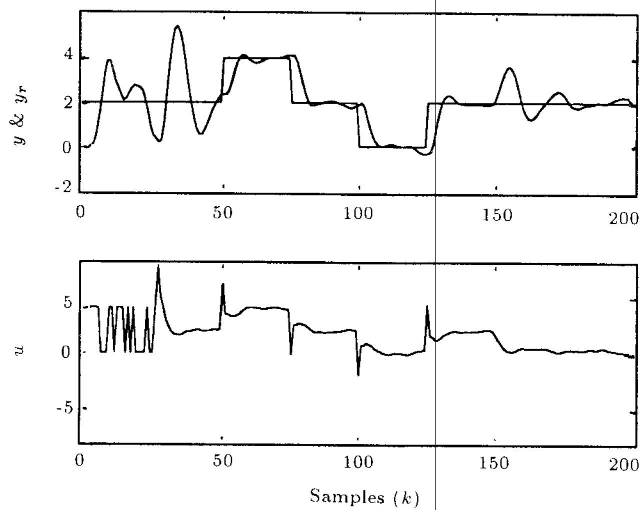

Figure 7. Response of process II using scheme I ($\phi_m = 63.5$).

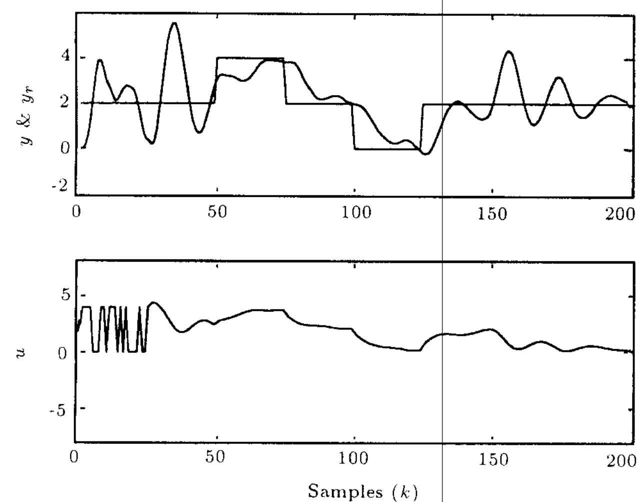

Figure 8. Response of process II using scheme II ($\alpha = 0.5$).

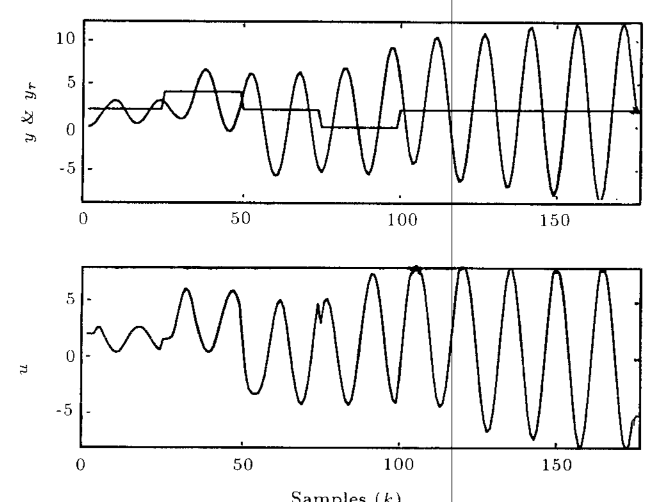

Figure 9. Response of process II using scheme III ($\beta = 2.0$).

