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Optimal Preview Control Design of an Active Suspension Based on a Full Car Model

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In this paper, an approach to the synthesis of optimal preview control laws for active vehicle suspension has been studied. A basic three-dimensional, seven-degree of freedom car riding model, subjected to two road inputs, is considered. To obtain an enhanced control scheme, the input senses and measures road irregularities, using the contactless sensors fixed on the vehicle front bumper. The suspension systems are optimized, with respect to ride comfort, road holding and suspension rattle space. The performance of active, active and delay and active and preview, are compared with a passive system by numerical simulation in the time domain. The results show that optimal preview control improves all aspects of system performance simultaneously, including acceleration of the vehicle, tire deflection and suspension rattle space, with less energy required to obtain that performance.

INTRODUCTION

Preview Control

Advanced suspension systems play a vital role in the performance of modern vehicles. These advanced suspensions are basically required to improve the compromise between many conflicting ride and handling measures of vehicle performance. They must support the vehicle body, keep the rider's comfort within permissible allowances, retain vehicle stability during various handling actions, control body and wheel attitude and minimize the vertical force variation of the road-to-tire contact. Ride comfort is related to reducing the acceleration of sprung mass that requires suspension forces to be small, while good road holding, which assures more stability needs large dynamic suspension forces.

The design of advanced suspension for vehicles is based on a large body of research. A large classified bibliography concerning the subject has been presented by Elbeheiry et al. [1]. Foag and Grubel [2] reflect that though passive suspensions have the advantages of being simple, rugged and not energy consuming, they are subject to important limitations. Further improvements in ride quality and handling performance can best be achieved by moving active suspensions. Arbitrary forces can be applied between the wheels and vehicle body through the use of actuators and passenger related benefits include the reduction of body movement in the vehicle's lateral, longitudinal and vertical directions. However, even with active suspension, the limitation remains that the servo control system must react very quickly to suppress disturbances that already have been encountered by the vehicle. In those cases where transients occur faster than the rate of response, some form of preparation strategy is clearly essential [3]. This preparation strategy implies the need for information describing disturbances before they are encountered by the vehicle. Use of this information has become known as preview control. The optimal preview control law consists of feed back and feed forward control terms. The feed back term is the same as that of the traditional LQ control algorithm and the feed forward term results from the preview input of the road. Recent developments in computer hardware and sensor and actuator technology, as well as decreasing their costs, have made the usage of active and preview suspension systems more practical and acceptable. Most research relating to optimal preview control of suspension systems, has considered two or four degrees of freedom for a quarter or half car model, respectively. A closer model to real life is a seven-

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degree of freedom model, studied in this research, which contains three-dimensional motion of the vehicle.

History

Early work in preview control relating to vehicles may be traced back to 1964 when Sheridan et al. [4] proposed two models for characterizing control schemes used by creatures in situations where preview information is available. Bender [5] proposed this idea and used the Wiener filter technique to find the optimal control law for a single degree of freedom vehicle model. This was the first detailed numerical investigation of the effectiveness of preview control on active vehicle suspension. Tomizuka [6-8] explained how preview control could be realized in practice. Sasaki et al. [9] and Iwata and Nakano [10] considered a two-dimensional half-car model, which is supported by front and rear suspension. With this degree of freedom model, both heave and pitch motion could be studied. Thompson et al. [11] used a state space approach and linear quadratic regulator theory and posed an optimal suspension design with a preview controller for a quarter model. Foag and Grubel [2] assumed a single scalar performance index as an adequate measure of system performance for a half-car model. Sharp [12] showed there is much to be gained from using the front wheels of a vehicle as a preview sensor for the rear wheels. Hrovat [13] concluded that preview control is most beneficial at higher speeds and/or on rougher roads. Yoshimura and Hosokawa [14] composed estimated state variables and the dynamic behavior of an irregular surface for the stochastic optimal control with preview. Nagiri et al. [15] confirmed preview effects on the response of the system experimentally, which can compensate the system delay of the active control suspension. Hac and Youn [16-18] formulated an analytical solution in a general form, using the effects of preview information on ride comfort, road holding, the working space of the suspension and the power requirements. Huisman et al. [19] applied a step function as road input and observed reduction in the working space and the maximum absolute acceleration of the sprung mass. Yeh and Tsao [20] studied a fuzzy preview control scheme and showed that satisfactory performance can be achieved. Sharp and Pilbeam [21] augmented a half-car model by a Pade filter to represent the wheel base travel time and established control laws, which minimized cost indices containing various weighting constants. Araki et al. [22] pointed out that front wheels are useful in estimate the disturbances, which are applied from the road surface to the rear wheels. Porkop and Sharp [23] presented a refined approach for the implementation of preview control in discrete time and used multiple preview inputs. Senthil and Norayanan [24] applied an optimal preview control algorithm to a two-degree of freedom model travelling with constant velocity on a randomly profiled road, which is modeled as a homogenous random process, being the output of a linear first order filter to white noise. Pilbeam and Sharp [25] showed that optimal preview control requires less power consumption. El-Demerdash and Crolla [26] studied the effects of component non-linearities on the ride performance of a hydro-pneumatic slow-active suspension and used the Pade approximation technique to represent the preview time. Van der Aa and Muijderman [27] considered two approaches to incorporate constraints on damper range, tire force and suspension travel in the design of controllers, to minimize peak values in the chassis acceleration. Mehra et al. [28] showed model predictive control, utilizing provided road information and incorporating all hard constraints on state, control and output variables. Thompson and Pearce [29] described the effects of preview on the performance for theoretical step type road input. Also, there are some other activities in this field, which have been studied over the past two decades.

SYSTEM MODEL AND PERFORMANCE CALCULATIONS

Mathematical Model

A seven-degree of freedom vehicle with controllable suspension is used in this study. A schematic representation of vehicle model suspension arrangements is shown in Figure 1.

In this paper, 3 degrees of freedom for the sprung mass (bouncing, rolling and pitching) and 4 degrees of freedom for the front and rear unsprung masses for their vertical motion are considered. The four actuators placed between the sprung mass, M_s, and the unsprung mass, M_{ui}, produce control force $u_i (i = 1, \cdots, 4)$, respectively. The dynamics of the actuators are neglected. I_{xx} and I_{yy} are moments of inertia of the sprung mass with respect to the central longitudinal and transversal axis, respectively. The left front and rear tire stiffness are denoted by k_{u1} and k_{u2} and for the right side are k_{u3} and k_{u4}. The damping effect of tires is negligible. Parameters k_{s1}, k_{s2} and c_{s1}, c_{s2} denote the left stiffness and damping ratios of passive suspension elements for the front and rear assemblies. k_{s3}, k_{s4} and c_{s3}, c_{s4} are similar parameters for the right side. These elements are included since they substantially reduce the active control forces and assure vehicle operation in the event of active system failure.

The variables $x_{01}, \cdots, x_{04}$ denote road irregularities under four wheels. It is assumed that, as in most practical situations, the rear tires travel over the same path as the front tires; hence, x_{02} is a delayed version of x_{01}, $x_{02} = x_{01}(t - \tau)$, and x_{04} is a delayed version

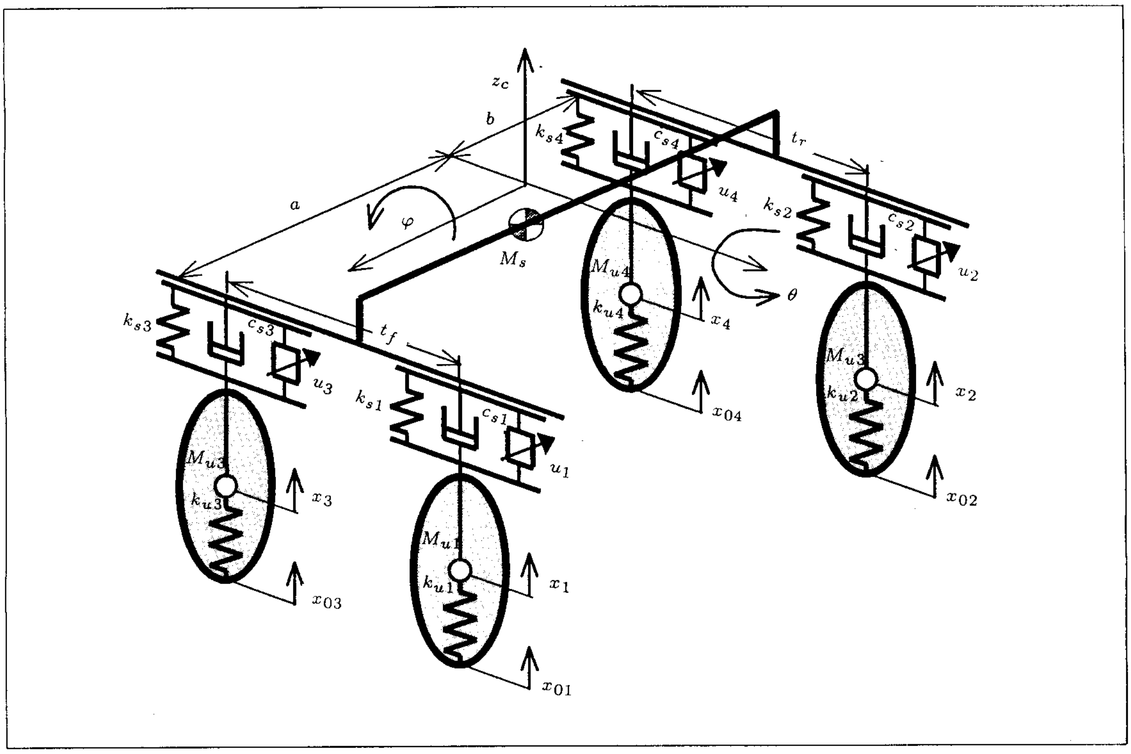

Figure 1. A seven-degree of freedom vehicle model.

of x_{03}, $x_{04} = x_{03}(t-\tau)$ where $\tau = \nu/(a+b)$, ν is the forward vehicle speed, and a+b is the vehicle wheelbase. Variables x_1 through x_4 are the vertical displacements of the front and rear wheels related to left and right sides. The body motion is described by the vertical position of the center of mass (bounce) z_c, the angle of rotation about the transversal axis (pitch) θ and the angle of rotation about the longitudinal axis (roll) φ, which are denoted by x_5, x_6 and x_7, respectively. The variables x_1 through x_7 are being considered with respect to their static equilibrium positions.

Assuming that the pitch and roll motions are small and the characteristics of all passive suspension elements are linear, the equations of motion can be evaluated by application of Newton's second law and its simplification as Euler's equation of motion for a rigid body [30] as follows:

$$\begin{aligned} &M_{u1}\ddot{x}_1 + c_{s1}\dot{x}_1 + (k_{u1}+k_{s1})x_1 - k_{u1}x_{01} \\ &\quad - c_{s1}(\dot{x}_5 - a\dot{x}_6 + 0.5t_f\dot{x}_7) \\ &\quad - k_{s1}(x_5 - ax_6 + 0.5t_f x_7) + u_1 = 0, \end{aligned} \tag{1}$$

$$\begin{aligned} &M_{u2}\ddot{x}_2 + c_{s2}\dot{x}_2 + (k_{u2}+k_{s2})x_2 - k_{u2}x_{02} \\ &\quad - c_{s2}(\dot{x}_5 - b\dot{x}_6 + 0.5t_r\dot{x}_7) \\ &\quad - k_{s2}(x_5 + bx_6 + 0.5t_r x_7) + u_2 = 0, \end{aligned} \tag{2}$$

$$\begin{aligned} &M_{u3}\ddot{x}_3 + c_{s3}\dot{x}_3 + (k_{u3}+k_{s3})x_3 - k_{u3}x_{03} \\ &\quad - c_{s3}(\dot{x}_5 - a\dot{x}_6 + 0.5t_f\dot{x}_7) \\ &\quad - k_{s3}(x_5 - ax_6 + 0.5t_f x_7) + u_3 = 0, \end{aligned} \tag{3}$$

$$\begin{aligned} &M_{u4}\ddot{x}_4 + c_{s4}\dot{x}_4 + (k_{u4}+k_{s4})x_4 - k_{u4}x_{04} \\ &\quad - c_{s4}(\dot{x}_5 - b\dot{x}_6 + 0.5t_r\dot{x}_7) \\ &\quad - k_{s4}(x_5 - bx_6 + 0.5t_r x_7) + u_4 = 0, \end{aligned} \tag{4}$$

$$\begin{aligned} &M_s\ddot{x}_5 + (c_{s1}+c_{s2}+c_{s3}+c_{s4})\dot{x}_5 \\ &\quad + (k_{s1}+k_{s2}+k_{s3}+k_{s4})x_5 \\ &\quad + (-ac_{s1}+bc_{s2}-ac_{s3}+bc_{s4})\dot{x}_6 \\ &\quad + (-ak_{s1}+bk_{s2}-ak_{s3}+bk_{s4})x_6 - c_{s1}\dot{x}_1 \\ &\quad - c_{s2}\dot{x}_2 - c_{s3}\dot{x}_3 - c_{s4}\dot{x}_4 - k_{s1}x_1 - k_{s2}x_2 - k_{s3}x_3 \\ &\quad - k_{s4}x_4 - (u_1+u_2+u_3+u_4) = 0, \end{aligned} \tag{5}$$

$$\begin{aligned} &I_{yy}\ddot{x}_6 + (-ac_{s1}+bc_{s2}-ac_{s3}+bc_{s4})\dot{x}_5 \\ &\quad + (-ak_{s1}+bk_{s2}-ak_{s3}+bk_{s4})x_5 \\ &\quad + (a^2c_{s1}+b^2c_{s2}+a^2c_{s3}+b^2c_{s4})\dot{x}_6 \\ &\quad + (a^2k_{s1}+b^2k_{s2}+a^2k_{s3}+b^2k_{s4})x_6 \\ &\quad + ac_{s1}\dot{x}_1 - bc_{s2}\dot{x}_2 + ac_{s3}\dot{x}_3 - bc_{s4}\dot{x}_4 \\ &\quad + ak_{s1}x_1 - bk_{s2}x_2 + ak_{s3}x_3 - bk_{s4}x_4 \\ &\quad + au_1 - bu_2 + au_3 - bu_4 = 0, \end{aligned} \tag{6}$$