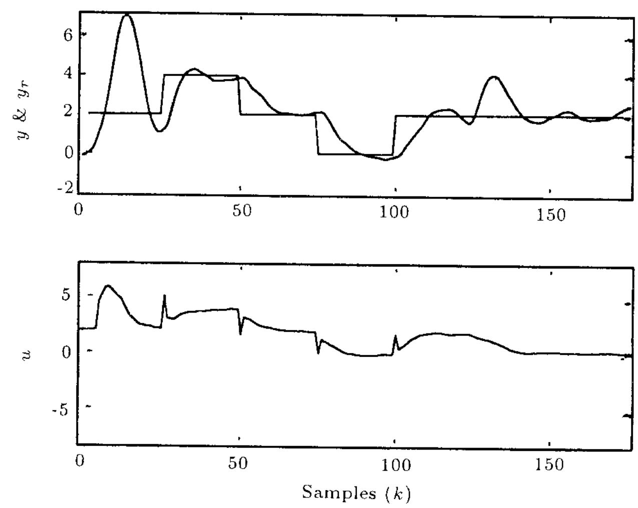

Figure 10. Response of process II using scheme IV ($\gamma = 0.7$).

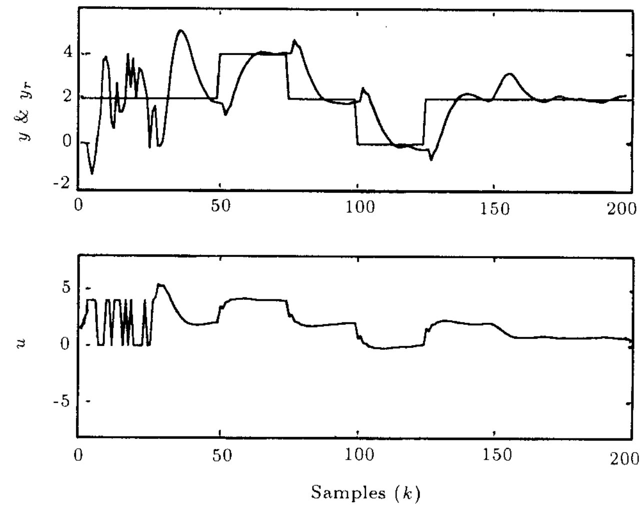

Figure 11. Response of process III using scheme I ($\phi_m = 56$).

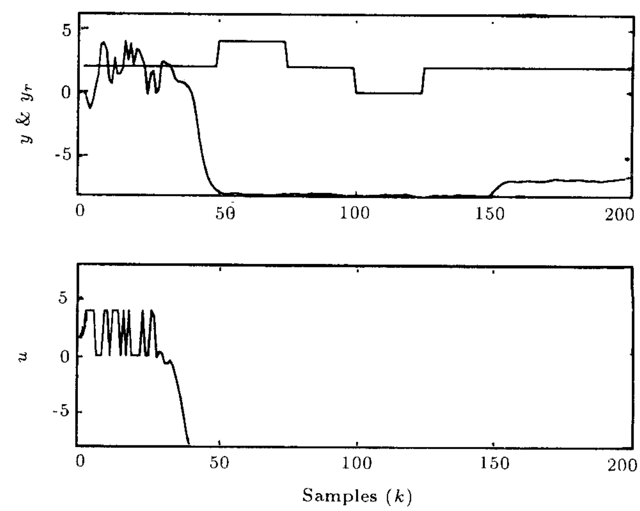

Figure 12. Response of process III using scheme II ($\alpha = 0.5$).

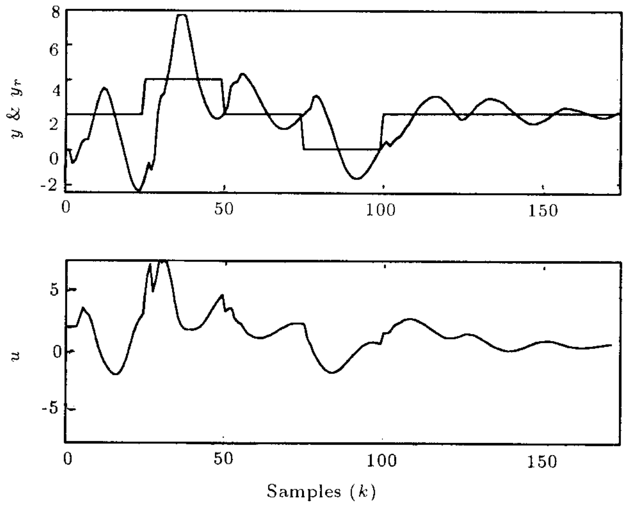

Figure 13. Response of process III using scheme III ($\beta = 3.5$).