$$I_{xx}\ddot{x}_7 + 0.25(t_f^2 c_{s1} + t_r^2 c_{s2} + t_f^2 c_{s3} + t^2 c_{s4})\dot{x}_7 + 0.25(t_f^2 k_{s1} + t_r^2 k_{s2} + t_f^2 k_{s3} + t_r^2 k_{s4})x_7 + 0.5(-t_f c_{s1}\dot{x}_1 - t_r c_{s2}\dot{x}_2 + t_f c_{s3}\dot{x}_3 + t_r c_{s4}\dot{x}_4) + 0.5(-t_f k_{s1} x_1 - t_r k_{s2} x_2 + t_f k_{s3} x_3 + t_r k_{s4} x_4) + 0.5(-t_f u_1 - t_r u_2 + t_f u_3 + t_r u_4) = 0, \tag{7}$$

where:

- x_1 vertical displacement of the left, front unsprung mass
- x_2 vertical displacement of the left, rear unsprung mass;
- x_3 vertical displacement of the right, front unsprung mass;
- x_4 vertical displacement of the right, rear unsprung mass;
- x_5 vertical displacement of the center of gravity of unsprung mass;
- x_6 pitch motion of the sprung mass;
- x_7 roll motion of the sprung mass.

Introducing the following state, the control input and disturbance input vectors:

$$x = [x_1, x_2, \cdots, x_{14}]^T, \quad u = [u_1, u_2, u_3, u_4]^T, \quad w = [x_{01}, x_{02}, x_{03}, x_{04}]^T, \tag{8}$$

with defining x_8 through x_{14} as the derivatives of x_1 through x_7, Equations 1 to 7 can be represented in the form of the state equations:

$$\dot{x} = Ax + Bu + Ew, \tag{9}$$

where A, B and E are constant matrices of dimensions $14 \times 14, 14 \times 4$, and 14×4, respectively.

Performance Index

The objective of the control is to improve the vehicle dynamical performance expressed in terms of ride comfort, running stability and traction characteristics. Ride comfort is related to car body acceleration that should be kept low. Some physical effects due to vibration are: Increasing oxygen uptake, pulmonary ventilation rate, increasing heart rate and increasing blood pressure.

It has been observed that in three directions i.e., up and down, side-to-side and front-to-rear, the vibrations should be decreased in order to reach ride comfort, although their effects on the human body are different.

For running stability or good handling characteristics, tire deflection, which is proportional to the dynamic tire-road contact force, should be small. In a full car model, all four wheels are being considered. Control forces are usually supplied by hydraulic actuators. It is desired to use lower power consumption for active control, so the control forces are considered in the performance index.

Consequently, the design of a vehicle suspension involves a compromise among conflicting goals. Optimizing with respect to ride comfort, suspension rattle space and road holding, the performance index to be minimized can be written as follows:

$$J = \lim \frac{1}{2T}\int_0^T \begin{bmatrix} 1 \\ \ddot{z}_c \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 \\ 0 & 0 & 0 & \rho_3 \end{bmatrix} \begin{bmatrix} 1 \\ \ddot{z} \\ \ddot{\theta} \\ \ddot{\varphi} \end{bmatrix} + \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}^T \begin{bmatrix} \rho_4 & 0 & 0 & 0 \\ 0 & \rho_5 & 0 & 0 \\ 0 & 0 & \rho_6 & 0 \\ 0 & 0 & 0 & \rho_7 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix}^T \begin{bmatrix} \rho_8 & 0 & 0 & 0 \\ 0 & \rho_9 & 0 & 0 \\ 0 & 0 & \rho_{10} & 0 \\ 0 & 0 & 0 & \rho_{11} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}^T \begin{bmatrix} \rho_{12} & 0 & 0 & 0 \\ 0 & \rho_{13} & 0 & 0 \\ 0 & 0 & \rho_{14} & 0 \\ 0 & 0 & 0 & \rho_{15} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, \tag{10}$$

where:

$$\ddot{z} = \ddot{x}_5 = \dot{x}_{12}, \quad \ddot{\theta} = \ddot{x}_6 = \dot{x}_{13}, \quad \ddot{\varphi} = \ddot{x}_7 = \dot{x}_{14}, \tag{11}$$

and:

$$s_1 = x_5 - x_1 + 0.5t_f x_7 - a x_6,$$
$$s_2 = x_5 - x_2 + 0.5t_r x_7 + b x_6$$
$$s_3 = x_5 - x_3 - 0.5t_f x_7 - a x_6,$$
$$s_4 = x_5 - x_4 - 0.5t_r x_7 + b x_6,$$
$$t_1 = x_1 - x_{01}, \quad t_2 = x_2 - x_{02},$$
$$t_3 = x_3 - x_{03}, \quad t_4 = x_4 - x_{04}, \tag{12}$$

and ρ_1 through ρ_{15} are the weighting constants reflecting the designer's preferences; several sets of weights can be used depending on the conditions of motion, such as velocity, road quality, vibration level acceleration, etc. In addition, with regard to unit parameters that are used in performance index (Equation 10), the values of weighting constants should be balanced. Two more generally used sets are road holding and ride comfort, which will be considered in this study. For expressing the performance index in a form that is quadratic in the state and input vectors, it is required to substitute the acceleration $\ddot{z}$, $\ddot{\theta}$ and $\ddot{\varphi}$ in Equation 10 using state Equation 9 and Notation 11 to have:

$$J = \lim_{T\to\infty} \frac{1}{2T} \int_0^T (x^T Q_1 x + 2x^T N u + u^T R u + 2x^T Q_{12} w + w^T Q_2 w) dt. \tag{13}$$

Q_1, R and Q_2 are symmetric, time-invariant weighting matrices and R is also positive definite. A symbolic computer program has been written to evaluate the constant matrices Q_1, N, R, Q_{12} and Q_2 with dimensions $14\times 14, 14\times 4, 4\times 4, 14\times 4$ and 4×4, respectively, because of the many algebraic calculations.

OPTIMAL PREVIEW CONTROL FOR SUSPENSION SYSTEMS

In this section, the optimal preview control problem for active suspension is formulated. A measure of system performance is important in using a linear optimal control theory. This performance measure takes the form of a quadratic cost function containing terms relating to the dynamic tire load variation, suspension working space, body acceleration, actuator activity, etc. Two cases will be considered for the performance index and state equations in the following.

In both cases, it is assumed that the road input $w(\tau)$ for $\tau \in [t, t+t_p]$, i.e., the preview information about $w(t)$ up to t_p time units ahead of t, is available.

The performance index is of the Form 13. The method of finding a continuous time optimal preview control law is given in [16,17]. The main result from [18] used here for a full car model with four inputs is as follows.

Theorem 1

Assume a system with state space Equation 9 and with preview time t_p, i.e., $w(\sigma), \sigma \in [t+t_p]$. The problem is to find a control law $u(t) = f\big(x(t), w(\sigma), \sigma \in [t+t_p]\big)$ that minimizes the quadratic performance index (Equation 13) with $w_2(t) = w_1(t-\tau)$ and $w_2(t) = w_2(t-\tau), \tau$ given. Let now consider the following notation:

$$A_n = A - BR^{-1}N^T, \quad Q_n = Q_1 - NR^{-1}N^T, \tag{14}$$

and assume that Q_n is nonnegative definite and factoring Q_n such that $Q_n = T^T T$; then, if the pair (A_n, B) is stabilizable and the pair (A_n, T) is detectable, the optimal preview control is given by:

$$u(t) = -R^{-1}\Big[(N^T + B^T P)x(t) + B^T r(t)\Big], \tag{15}$$

where P is the positive definite solution of the algebraic Riccati equation:

$$PA_n + A_n^T P - PBR^{-1}B^T P + Q_n = 0. \tag{16}$$

The vector $r(t)$ is given by:

$$\dot{r}(t) = -A_c^T r(t) - \big(PD + Q_{12}\big) w(t), \quad r(T=\infty) = 0, \tag{17}$$

where:

$$A_c = A - BR^{-1}(N^T + B^T P) = A_n - BR^{-1}B^T P, \tag{18}$$

is the closed loop system matrix and asymptotically stable. Proof of this result has been presented in [18].

Theorem 2

The performance index, which is required to be minimized, is:

$$J = \lim_{T\to\infty} \frac{1}{2T} \int_0^T \big(y^T Q_1 y + 2x^T N u + u^T R u + 2x^T Q_{12} w + w^T Q_2 w\big) dt, \tag{19}$$

under the constraints:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t), \quad x(t) = x_t \\ y(t) &= cx(t) + Du(t) + Fv(t). \end{aligned} \tag{20}$$

This case occurs when some errors exist in the output measurements and is a general form of linear state equations. Knowing the road surface in the time interval $[t, t+t_P]$, the problem is finding a control law $u(t)$ that minimizes the performance index (Equation 19) for the system described by Equation 20. The solution with its proof is presented in Appendix A.

From Equation A12 it is seen that the input u is composed of a feedback, a feed forward and the measurement error term. The feedback term is exactly the same as that of the traditional LQ control algorithm when no preview is available. The feed forward term has preview information, with respect to road input from the present time t up to t_P time units beyond t. The measurement error term is created by the instruments that measure the output parameters. A schematic block is illustrated in Figure 2, where:

$$K_1 = -R_d^{-1}\Big[\big(B^T P + D^T Q_1 C + N^T\big)\Big],$$

$$K_2 = -R^{-1}B^T,$$

$$K_3 = -R_d^{-1} D^T Q_1 F,$$

and:

$$R_d = R + D^T Q_1 D. \tag{21}$$

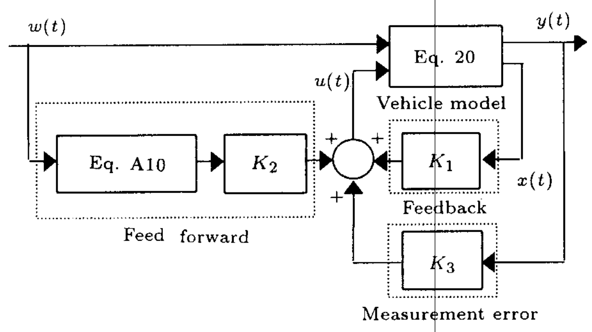

Figure 2. The three terms involved in calculating the input vector u.

NUMERICAL SIMULATION AND ANALYSIS

Full Vehicle Model Preview Control

The vehicle model used in this study is a seven-degree of freedom full car model defined in the previous section and shown in Figure 1. The numerical data used for the vehicle model simulation are given in Table 1, which was also used by Chalasani [31].

To illustrate the adaptive capability of an active suspension, two sets of weighting constants in the performance index (Equation 13) were used. They are given in Table 2, together with the closed loop system poles. The first set of weighting corresponds to a design in which ride comfort is preferred to road holding, while, in the second case, road holding is more heavily weighted. It can be seen that in the second case, the system poles have larger magnitudes and are shifted further to the left in the complex plane, resulting in a stiffer, faster responding suspension.

The type of road to be considered is a hole followed by a bump. It is applied to a vehicle in two cases to observe the effects of vehicle responses. First, the vehicle passes over it on both sides and second, the condition when one side (say the left side of the vehicle) has conflict with the hole and bump.

Figure 3 shows the effect of preview time on the performances of an active suspension, in which the time

Table 1. Parameter values of the vehicle car model.

Sprung Mass		Unsprung Mass	
Parameters	**Values**	**Parameters**	**Values**
M_s	1400 kg	M_{ul}, M_{u3}	40.0 kg
I_{xx}	460 kg.m^2	M_{u2}, M_{u4}	35.5 kg
I_{yy}	2460 kg.m^2	k_{s1}, k_{s3}	19960 N/m
t_f	1.522 m	k_{s2}, k_{s4}	17500 N/m
t_r	1.510 m	c_{s1}, c_{s3}	1290 N.s/m
a	1.011 m	c_{s2}, c_{s4}	1620 N.s/m
b	1.803 m	$k_{u1} \sim k_{u4}$	175500 N/m

Table 2. Weighting constants for both design and the corresponding closed loop system poles.

Ride Comfort			Road Holding		
Weighting Constants		**Eigenvalues**	**Weighting Constants**		**Eigenvalues**
1	3	-28.22+72.25i	1	3	-67.17+94.50i
2	8	-28.22 -72.25i	2	8	-67.17 -94.50i
3	1	-10.77+67.53i	3	1	-33.24+74.65i
4	1200	-10.77 -67.53i	4	18000	-33.24 -74.65i
5	1400	-9.23+70.13i	5	19500	-25.55+71.73i
6	1200	-9.23 -70.13i	6	18000	-25.55 -71.73i
7	1400	-8.59+67.64i	7	19500	-25.26+73.56i
8	12000	-8.59 -67.64i	8	200000	-25.26 -73.56i
9	11000	-5.31+6.14i	9	195000	-8.07+9.80i
10	12000	-5.31 -6.14i	10	200000	-8.07 -9.80i
11	11000	-4.00+5.07i	11	195000	-7.14+8.76i
12	1e-6	-4.00 -5.07i	12	1e-6	-7.14 -8.76i
13	1e-6	-3.51+4.52i	13	1e-6	-5.48+7.09i
14	1e-6	-3.51 -4.52i	14	1e-6	-5.48 -7.09i
15	1e-6		15	1e-6	

delay between front and rear axles was accounted for in the controller design. All values are measured in terms of the mean square and, relative to these, of an active suspension without preview and are simulated up to 0.2 seconds. Also different preview time can mean different vehicle velocity.