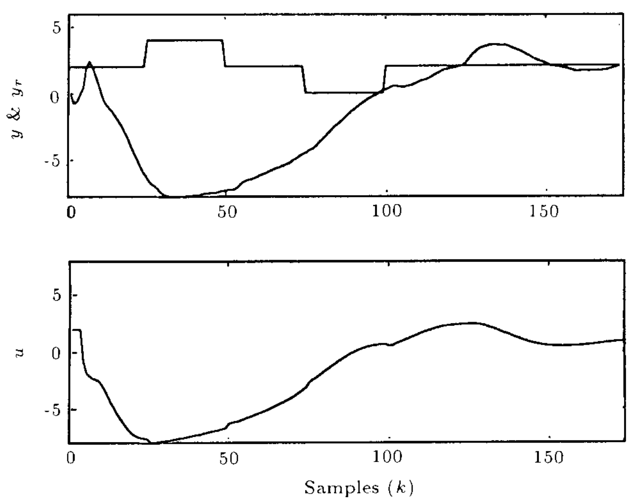

Figure 14. Response of process III using scheme IV ($\gamma = 0.74$).

to the unit circle. The performances of all schemes are degraded, as the system poles become closer to the unit circle. It should be mentioned that the third and fourth schemes are more sensitive to the pole locations and for poles very close to the unit circle, the closed loop system may become unstable.

Process Model Changes

In this part, the effects of process model changes on the performances of the control schemes are verified. The simulated models and their corresponding time intervals are presented in Table 3. In all schemes, the dead time is assumed to be two samples and for schemes I and II the degrees of $B(q^{-1})$ and $A(q^{-1})$ polynomials are set equal to four and two respectively.

The simulation results are illustrated in Figures

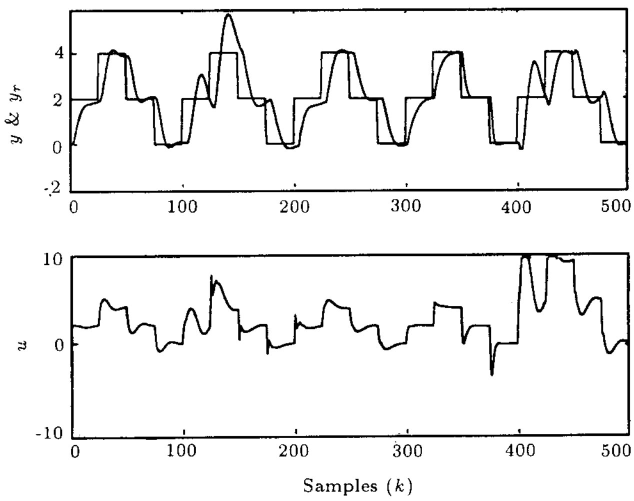

Figure 15. Closed loop response using scheme I ($\phi_m = 60$).

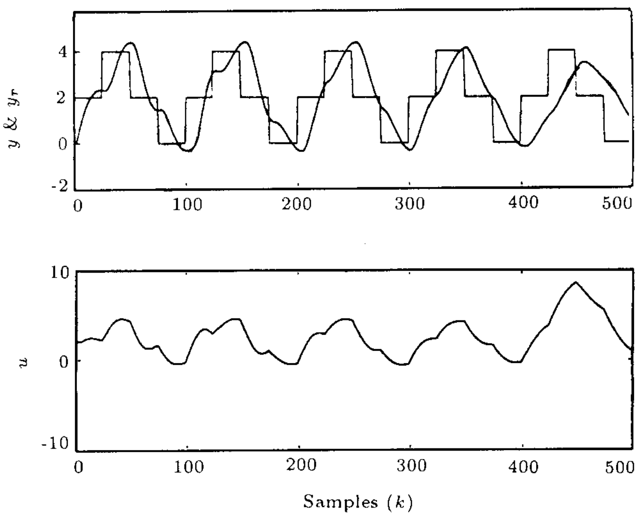

Figure 16. Closed loop response using scheme II ($\alpha = 0.27$).