It is important to notice that sufficient preview improves all aspects of system performance simultaneously. In view of input type and design preferences relating to ride comfort and road holding, two cases have been considered in Figures 3a and b. About 0.1 sec for input to one side of the preview has been required to achieve most of the possible benefits. The performance index and its components for 0.12 sec preview can be rationally calculated to let their achieve significant performance improvement.

To facilitate a direct comparison among various cases, the components of the performance index, containing suspension rattle space, tire deflection and acceleration of the vehicle in three dimensions, were evaluated. These conditions are active, active and delay, active and preview, which were normalized by dividing them by the corresponding values obtained for the passive system. The results are collected in Table 3 for the four above cases. The preview time was considered 0.12 sec. Incorporation of time delay between the front and rear wheels was evaluated by comparing it with the active case and the active with the preview case. The performance index of each

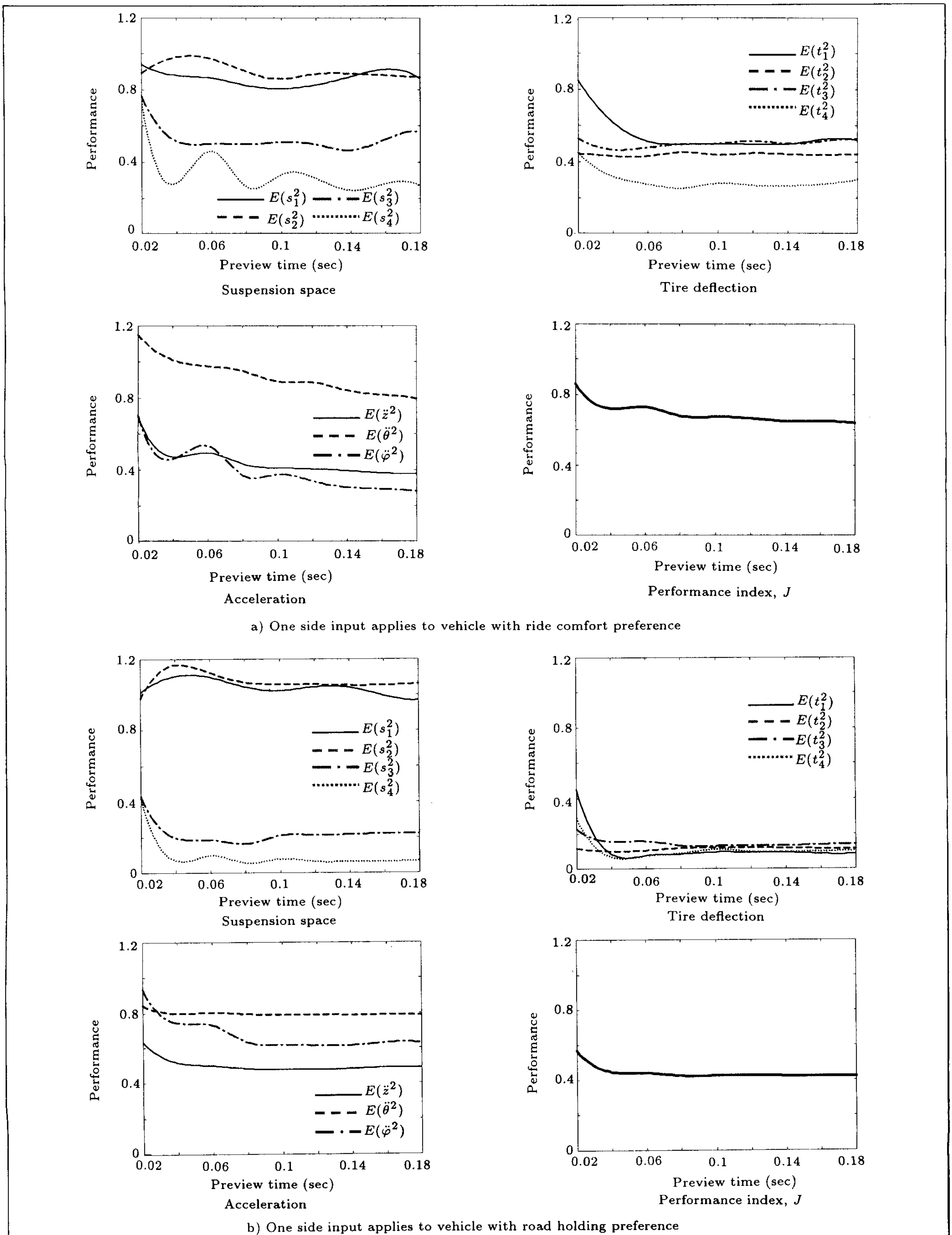

Figure 3. The effect of preview time on the relative preference where the active system corresponds to a value of 1.

Table 3. Components of performance index (considering the passive system, corresponds to 100 percent).

a) Both Sides Input, Ride Comfort												
System	$E(\ddot{z}_c^2)$	$E(\ddot{\theta}^2)$	$E(\ddot{\varphi}^2)$	$E(s_1^2)$	$E(s_2^2)$	$E(s_3^2)$	$E(s_4^2)$	$E(t_1^2)$	$E(t_2^2)$	$E(t_3^2)$	$E(t_4^2)$	J
Passive	100	100	100	100	100	100	100	100	100	100	100	100
Active	49.6	47.5	100	111.0	141.0	111.0	1370.0	149.0	245.0	159.0	245.0	72.7
Active & delay	21.1	52.4	100	108.0	103.0	114.0	105.0	156.0	96.2	159.0	101.0	58.9
Active & preview	20.3	39.4	100	78.3	98.3	84.6	100.0	66.6	93.3	65.7	100.0	48.3
b) Both Sides Input, Road Holding												
System	$E(\ddot{z}_c^2)$	$E(\ddot{\theta}^2)$	$E(\ddot{\varphi}^2)$	$E(s_1^2)$	$E(s_2^2)$	$E(s_3^2)$	$E(s_4^2)$	$E(t_1^2)$	$E(t_2^2)$	$E(t_3^2)$	$E(t_4^2)$	J
Passive	100	100	100	100	100	100	100	100	100	100	100	100
Active	176.0	128.0	100	73.6	90.3	74.0	91.7	72.7	83.0	73.1	82.1	98.1
Active & delay	121.0	93.5	100	71.1	68.5	73.9	67.6	73.7	8.9	73.5	9.7	67.7
Active & preview	68.1	72.9	100	61.7	60.9	63.9	61.1	5.8	9.4	5.5	10.2	37.5
c) One Side Input, Ride Comfort												
System	$E(\ddot{z}_c^2)$	$E(\ddot{\theta}^2)$	$E(\ddot{\varphi}^2)$	$E(s_1^2)$	$E(s_2^2)$	$E(s_3^2)$	$E(s_4^2)$	$E(t_1^2)$	$E(t_2^2)$	$E(t_3^2)$	$E(t_4^2)$	J
Passive	100	100	100	100	100	100	100	100	100	100	100	100
Active	50.7	47.9	31.2	101.0	123.0	212.0	154.0	126.0	140.0	550	482	62.8
Active & delay	21.4	53.0	12.0	99.4	109.0	99.3	71.4	131.0	64.8	263	221	48.8
Active & preview	20.9	39.6	10.2	79.2	104.0	127.0	48.6	64.1	64.2	335	141	40.8
d) One Side Input, Road Holding												
System	$E(\ddot{z}_c^2)$	$E(\ddot{\theta}^2)$	$E(\ddot{\varphi}^2)$	$E(s_1^2)$	$E(s_2^2)$	$E(s_3^2)$	$E(s_4^2)$	$E(t_1^2)$	$E(t_2^2)$	$E(t_3^2)$	$E(t_4^2)$	J
Passive	100	100	100	100	100	100	100	100	100	100	100	100
Active	177.0	128.0	134.0	59.4	71.9	172.0	197.0	74.4	67.2	329	344.0	97.7
Active & delay	122.0	93.2	81.4	52.5	67.7	149.0	85.5	62.5	12.4	130	192	63.9
Active & preview	68.5	73.0	53.3	59.1	72.6	38.1	11.2	6.0	6.8	416.0	330.0	34.5

case, for three conditions with respect to passive, is calculated and shown in the last columns of Table 3. The vehicles transient responses to a hole in the road followed by a bump were simulated. In order to show a presentation of wheel tracking performance; ride comfort and road holding preferences are drawn in Figures 4 and 5.

When both sides of the vehicle pass over similar inputs, the reaction of the left and right side of the vehicle is approximately similar. But when one side (say the left) encounters road input, the reaction of the vehicle for left and right sides is different. Figures 4 and 5 show the response of the left side of the vehicle when its left wheels pass over a hole/ bump input. In order to see the energy consumption, the control forces for all mentioned cases are plotted in Figures 6 and 7. All the above curves are simulated up to 1.2 sec, i.e., suitable for preview time consideration.

Results and Analysis

The advantages of preview control can be seen from the curves and tables presented in the above section. With the help of Figure 3, that shows the effects of preview time on preview control, it can be deduced that, usually, a minimum preview time is required to achieve most of the possible benefits. The effective preview time for ride comfort is about 0.1 sec while about 0.04 sec is enough for road holding, which is due to larger magnitude of road part of the eigenvalues

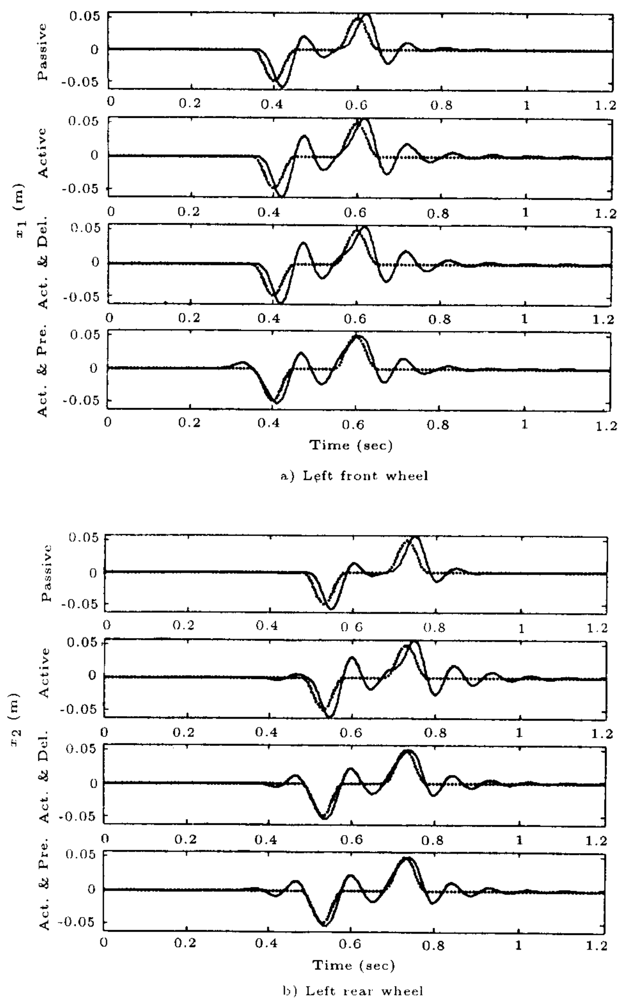

Figure 4. Wheel tracking performance for left side input with ride comfort preference (solid line) and road input (dotted line).

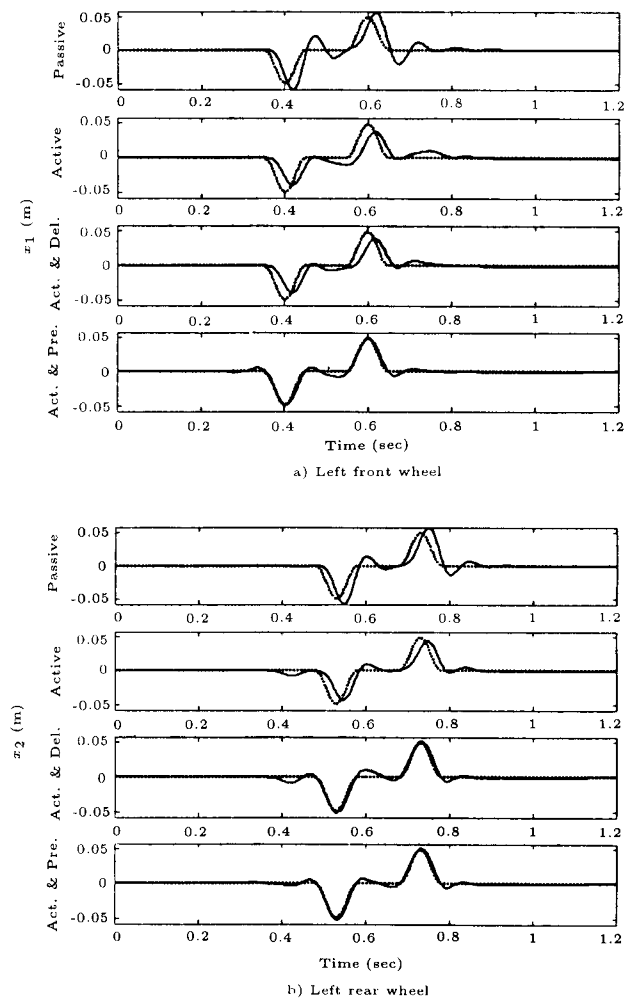

Figure 5. Wheel tracking performance for left side input with road holding preference (solid line) and road input (dotted line).

related to road holding as shown in Table 2. On the other hand, the preview time larger than 0.2 sec does not considerably assist the components of performance index to improve their properties.

For maximum performance, tire deflection needs less preview time than acceleration; about 0.15 for acceleration in the ride comfort and 0.08 sec in the road holding case, while 0.05 sec is enough for deflection in the two cases. It can be asserted that the preview information improves performance index and approximately all its elements.