15 to 18. As can be seen from the reults, scheme I has the best performance. The performance of scheme II is acceptable but it is more sensitive to the model mismatch. In order to decrease its sensitivity, the design parameter (α) should be increased which results in a sluggish response. The performances of schemes III and IV are not satisfactory.

EXPERIMENTAL RESULTS

To evaluate the performances of the aforementioned schemes, they are applied to a bench scale plate heat exchanger. The schematic diagram of the set-up is shown in Figure 19. The temperature of the exit stream $y(t)$ is controlled by adjusting the flow rate of hot water $u(t)$. The process can be modeled by a second order model plus dead time. For schemes I and II, the degrees of $B(q^{-1})$ and $A(q^{-1})$ polynomials are set equal to one and two respectively. The process time delay is assumed to be three. Experiments were

Table 3. Simulated process models.

Samples	Continuous Model	Discrete Model
0-100	$\frac{e^{-S}}{(3S+1)(3.53S+1)}$	$\frac{q^{-2}(0.0386+0.0314q^{-1})}{1-1.47q^{-1}+0.54q^{-2}}$
100-200	$\frac{e^{-4S}}{(3S+1)(3.53S+1)}$	$\frac{q^{-5}(0.0386+0.0314q^{-1})}{1-1.47q^{-1}+0.54q^{-2}}$
200-300	$\frac{e^{-4S}}{(6.53S+1)}$	$\frac{0.142q^{-5}}{1-0.86q^{-1}}$
300-400	$\frac{e^{-S}}{(6.53S+1)}$	$\frac{0.142q^{-2}}{1-0.86q^{-1}}$
400-500	$\frac{(0.5-S)e^{-S}}{(3S+1)(3.53S+1)}$	$\frac{q^{-2}(-0.05+0.08q^{-1})}{1-1.47q^{-1}+0.54q^{-2}}$

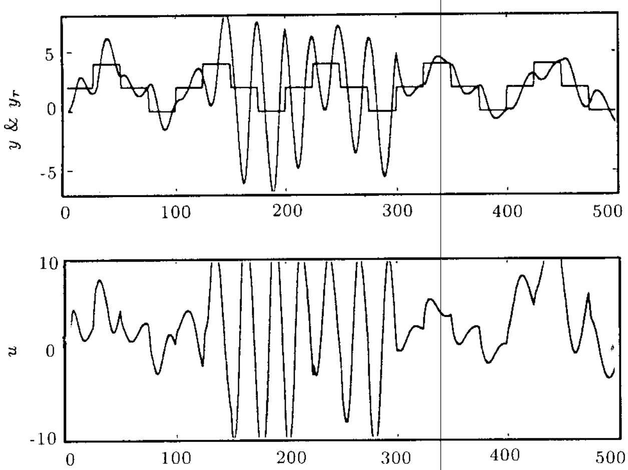

Figure 17. Closed loop response using scheme III ($\beta = 1.2$).

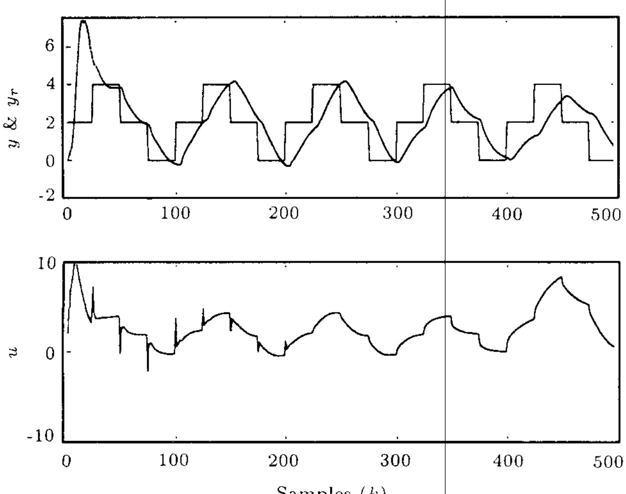

Figure 18. Closed loop response using scheme IV ($\gamma = 0.87$).