When one attends to the results collected in Table 3, it is noted that optimization in the case of ride comfort has better results than road holding on the acceleration of the sprung mass. It means $\ddot{\theta}, \ddot{\varphi}$ and $\ddot{z}$ have been reduced more with active control of the suspension system. On the contrary, tire deflection and suspension act more effectively in road holding than ride comfort. The vehicle transient responses to a hole in the road followed by a bump, simulated in Figures 4 and 5, show that optimal control for both types of input improves from active control to active control with delay and then to control with preview, respectively, with respect to the passive system. In total, the performance index optimized better in the road holding in all three active controls, rather than ride comfort. Also, it can be seen from Table 3 and Figures 4 and 5, that active control with preview has suitable results for approximately all acceleration directions, tire deflection and suspension space for both types of input. Considering the input type, it can be mentioned that $\ddot{\varphi}$ has a better reaction for a one side input, which is almost predictable because of the reduction of acceleration about the longitudinal axis in this type of input. Tire deflection for a one side input increases at the opposite side of an applying force but, nevertheless, has a small magnitude. The suspension space reduction is better at the opposite side of an applying force for one side input rather than for two sides input. Consequently, optimization for

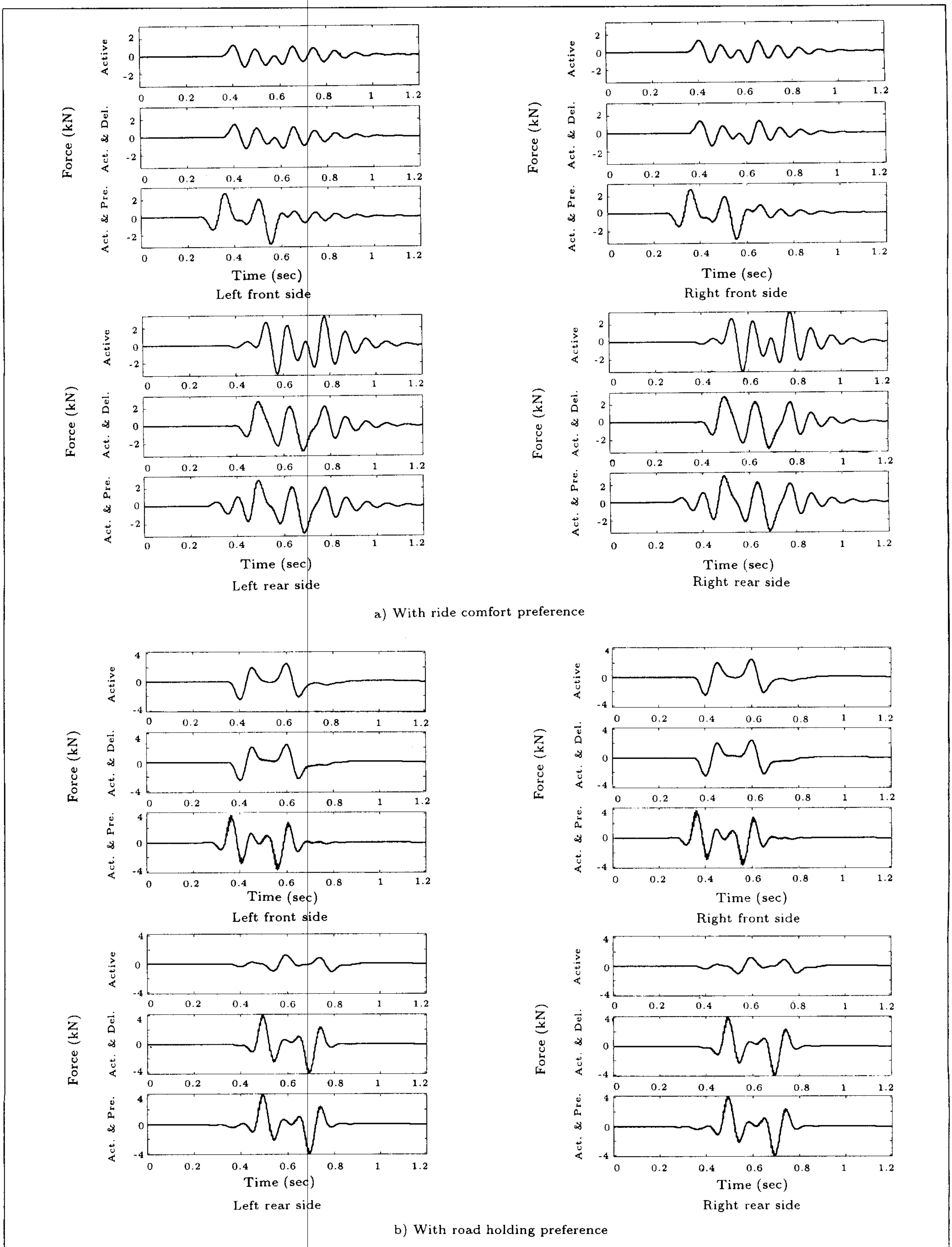

Figure 6. Active control forces for both sides road input.