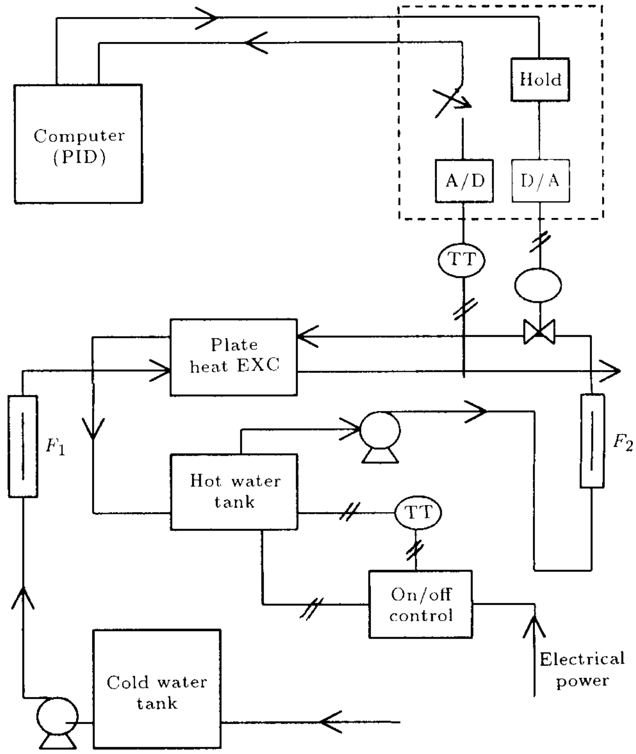

Figure 19. Experimental set-up.

performed with sampling rate of 10 seconds. Two set-point changes with magnitude of 15 and 10°C followed by a load change with magnitude of 100 cm^3/min in flow rate of cold stream were applied. The results are demonstrated in Figures 20 to 23. As can be seen from the results, scheme I has the best performance, scheme II has an acceptable performance and the performances of schemes III and IV are not satisfactory. These results are consistent with the results obtained through simulations.

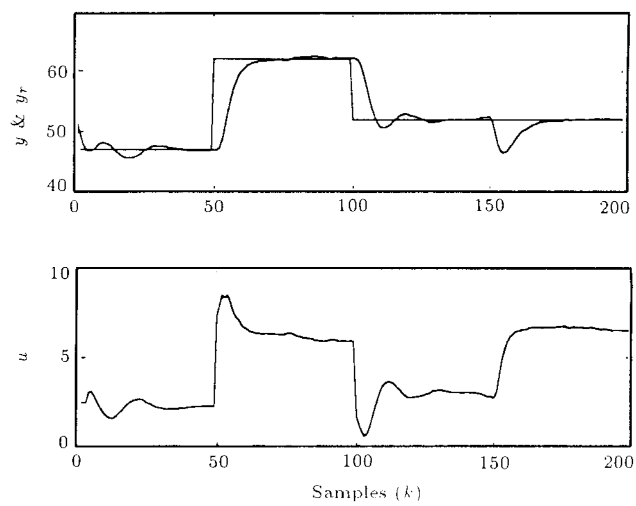

Figure 20. Experimental response using scheme I ($\phi_m = 65$).

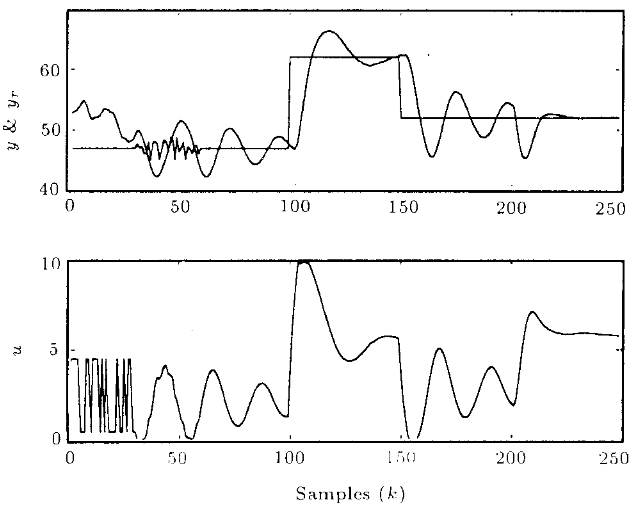

Figure 21. Experimental response using scheme II ($\alpha = 1.2$).

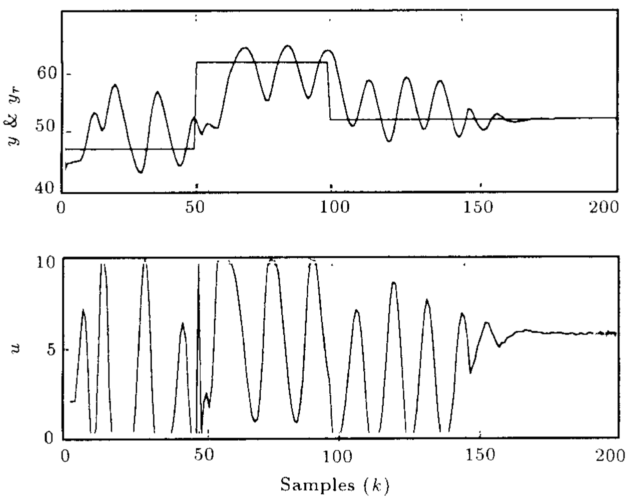

Figure 22. Experimental response using scheme III ($\beta = 1.2$).

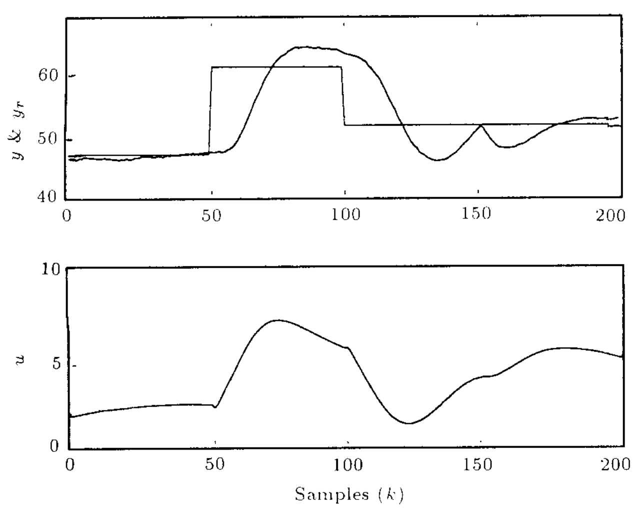

Figure 23. Experimental response using scheme IV ($\gamma = 0.4$).

CONCLUSION

In this article, the performances of four adaptive schemes are compared through simulation and experimental studies. Simulation results demonstrate that control scheme I has the best performance for overdamped, underdamped and non-minimum phase systems and is more robust to model mismatch. Scheme II has an acceptable performance, however, the performances of schemes III and IV are not satisfactory. Experimental study on a bench scale plate heat exchanger, which is an overdamped system, confirms the results obtained through computer simulations for such processes. Simulation results indicate that the performances of all schemes degrade as system poles become closer to the unit circle.

NOMENCLATURE

$A(.), B(.)$	polynomials in q^{-1}
a_i, b_i	coefficients of polynomials $A(.)$ and $B(.)$
d	time delay
$E(.)$	expected value
e	controller input ($e = y_r - y$)
$G(.)$	polynomial in q^{-1}
g_i	coefficients of polynomial $G(.)$
h	sampling time
m	degree of $B(.)$ polynomial
n	degree of $A(.)$ polynomial
q^{-1}	backward shift operator
S	Laplace transform variable
T_r	rise time
u	process input
y	process output
y_r	set-point
z	Z-transform variable

Greek Letters

α, β	weighting factor for the control efforts
γ	model reference pole
λ	forgetting factor
ϕ_m	phase margin
ν	bounded unmeasured disturbances

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