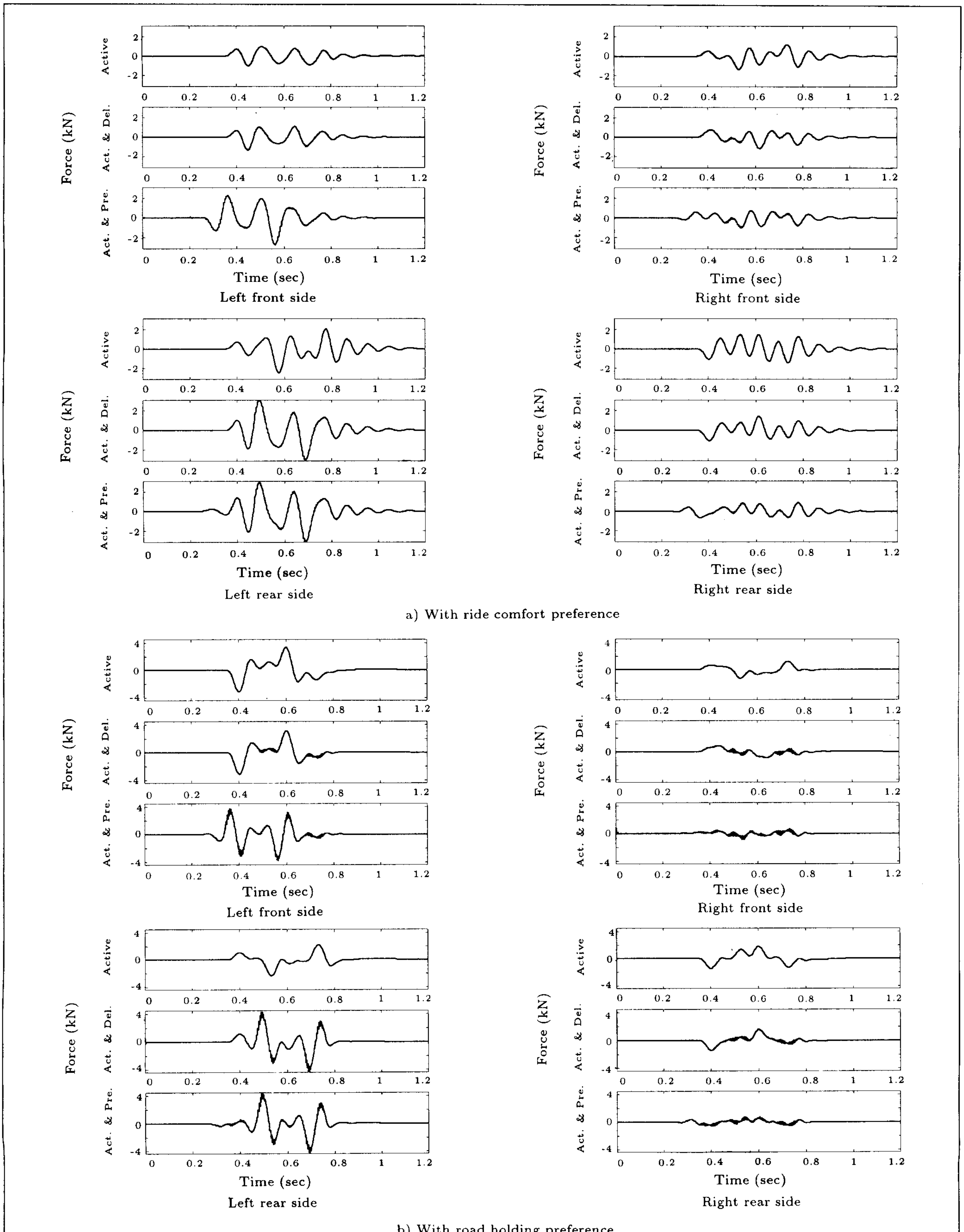

Figure 7. Active control forces for left side road input.

active control with preview has better results for one side input that is more practical than for two sides input.

Control force properties obtained for ride comfort and road holding are plotted in Figures 6 and 7. It is observed that less control force in a greater period of time is needed in ride comfort than in road holding. Also, the type of input is effective on the rate of control force. When the input is applied to one side, less force is needed at the opposite side than at the side of the applying force. In all cases, the rate control force applied to rear wheels is more than that for the front wheels. So, stronger or stiffer actuators are needed for rear wheels.

CONCLUSION

In this study, a linear optimal preview control problem of vehicle suspension for a full car model has been formulated and solved as a continuous time problem. The approach is based on the linear quadratic control theory and takes into account the road input information a little time ahead of the front wheels and the time delay between the front and rear wheels. It yields a full state feedback control law and feed forward information to optimize the performance of the suspension system.

The performance index and its components of the system were evaluated in the time domain for a seven-degree of freedom model showing good potential preview in improving almost all aspects of system performance. While traveling on a hole and bump profiled road, the presence of preview information reduces body acceleration, suspension working space, tire deflection and control force at the same time.

Numerical simulation has shown that the results improve from active control to active control with delay and then to the active control with preview, respectively, for two cases of input: one side and two sides inputs.

Furthermore, the simulation shows that optimal performance is achieved with a minimum preview time depending on the components, preferences and type of input. Also, some greater preview time doesn't establish any improvement on the mentioned properties.

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APPENDIX

Proof of Theorem

Calculus of variation is used. The Hamiltonian H for this problem is:

$$H = \frac{1}{2}y^T Q_1 y + x^T N y + \frac{1}{2}u^T R u + x^T Q_{12}\nu + \frac{1}{2}\nu^T Q_2 \nu + \lambda^T(Ax + Bu + E\nu), \tag{A1}$$

where $\lambda(t) \in R^n$ is a vector of Lagrange multipliers . The necessary conditions for the optimum are:

$$H_u = 0, \quad H_x = -\dot{\lambda}, \quad \lambda(T) = P_T(x^T),$$

(where H_u, H_x denote gradients of H with respect to u, x, respectively). The minimization of the performance index under Constraints 20 results in the following equations:

$$u(T) = -R_d^{-1}\left[B^T\lambda(t) + (D^T Q_1 C + N^T)x(t) + D^T Q_1 F w(t)\right], \tag{A2}$$

$$\dot{\lambda}(t) = -A_d^T\lambda(t) - C_d x(t) - f_\lambda(t) - Q_{12}\nu(t),$$

$$\lambda(t + t_p) = 0, \tag{A3}$$

$$\dot{x}(t) = A_d x(t) - B_d\lambda(t) + f_x(t), \quad x(t) = x_t, \tag{A4}$$

where A_d, B_d, C_d, f_λ and f_x are defined by:

$$R_d = R + D^T Q_1 D,$$

$$A_d = A - BR_d^{-1}(D^T Q_1 C + N^T),$$

$$B_d = BR_d^{-1}B^T,$$

$$Q_d = Q_1 - Q_1 D R_d^{-1} D^T Q_1,$$

$$N_d = C^T Q_1 D R_d^{-1} + N R_d^{-1} D^T Q_1 C + N R_d^{-1} N^T,$$

$$C_d = C^T Q_d C - N_d,$$

$$f_\lambda = (C^T Q_d - N R_d^{-1} D^T Q_1)Fw,$$

$$f_x = E\nu - BR_d^{-1}D^T Q_1 F w. \tag{A5}$$

To solve the linear structure of Equations A3 and A4, a new vector is introduced, defined by:

$$r(t) = \lambda(t) + Px(t), \tag{A6}$$

where $P(T) = P_T, r(T) = 0, P(t) \in R^{n\times n}$ and $r(t) \in R^n$ is a vector dependent on $\nu(t)$. This assertion can be strictly verified by treating Equations A3 and A4 as a one state equation with the state vector $col[x, \lambda]$, writing the solution in terms of the transmission matrix and eliminating $\lambda(T)$ and $x(t)$.

Upon differentiation of Equation A6 with respect to t and using Equation A4 to eliminate $\dot{x}$, one gets:

$$\dot{\lambda} = (\dot{P} + PA_d - PB_d P)x - PB_d r + Pf_x + \dot{r}. \tag{A7}$$

Comparing this with Equation A3 and eliminating λ, one gets, after some manipulation:

$$(\dot{P} + PA_d - PB_d P + C_d + A_d^T P)x = -\dot{r} + (PB_d - A_d^T)r - Pf_x - f_\lambda - Q_{12}\nu. \tag{A8}$$

Since $x(t)$ and $\nu(t)$ are arbitrary vectors, Equation A8 is possible only when the algebraic Riccati equation:

$$A_d^T + PA_d - PB_dP + C_d = 0, \tag{A9}$$

holds, where P is the symmetric positive definite solution of Equation A9. Then, Equation A8 can be written as:

$$\begin{aligned} \dot{r}(t) &= -A_G^T r - Pf_x - f_\lambda - Q_{12}v, \\ r(t+t_p) &= Px(t+t_p), \end{aligned} \tag{A10}$$

and Equation A4 as:

$$\dot{x}(t) = A_g x - B_d r + f_x, \tag{A11}$$

where $A_g = A_d - B_dP$. Equation A2 becomes:

$$\begin{aligned} u(t) = &- R_d^{-1}\left[(B^TP + D^TQ_1C + N^T)\right]x \\ &- R_d^{-1}B^Tr - R_d^{-1}D^TQ_1Fw \end{aligned} \tag{A12}$$